Robust fleet planning under stochastic demand

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by

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Preface

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Background Before deciding on a research topic I knew that in order to make a research project a success you need to know what you want to research and why it is relevant, choose a research topic that triggers your curiosity and make sure that your supervisors are aligned. Knowing the importance of the above, I started out by spending some time on reviewing the literature in my research field; optimization in air transportation. Through discussion with my supervisors I found that my interest was geared towards optimization models for airline fleet planning and how to consider the stochastic nature of these problems.

After hours of reading literature, thinking how to approach the problem and solve its sub-problems, discussing with peers and supervisors, and spending countless hours on coding, I can now say that the research is finalized, to the extent to which research can be finalized.. This experience definitely contributed towards the improvement of my research skills.

Acknowledgements I would like to thank my supervisors Bruno Santos and John-Paul Clarke for gently steering the project and mentoring me, always being critical, supporting me and giving me confidence, for learning me how to think creatively without restrictions in an engineering environment and for providing me with great memories of the discussions we had with the three of us. The extraordinary knowledge-sharing environment at the Air Transportation Lab at Georgia Tech is something that I will remember for a long time and I want to thank the lab for providing me with such a great experience. Furthermore, I want to thank the Justus and Louise van Effen foundation for enabling my period abroad at Georgia Tech. Lastly, I would like to provide credits for the cover page picture which is posted on the internet by Benjamin Zhang from Business Insider (Zhang, 2015). In my opinion this picture is relevant because it shows what happens if fleet planning is not done properly.

Abstract

Motivation and problem statement

Airline fleet planning is the most strategic long-term consideration in airline planning and can profoundly impact the airlines' financial performance and operational flexibility. The fleet is deployed over a long-term planning horizon over which uncertainty will materialize both on the revenue side (e.g. stochastic demand) as well as on the cost side (e.g. fuel price volatility). This makes the investment in a fleet of aircraft a highly capital-intensive long-term commitment, which bears inherent risk. Consequently, there is a need for airlines to have a robust fleet that is resilient and flexible to this uncertainty in terms of profit generating capability. This research considers the long-term stochastic nature of demand in an attempt to capture the uncertainty associated with demand realization.

From an academic point of view, the complexity of the problem is defined by the development of a multi-year optimization model subject to uncertainty. Since both optimization models and models that explore the evolution of stochastic variables tend to be computationally demanding, the challenge is to combine these methodologies into one fleet planning modeling framework that is capable to obtain meaningful results while ensuring reasonable computation times.

Research objective and methodology

The research objective is to develop an innovative airline fleet planning concept that is capable to consider the long-term stochastic nature of air travel demand while generating meaningful results in reasonable computation times. The proposed methodology aims to identify robust fleets through the adoption of a portfolio of fleets (each of different size or composition) and a three-step modeling framework.

First, the long-term stochastic nature of demand for each origin-destination pair under consideration is modeled as a mean reverting Ornstein-Uhlenbeck process. The future evolution of demand is explored using a Monte Carlo simulation and sampled into demand sample values.

Second, a weekly flight frequency aircraft type assignment optimization model is iterated for each combination of fleet from the portfolio and demand sample value. Consequently a value matrix is filled with operating profits per fleet, per time period, per demand sample value.

Third, scenarios are generated through the value matrix across the planning horizon using discrete-time Markov chain transition probability matrices. Each scenario contains a sequence of annual operating profits, which are discounted to a single net present value (NPV). By generating numerous scenarios, a distribution of NPVs is obtained based on the evolution of operating profit across the planning horizon across the range of stochastic demand.

Results

Ultimately, the methodology generates two types of results with varying level of detail. The first result is a distribution of NPVs of operating profit across the planning horizon across the range of stochastic demand for each fleet in the portfolio. This provides insight in the magnitude and uncertainty of NPVs across the multi-year planning horizon, and how they relate to the required fleet investment.

The second result is a large data table that contains all financial and non-financial performance metrics per fleet, per time period, per demand sample value. This vast amount of both financial and non-financial data can be used to unravel the underlying factors that drive the distribution of profitability; what are the aircraft utilizations of the different aircraft types in the fleet? What is the average network load factor? How many passengers are spilled? What is the spilled revenue? How many OD pairs are served? What percentage of the passengers is transported nonstop? What are the weekly operating cost and ownership cost? What is the routing network? This vast amount of detailed information can be used for subsequent detailed analysis.

The information that stems from these two types of results enables explicit comparison between fleets on both financial and non-financial performance metrics across different realizations of stochastic demand across the planning horizon. A case study is presented and serves as proof of concept.

Results for a small case study are generated in less than one hour and it is estimated that a real world case study requires a computation time of approximately 10 hours. Moreover, it is made explicit how the computation time increases with increasing problem size.

Limitations and conclusions

Two major limitations are identified. First, the portfolio of fleets contains only a small subset of the theoretically possible collection of different fleet sizes and compositions, and consequently optimality cannot be guaranteed. Second, the impact of competition is neglected in the methodology due to the absence of a market share model and the assumption of fixed market shares for each origin-destination pair over time.

It is concluded that the proposed methodology has the potential to identify robust fleet plans by providing insight into the operating profit generating capability of different fleets in terms of size and composition across a multi-year planning horizon under stochastic demand. Furthermore, the fleets can be subject to explicit comparison using a vast amount of financial and non-financial performance metrics. Moreover, it is estimated that the methodology can generate results for a real world case study in approximately 10 hours. Consequently, it is concluded that the proposed methodology is capable to generate meaningful results in reasonable computation times.

Contents

List of Figures

List of Tables

Symbols and abbreviations

Symbols

λ	Speed of mean reversion
μ	Long-term average air travel demand growth rate
σ	Standard deviation of historical estimation error
a	Y-intercept of the linear least squares regression
b	Slope coefficient of the linear least squares regression
N	Set of airports
K	Set of aircraft types
H	Set of airports that have a hub function
o, d	airports - used for describing passenger flow between airports
h	airports - used for describing airports that also have a hub function
i, j	airports - used for describing aircraft flow between airports
Y	Years in the planning horizon: 1, ..., Y
F	Fleets in the portfolio: 1, ..., F
S	Sample values: 1, ..., S
M	Number of OD demand matrices per year: 1, ..., M
N	Airports: 1, ..., N
H	Hubs: 1, ..., N
Z	Number of OD pairs under consideration: 1, ..., Z
K	Aircraft types under consideration: 1, ..., Z
D	Number of Monte Carlo simulation runs
B	Number of scenarios

Abbreviatons

ANLF	Average Network Load Factor
ASM	Available Seat Mile
CASM	Cost per ASM
DTMC	Discrete-time Markov chain
FAM	Fleet Assignment Model
FCP	Fleet Composition Problem
GDP	Gross Domestic Product
ILP	Integer Linear Programming
LP	Linear Programming
MILP	Mixed Integer Linear Programming
NPV	Net Present Value
OD	Origin-Destination
OR	Operations Research
Pax	Passengers
QSI	Quality Service Index
ROIC	Return On Invested Capital
RPM	Revenue Passenger Mile
TAT	Turnaround time
WACC	Weighted Average Cost of Capital

1 Introduction

This document serves as a final report of the MSc. graduation thesis which is part of the curriculum in Aerospace Engineering at Delft University of Technology. The research topic of this thesis is an innovative methodology for robust airline fleet planning under stochastic demand. This chapter serves as an introduction and details the historical financial performance of airlines, the airline planning process and an introduction to the fleet planning problem.

Historical financial performance of airlines

Airlines are among the poorest performers when it comes to providing return on invested capital (ROIC). Between 2004 and 2011, North American airlines annually returned 4.1% to their investors against an average weighted cost of capital (WACC) of 7.5% (Pearce, 2013), essentially destroying value. In 2012, an average net profit per passenger of $2.26 remained from an average revenue per passenger of $228.26 (Pearce, 2013), which results in a meager average net profit margin of 1.1%.

Historically, airlines were government-owned for multiple reasons other than the rate of return on invested capital such as national economic development and job creation and security (Gibson, 2010, p.110). Due to deregulation and the privatization of many airlines over the years, the air transport market has commoditized and competition has increased significantly causing price competition and downward pressure on profit margins (Pearce, 2013). Still, in despite of the trend of deregulation, the airline industry continues to be one of the most *"regulated deregulated industries"* (Belobaba et al., 2009).

