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Robust Economic MPC for a Power Management Scenario with Uncertainties

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Abstract— **This paper presents a novel incorporation of probabilistic constraints and Second Order Cone Programming (SOCP) with economic Model Predictive Control (MPC). Hereby the performance of the controller is robustyfied in the presence of both model and forecast uncertainties. Economic MPC is a receding horizon controller that minimizes an economic objective function and we have previously demonstrated its usage to include a refrigeration system as a controllable power consumer with a portfolio of power generators such that total cost is minimized. The main focus for our work is power management of the refrigeration system. Whereas our previous study was entirely deterministic, models of e.g. supermarket refrigeration systems are uncertain, as are forecasts of outdoor temperatures and electricity demand. The linear program we have formulated does not cope with uncertainties and thus it is, liable to drive an optimal solution to an infeasible or very expensive solution. The main contribution of this paper is the Finite Impulse Response (FIR) formulation of the system models, allowing us to describe and handle model uncertainties in the framework of probabilistic constraints. Our new solution using this setup for robustifying the economic MPC is demonstrated by simulation of a small conceptual example. The scenario is primarily chosen to illustrate the effect of our proposed method in that it can be compared with our previous deterministic simulations.**

I. Introduction

In [1] we introduced economic MPC to control a number of independent dynamic systems that must collaborate to minimize the overall cost of satisfying the cooling demand for some goods while meeting market demands for power at all times. Our control strategy is an economic optimizing model predictive controller, economic MPC. MPC for constrained systems has emerged during the last 30 years as the most successful methodology for control of industrial processes [2]. MPC is increasingly being considered for refrigeration systems [3], [4] and for power production plants [5]. MPC based on optimizing economic objectives has only recently emerged as a general methodology with efficient numerical implementations and provable stability properties [6]–[8]. Our proposed economic MPC controller has previously been formulated in a deterministic setting and the contribution of this paper is to put our strategy into a more realistic scenario where different uncertainties always affect the system. Thus, this paper provides a novel extension to the economic MPC to provide robust performance in the presence of both forecast and model uncertainties. This is done in a way similar to [9] where energy consumption for climate control is minimized under influence of uncertain weather predictions but also the ability to handle model uncertainties in the closed-loop MPC is an important issue in this paper.

The Smart Grid is the future intelligent electricity grid and is intended to be the smart electrical infrastructure required to increase the amount of green energy significantly. The Danish transmission system operator (TSO) has the following definition of Smart Grids which we adopt in this work: "Intelligent electrical systems that can integrate the behavior and actions of all connected users - those who produce, those who consume and those who do both - in order to provide a sustainable, economical and reliable electricity supply efficiently" [10]. A larger share of intermittent stochastic power-generating sources such as wind turbines makes it difficult to balance demand and supply of electricity in a flexible and cost-efficient manner. To account for this we previously introduced large power consumers, such as cold rooms, or an aggregation of a number of like consumers such as supermarket systems, with the ability to adjust the power consumption profile to the power supply. Due to the large thermal capacity of cold rooms, their consumption of electricity can, to some degree, be shifted in time to benefit the overall system. The thermal capacity in the refrigerated goods is then utilized to store "coldness" such that the refrigeration system can increase cooling when there is an over production of energy and then lower its consumption at other times. The temperature is allowed to vary within certain bounds, which have no impact on food quality. We exploit that the dynamics of the temperature in the cold room are rather slow while the power consumption can be changed rapidly. [9], [11], [12] also utilized load shifting capabilities to reduce total energy consumption.

Several works exist that consider constrained model predictive control (MPC) in the presence of uncertainty [13]. In many applications distributions can be quantified for uncertainty and if this information is ignored (e.g. by defining worst-case costs and invoking constraints over all uncertainty realizations) it can lead to conservative results, and the need for a stochastic extension to constrained MPC is clear [14]. Taking expected values of the cost provides an obvious way to utilize probabilistic information [15]. However constraints often admit a probabilistic formulation too, e.g. a variable should not exceed a certain bound with a given probability.

