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Robust Fault Estimation for Stochastic Takagi-Sugeno Fuzzy Systems

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Abstract— Nowadays, industrial plants are calling for high-performance fault diagnosis techniques to meet stringent requirements on system availability and safety in the event of component failures. This paper deals with robust fault estimation problems for stochastic nonlinear systems subject to faults and unknown inputs relying on Takagi-Sugeno fuzzy models. Augmented approach jointly with unknown input observers for stochastic Takagi-Sugeno models is exploited here, which allows one to estimate both considered faults and full system states robustly. The considered unknown inputs can be either completely decoupled or partially decoupled by observers. For the un-decoupled part of unknown inputs, which still influence error dynamics, stochastic input-to-state stability properties are applied to take nonzero inputs into account and sufficient conditions are achieved to guarantee bounded estimation errors under bounded unknown inputs. Linear matrix inequalities are employed to compute gain matrices of the observer, leading to stochastic input-to-state-stable error dynamics and optimization of the estimation performances against un-decoupled unknown inputs. Finally, simulation on wind turbine benchmark model is applied to validate the performances of the suggested fault reconstruction methodologies.

Keywords—robust fault estimation; stochastic Takagi-Sugeno systems; unknown input observer; stochastic input-to-state stability

I. INTRODUCTION

Driven by the increasing demands for productivity, the level of complexity and technical processes of industrial systems are accordingly growing. This leads to the systems susceptible to unexpected deviations which have significant impacts on safety, economy, and environment. In order to monitor these abnormal conditions in their earlier occurrence and maintain high safety and reliability standards in practical plants, extensive and comprehensive investigations about fault diagnosis for control systems have been developed during the past decades [1-5]. Nonlinear phenomena, which constitute an intrinsic characteristic of many practical systems, could lead to quite complex analytic solutions. Additionally, motivated by the widespread presence of random factors in the operation of systems and the inevitable stochastic error in measurement, stochastic systems formulated in Itô-type stochastic differential equations have drawn considerable attention. Therefore, fault diagnosis for stochastic nonlinear systems, which can represent the realistic system properties more practically, has been a hot issue in both industry and academia recently [6, 7]. Fault

estimation, also called fault reconstruction, is an advanced fault diagnosis approach, for its capability to track faulty features, and provide auxiliary full state estimation at mean time. As the performance of a control system is inevitably affected by various perturbations, robustness has had a vital status for a fault estimation module. The interest of unknown input observer stems from the great advantage to decouple unknown perturbations, and has widespread applications on robust fault diagnosis. As a result, unknown input observer-based fault reconstruction can take both advantages, thus have been dedicated to robust monitoring of many engineering systems [8-10]. One constraint of this technique is the completely decoupled assumption of unknown inputs, which fails to meet wide range of realistic plants. Our early work [11], represents the first attempt to introduce unknown input observer-based fault estimation into linear systems and Lipschitz nonlinear systems subject to partially decoupled unknown inputs, associated with linear matrix inequalities to optimize the estimation performance. The work has been extended to stochastic nonlinear systems satisfying quadratic inner-bounded conditions in [12]. However, it is worth noting that for general forms of stochastic nonlinear systems, this issue meet challenges due to lack of systematic way to settle observer gains for such complex systems, thus remains to be an area to be explored.

Takagi-Sugeno (T-S) fuzzy logics, which are capable of providing the approximation to stochastic processes in mean square sense with arbitrary accuracy, have been widely used as a convenient tool for handling nonlinearities. Early pioneer works for this idea can be traced back to [13], where the continuous-time nonlinear dynamics was linearized at several valid operating points and the overall model was obtained by a fuzzy “blending” of these local linear models through appropriate membership functions. This kind of simplified structure provides approach for monitoring nonlinear processes. Following the ideas of T-S fuzzy modelling schemes, tremendous research efforts have been dedicated to fault diagnosis of stochastic T-S fuzzy systems. Specifically, [14] presented a robust observer for robust fault estimation of T-S fuzzy models; T-S fuzzy unknown input observers were investigated in [15] to decouple the influences from unknown disturbances; [16] utilized unknown input observer to solve state estimation problem for T-S fuzzy models; [17] designed sliding mode observers for stochastic T-S fuzzy systems for

fault estimation. Nevertheless, to the best of the authors' knowledge, no effort has been devoted to do unknown input observer-based fault reconstruction for stochastic nonlinear systems in presence of partially decoupled unknown inputs.

Since traditional global asymptotically stability in probability fails to analyze and process the state responses of the system with disturbances, stochastic input-to-state stability, was originally introduced in [18], to provide a natural setting for classification and parameterization of finite-energy inputs into finite-energy states, and has been extensively investigated by many scholars for stochastic systems. In particular, [19] and [20] addressed sufficient conditions of stochastic input-to state stability for nonlinear systems combined with Lyapunov theory; weighted fuzzy observer was proposed for discrete-time nonlinear systems based on this property; [22] solved H_∞ filtering problem for stochastic T-S fuzzy plants with time-varying delays with the aid of stochastic input-to-state stability. Therefore, the research of stochastic input-to-state stability-based robust fault reconstruction is of great meaning for stochastic nonlinear systems.