A report by IATA (Pearce, 2013) attempts to unravel the underlying factors for the poor financial performance of airlines, with the help of Porter's five forces (Porter, 1979). It is suggested that the poor performance can be explained by a complex combination of multiple factors that define the airline industry dynamics, such as; bargaining power of suppliers (i.e. aircraft and engine manufacturers, labor unions), bargaining power of buyers (i.e. customers), relatively easy market entrance conditions, regulation, commoditization of air transportation, a fragmented industry structure and problems with the air transport value chain (Pearce, 2013; Porter, 1979). Moreover, the cyclical nature of demand causes cyclical profitability and since fuel cost form a significant portion of airline operating cost, volatility in fuel prices can significantly impact the evolution of year-to-year operating profits and could contribute to a critical financial state of the airline or even bankruptcy (Carter et al., 2006). Furthermore, airline orders for new aircraft tend

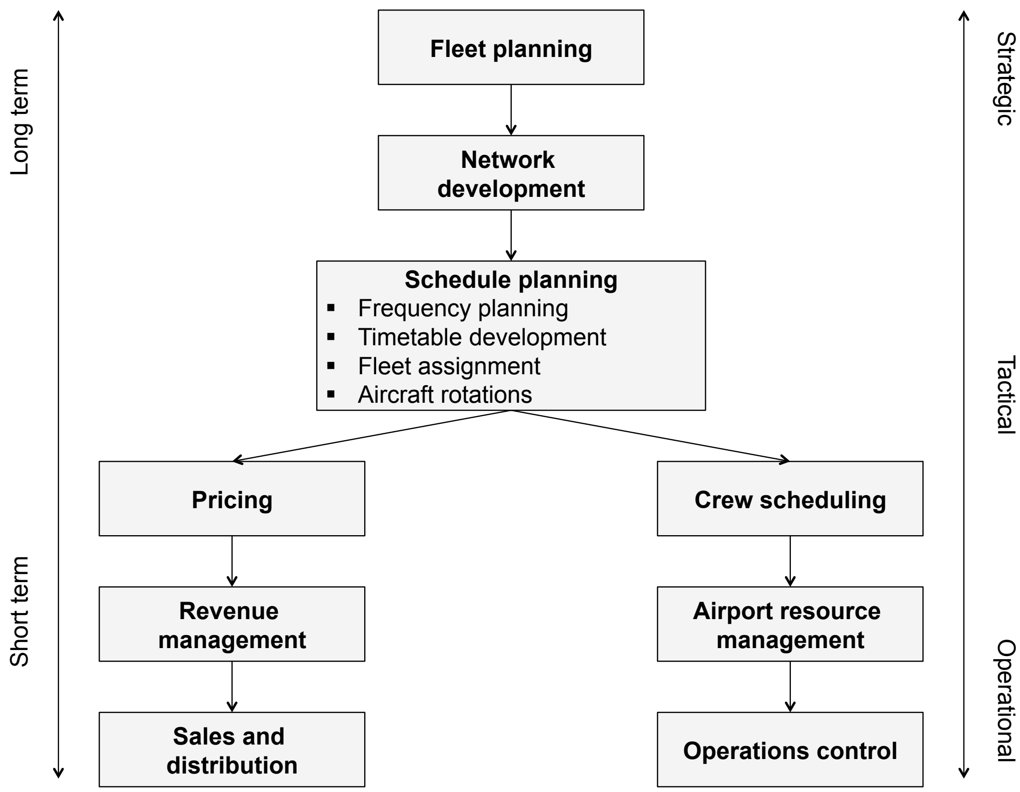


Figure 1.1: A schematic overview of the airline planning process adapted from Barnhart (2003)

to be synchronized with years of high profit, and due to the lead time between the order and delivery of aircraft, these aircraft are often delivered in periods of downturn of the business cycle which causes overcapacity (Clark, 2007; Gibson, 2010; Stonier, 1999). The combination of all these factors results in a persistently low profit margin, an inability to meet return requirements (i.e. ROIC lower than WACC), and a high risk of bankruptcy due to year-to-year volatility in demand and fuel prices.

Because of the shift from government-owned to privatized airlines, there is a need to make the airline industry attractive to investors by increasing the returns on invested capital to levels at or above the average weighted cost of capital. Ultimately this boils down to increasing and optimizing the effectiveness of the airline's asset base (i.e. optimizing the fleet plan) to produce the right level of operating profits, which could be realized by making use of complex mathematical fleet planning optimization models.

Airline planning process

The airline planning process consists of multiple steps that are performed sequentially or integrally (Barnhart and Cohn, 2004; Lohatepanont and Barnhart, 2004). These steps can be categorized based on the time span between the point of decision-making and the day of operations, which can range from long-term to mid-term and short-term. Alternatively, the different steps can be categorized from a business perspective, with decisions having an impact on a strategic, tactical and operational level. All the steps that are involved in airline planning are visualized in Figure 1.1.

The fleet planning problem involves the management of the fleet size and composition of an airline over time by deciding on matters such as: how many aircraft to acquire, which aircraft types to acquire, when to acquire them, when to dispose them and decisions regarding leasing or buying. Often, fleet planning decisions are closely tied to decisions on network development

(Belobaba et al., 2009, p. 162), which deals with the question on which markets (i.e. origin-destination pairs) to serve and which routing network to employ (e.g. hub-and-spoke or point-to-point). The schedule planning step deals with decisions on the frequency offered on routes (i.e. frequency planning), the departure and arrival times of flights (i.e. timetable development), the assignment of aircraft types (i.e. fleet assignment), the assignment of specific aircraft (which are specified by their *tail numbers*) to a series of flight legs to and from maintenance locations (i.e. aircraft rotations). Crew assignment deals with assigning both cockpit and cabin crew to a series of flight legs (i.e. crew scheduling). Decisions regarding pricing and revenue management are concerned with setting price levels per fare class as well as trying to achieve the right balance of accepting low fare, early booking leisure passenger while protecting enough seats for the higher fare, later booking business passengers (Swan, 2002). Ultimately decisions have to be made on the day of operations regarding disruptions and recovery (i.e. operations control).

Intuitively, it can be understood that making all the decisions regarding the fleet composition, routing network and schedule design might be a tedious job, if not impossible, to do by hand. Furthermore, it is even more difficult to tie all these decisions together in order to maximize profit on an airline level. The size and complexity of these problems have driven advancements in *operations research* (Barnhart and Cohn, 2004; Gabrel et al., 2014), which is a scientific research field that aims to provide solutions to business problems by mathematically formulating them as optimization models (Hillier and Lieberman, 2010). The extent to which these sophisticated models have been developed in theory and used in practice differs per planning step. The holy grail of a single tool in which each and every decision is combined from long-term to short-term is yet to be developed.

The fleet planning problem

Fleet planning is the most strategic long-term consideration in airline planning and can profoundly impact the airlines' financial performance and operational flexibility. Investing in an aircraft fleet is a highly capital-intensive long-term commitment which bears inherent risk because the fleet is deployed over a long-term planning horizon over which uncertainty will materialize, both on the revenue side (e.g. stochastic demand) as well as on the cost side (e.g. fuel price volatility). Consequently there is a need for airlines to have a robust fleet that is resilient and flexible to this uncertainty in terms of profit generating capability. Figure 1.2 presents numerous measures that can be taken to achieve robustness.

There is an increasing trend towards aircraft leasing because of the flexibility benefits and reduced up-front investment cost. Leasing comes at an operational cost for the airline however, due to a compensation for the incurred risk that is transferred to the leasing company. Figure 1.2 also highlights that airlines can potentially avoid these flexibility cost by creating flexibility in an alternative way: having the flexibility built into the fleet composition itself. This research presents an innovative methodology that aims to achieve such robust fleets.