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[16] and [17] considered MPC with probabilistic constraints with the cost based on the expected value of a linear function of the states. In the former the implementation of probabilistic constraints can be conservative due to the use of statistical confidence ellipsoidal approximations, whereas the latter uses affine disturbance feedback. [18] and [19] demonstrate that probabilistic linear constraints can be written as second-order cone (SOC) constraints that are convex provided the probability involved is greater than 0.5. Probabilistic constraints are also introduced in [20] for model uncertainties and in [21] for uncertain disturbances. Both works confine the analysis to open loop optimization whereas [22] uses SOCP methods to calculate steady-state targets for MPC under uncertainty. In [23] a fast algorithm for MPC with probabilistic constraints is presented. For power management scenarios e.g. [24] proposed a risk-constrained stochastic programming for signing day-ahead contracts under uncertain price forecasts and in [25] a stochastic mixed-integer program is proposed for the scheduling of reserves by demand response under forecast uncertainty and random outages of generating units and transmission lines.

This paper is organized as follows. Section II introduces economic MPC and illustrates the problem with linear programming for uncertain systems. In section III we explain the different sources of uncertainty and reformulate both the model and forecast uncertainties to fit into solutions with probabilistic constraints. Section IV describes the models, assumptions and scenarios used for our case study, and the results are provided in section V. We give conclusions in Section VI.

II. Economic MPC for Linear Systems

In this section we describe the economic MPC for linear systems. The Economic MPC minimizes an economic cost directly as opposed to minimizing the deviation from a set-point in some norm. We consider continuous variables only and the resulting optimal control problem is formulated as a linear program. The solution of this program is implemented on the system in a receding horizon manner.

A. Distributed Independent System

In this paper, we consider a distributed independent system represented in continuous time as:

$$Y_i(s) = G_{yu,i}(s)U_i(s) + G_{yd,i}(s)D_i(s) \quad i \in \mathcal{P} \tag{1a}$$
$$Z_i(s) = G_{zu,i}(s)U_i(s) + G_{zd,i}(s)D_i(s) \quad i \in \mathcal{P} \tag{1b}$$

with $i \in \mathcal{P} = \{1, 2, \dots, P\}$ being an index referring to each plant. $U \in \mathbb{C}^{n_u}$ is the manipulable variables, $D \in \mathbb{C}^{n_d}$ is known disturbances, $Y \in \mathbb{C}^{n_y}$ is the outputs associated with a cost, and $Z \in \mathbb{C}^{n_z}$ is the outputs associated with output constraints. G_{yu}, G_{yd}, G_{zu}, and G_{zd} are transfer function matrices of compatible size.

The set of plants, $\mathcal{P}$, consists of controllable producers (e.g. conventional power plants), $\mathcal{S}_C$, non-controllable producers (e.g. wind farms), $\mathcal{S}_{NC}$, controllable consumers (e.g. large cooling houses as in this paper), $\mathcal{D}_C$ and non-controllable consumers, $\mathcal{D}_{NC}$. We combine the effect from all non-controllable units in the net power demand signal r. In this signal we model changes in e.g. wind speed as step-like changes. The dynamically independent plants must collaborate to meet a common objective i.e. satisfy the market demand for the goods they produce. The optimal control problem defining the economic MPC for (1) may then be stated as the block-angular linear program:

$$\min_{\{x,u,y,z\}} \quad \phi = \sum_{i\in\mathcal{S}} \left(\sum_k c'_{u,i} u_{i,k} + c'_{y,i} y_{i,k} \right) \tag{2a}$$
$$s.t. \quad \sum_{i\in\mathcal{S}_C} y_{i,k} - \sum_{i\in\mathcal{D}_C} y_{i,k} \geq r_k \tag{2b}$$
$$x_{i,k+1} = A_i x_{i,k} + B_i u_{i,k} + E_i d_{i,k} \tag{2c}$$
$$y_{i,k} = C_i x_{i,k} + D_i u_{i,k} + F_i d_{i,k} \tag{2d}$$
$$z_{i,k} = C_{z,i} x_{i,k} + D_{z,i} u_{i,k} + F_{z,i} d_{i,k} \tag{2e}$$
$$u_{\min,i} \leq u_{i,k} \leq u_{\max,i} \tag{2f}$$
$$\Delta u_{\min,i} \leq \Delta u_{i,k} \leq \Delta u_{\max,i} \tag{2g}$$
$$z_{\min,i} \leq z_{i,k} \leq z_{\max,i} \tag{2h}$$