This paper addresses robust fault estimation methodologies for stochastic T-S fuzzy models affected by partially decoupled unknown inputs based on stochastic input-to-state stability theory. Firstly, unknown input observers are designed for augmented expressions of local models to estimate states and faults simultaneously, and the global observer is obtained by convex combination of these individual observers; then stochastic input-to-state stability is employed to analyse the convergence of error dynamic against un-decoupled unknown inputs; linear matrix inequality (LMI) strategy is combined to optimize the estimation performance by attenuating the influences from un-decoupled unknown inputs and Brownian motion; finally simulation on a wind turbine model is dedicated to illustrate the effectiveness of proposed methods.

The rest of paper is organized as follows: Section II states some preliminary knowledge and definitions which are essential foundation of further analysis; Section III presents the methodologies for robust fault reconstruction for stochastic T-S fuzzy systems; the theory is applied to simulation on wind turbine systems in Section IV to show its validation, followed by Section V to conclude the work.

II. PRELIMINARIES AND PROBLEM FORMULATION

Notation: throughout this paper, \mathcal{R}^n and $\mathcal{R}^{n \times m}$ always stand for n -dimensional Euclidean space and $n \times m$ real matrices, while \mathcal{R}^+ is the set of all nonnegative real numbers. I_n represents identity matrix with dimension of $n \times n$, and 0 denotes a scalar zero or a zero matrix with appropriate zero entries. For any given vector $x = (x_1, x_2, \dots, x_n) \in \mathcal{R}^n$, $|x|$ refers to its Euclidean norm defined by $|x| = (\sum_{i=1}^n x_i^2)^{1/2}$, and $|x|_{Tf} = (\int_0^{Tf} x^T(\tau) x(\tau) d\tau)^{1/2}$, while $\|x\|$ denotes its super norm. $\mathbb{E}(x)$ represents mathematical expectation of x and \forall means for all. The superscript T represents the transpose of matrices or vectors. The notation $X > 0$ indicates that the symmetric X is positive definite. Besides, in a large matrix expression, $\begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix}$.

Consider stochastic T-S fuzzy models suffering from faults and unknown inputs in the form of Itô-type differential equations as follows:

IF μ_1 is M_{1i} and ... μ_q is M_{qi} , THEN

$$\begin{cases} dx(t) = [A_i x(t) + B_i u(t) + B_{di} d(t) + B_{fi} f(t)] dt \\ \quad + W_i x(t) dw(t) \\ y(t) = C_i x(t) + D_{fi} f(t) + D_{di} d_w(t) \end{cases} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ represents unmeasurable state vector with measurable initial value of $x_0 \in \mathcal{R}^n$ at initial time t_0 ; $w(t)$ is a standard one-dimensional Brownian motion on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq t_0}, \mathcal{P})$, with Ω being a sample space, \mathcal{F} being a σ -field, $\{\mathcal{F}_t\}_{t \geq t_0}$ being a filtration and \mathcal{P} being a probability measure. $w(t)$ satisfies $\mathbb{E}[w(t)] = 0$ and $\mathbb{E}[w^2(t)] = t$. $u(t) \in \mathcal{R}^m$ stands for control input vector and $y(t) \in \mathcal{R}^p$ is measurement output vector; $d(t) \in \mathcal{R}^{l_{du}}$, $d_w(t) \in \mathcal{R}^{l_{dw}}$ are bounded unknown input vectors from plant and sensors, respectively; $f(t) \in \mathcal{R}^{l_f}$ includes faults from both actuators and sensors. Without loss of generality, the trends of faults are assumed either to be incipient or abrupt, which means the second-order derivatives of their means should be zero piecewise. For faults whose second order derivatives of means are not zero but bounded signals, the bounded signals could be regarded as a part of unknown inputs. $i = 1, 2, \dots, r$, and r is the total number of local models, M_{ji} are fuzzy sets and decision vector μ involves all individual premise variables μ_j , $j = 1, 2, \dots, q$. $A_i, B_i, B_{di}, B_{fi}, C_i, D_{di}, D_{fi}$ can be obtained by linearization or identification of nonlinear systems. In the rest of paper, the symbol t in vectors will be omitted for the simplicity of presentation. By using the standard fuzzy blending method, the global model of system (1) can be inferred as:

$$\begin{cases} dx = \sum_{i=1}^r h_i(\mu) [(A_i x + B_i u + B_{di} d + B_{fi} f) dt \\ \quad + W_i x dw] \\ y = \sum_{i=1}^r h_i(\mu) (C_i x + D_{fi} f + D_{di} d_w) \end{cases} \quad (2)$$

where $h_i(\mu)$ are weighting functions, following the convex sum properties: $\sum_{i=1}^r h_i(\mu) = 1$ and $0 \leq h_i(\mu) \leq 1$.