From a scientific perspective, mathematical optimization models have found successful real world implementations in some areas of airline planning; e.g. the implementation of fleet assignment models resulted in significant cost efficiency and operating margin improvements (Abara, 1989; Rushmeier and Kontogiorgis, 1997; Subramanian et al., 1994). However, there is scant literature on detailed fleet planning optimization models predominantly because it is considered very challenging to deal with the long-term uncertainty that is inherent to the long-term nature of the fleet planning problem (Belobaba et al., 2009). It is postulated that a fleet planning methodology that *is* capable to appropriately consider the stochastic nature of demand could potentially yield more robust airline fleets. The innovative methodology presented in this thesis aims to fill that literature gap.

The motivation behind this fleet planning research project stems from the combination of its elements; it is an optimization problem that spans a multi-year period and is subject to uncertainty. These properties make it challenging to obtain meaningful results in reasonable compu-

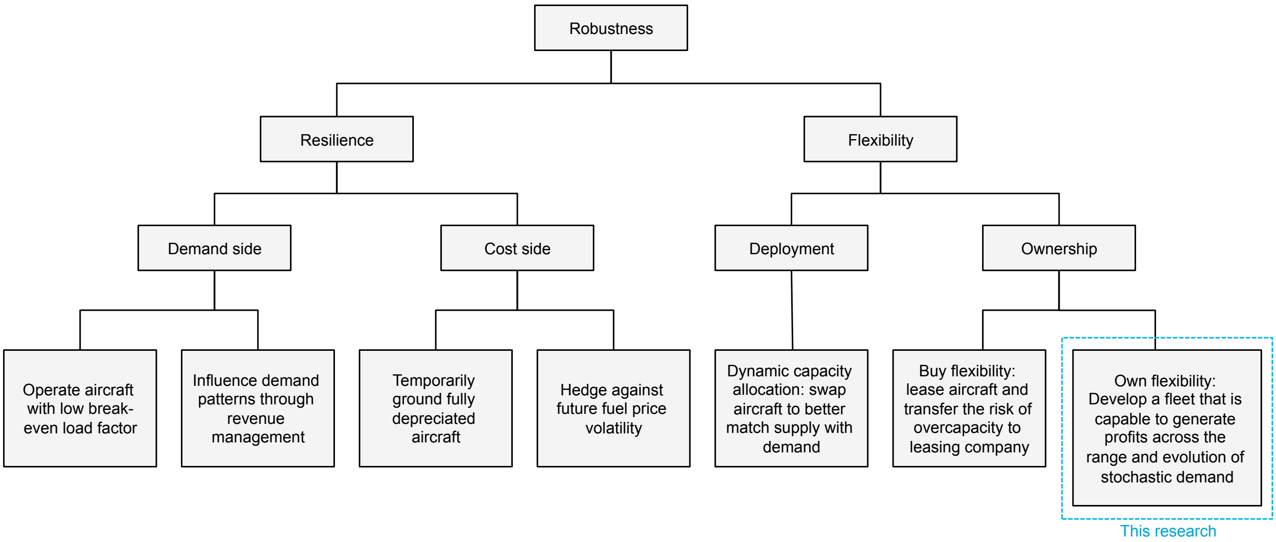


Figure 1.2: Robustness breakdown into resilience and flexibility (Clarke, 2004), and a further breakdown provided by the author

tation times and therefore it is considered interesting to develop a new methodology to address the problem and test its validity.

The research objective of this thesis is to develop a fleet planning concept that considers the long-term stochastic nature of air travel demand and is capable to generate meaningful results in reasonable computation times. Results are considered meaningful if they allow for the explicit comparison of both financial and non-financial performance metrics of different fleets. To achieve this objective the proposed methodology adopts a portfolio of fleets and uses an optimization model that simultaneously considers network development and frequency planning. This allows for the explicit comparison of the profit generating capability of each fleet from the portfolio across a long-term planning horizon across numerous realizations of stochastic demand.

Report structure

The state of the art in the body of knowledge of fleet composition and fleet assignment models is presented in Chapter 2. A project plan, including a detailed problem statement, research objective and project scope is detailed in Chapter 3. The overarching solution methodology is presented in Chapter 4. It encompasses a detailed elaboration of the three models that together shape the methodology; the stochastic demand forecasting model, the fleet assignment optimization model and the scenario generation model. A case study serves as proof of concept and is presented in Chapter 5. It contains an in-depth analysis of the results of each model and synthesizes the results at the level of the overarching methodology. Chapter 6 present the verification and validation of the methodology. The conclusions, limitations and recommendations for future work are presented in Chapter 7. The appendices contain input data and an extensive collection of results.

2

Literature review

The aim of this chapter is to assess the state of the art in the literature on the airline fleet planning problem. The relations, gaps, advantages and disadvantages between different findings in the body of knowledge are synthesized and used as a context in which the research project objective is embedded.

The research fields of fleet planning problems and fleet assignment problems are both investigated. This is done because the goal of the research is to address the fleet planning problem while the methodology that is used to address the problem contains a fleet assignment optimization model that optimally assigns each fleet from the portfolio of fleets across the planning horizon across the range of stochastic demand.

A historical overview and state of the art in fleet composition and fleet assignment models is presented in Sections 2.1 and 2.2, respectively. Both sections elaborate on the different models that were developed in the respective research fields, the methods that were used, the way in which demand was modeled, the different objective functions and constraints that were implemented and the optimization algorithms that were adopted. Section 2.3 aims to identify useful insights from other research fields such as rail car and maritime fleet planning models, while Section 2.4 provides an overview of best practices with regard to fleet planning in the airline industry. Finally, Section 2.5 presents the conclusions of this chapter.

2.1 Fleet composition problems

This section aims to synthesize the body of knowledge on airline fleet planning models. Figure 2.1 provides a historical overview of some key contributions in this research field.

One of the earliest contributions to fleet planning models stem from Schick and Stroup (1981) and Shube and Stroup (1975), which developed a multi-year linear programming model to obtain optimal fleet compositions. The objective function aims to minimize operating cost, ownership cost, cost associated with overcapacity and cost of acquisitions (i.e. debt), while satisfying a set of constraints; demand satisfaction, minimum and maximum flight frequencies per route, aircraft balance constraints and purchase constraints related to aircraft availability and maximum indebtedness. This approach of solving mathematical models using linear programming was fairly new in the respective period and the main conclusions of Schick and Stroup (1981) revolved about the relevance of such a computerized fleet planning model to the airline indus-

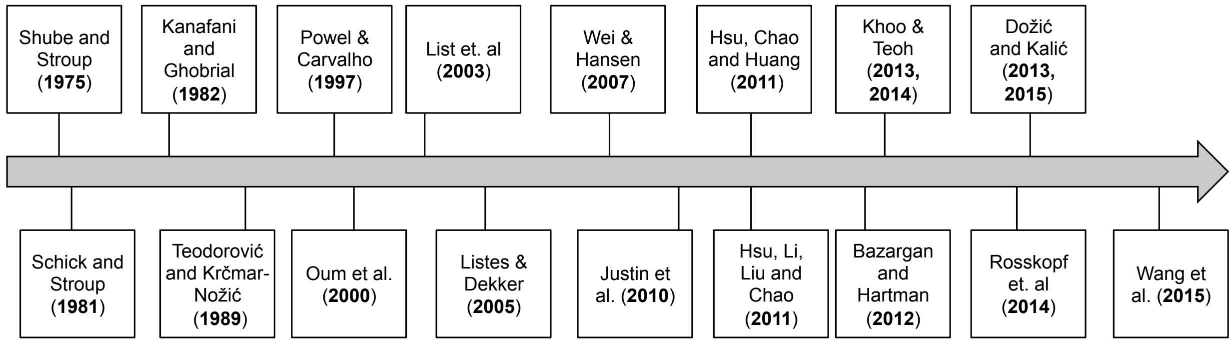


Figure 2.1: Timeline of the most relevant literature in fleet composition problems

try. It is indicated that although the model returns solutions that are realistic, in the sense that the fleet mix is planned over a multi-year period and satisfies demand and capacity constraints as well as imposed frequency requirements, the question is if the airline industry, which traditionally tackled the fleet planning problem in a bottom-up manual approach, is ready for such a top-down computerized approach using linear programming.

A basic fleet composition problem The starting point of a basic fleet planning problem is to assume that future demand per route is known (i.e. deterministic). The goal is to decide which aircraft to buy and how to optimally assign the aircraft to specific routes, considering the demand per route and the specific aircraft characteristics such as ownership cost, cruise speed, seating capacity and block times. The basic model for the fleet planning problem proposed by Santos (2013) is presented below and has a profit maximizing objective function that is based on revenue and ownership cost.