with $i \in \mathcal{P}$ and $k \in \mathcal{T}$. $\mathcal{T} \in \{0, 1, \dots, N\}$. The objective function (2a) says that the total cost of production from all the power plants in the time horizon considered must be minimal. (2b) couples the independent plants by requiring that the supply exceeds the demand. This is not a realistic constraint for controlling an entire Smart Grid where supply and demand have to balance at all times. But for the illustration of the effect gained from including controllable consumers, this simplification does not change the solution. (2c)-(2e) are the discrete-time state space realization of (1), (2f) constitutes the input constraints and (2g) is a constraint on the rate of movement ($\Delta u_k = u_k - u_{k-1}$). The output constraints are represented by (2h).

The supply-demand constraint (2b) and the output constraints (2h) may not be feasible for every disturbance and initial state scenario. In such situations (2) may be modified to a feasible linear program by representing (2b) and (2h) as soft constraints with large constraint violation penalties.

(2) can be formulated as an instance of a linear program which may be solved efficiently using Dantzig-Wolfe decomposition [26].

B. Linear Programs and Control with Uncertainty

The optimum of a linear program is an extreme point, as illustrated in Fig. 1. This property of linear programs leads to either dead-beat or idle control when linear programs are used to solve model predictive control problems with an ℓ_1-penalty [27]. For economic MPC the fact that the optimum is an extreme point implies that even small perturbations in the data or the disturbances may change the otherwise optimal solution to an infeasible or very expensive solution. Uncertainties are always present in real systems and the solution we presented in [1] is therefore entirely conceptual. Exemplified by the power production case, an optimal solution is one where the amount of power produced exactly matches the consumption. However, due to the optimization relying on a prediction of power production from non-controllable producers, there is a risk of power shortage if e.g. the estimate of power from non-controllable producers was overstated. Since this situation is very expensive, a more desirable solution would be to produce just enough extra power to leave room for most of the effect from uncertainties. Another scenario would be the cold room temperature that, in an energy context, optimally aims at the upper limit, causing the foodstuff to be damaged if the surrounding temperature gets higher than predicted or if the real dynamics of the refrigeration system are slightly different than modeled. This has previously been handled by adding a somewhat arbitrary amount of back-off from the calculated optimal point. In this work we want to introduce a confidence interval such that the solution accounts for the amount of uncertainty. The tubes in Fig. 1 illustrate this.

III. Economic MPC with Probabilistic Constraints

As pointed out above the optimal solution to a deterministic LP is not always optimal, nor feasible, in the stochastic case. Therefore we describe means to handle the uncertainties in both forecasts and in the models of the system. We are using assumptions of the uncertainty belonging to certain distribution functions and define the confidence level (probability) that the constraints should hold with. The probabilistic constraints are then reformulated as their deterministic counterparts.