It should be pointed out that unknown inputs usually consist of parameter perturbations, exogenous disturbances, measurement errors, and other uncertainties. As a T-S fuzzy representation is an approximate of the real model, and the modelling errors can also be regarded as a part of unknown inputs d and d_w . In this way, plant (2) can describe a wide range of engineering systems in presence of faults and extra disturbances. The main objective of this paper is to design a robust fault reconstruction scheme for system (2) which can provide estimations of the trends of full system states and considered faults simultaneously while reduce the influences of unknown inputs and Brownian motion. Before presenting our techniques, the following definitions are introduced as preliminaries:

Definition 1 [23]. A function $\gamma: \mathcal{R}^+ \rightarrow \mathcal{R}^+$ is said to be a generalized \mathcal{K} -function if it is continuous with $\gamma(0) = 0$, and satisfies

$$\begin{cases} \gamma(\sigma_1) > \gamma(\sigma_2), \text{ if } \gamma(\sigma_1) \neq 0 \\ \gamma(\sigma_1) = \gamma(\sigma_2) = 0, \text{ if } \gamma(\sigma_1) = 0, \forall \sigma_1 > \sigma_2 \geq 0 \end{cases} \quad (3)$$

\mathcal{K}_∞ is the subset of \mathcal{K} -functions that are unbounded. **Definition 2** [23]. A function $\beta: \mathcal{R}^+ \times \mathcal{R}^+ \rightarrow \mathcal{R}^+$ is said to be a generalized \mathcal{KL} -function if for each fixed $t \geq 0$, the function $\beta(s, t)$ is a generalized \mathcal{K} -function, and for each fixed $s \geq 0$, it decreases to zero as $t \rightarrow T$ for some constant $T > 0$.

Definition 3 [24]. A system is said to be stochastic input-to-state stable, if $\forall \varepsilon > 0$, there exist functions $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}_\infty$, such that for continuous bounded input v and any initial condition $x(t_0) = x_0$, we have for $\forall t \geq t_0, \forall x_0 \in \mathcal{R}^n$,

$$P\{|x(t)| \leq E[\beta(|x_0|, t - t_0)] + E[\gamma(\|v\|)]\} \geq 1 - \varepsilon \quad (4)$$

As mentioned in our previous work [12], globally asymptotically stability in probability defined in [25] is a specific situation of stochastic input-to-state stability when input $v = 0$.

III. ROBUST FAULT ESTIMATION SCHEME

In this section, the methodology to design a robust fault reconstruction approach for system (2) is presented. Throughout this paper, $B_d = [B_{d1} \ B_{d2}]$, $d = [d_1 \ d_2]^T$, $d_1 \in \mathcal{R}^{l_{d1}}$ and $d_2 \in \mathcal{R}^{l_{d2}}$. We assume that d_1 rather than d_2 can be decoupled, which means B_{d1} is of full column rank whereas B_d is not.

In order to estimate the trends of faults and system states at the same time, an auxiliary system is constructed as follows, by considering the faults as augmented system states:

$$\begin{cases} d\bar{x} = \sum_{i=1}^r h_i(\mu) [(\bar{A}_i \bar{x} + \bar{B}_i u + \bar{B}_{di} d) dt + \bar{W}_i \bar{x} dw] \\ y = \sum_{i=1}^r h_i(\mu) (\bar{C}_i \bar{x} + D_{di} d_w) \end{cases} \quad (5)$$

where

$$\begin{aligned} \bar{n} &= n + 2l_f, \bar{x} = [x^T \quad \dot{f}^T \quad f^T]^T \in \mathcal{R}^{\bar{n}} \\ \bar{A}_i &= \begin{bmatrix} A_i & 0_{n \times l_f} & B_{fi} \\ 0_{l_f \times n} & 0_{l_f \times l_f} & 0_{l_f \times l_f} \\ 0_{l_f \times n} & I_{l_f} & 0_{l_f \times l_f} \end{bmatrix} \in \mathcal{R}^{\bar{n} \times \bar{n}}, \\ \bar{B}_i &= [B_i^T \quad 0_{m \times l_f} \quad 0_{m \times l_f}]^T \in \mathcal{R}^{\bar{n} \times m} \\ \bar{B}_{di} &= [B_{di}^T \quad 0_{l_d \times l_f} \quad 0_{l_d \times l_f}]^T \in \mathcal{R}^{\bar{n} \times l_d} \\ \bar{C}_i &= [C_i \quad 0_{p \times l_f} \quad D_{fi}] \in \mathcal{R}^{p \times \bar{n}} \\ \bar{W}_i &= \begin{bmatrix} W_i & 0_{n \times l_f} & 0_{n \times l_f} \\ 0_{l_f \times n} & 0_{l_f \times l_f} & 0_{l_f \times l_f} \\ 0_{l_f \times n} & 0_{l_f \times l_f} & 0_{l_f \times l_f} \end{bmatrix} \in \mathcal{R}^{\bar{n} \times \bar{n}} \end{aligned}$$