Sets

F	Set of flights
K	Set of aircraft
N	Set of airports

Index

i	Index for flights
k	Index for aircraft fleet type
j	Index for airport

Parameters

p_i^k	Revenue per flight i per aircraft type k
C^k	Owning cost per aircraft type k
$time_i^k$	Flight time per flight i per aircraft type k
BT^k	Block time per aircraft type k
$Budget$	Aircraft investment budget

Decision variables

z_i^k	number of flights i per aircraft type k
AC^k	number of aircraft per aircraft type k

Objective function

$$\text{Maximize} \sum_{k\in K}\sum_{i\in L} p_i^k \cdot z_i^k - \sum_{k\in K} C^k \cdot AC^k \tag{2.1}$$

Subject to

$$\sum_{k \in K} z_i^k = 1 \qquad \text{for } i \in F \tag{2.2}$$

$$\sum_{i \in L} z_i^k \cdot time_i^k \leq BT^k \cdot AC^k \qquad \text{for } k \in K \tag{2.3}$$

$$\sum_{i \in arr(j)} z_i^k = \sum_{i \in dep(j)} z_i^k \qquad \text{for } k \in K, j \in N \tag{2.4}$$

$$\sum_{k \in K} C^k \cdot AC^k \leq Budget \tag{2.5}$$

$$z_i^k \in Z^+ \qquad \text{for } i \in F,\ k \in K \tag{2.6}$$

$$AC^k \in Z^+ \qquad \text{for } k \in K \tag{2.7}$$

Where the objective function (Equation 2.1) maximizes profit by optimizing which aircraft to buy and how to assign them to flights. Equation 2.2 is a flight coverage constraint that ensures that one aircraft type is assigned per flight. The aircraft productivity constraint is given by Equation 2.3 and ensures that the operation hours per aircraft type do not exceed the given block times. The set of flights that arrive at and depart from an airport j is given by $arr(j)$ and $dep(j)$, respectively. The continuity constraint equation (Equation 2.4) ensures aircraft continuity by equating the number of aircraft that arrive at and depart from an airport. An aircraft investment budget constraint is represented by Equation 2.5. Equations 2.6 and 2.7 ensures that both decision variables, z_i^k and AC^k, can take on only integer values. Again, it is noted here that the presented mathematical formulation is a simplified example that can be extended for example through introducing additional terms that account for operating cost and allow for passenger flows through hubs.

Optimizing frequency Kanafani and Ghobrial (1982) presented a similar approach to Schick and Stroup (1981), but with a focus on optimizing aircraft utilization through frequency assignment by matching specific aircraft characteristics (e.g. small short-haul aircraft and large long-haul aircraft) to the demand characteristics of a particular route network.

This focus on frequencies was expanded by Teodorović and Krčmar-Nožić (1989) who developed a multi-criteria nonlinear integer model that aims to return the optimal level of flight frequencies when maximizing for profit, number of passengers flown and minimizing disruptions. As part of the method a market share model is assumed that is dependent only on the offered flight frequencies of competing airlines, and thereby fails to include other aspects such as for example fare levels. Teodorović and Krčmar-Nožić (1989) notes that the problem is a large combinatorial problem of which an optimal solution cannot be found; therefore a Monte Carlo simulation is employed which randomly generates solutions and allows for choosing the best solution among the feasible solutions.

Integer solutions A consideration in fleet planning models is whether or not to force the decision variables to take on integer values (i.e. numbers without a fractional part) instead of real values (i.e. numbers with a fractional part). Integer solutions, are more intuitive to understand in practice and more realistic but also more demanding in terms of computing power and computation times. In order to try to include the integrality constraint in a fleet management optimization model without significantly extending computation times, Powell and Carvalho (1997) model the multi-commodity network flow problem as a dynamic control problem. The approach allows for heterogeneous fleet composition which expands one of their earlier contributions that was restricted to optimizing for homogeneous fleets. It is argued that by using a logistics queuing network (LQN) approach to arrive at integer solutions rather than applying linear relaxation to a linear programming problem, solutions are obtained that are on average within a 3.5% optimality gap. Moreover this approach has the capability to solve larger problems, which better resemble reality, with smaller computation times. For a more thorough overview about solution approaches for LP and MIP problems, such as branch-and-bound, branch-and-price, branch-and-cut and column generation the reader is referred to dedicated literature provided by Barnhart et al. (1994) and Barnhart, Johnson, Nemhauser, Savelsbergh and Vance (1998).

Buying versus leasing A major consideration in fleet planning is whether to buy or lease aircraft in the fleet. Oum et al. (2000) address this aspect of the fleet composition problem by focusing on the optimal lease/own mix for airlines that experience cyclical and stochastic demand. A formulation is proposed for the cost trade-off between owning an aircraft, which yields reduced capital cost and increased expected cost of overcapacity, as opposed to leasing an aircraft. Through a case study on 23 airlines in the period 1986-1993 it is observed that the optimal portion of leased aircraft with respect to all the aircraft in the fleet lies between 40% and 60%. It is concluded that by noting that aircraft lease contracts act as a means for risk sharing between airlines, which have reduced risk by increased flexibility in capacity management while they pay a risk premium to the leasing companies for the transfer of incurred risk.

Bazargan and Hartman (2012) approaches the fleet planning problem from the same perspective and proposes a binary-integer linear programming model for aircraft replacement strategy. The objective function minimizes the total discounted cost of buying, leasing, operating and maintaining aircraft over a certain planning horizon of 10 years. Moreover it includes a cost term that represents additional costs associated with owning aircraft, such as spare parts, hangars and crew training. Furthermore there is a term that accounts for the sale of aircraft. Five observations are made from the results, that apply to both of the case studies that were performed: new aircraft are favored over old aircraft irrespective of buying/leasing decisions; solutions with short-term leases are favored; old aircraft are to be sold; fleet diversity is discouraged; and leasing is preferred over buying. The latter observation is consistent with other studies (Hsu, Li, Liu and Chao, 2011; Oum et al., 2000). Although a method is proposed that incorporates quite some terms in the objective function and constraints, the contribution fails to account for uncertainty in demand or any other factor, rather a sensitivity analysis is performed on lease and buy prices (plus or minus 50%). When analyzing the magnitude of these different cost terms it is observed that operation and maintenance cost are the major cost drivers when evaluated over the long term. The results of a case study indicate a strategy towards leasing new aircraft of common aircraft types over the short term and moreover shows that aircraft with a higher purchase price and a higher operating efficiency are preferred over aircraft that are less expensive to acquire but more costly to operate.

Dynamic capacity allocation In an effort to account for stochastic demand in the fleet composition problem, Listes and Dekker (2005) proposes a two-stage stochastic programming model for fleet composition optimization. In a proof-of-concept it is proposed to add robustness to the fleet planning decision by including stochastic demand and using the concept of demand driven dispatch, as introduced by Berge and Hopperstad (1993). The latter concept acknowledges the existence of uncertainty in future demands when decisions about fleet compositions or initial fleet assignments are made, and tries to accommodate that uncertainty by having aircraft of different sizes within the same crew-compatible family in the fleet, so that they can be swapped when more information about the actual demand becomes available close to the day of operation. Moreover, it is noted that when the stochastic model is solved with integrality constraints the optimality gap is smaller than 0.5%, which is comparable to the order of magnitude of the optimality gaps that result from linear relaxation in deterministic models. Although Listes and Dekker (2005) makes a profound step forward when it comes to considering for stochastic demand in the fleet planning decision, the approach is limited due to a sole focus on short cycle variations in demand that are to be solved using re-assignment. The approach fails to account for the longer-term uncertainty in demand that is characteristic to fleet planning. Moreover, the fleet assignment model in the second stage is based on deterministic demand. A scenario aggregation solution algorithm is used to solve the fleet composition problem in the first stage. The assumption is made that demand is independent and follows a normal distribution, which is discretized into a set of scenarios using descriptive sampling.