First we define the system model in Finite Impulse Response (FIR) form:

$$y_k = b_k + \sum_{i=0}^{k} H_i u_{k-i}, \; H_i = \begin{cases} D & for\ i=0 \\ CA^{i-1}B & for\ i>0 \end{cases} \quad (3)$$

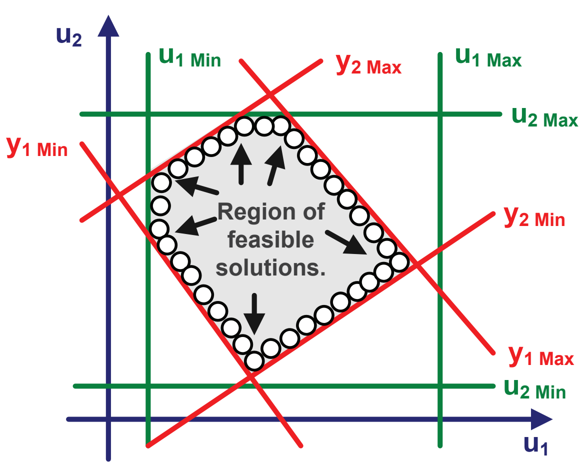


Fig. 1. Example of LP with two inputs and two outputs. Boundaries of the feasible region are illustrated with green for input constraints and red for output constraints. The arrows indicate possible optimal solutions and the circles illustrate the confidence interval around a solution caused by uncertainty.

where b_k is a bias term. Next, the stochastic optimization problem is defined as (boldface variables are uncertain):

$$\min E\left\{\sum_{k=0}^{N} \mathbf{c_k}' u_k\right\} \quad (4a)$$

$s.t.$

$$u_{min} \le u_k \le u_{max} \quad (4b)$$

$$Prob\{\mathbf{y_k} \ge \mathbf{r_k}\} \ge 1-\alpha, \quad \alpha \in [0;1] \quad (4c)$$

$$\mathbf{y_k} = b_k + \sum_{i=1}^{k} \mathbf{H_i} u_{k-i} + \sum_{i=1}^{k} \mathbf{H_{D,i}} \mathbf{d_{k-i}} \quad (4d)$$

where $\mathbf{r}$ is a reference trajectory, $\mathbf{d}$ a disturbance, $1-\alpha$ the confidence level for the constraint, and:

$$\begin{array}{llll} 1) & \mathbf{c_k} \sim N(\bar{c_k}, \sigma_c^2) & 2) & \mathbf{r_k} \sim N(\bar{r_k}, \sigma_r^2) \\ 3) & \mathbf{H_i} \sim N(\bar{H}_i, \Sigma_H^2) & 4) & \mathbf{H_{D,i}} \sim N(\bar{H}_{D,i}, \Sigma_H^2) \\ 5) & \mathbf{d_k} \sim N(\bar{d_k}, \sigma_d^2) & & \end{array} \quad (5)$$

1) and 2) are forecast uncertainties, 3) and 4) describe model uncertainties while 5) is uncertainty in the disturbances.

A. Forecast Uncertainty

1) Uncertainty in price, $\mathbf{c_k}$: Since we are minimizing the expected value of the objective function we use the certainty equivalent description and substitute $\bar{c_k}$ with c_k.

2) Uncertainty in the reference, $\mathbf{r_k}$: The probability constraint is reformulated as a deterministic constraint:

$$Prob\{Y \ge \mathbf{R}\} \ge 1-\alpha \quad (6a)$$

$$\frac{\mathbf{R}-\bar{R}}{\sigma_r} \sim N(0,1) \Rightarrow \Phi\left(\frac{Y-\bar{R}}{\sigma_r}\right) \ge 1-\alpha \quad (6b)$$

$\Phi(x)$ is the cumulative distribution function (CDF) of a zero mean unit variance Gaussian random variable x

$$Y \ge \bar{R} + \sigma_r \Phi^{-1}(1-\alpha) \quad (7)$$

Hence a security margin is added to $\mathbf{r_k}$ resulting in a back-off from the optimal (in the deterministic case) boundary. This strategy is closely related to the affine feedback methods described e.g. in [28] and [17].