To reduce the level of complication for observer design, we can choose any \bar{C} from \bar{C}_i as the output coefficient, and the differences between other local outputs with the selected one are regarded as measurement perturbations. In this way, systems (5) is equivalent to the following expression:

$$\begin{cases} d\bar{x} = \sum_{i=1}^r h_i(\mu) [(\bar{A}_i \bar{x} + \bar{B}_i u + \bar{B}_{di} d) dt + \bar{W}_i \bar{x} dw] \\ y = \sum_{i=1}^r h_i(\mu) [\bar{C} \bar{x} + (\bar{C}_i - \bar{C}) \bar{x} + D_{di} d_w] \end{cases} \quad (6)$$

By letting $d_s = \sum_{i=1}^r h_i(\mu) [(\bar{C}_i - \bar{C}) \bar{x} + D_{di} d_w]$, system (6) can be simplified as:

$$\begin{cases} d\bar{x} = \sum_{i=1}^r h_i(\mu) [(\bar{A}_i \bar{x} + \bar{B}_i u + \bar{B}_{di} d) dt + \bar{W}_i \bar{x} dw] \\ y = \bar{C} \bar{x} + d_s \end{cases} \quad (7)$$

In system (7), the component of state vector \bar{x} involves both original state x and fault f . By designing unknown input observer in the following form

$$\begin{cases} \dot{\hat{z}} = \sum_{i=1}^r h_i(\mu) [R_i \bar{z} + T \bar{B}_i u + (K_{i1} + K_{i2}) y] \\ \hat{x} = \bar{z} + H y \end{cases} \quad (8)$$

where \bar{z} stands for its state vector and \hat{x} is the estimation of \bar{x} , simultaneous estimation of x and f can be achieved. Matrices R_i, T, K_{i1} and K_{i2} , $i = 1, 2, \dots, r$, are observer gains to be designed such that \hat{x} is close enough to \bar{x} . Obviously, the global unknown input observer is also a fuzzy aggregation of a set of local observers with the same weights of (2).

Defining estimation error to be $\bar{e} = \bar{x} - \hat{x}$, and subtracting (8) from (7) leads to the following error dynamic:

$$\begin{aligned} d\bar{e} &= \sum_{i=1}^r h_i(\mu) \{(\bar{A}_i - H\bar{C}\bar{A}_i - K_{i1}\bar{C})\bar{e} + (\bar{A}_i - H\bar{C}\bar{A}_i \\ &\quad - K_{i1}\bar{C} - R_i)\bar{z} + [(\bar{A}_i - H\bar{C}\bar{A}_i - K_{i1}\bar{C})H - K_{i2}]y \\ &\quad + [(\bar{I}_{\bar{n}} - H\bar{C}) - T]\bar{B}_i u + (\bar{I}_{\bar{n}} - H\bar{C})\bar{B}_{di} d_1 \\ &\quad + (\bar{I}_{\bar{n}} - H\bar{C})\bar{B}_{di} d_2 - K_{i1} d_s - H\dot{d}_s\} dt \\ &\quad + (\bar{I}_{\bar{n}} - H\bar{C})\bar{W}_i \bar{x} dw \end{aligned} \quad (9)$$

If the observer gains satisfy the following conditions:

$$(\bar{I}_{\bar{n}} - H\bar{C})\bar{B}_{di} = 0 \quad (10)$$

$$R_i = \bar{A}_i - H\bar{C}\bar{A}_i - K_{i1}\bar{C} \quad (11)$$

$$T = \bar{I}_{\bar{n}} - H\bar{C} \quad (12)$$

$$K_{i2} = R_i H \quad (13)$$

the state estimation error can be reduced to

$$d\bar{e} = \sum_{i=1}^r h_i(\mu) \{[R_i \bar{e} + (\bar{I}_{\bar{n}} - H\bar{C})\bar{B}_{di} d_2 - K_{i1} d_s \\ - H\dot{d}_s] dt + (\bar{I}_{\bar{n}} - H\bar{C})\bar{W}_i \bar{x} dw\} \quad (14)$$

In order to meet the conditions (10) to (13), we have the following assumptions:

$$(1) \text{rank}(\bar{C}[\bar{B}_{d11} \ \bar{B}_{d21} \ \dots \ \bar{B}_{dr1}]) =$$

$$\text{rank}([\bar{B}_{d11} \ \bar{B}_{d21} \ \dots \ \bar{B}_{dr1}]);$$

$$(2) \text{For } \forall i, \begin{bmatrix} A_i & B_{fi} & B_{di} \\ C & D_{fi} & 0 \end{bmatrix} \text{ is of full column rank;}$$

$$(3) \text{For } \forall i, \text{rank} \begin{bmatrix} sI_n - A_i & B_{di} \\ C & 0 \end{bmatrix} = n + l_{di}.$$