Including competitive elements In an effort to capture the effect of the competitive element of the airline industry on individual fleet planning decisions of airlines, Wei and Hansen (2007) proposes a game theory model that aims to find the airline's choices in terms of aircraft size and frequency under a profit maximization objective. A duopoly market is assumed with only non-stop city pair services in which airlines obtain market share through frequency share, but neglect the effect of fare levels, which is detrimental to the approach of capturing competitiveness. Although this approach is not a full-fledged computerized LP optimization model, it does provide insight in strategic considerations on competition that are also very important in fleet planning decisions and are often not captured in mathematical optimization models. Wang et al. (2015) approach the fleet planning problem from the same angle, aiming to include the effect of competitive nature of the industry in aircraft size and frequency decisions. However it is proposed to adopt a multi-objective, profit maximizing objective function and use Monte Carlo simulation as a solution heuristic.

A manufacturers perspective Justin et al. (2010) provides insight in the fleet planning literature from yet another perspective. While airlines are seeking for the optimal set of aircraft to have in their fleet, aircraft manufacturers are the developers and suppliers of this costly equipment. Aircraft research and development programs are highly expensive, take a long time and bear a high risk. Manufacturers inform themselves on three project critical elements on which a project continuation will hinge (Justin et al., 2010): what is the value of a particular future aircraft to the airline as a customer, what are the other aircraft that are available in the market (i.e. competing products), and what will the R&D project cost? Ultimately, these three elements will determine the profitability of a development program for the manufacturer. Justin et al. (2010) perform the first mentioned analysis; an aircraft valuation performed by manufacturers through the eyes of the future customers, i.e. the airlines. This airline perspective entails the evaluation of aircraft over the entire network of routes, which is noted to be a contribution against the traditional route-based analysis.

Stochastic programming Hsu, Li, Liu and Chao (2011) proposes an optimal replacement schedule for airline fleets using a stochastic dynamic programming model which is solved using backward computing. A grey topological forecasting method combined with Markov-chain is used to model the stochastic demand on a market level and a market share estimation that is only frequency dependent to calculate demand on an airline level. From the results it is observed that high volatility in demand drives fleet planning decisions to favor leasing over buying. The objective function minimizes the cost of three cost terms per period over a multi-period planning horizon. These cost terms are operating cost, aircraft replacement cost and a penalty cost which arises from the potential difference between forecasted and actually realized demand. Operating cost are assumed to be dependent on aircraft *status*, which is defined as: aircraft age, type and mileage travelled. A sensitivity analysis is performed on the aircraft age and average lease cost. Although a profound step forward is made using the sophisticated demand forecasting method that accounts for the cyclical demand, Hsu, Li, Liu and Chao (2011) note that this method still lacks the influence of non-cyclical (i.e. random) variations in demand as result of terrorist attacks and aircraft accidents.

Khoo and Teoh (2013) also note the drawback of earlier attempts in literature to include the cyclical nature of stochastic demand, which still neglect the presence of unexpected events, which is referred to as *"the possibility of unexpected events that could take place unexpectedly"*. In order to capture the latter element it is proposed to formulate of a stochastic demand index (SDI). This SDI is developed in multiple steps by identifying a range of possible unexpected events such as disease outbreaks and natural disasters, as well as the probability distributions of these situations based on their historical occurrence. Then the occurrence of all these uncertain events is modeled using a Monte Carlo simulation and combined with a traditional demand forecast that does not account for uncertain events in order to arrive at a single SDI for each operating period, which is used as an input to a fleet management optimization model.

Strategic alliances Building on to an earlier contribution (Hsu, Li, Liu and Chao, 2011), Hsu, Chao and Huang (2011) expands that model by applying it to fleet planning decisions that are evaluated through the lens of strategic alliances. The research objective is to let the expected profit of the strategic airlines converge to each other through interactive-bargaining negotiations on their aggregate fleet planning decisions. In essence such collaboration is aimed at reducing the cost associated with overcapacity by one airline under the assumption that the other airline is interested in temporarily leasing that capacity from its strategic alliance partner at a discount rate with respect to leasing companies. However, the results should be evaluated through the context of the assumptions that were made. Aircraft purchase discounts and the second-hand aircraft market were neglected in this study and could very well impact the solutions significantly.

An environmental perspective Although most of the literature is aimed at the financial aspect of fleet planning by maximizing revenue, minimizing cost, or both, some research is conducted that also incorporates the environmental aspect of fleet planning. Rosskopf and Luetjens (2012) and Rosskopf et al. (2014) try to balance the trade-off between optimizing economical performance and environmental impact by proposing a fleet optimization model, which is called *FLOP*, for fleet composition and assignment. The results of a case study indicate that a quantified trade-off could be made between a 6-7% reduction in emissions (NO_x) at the cost of a 3% loss in financial performance, predominantly realized by replacing old aircraft with new aircraft.

Khoo and Teoh (2014) include environmental performance that is related to noise, emissions and fuel consumption in a single indicator which is called the *Green Fleet Index* (GFI). The objective function minimizes both the GFI and maximizes for profit using a bi-objective dynamic programming model. Furthermore a set of 8 constraints is included to reflect reality: budget, demand, parking (i.e. based on aircraft geometry), sales of aircraft, order delivery, aircraft range, fleet commonality, lead times and selling time constraints. It is noted that while both objectives seem conflicting, they can be achieved simultaneously by increasing load factors.

Fuzzy logic Dožić and Kalić (2013) proposes a two-stage model for fleet planning. The appropriate fleet mix is determined in the first stage with the use of fuzzy logic. The minimum level of aircraft per type in order to cover a flight schedule is determined in the second stage using both a sequential and a simultaneous heuristic algorithm for fleet assignment. As an assumption all aircraft types are categorized in two classes: small and medium sized aircraft. Three categories of stage length as well as three categories of demand patterns are adopted. Later, Dožić and Kalić (2015) introduced a three-stage model for fleet planning. Again, fuzzy logic is applied in the first stage to determine the fleet composition. The second stage determines the minimum level of aircraft per type and in the third stage the even swaps method is applied for aircraft type trade-off considerations.

2.2 Fleet assignment models

The fleet assignment model (FAM) aims to match supply with demand by assigning aircraft types of a given and fixed fleet to scheduled flights with the objective to maximize operating profit or minimize operating cost.

While fleet composition problems are more of a strategic nature since they consider which aircraft to buy, fleet assignment problems are more of a tactical nature since they aim to optimize the utilization of a given fleet that consists of aircraft that already have been bought.

Inputs to the FAM are the demand forecast and the number and type of aircraft in the fleet. These fleet assignment optimization models have been applied in an industry setting with profound success stories with examples including a 1.4% increase in operating margin at American Airlines (Abara, 1989), and operating cost savings of $15 million at US Airways (Rushmeier and Kontogiorgis, 1997) and $100 million at Delta (Subramanian et al., 1994). Figure 2.2 provides a brief historical overview of some key contributions to the body of knowledge of fleet assignment models.