B. Model and Disturbance Uncertainty

3), 4) and 5) lead to stochastic programming which is described in this section. We formulate the system using the FIR description in Eq. (3)-(5):

$$\mathbf{Y} = \begin{bmatrix} \mathbf{C} & \mathbf{\Gamma} \end{bmatrix} \begin{bmatrix} U_{past} \\ U \end{bmatrix} + \begin{bmatrix} \mathbf{C_D} & \mathbf{\Gamma_D} \end{bmatrix} \begin{bmatrix} \mathbf{D_{past}} \\ \mathbf{D} \end{bmatrix} \quad (8)$$

and the optimization problem as:

$$\min_U \quad \phi = E\left\{\sum_{k=0}^{N-1} \mathbf{c_{u,k}} u_k\right\} \quad (9a)$$

$s.t.$

$$\mathbf{y_k} = \begin{bmatrix} \mathbf{C_k} & \mathbf{\Gamma_k} \end{bmatrix} \begin{bmatrix} U_{past} \\ u_k \end{bmatrix} + \begin{bmatrix} \mathbf{C_{D,k}} & \mathbf{\Gamma_{D,k}} \end{bmatrix} \begin{bmatrix} \mathbf{D_{past}} \\ \mathbf{d_k} \end{bmatrix} \quad (9b)$$

$$Prob\{\mathbf{y_k} \ge r_k\} \ge 1-\alpha, \quad k = 1, 2, \cdots, N \quad (9c)$$

where $\mathbf{C_k}$ and $\mathbf{\Gamma_k}$ are rows from the corresponding matrices in Eq. (10) and subscript "past" indicates the previous signals corresponding to the number of coefficients in the FIR model.

$$\begin{bmatrix} \Gamma_{N\times N} & | & C_{N\times(n-1)} \end{bmatrix} =$$
$$\left[\begin{array}{cccccc|cccc} H_1 & 0 & \cdots & \cdots & \cdots & 0 & H_n & \cdots & \cdots & H_2 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \vdots \\ H_n & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ & & & & & & 0 & \cdots & 0 & H_n \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & H_n & \cdots & H_1 & 0 & \cdots & \cdots & 0 \end{array}\right] \tag{10a}$$

$$U_{past} = \begin{bmatrix} u_{-(n-1)} \\ \vdots \\ u_{-1} \end{bmatrix}, \quad U = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} \tag{10b}$$

The statistical properties of the resulting output $\mathbf{y_k}$ can be described as:

$$\mathbf{Y_U} \sim N(\bar{Y}_U, \Sigma_{Y_U}), \quad \mathbf{Y_D} \sim N(\bar{Y}_D, \Sigma_{Y_D}) \tag{11a}$$
$$\mathbf{Y} = \mathbf{Y_U} + \mathbf{Y_D}, \quad \mathbf{Y} \sim N(\bar{Y}_U + \bar{Y}_D, \Sigma_{Y_U} + \Sigma_{Y_D}) \tag{11b}$$

where:

$$\bar{Y}_{U,k} = \begin{bmatrix} \bar{C}_k & \bar{\Gamma}_k \end{bmatrix} \begin{bmatrix} U_{past} \\ U \end{bmatrix} \tag{12a}$$
$$\Sigma_{Y_U,k} = \begin{bmatrix} U_{past} & U \end{bmatrix} \begin{bmatrix} \Sigma_{C,k} & 0 \\ 0 & \Sigma_{\Gamma,k} \end{bmatrix} \begin{bmatrix} U_{past} \\ U \end{bmatrix} \tag{12b}$$

The product of the two normally distributed variables coming from the model uncertainties and the uncertain disturbance respectively can be described by an approximate normal distribution with the following properties [29]:

$$\bar{Y}_{D,k} \approx \begin{bmatrix} \bar{C}_{D,k} & \bar{\Gamma}_{D,k} \end{bmatrix} \begin{bmatrix} \bar{D}_{past} \\ \bar{D} \end{bmatrix} \tag{13a}$$
$$\Sigma_{Y_D,k} \approx \begin{bmatrix} \bar{D}_{past} & \bar{D} \end{bmatrix} \begin{bmatrix} \Sigma_{C_D,k} & 0 \\ 0 & \Sigma_{\Gamma_D,k} \end{bmatrix} \begin{bmatrix} \bar{D}_{past} \\ \bar{D} \end{bmatrix} + \begin{bmatrix} \bar{C}_{D,k} & \bar{\Gamma}_{D,k} \end{bmatrix} \Sigma_D \begin{bmatrix} \bar{C}_{D,k} \\ \bar{\Gamma}_{D,k} \end{bmatrix} \tag{13b}$$