According to [11, 26], the above assumptions are to ensure that for each local model, equation (10) can be solved, and one solution of H can be obtained as

$$H^* = \bar{B}_M [(\bar{C}\bar{B}_M)^T (\bar{C}\bar{B}_M)]^{-1} (\bar{C}\bar{B}_M)^T \quad (15)$$

where B_M is of full column rank, obtained by a non-singular matrix M such that

$$(\bar{B}_{d11} \ \bar{B}_{d21} \ \dots \ \bar{B}_{dr1})M = (\bar{B}_M \ 0) \quad (16)$$

Moreover, the model is observable.

Under these necessary assumptions, d_1 has been decoupled by settling H , however, d_2, d_s and the Brownian motion still affect the error dynamic. As a good observer should lead to the convergence of error \bar{e} , the design of robust fault estimation scheme is converted into attenuating the influences of un-decoupled unknown inputs and the Brownian motion.

Global asymptotically stability in probability can be used to analyze the convergence system (14) in absence of disturbances considering that a small enough perturbation should not destroy the stability properties. However, for robust fault estimation, understanding how large can the perturbations change the phase portrait is beneficial to enhance estimation performances. Thus, stochastic input-to-state stability is employed in this paper to study the convergence of error

dynamic (14). For bounded unknown inputs, our target is to design observer gains such that estimation error can be mapped around equilibrium around a certain distance, which is a function of the unknown inputs.

Similarly to traditional global asymptotic stability in probability, a dissipative function should be defined to obtain the stochastic input-to-state stable property.

Definition 4: A function V is called a stochastic input-to-state stability-Lyapunov function if there exist \mathcal{K}_∞ functions $\psi_1, \psi_2, \psi_3, \psi_4$ such that

$$(i) \psi_1(|x|) \leq V(x) \leq \psi_2(|x|) \quad (17)$$

$$(ii) \mathcal{L}V(x) \leq -\psi_3(|x|) + \psi_4(\|v\|) \quad (18)$$

where \mathcal{L} is the infinitesimal generator of V .

In our previous work [12], sufficient proof has been provided of the following lemma.

Lemma [12]: A system is stochastic input-to-state stable if there exist a stochastic input-to-state stability-Lyapunov function.

Based on the above definition and Lemma, the following theorem is addressed as sufficient conditions for checking the stochastic input-to-state stability properties of error dynamic (14), and achieve robustness to d_2, d_s and Brownian motion.

Theorem: For system (2), there exists a fuzzy unknown input observer in the form of (8), resulting in a stochastic input-to-state stable error dynamic (14) which satisfies $|\bar{e}|_{Tf} \leq \bar{\gamma}|v|_{Tf}$, if $\forall i$, there exist positive definite matrices P and \bar{P} , matrices Y_i , such that

$$\begin{bmatrix} \Lambda_i & (I_{\bar{n}} - H\bar{C})\bar{B}_{di2} & -Y_i & -PH & 0 \\ * & -\gamma^2 I_{ld2} & 0 & 0 & 0 \\ * & * & -\gamma^2 I_s & 0 & 0 \\ * & * & * & -\gamma^2 I_s & 0 \\ * & * & * & * & \Pi_i \end{bmatrix} < 0 \quad (19)$$

where $\Lambda_i = I_{\bar{n}} + \bar{A}_{i1}^T P + P\bar{A}_{i1} - \bar{C}^T Y_i^T - Y_i \bar{C} + \bar{P}$, $Y_i = PK_{i1}$, $\bar{A}_{i1} = (I_{\bar{n}} - H\bar{C})\bar{A}_i$, $\Pi_i = \bar{W}_i^T T^T P T \bar{W}_i - \gamma^2 I_{\bar{n}}$ $i = 1, 2, \dots, r$, $v = [d_2 \ d_s \ \dot{d}_s \ \bar{x}]$, and $\bar{\gamma}$ is a performance index indicating the level of noise attenuation.