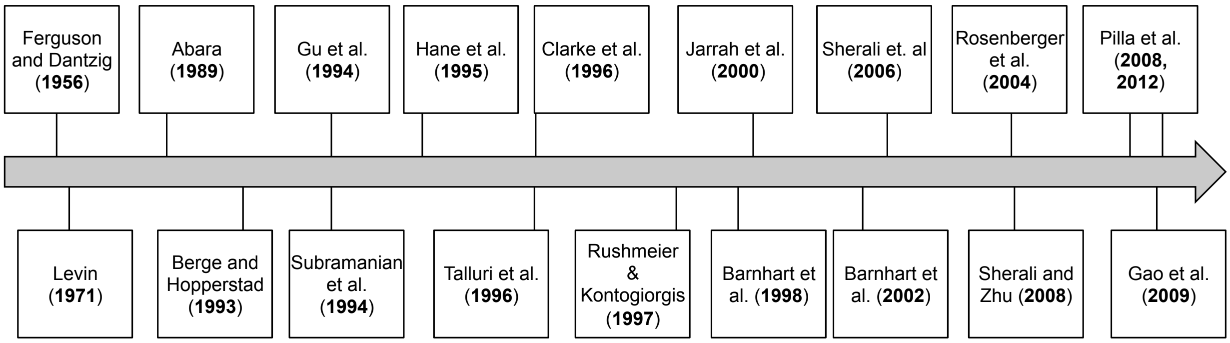


Figure 2.2: Timeline of the most relevant literature in fleet assignment models

A historical introduction Ferguson and Dantzig (1956) presented one of the earliest contributions to FAM that are solved using linear programming. It employed a profit maximization objective function to a combined problem of fleet assignment and aircraft routing under uncertain demand, by assuming a hypothetical demand distribution. Abara (1989) proposed an integer linear programming problem (ILP) and regards the FAM as a multi-commodity network flow problem. The model has 4 basic constraints; flight coverage, continuity, schedule balance and aircraft count, and a fifth constraint that can be modified based on the problem at hand. The objective function can take on three forms, either minimizing cost, maximizing profit or maximizing aircraft utilization. The results from a case study at American Airlines include a reduction in operating cost by 0.4%, a 1.4% increase in operating margin and an increased daily aircraft utilization by one hour. It is noted that even small problems are of an extensive scale in terms of the LP-matrix; a problem with 400 flights per day, 60 city-pairs and 3 aircraft types results in a LP-matrix with 1.800 rows and 6.300 columns. In order to obtain integer solutions the problem was first solved using LP relaxation, followed by fixing of variables and then solved as a mixed-integer program (MIP). Abara (1989) observed that the continuous (non-integer) solution usually already consists of predominantly integer variables but that it could still take quite some iterations to solve. It was found that computation times are dependent on the number of aircraft types in the fleet and ranged between 2 to 60 minutes for 2 to 4 aircraft types, respectively.

Reducing computation times The high computational complexity of the FAM and the desire for integer solutions drives the computation times and required computational power. In an effort to tackle this problem, Hane et al. (1995) proposes a FAM that is solved with various methods with the goal of reducing computation times. These methods include an interior-point algorithm, dual steepest edge simplex, cost perturbation, model aggregation, branching on set-partitioning constraints and prioritizing the order of branching. When comparing the results with a traditional branch-and-bound method it is found that the approach is able to obtain solutions more than two orders of magnitude faster, while the results remain within a 0.2% optimality gap.

Sherali et al. (2006) provides an extensive literature review on FAM and note that two different approaches are used in the mathematical formulation of the flight network to which the FAM is applied. Abara (1989) uses *connection networks* in which the arcs represent connections, whereas Hane et al. (1995) use a *time-space network* wherein arcs represent flight legs. The advantage of the latter approach is that less decision variables are present in the model, because the number of feasible flight legs is much less than the number of feasible connections, which makes the problem less demanding in terms of computational power or time. The time-space network approach makes uses of three sets of arcs: ground arcs, flight arcs and overnight arcs.

The basic FAM Hane et al. (1995) proposed a basic FAM which is based on a time-space-network. A simplified version of the basic FAM (Bazargan, 2004) is presented below:

Sets

F	Set of flights
K	Set of fleet types
C	Set of last-nodes, representing all nodes with aircraft grounded overnight at an airport in the network
M	Number of nodes in the network

Index

i	Flight index
k	Fleet type index
j	Node index

Parameters

$C_{i,k}$	Cost of assigning fleet type k to flight i
N_k	Number of available aircraft in fleet type k

$$S_{i,j} = \begin{cases} +1 & \text{if flight i is an arrival at node j} \\ -1 & \text{if flight i is a departure from node j} \end{cases}$$

Decision variables

$$x_{i,k} = \begin{cases} 1 & \text{if flight i is assigned to fleet type k} \\ 0 & \text{otherwise} \end{cases}$$

$G_{j,k}$ number of aircraft of fleet type k on ground at node j (integer)

Objective function

$$\text{Minimize} \sum_{k \in K} \sum_{i \in F} c_{i,k} x_{i,k} \tag{2.8}$$

Subject to

$$\sum_{k \in K} x_{i,k} = 1 \qquad \text{for } i \in F \tag{2.9}$$

$$G_{j-1,k} + \sum_{i \in F} S_{i,j} x_{i,k} = G_{j,k} \qquad \text{for } j \in M,\ k \in K \tag{2.10}$$

$$\sum_{j \in C} G_{j,k} \leq N_k \qquad \text{for } k \in K \tag{2.11}$$

$$x_{i,k} \in \{0, 1\} \qquad \text{for } i \in F,\ k \in K \tag{2.12}$$

$$G_{j,k} \in Z^+ \qquad \text{for } j \in M,\ k \in K \tag{2.13}$$

Where the objective function (Equation 2.8) minimizes the total cost of assigning the aircraft types to the flights. The flight-cover constraint (Equation 2.9) ensures that each flight is covered by one aircraft type. The aircraft balance constraint (Equation 2.10) ensures aircraft balance at the nodes by summing the number of aircraft per type before the node ($G_{j-1,k}$) plus the number of aircraft per type that arrive at the node ($S_{i,j} = +1$) minus the number of aircraft per type that depart from the node ($S_{i,j} = -1$) and equating the sum of these with the number of aircraft per type at the node ($G_{j,k}$). Equation 2.11 is the availability constraint and ensures that the sum of the number of aircraft per type that are assigned does not exceed the number of available aircraft per type in the fleet. Constraints 2.12 and 2.13 ensure that the decision variables $x_{i,k}$ and $G_{j,k}$ take on binary and positive integers values, respectively.

Improvements to the basic FAM Subramanian et al. (1994) proposes a model that is based on the model provided by Hane et al. (1995). The objective function minimizes operational cost and cost associated with spill. The model is named *"Coldstart"*, to indicate the contrast with a *"Warmstart"* approach that was used by Delta Airlines, which entailed a manual fleet assignment by the fleet planners, that was optimized afterwards by performing local swaps heuristics. The mixed-integer programming (MIP) model is rather large with 40.000 integer variables, 20.000 binary variables and 40.000 constraints. The impact of different solution techniques to this large-scale problem is highlighted by noting that a monthly schedule was solved in 45 iterations in 43 minutes when using the interior-point algorithm like Hane et al. (1995) while it took over 300 thousand iterations and 19 hours to solve the problem using a primal-simplex approach on the same workstation. Another warm start approach is proposed by Talluri (1996), however in the approach the initial fleet assignment is performed using a basic FAM as presented by Hane et al. (1995). Then, to that initial assignment a swapping heuristic is introduced that can be used to swap between different aircraft types at overnight stations while satisfying all the (coverage/flow balance/aircraft count) constraints. Jarrah et al. (2000) identifies the similar need of incrementally fine-tuning an initial assignment in order to include business judgment calls in the assignment that could not be captured in the model, which is called *re-fleeting*. Gu et al. (1994) elaborates the complexity of the FAM and shows that the problem is NP-hard when three aircraft types are employed which means that the problem is computationally complex to solve. This problem is addressed by Hane et al. (1995) through applying three preprocessing steps that reduce the size of the flight network, thereby making the problem less computationally demanding.

Extensions to the basic FAM Next to the efforts that aimed to overcome some of the computational difficulties associated with the FAM, other contributions in literature aimed at extending the FAM by capturing more elements in the model, or extending it to other scheduling steps. Clarke et al. (1996) extends the basic FAM by capturing maintenance and crew considerations into the model. Barnhart, Boland, Clarke, Johnson, Nemhauser and Shenoi (1998) proposes a *string*-based weekly FAM that simultaneously solves the fleet assignment and aircraft routing problem, whereby strings refer to a sequence of flights between maintenance stations.

A branch-and-bound solution approach is adopted that uses column generation to solve the problem. In an effort to capture the network effects, by acknowledging that demand is origin-destination (O-D) based instead of flight leg based, Barnhart et al. (2002) proposes an approach which is called itinerary-based fleet assignment model (IFAM). A linear relaxation and a branch-and-bound solution approach are applied to solve the ILP. Rosenberger et al. (2004) aims to incorporate robustness into the FAM, by reducing schedule disruptions that stem from delays and cancellations by having more flight strings that start and end at the same of hub. Gao et al. (2009) proposes an integrated model for fleet assignment and robust crew scheduling. Crew base purity and fleet purity are included in the approach and it is analyzed how these elements relate to crew cost and schedule robustness to disruptions.