Hence, using that $(\mathbf{y_k} - \bar{y}_k)/\Sigma_{y,k}^{1/2} \sim N(0,1)$, the probabilistic constraint can be reformulated as follows:

$$Prob\{\mathbf{y_k} \geq r_k\} \geq 1 - \alpha \tag{14a}$$
$$\Phi^{-1}(\alpha) \left\| \Sigma_*^{1/2} \begin{bmatrix} *_{past} \\ * \end{bmatrix} \right\|_2 + \bar{y}_k \geq r_k \tag{14b}$$

where the '*' indicates that the norm is taken of the vector formed by all the quadratic terms described in Eq. (12b) and (13b). Hence, in a MIMO case where $\mathbf{y_k}$ is the sum of two independent outputs, the vector in the norm would simply contain an element from each of the outputs. This is easily realized by the property $\sqrt{a^2+b^2+c^2} = \|[a\ b\ c]\|_2$. The constraint in Eq. (14b) has the form of a second order cone and the solution to the optimization problem constrained by Eq. (14b) can be computed using SOCP as in [18], [19].

In summary, uncertain model descriptions alone or in combination with uncertain disturbances lead to second order cone constraints, while an uncertain reference just adds a margin to the boundary. These two cases can of course easily be combined.

IV. Simple Power Management Scenario

The case study used in this paper includes two controllable power generators and one power consumer. The power consumer is a cold room for which we provide a simple model. This case study is identical to the one presented in [1] to illustrate the properties and potential of economic MPC in managing the power production and consumption in a distributed energy system. The novelty in this paper is the inclusion of a realistic scenario with uncertainties in both models and forecasts and the means to handle such as described in the previous sections. We use the Economic MPC implementation with probabilistic constraints formulated as an SOCP to calculate the cost-optimal control in presence of uncertainties with known probability distribution functions.

A. Controllable Power Generators

In [30] simple models for power generators are provided. In this paper we adopt these models which are of the form:

$$\phi_i = \sum_{k\in\mathcal{T}} c_i' u_{i,k} \tag{15a}$$
$$Y_i(s) = G_i(s)U_i(s) \qquad G_i(s) = \frac{1}{(\tau_i s + 1)^3} \tag{15b}$$
$$u_{\min,i} \leq u_{i,k} \leq u_{\max,i} \tag{15c}$$
$$\Delta u_{\min,i} \leq \Delta u_{i,k} \leq \Delta u_{\max,i} \tag{15d}$$

to model two conventional power generators where u_i is the power set-point for the i-th generator. (15a) represents the costs of producing power from a given power generator. Power generator 1 is cheap and slow, $(c_1, \tau_1, u_{\min,1}, u_{\max,1}, \Delta u_{\min,1}, \Delta u_{\max,1}) = (1, 20, 0, 15, -1, 1)$. Power generator 2 is expensive and fast, $(c_2, \tau_2, u_{\min,2}, u_{\max,2}, \Delta u_{\min,2}, \Delta u_{\max,2}) = (2, 10, 0, 15, -3, 3)$. The model in Eq. (15) describes the closed-loop system with internal controllers and is therefore quite simple without the lower level complexity of the generators. The model has been validated against experimental data at DONG Energy, Denmark.