Proof: based on the lemma, the proof involves establishing a dissipation inequality via a suitable storage function. Here, we choose the function as $V(\bar{e}) = \bar{e}^T P \bar{e}$, and it is not hard to obtain that:

$$\lambda_{\min}(P)|\bar{e}|^2 \leq V(\bar{e}) \leq \lambda_{\max}(P)|\bar{e}|^2$$

which means it meets (17). Taking the infinitesimal operator along the state trajectories of dynamic (14), by using the Itô formula, $\mathcal{L}V(\bar{e})$ can be calculated as:

$$\begin{aligned} \mathcal{L}V(\bar{e}) &= \sum_{i=1}^r h_i(\mu) [\bar{e}^T P (R_i \bar{e} + T \bar{B}_{di2} d_2 - K_{i1} d_s - H \dot{d}_s) \\ &\quad + (R_i \bar{e} + T \bar{B}_{di2} d_2 - K_{i1} d_s - H \dot{d}_s)^T P \bar{e} \\ &\quad + \bar{x}^T \bar{W}_i^T T^T P T \bar{W}_i \bar{x}] \\ &= \sum_{i=1}^r h_i(\mu) [\bar{e}^T (\bar{A}_{i1}^T P + P\bar{A}_{i1} - \bar{C}^T Y_i^T - Y_i \bar{C}) \bar{e} \\ &\quad + 2\bar{e}^T P T \bar{B}_{di2} d_2 - 2\bar{e}^T P K_{i1} d_s - 2\bar{e}^T P H \dot{d}_s \\ &\quad + \bar{x}^T \bar{W}_i^T T^T P T \bar{W}_i \bar{x}] \end{aligned} \quad (20)$$

Adding and subtracting $\bar{e}^T \bar{P} \bar{e} - \bar{\gamma}^2 v^T v$ yields:

$$\mathcal{L}V(\bar{e}) = [\bar{e}^T \quad v^T] (\sum_{i=1}^r h_i(\mu) \Psi_i) \begin{bmatrix} \bar{e} \\ v \end{bmatrix} - \bar{e}^T \bar{P} \bar{e} + \bar{\gamma}^2 v^T v \quad (21)$$

where

$$\Psi_i = \begin{bmatrix} \Theta_i & (I_{\bar{n}} - H\bar{C})\bar{B}_{di2} & -Y_i & -PH & 0 \\ * & -\gamma^2 I_{ld2} & 0 & 0 & 0 \\ * & * & -\gamma^2 I_s & 0 & 0 \\ * & * & * & -\gamma^2 I_s & 0 \\ * & * & * & * & \Pi_i \end{bmatrix}$$

and $\Theta_i = \bar{A}_{i1}^T P + P\bar{A}_{i1} - \bar{C}^T Y_i^T - Y_i \bar{C} + \bar{P}$. LMI (19) implies that $\Psi_i < 0$, which means

$$\mathcal{L}V(\bar{e}) \leq -\bar{e}^T \bar{P} \bar{e} + \bar{\gamma}^2 v^T v \quad (22)$$

Since \bar{P} is positive, a positive scale $\bar{\lambda}$ can be found such that

$$\mathcal{L}V(\bar{e}) \leq -\bar{\lambda} \bar{e}^T \bar{e} + \bar{\gamma}^2 v^T v \quad (23)$$

Therefore, $V(\bar{e})$ is a stochastic input-to-state stability-Lyapunov function with $\psi_3(\bar{e}) = \bar{\lambda}|\bar{e}|^2$ and $\psi_4(\|v\|) = \bar{\gamma}^2\|v\|^2$. According to the Lemma, dynamic (14) is stochastic input-to-state stable.

We are now in a position to attenuate the influences of v on estimation error. Choose $\bar{\gamma}$ as a performance index, and then we define:

$$\Gamma = \mathbb{E} \left(\int_0^{Tf} (\bar{e}^T \bar{e} - \bar{\gamma}^2 v^T v) dt \right) \quad (24)$$

It follows that:

$$\begin{aligned} \Gamma &= \mathbb{E} \left\{ \int_0^{Tf} [\bar{e}^T \bar{e} - \bar{\gamma}^2 v^T v + \mathcal{L}V(\bar{e})] dt \right\} - \mathbb{E} \left[\int_0^{Tf} \mathcal{L}V(\bar{e}) dt \right] \\ &= \mathbb{E} \int_0^{Tf} \sum_{i=1}^r h_i(\mu) [\bar{e}^T (I_{\bar{n}} + \bar{A}_{i1}^T P + P\bar{A}_{i1} - \bar{C}^T Y_i^T - Y_i \bar{C} \\ &\quad + \bar{P}) \bar{e} - \bar{e}^T \bar{P} \bar{e} + 2\bar{e}^T P T \bar{B}_{di2} d_2 - 2\bar{e}^T P K_{i1} d_s \\ &\quad - 2\bar{e}^T P H \dot{d}_s + \bar{x}^T \bar{W}_i^T T^T P T \bar{W}_i \bar{x}] dt - \mathbb{E} \int_0^{Tf} \mathcal{L}V(\bar{e}) dt \\ &\leq \mathbb{E} \left\{ \int_0^{Tf} [\bar{e}^T \quad v^T] (\sum_{i=1}^r h_i(\mu) \Omega_i) \begin{bmatrix} \bar{e} \\ v \end{bmatrix} dt \right\} \\ &\quad - \mathbb{E} \left[\int_0^{Tf} \mathcal{L}V(\bar{e}) dt \right] \end{aligned} \quad (25)$$