Coping with uncertain demand in the FAM One of the earlier contributions in which stochastic demand is incorporated is provided by Jacobs et al. (1999), which presents a combined model for FAM and revenue management which is solved as an MIP using Benders decomposition method. Listes and Dekker (2005) also considers stochastic demand in the FAM, but by making use of the concept of demand driven dispatch (Berge and Hopperstad, 1993). The aim is however to use such an approach from a fleet planning perspective, therefore a review of this contribution can be found in Section 2.1. A similar approach of introducing flexibility to cope with demand uncertainty is proposed by Sherali and Zhu (2008). A two-stage stochastic MIP for FAM (SPFAM) is used where family type fleet assignment is performed in the first stage and aircraft types within a family are allocated in the second stage, which resembles the strategy provided by Berge and Hopperstad (1993). A solution technique is used which is based on Benders decomposition method and from the results a potential 1.7% increase in profits is observed. Solution times may take as long as 4-12 hours. In a first contribution Pilla et al. (2008) also proposes a two-stage stochastic programming approach for the FAM that makes use of demand driven dispatch. The difficulty of solving a two-stage SP with the traditionally used Benders decomposition method is noted however. The aim is to produce a more tractable solution method by fitting a multivariate adaptive regression splines (MARS) approximation to the expected profit function. Later, Pilla et al. (2012) builds on to the initial contribution by proposing a cutting plane algorithm to optimize the MARS approximation function, which is referred to as *MARS-CP*.

2.3 Insights from other research fields

This section provides insight to fleet planning literature in other industries than airline fleet planning, such as fleet management problems in the rail and maritime industries.

Maritime Jin and Kite-Powell (2000) investigates the fleet management problem with an application to vessels by evaluating vessel utilization, acquisition of new vessels and disposal of old ships. The contribution is based on evaluating the disposal of old vessels and acquisition of new vessels separately by linking the trade-offs of these decisions to the vessel utilization. This allows for solutions with varying fleet size over time. In an effort to introduce the theory of robust optimization into the world of maritime planning, Wang et al. (2012) proposes a robust optimization model for liner ship fleet planning in which both the expected value of profits as well as the variance in these variables are evaluated, based on a given route network. In order to capture the uncertainty of demand, a limited and discrete set of deterministic demand scenarios with deterministic probabilities is assumed, which is comparable to the approach proposed by Listes and Dekker (2005).

Rail car Bojović (2002) proposes a method that aims to find the optimal fleet size of rail freight car, in terms of minimizing cost while satisfying demand. The demand is modeled by assuming it is the sum of a deterministic demand component that follows a normal distribution and stochastic demand component that is represented by a Gaussian random process (i.e. white noise). Moreover, Bojović (2002) provides an extensive literature review on vehicle fleet sizing problems.

Sayarshad and Ghoseiri (2009) also proposes an optimization model for the rail car fleet sizing and allocation problem. A simulated annealing algorithm is used as solution heuristic to solve the problem. Although deterministic demand and travel times are assumed, it hints towards the relevance of future work that includes stochastic variables. In a later contribution, Sayarshad and Tavakkoli-Moghaddam (2010) does just that by including stochastic demand in the same rail car fleet sizing and allocation problem. Again, a simulated annealing heuristic is used to solve the two-stage stochastic program. In a simple example case it is assumed that the demand is normally distributed and 5 demand scenarios are employed, each with probability of 0.2. Another effort to include stochastic demand in the rail car fleet sizing and allocation is provided by Milenković and Bojović (2013), which proposes a fuzzy random linear dynamic model as an alternative approach to the probability theory-based approach proposed by Sayarshad and Tavakkoli-Moghaddam (2010). Milenković and Bojović (2013) notes that rail car demand may fluctuate as much as 50% on a weekly basis and the uncertain demand is modeled as fuzzy random variables.

Other Fleet sizing problems are considered in a whole range of industries other than airline, rail or maritime. List et al. (2006) details a two-stage stochastic programming model for the investment under uncertainty in equipment that is to be used for the transport of radio active wastes from weapon factories to disposal sites. Three sources of uncertainty are identified; uncertainty about the quantity of the waste, uncertainty about the waste processing rates and uncertainty about the certification of packages. The first stage deals with the variables regarding investments in transportation equipment (e.g. trucks, packages), while the second stage deals with minimizing the operation cost of transportation. In order to unravel the difference between a deterministic (expected value), stochastic programming and robust optimization approach all three of these approaches are used. It is observed that with respect to the deterministic case, stochastic programming results in solutions in which 27% more trucks are acquired, whereas robust optimization yields a solution in which 54% more trucks are acquired with respect to the deterministic case. Moreover two risk terms are introduced (financial risk and political risk) in the robust optimization method so that these two risks can be part of a trade-off and the effects of such a trade-off can be evaluated over the equipment investment decisions under uncertainty. In an earlier contribution, List et al. (2003) provides a robust optimization approach for fleet planning under uncertainty in which the trade-off between risk and investment is made explicitly. It is noted that traditional robust optimization methods make use of a mean-variance trade-off in the approach, while this could yield inefficient solutions when applied to fleet planning models. Therefore, List et al. (2003) proposes to focus on a one-sided risk measure instead of variance in an effort to control the likelihood that the objective function exceeds a certain value. The latter value can be adjusted so that trade-offs between cost and risk can be explicitly made.

Another research that highlights the difference between a stochastic programming and robust optimization approach for a transportation problem under uncertainty is provided by Maggioni et al. (2014). The problem deals with the transportation planning of a resource, gypsum, from a set of supply points to a set of demand points. The objective is to minimize total cost which is a function of transportation cost and buying cost, while accounting for uncertainty of two variables: demand for gypsum by cement factories and buying cost from external sources. By comparing the stochastic programming and robust optimization results it is observed that the robust optimization approach returns higher values for the objective function, while the stochastic programming requires a higher computational complexity. This is result is in line with the findings of List et al. (2006) and also in line with the expectation about robust optimization, which is a more conservative, risk-averse approach than stochastic programming and thus will yield larger solution values in case of minimization problems.

Fleet sizing and resource allocation problems are also broadly discussed in the research fields of road transportation, energy systems, production planning, supply chain analysis and a plethora of other industry applications. A thorough review is considered out of scope.

2.4 Industry best practices

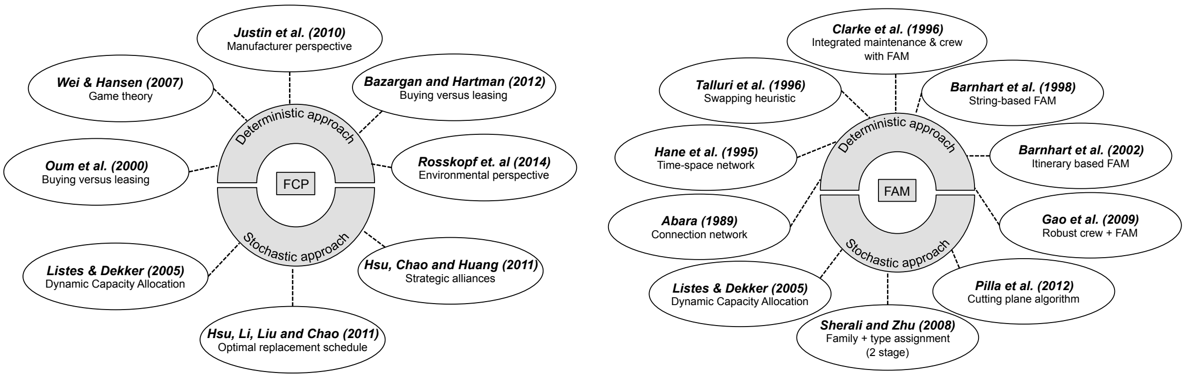
While scientific literature approaches the fleet planning problem in a rigorous mathematical manner from an airline network point of view, it appears that airline decision making is predominantly based on more practical and lower level trade-offs. It seems that airlines are not approaching the fleet planning decisions from a perspective of what the optimal fleet composition would be to cater a range of demands over the entire network. The main focus seems to be fleet renewal or capacity expansion based on route-based analysis, as opposed to a network view. Airlines are restricted to practical constraints such as aircraft purchase slots and delivery lead times which are often not captured in scientific models. Moreover, the aircraft acquisition consideration is mainly driven by availability of cash in the company (i.e. cash flow) or the option to raise cash externally (i.e. debt).