B. Simple Cold Room

The energy balance for the cold room is

$$mc_p \frac{dT_{cr}}{dt} = Q_{load} - Q_e \tag{16}$$

with

$$Q_{load} = (UA)_{amb-cr}(T_{amb} - T_{cr}) \tag{17a}$$
$$Q_e = (UA)_{cr-e}(T_{cr} - T_e) \tag{17b}$$

T_{cr} is the temperature in the cold room which must be kept within certain bounds, $T_{cr,\min} \leq T_{cr} \leq T_{cr,\max}$. T_e is the evaporation temperature of the refrigerant. It can be controlled by the compressor work and must satisfy $T_{cr} \geq T_e$. T_{amb} is the ambient temperature. UA is the heat transfer coefficient. m and c_p are the mass and the overall heat capacity of the refrigerated goods, respectively. The energy consumed by the refrigeration system is work performed by the compressors: $W_C = \eta Q_e$. η is the coefficient of performance. In this work η is assumed to be constant. Consequently

$$W_C(s) = \frac{a - bs}{\tau s + 1} T_e(s) + \frac{\alpha K_d}{\tau s + 1} T_{amb}(s) \tag{18a}$$

$$T_{cr}(s) = \frac{K_u}{\tau s + 1} T_e(s) + \frac{K_d}{\tau s + 1} T_{amb}(s) \tag{18b}$$

with $Y_3 = W_C$, $Z_3 = [T_{cr}; T_{cr} - T_e]$, $U_3 = T_e$, $D_3 = T_{amb}$. The constraints are

$$T_{cr,min} \leq T_{cr} \leq T_{cr,max} \tag{19a}$$

$$0 \leq T_{cr} - T_e \leq \infty \tag{19b}$$

$$T_{e,min} \leq T_e \leq T_{e,max} \tag{19c}$$

Thus, the refrigeration system can be modeled in a form compatible with the economic MPC for linear systems. The model here is somewhat simplified, especially the assumption for (18). However the resulting dynamics are well suited for illustrating the conceptual case in this paper.

C. Supply and Demand

The production by the power generators, $y_{1,k} + y_{2,k}$, must exceed the demand for power by the cooling house and the other consumers

$$y_{1,k} + y_{2,k} \geq y_{3,k} + r_k \quad k \in \mathcal{T} \tag{20}$$

We model wind farms as instantaneously changing systems and include the effect of their power production in the exogenous net power demand signal, r_k. This is seen in the case study in Fig. 2.