where

$$\Omega_i = \begin{bmatrix} \Lambda_i & (I_{\bar{n}} - H\bar{C})\bar{B}_{di2} & -Y_i & -PH & 0 \\ * & -\gamma^2 I_{ld2} & 0 & 0 & 0 \\ * & * & -\gamma^2 I_s & 0 & 0 \\ * & * & * & -\gamma^2 I_s & 0 \\ * & * & * & * & \Pi_i \end{bmatrix}$$

in which $\Lambda_i = I_{\bar{n}} + \bar{A}_{i1}^T P + P\bar{A}_{i1} - \bar{C}^T Y_i^T - Y_i \bar{C} + \bar{P}$.

Under zero initial condition $\bar{e}(0) = 0$,

$$\mathbb{E} \left(\int_0^{Tf} \mathcal{L}V(\bar{e}) dt \right) = V(\bar{e}) > 0 \quad (26)$$

thus $\Omega_i < 0$ leads to $\Gamma < 0$, i.e. LMIs (19) is sufficient to make (14) satisfy

$$|\bar{e}|_{Tf} \leq \bar{\gamma}|v|_{Tf} \quad (27)$$

As a result, for given performance index $\bar{\gamma}$, LMIs (19) can make sure the estimation error to convergence to equilibrium within a certain distance, and the error to be reduced to certain value.

It is obvious that conditions in the above theorem can also lead to a proof of global asymptotic stability in probability by choosing \bar{P} specially to be null matrix.

IV. SIMULATION

In this section, the obtained results are applied on wind turbine benchmark model represented by T-S fuzzy logic. According to [26], the nonlinear benchmark wind turbines introduced in [27] can be well modelled by T-S fuzzy logics. Here, 18 IF-THEN rules can be chosen to approximate the original wind turbine model. Then the system can be described

in the form of plant (2). The actuator fault taken into account occurs in reference of generator torque with the following value:

$$\begin{cases} 0 & 0s \leq t < 1500s \\ -0.2(t - 1500) & 1500s \leq t < 2000s \\ -100 + \sin(0.1t) & 2000s \leq t < 2500s \\ 0.2(t - 2500) - 100 & 2500s \leq t < 3000s \\ 0 & t \geq 3000s \end{cases} \quad (28)$$

And the sensor fault is assumed to be 20% decrease of the first output. The estimation results of full system states and concerned faults are shown in Figs. 1-8.

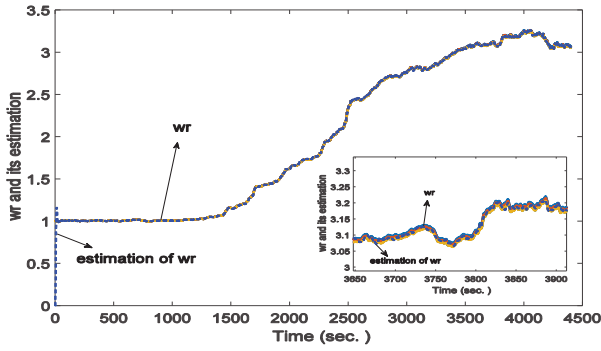


Fig. 1. ω_r and its estimation

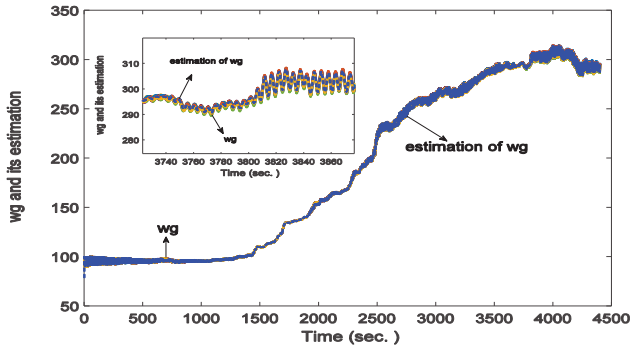


Fig.2. ω_g and its estimation

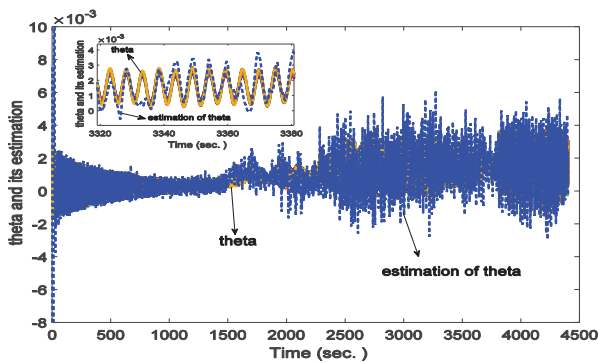


Fig.3. θ_Δ and its estimation

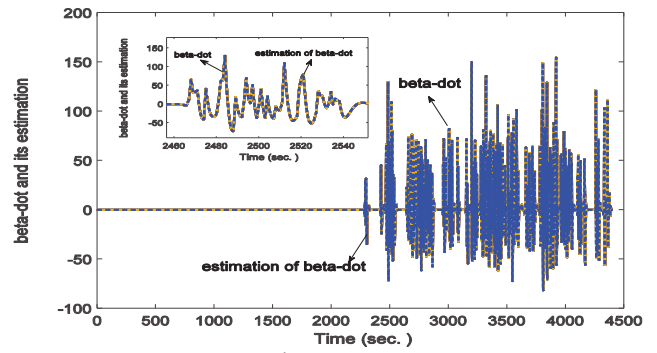


Fig.4. β and its estimation

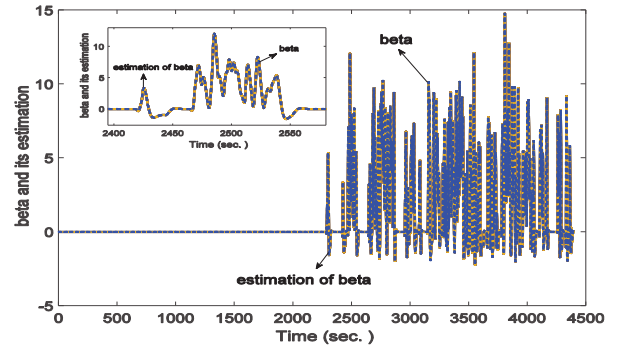


Fig.5. β and its estimation

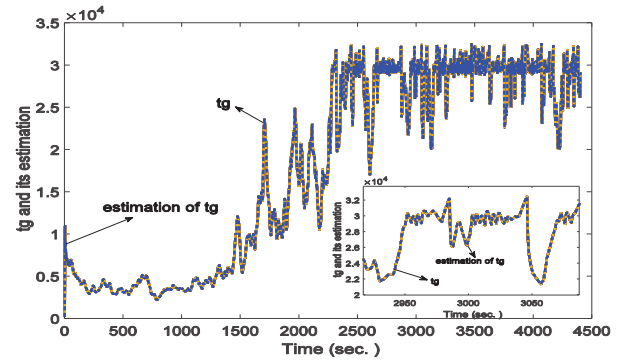


Fig.6. τ_g and its estimation

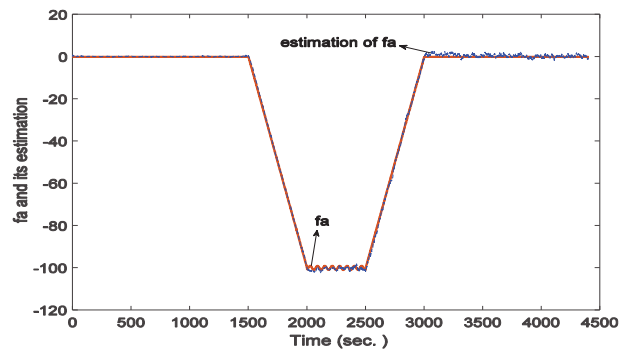


Fig.7. f_a and its estimation

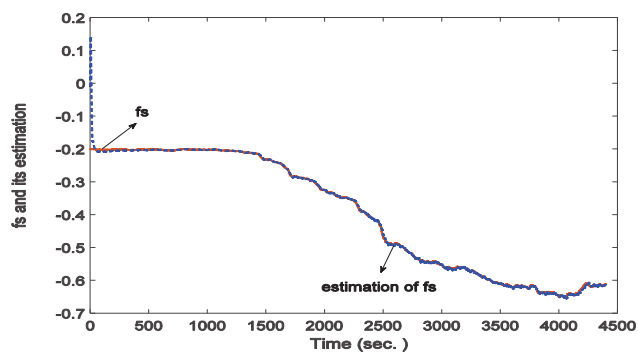


Fig.8. f_s and its estimation

As shown in the above figures, the estimation of both system states and considered faults are excellent. For given performance index, the estimation errors can converge to equilibrium of a certain distance determined by the index, and the un-decoupled unknown inputs are attenuated by LMIs successfully.

V. CONCLUSION

In this work, a novel robust fault estimation technique is developed for stochastic T-S fuzzy systems in presence of unknown inputs. Based on a fuzzy combination of a set of local unknown input observers, an augmented state which includes the original system states and considered faults can be estimated simultaneously while a part of unknown inputs can be decoupled. The un-decoupled process disturbances can be attenuated by solving LMIs which can guarantee that under bounded unknown inputs, the error trajectories is also bounded within a certain value determined by a performance index. The simulation study for a 4.8 MW wind turbine model has well demonstrated the effectiveness of the proposed fault estimation technique. It is encouraged to develop resilient control techniques [28] for wind turbine systems characterized by nonlinear stochastic Brownian systems.

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