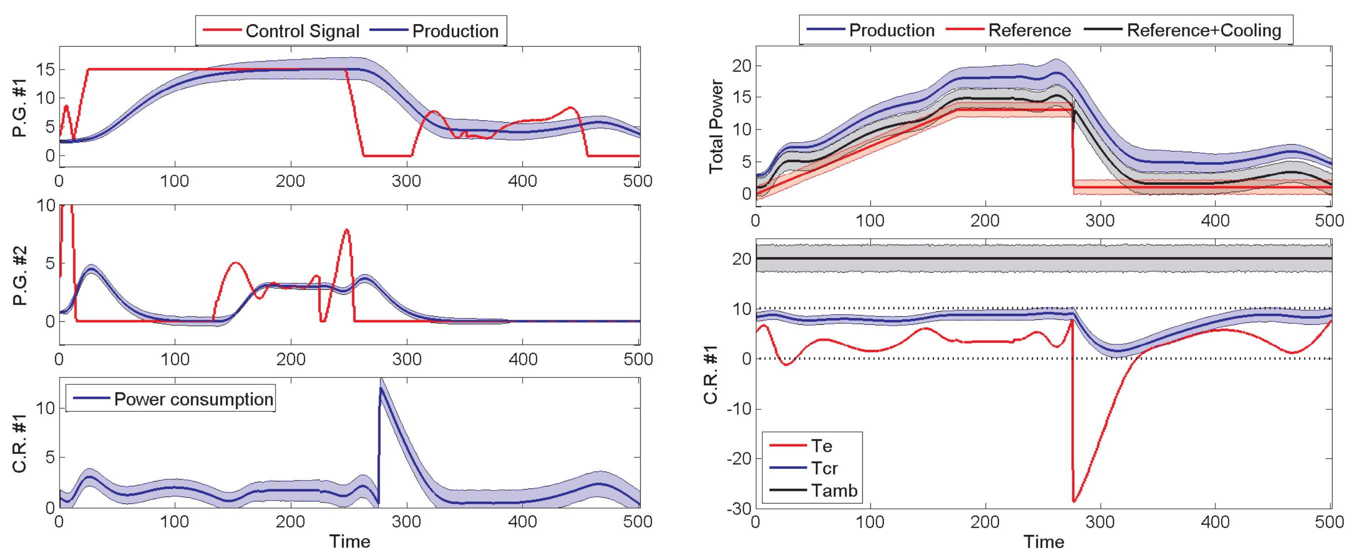
D. Uncertainty

In our scenario the models of power plants and refrigeration systems are not perfectly known and an uncertain FIR as in Eq. (3)-(5) is used for the system models. The temperature surrounding the cold room ($\mathbf{T_{amb}}$) is stochastic as is the reference ($\mathbf{r}$). The latter is caused by the predictions of both non-controllable consumption and non-controllable production being uncertain. We have already seen how the price (c) can be assumed as deterministic without changing the solution.

V. Results

In [1] we have demonstrated the significant savings gained by including controllable consumers in the setup. Hence, we will only consider the improved ability to handle uncertainties without unnecessary high costs or severe violation of constraints.

Using Yalmip [31] we have simulated the scenario described in the previous section. The constraints on the cold room temperature and on balancing supply and demand are formulated as probability constraints and implemented with SOCP as described in section III. A simulation scenario is provided in Fig. 2. From the figure it is noted how the refrigeration system is utilized to balance the power demand such that extra power is used when it is freely available and less is used at other times. This is further elaborated on in [1]. But what is more important for the



(a) Power productions / consumption. P.G. #1 and 2 show the power productions from the two power plants (blue) and their power set-points (red). C.R. #1 is power consumption in the cold room.

(b) "Total Power" shows total power production (blue) versus the reference consumption (with (black) and without (red) refrigeration). C.R. #1 shows the temperature in the cold room T_{cr} and the control signal T_e. ($T_{cr,min}$, $T_{cr,max}$) and outdoor temperature, T_{amb}, are shown with dotted and solid black respectively.

Fig. 2. Simulation of Power Management scenario. $\alpha = 0.5$, $H_i \sim N(\bar{H}_i, 0.0055^2)$, $r_k \sim N(\bar{r}_k, 0.7071^2)$, $T_{amb} \sim N(\bar{T}_{amb}, 1.7321^2)$. The shaded bands show the 95% confidence interval from 10,000 random instances.

work presented here are the confidence intervals shown as shaded areas around each of the trajectories. The solid lines are the expected outcome, while the shaded areas are created by 10,000 simulations with random instances of the noise descriptions. The 95% percentile was used both in the SOCP formulation and for plotting the shaded areas. It is easily seen how the amount of back-off from the boundaries is just enough to account for the 95% confidence interval of the uncertainty descriptions for the system. This is particular clear in Fig. 2(b), where the total production is above the total consumption, T_{cr} stays within the boundaries specified and $T_e \leq T_{cr}$ is satisfied. All with 95% probability.

Regarding the uncertainty of the predictions of outdoor temperature and power demand in a closed-loop scenario, a variance that increases over the prediction horizon could be chosen such that the short-term predictions are more certain than those at the end of the horizon. Furthermore the disturbances could be measured at each time step, minimizing the uncertainty in the vector of past disturbances to the level related to doing the measurement.

VI. Conclusion

In this paper we have extended our analysis of economic MPC for a Power Management scheme in which we included a refrigeration system with thermal storage capabilities as a controllable power consumer in order to minimize the total cost. We have presented a novel formulation including uncertainties from both system models and forecasts in the framework of probabilistic constraints. Thereby our previous economic MPC based on linear programming has evolved into a stochastic economic MPC that can be implemented as a convex SOCP. The concept was demonstrated in a conceptual case as an efficient way to treat uncertainties in the system. Therefore, the proposed economic MPC controller can now be implemented in a realistic scenario with robust performance guarantees.

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