Clark (2007, p.41) indicates that a whole range of commercial decision support tools is available that can be broadly classified in two categories: tools that focus on ownership of a certain fleet composition and tools that focus on optimizing the fleet deployment. Commercial software is available from aircraft manufacturers, airline consulting firms and dedicated airline planning software developers assist airlines in decision making with regard to fleet planning. It seems however that the software does not contain sophisticated methods such as stochastic programming in order to consider stochastic demand. It appears that airlines perform sensitivity analysis with best, worst and base case demand scenarios in order to grasp the uncertainty of stochastic demand.

2.5 Concluding remarks

Figure 2.3a and 2.3b provide an overview of the different perspectives and solution methodologies to the fleet composition problems and fleet assignment models in literature. Through observing these contributions different methods and modeling techniques are identified from literature that could be used to optimize the fleet plan or fleet assignment and deal with stochastic demand, with examples including; sensitivity analysis, Monte Carlo simulation, stochastic programming, robust optimization, dynamic capacity allocation, real options analysis and value at risk. On an aggregate level it could be observed that these techniques differ from each other by providing a different balance in; which elements are captured in the model to get the desired level of realistic solutions, the problem size and computational complexity of the resulting problems and the solution techniques that are used to solve them, the balance between finding (near-)optimal solutions in the feasibility space in reasonable computation times as well as the search for a trade-off between risk and cost. Consequently it can be stated that there is a desire to develop optimization models that are capable to capture elements that reflect the appropriate level of reality, deal with the complexity that stems from that approach by having appropriate solution techniques and to deliver optimal or near-optimal results in reasonable computation times.

A trend to capture more realistic features in the fleet composition problem is characterized by the trade-off between buying and leasing, game theory in a competitive market and the influence



(a) Different perspectives on the FCP (b) Extensions and improvements of the FAM

Figure 2.3: An overview of deterministic and stochastic fleet composition problems (FCP) and fleet assignment models (FAM) in the body of knowledge

of strategic alliances on fleet planning. In the research field of fleet assignment models there is a trend towards finding appropriate solution techniques to deal with integrality constraints, reducing computation times and extending the FAM to other airline planning steps such as crew assignment and aircraft routing.

Although the number of contributions that consider stochastic demand is not yet in the majority, a clear research trend can be identified that aims to find an effective way to deal with uncertainty in reasonable computation times. This can be considered challenging however, since the deterministic counterparts already are consuming in terms of computation power and times.

As a vision, it can be formulated that when aiming to effectively consider stochastic demand in long-term fleet planning models within reasonable computation times there are some considerations to keep in mind; it is likely that the problem itself should be simplified; a small number of scenarios should be adopted; the computation power of the computer has to be increased; a new, quick and simple modeling technique has to be developed that effectively exploits the topology of the mathematical model for these kinds of problems; or a combination of all the aforementioned.

To conclude it can also be observed that with respect to fleet composition models there is supposedly a significant gap between the sophisticated optimization models that are discussed in literature and the relatively basic approach that is adopted by airlines in practice.

3

Project plan

This chapter details the problem statement, research objective, scope, and contribution. It is important to emphasize that the research project design was not laid out during a one time brainstorming and project definition session but rather is the result of an iterative-parallel design strategy that encompasses continuous redefinition of the project scope. The presented problem statement and proposed solution method are the result of that iterative design strategy.

3.1 Problem statement

The problem statement is defined by describing it both from an industry perspective as well as a scientific perspective.

- Industry perspective: the industry problem is defined by the poor financial performance of airlines;
  - Airlines have low operating and net profit margins and consistently fail to meet investors expectations in terms of return on invested capital (ROIC) with respect to weighted average cost of capital (WACC).
  - This poor financial performance ultimately is the result of a complex set of underlying business characteristics that define airline profitability.
  - As result of the low profit margins, airlines have a small margin for error when it comes to operating their business profitably by balancing supply with demand through optimally utilizing their asset base (i.e. aircraft fleet). This problem is severed by the fact that airlines are subject to cyclicality and uncertainty on both the revenue side (i.e. 10 year business cycles, seasonality throughout the year) and on the cost side (i.e. fuel price volatility), as well as the perishable nature of the product (i.e. an unsold aircraft seat for a specific flight cannot be sold anymore after the flight has departed).
- Scientific perspective: from a scientific perspective the problem statement is defined as the challenge to develop airline fleet planning optimization models that consider the evolution of uncertainty across the long-term planning horizon. Airline fleet planning optimization is particularly challenging because;

- It spans a long-term planning horizon over which uncertainty materializes which has the potential to profoundly impact the evolution of profitability across the planning horizon. Simulation models that explore the uncertainty over time can be computationally demanding. The same holds for optimization models that optimize the allocation of capacity to demand. This makes it challenging to combine these two methodologies into one overarching solution methodology that deals with optimization under uncertainty.
- Fleet planning decisions are closely tied to airline network decisions. This adds a substantial layer of detail to fleet planning decisions since it requires the evaluation of route profitability per aircraft type for all potential routes and all aircraft types under consideration. An apparent real world example of the impact of the routing network on the fleet composition is observed when comparing the fleets of low cost carriers that operate a point-to-point network with one or two aircraft types (i.e. thereby exploiting the benefits of fleet commonality) versus legacy carriers that operate a hub-and-spoke network with a diverse fleet composition to cater the need for different capacity levels in their network (Belobaba et al., 2009).

To summarize, from a scientific perspective it is a challenge to obtain meaningful results in reasonable computation times in airline fleet planning optimization models that capture the stochastic nature of demand across a long-term planning horizon. From an industry perspective there is a need to improve the consistent poor financial performance of airlines.

3.2 Research objective

The formulation of the research objective is grounded in the problem statement, which is detailed in Section 3.1. The research objective is;

To analyze the potential benefit of a portfolio-based airline fleet planning concept, by developing a model which considers the stochastic nature of air travel demand and aims to generate meaningful results in reasonable computation times.

In order to define the methodology that is suitable to achieve the research objective, it is important to demarcate the concepts of *meaningful results* and *reasonable computation times* in the context of this research. Therefore, these two concepts are detailed over the next two paragraphs.

Meaningful results During the demarcation of the project scope it is decided what kind of results are considered meaningful. Ultimately, the goal is to identify which fleet displays *the most desired behavior* across the uncertain planning horizon. What type of behavior is considered desirable depends on the strategic vision the airline has with regards its network, the type of business model it adopts as well as the risk profile it is seeking to pursue. In this research results are considered to be meaningful if they allow for the (financial and non-financial) comparison of the performance of different fleets across the planning horizon under stochastic demand. Therefore, in the context of this research meaningful results are results that allow to:

- Compare how different fleets (both in size and composition) generate cash flow across multiple years in the planning horizon across multiple realizations of stochastic demand.
- Compare non-financial performance metrics such as; network load factors, aircraft utilization per aircraft type and metrics that describe the routing network (e.g. the number of passengers that are transported using a nonstop flight or with a connecting flight through a hub).

Based on the premise that more informed decision making can be achieved when different alternatives can be evaluated, especially acknowledging that not all elements of influence in fleet planning can be captured in the model, it is considered meaningless if the result of the methodology is a single output that states which fleet composition and size is supposedly optimal, even if uncertainty is taken into account in order to arrive at that result. Consequently, the methodology should be set up in such a way that it allows for these explicit comparisons to take place. This is achieved by the adoption of a portfolio of fleets, each different in terms of size or composition. The deployment of each fleet from the portfolio is evaluated across the planning horizon across multiple realizations of stochastic demand, which allows for comparisons to take place.

Reasonable computation times Since fleet planning is one of the most strategic long-term decisions in airline planning, it is considered acceptable if solutions can be generated within two hours to get some basic results for a limited set of OD pairs and fleets under consideration.

This view is extended by noting that the term *reasonable* is subject to interpretation. Therefore it is considered valuable if the methodology harvests insight into how the meaningfulness of results scales with computation times. Armed with that insight, fleet planning decision makers in industry can make explicit trade-offs with regards to the level of detail they wish to consider and the corresponding computation times to get to a solution. Below is a list of elements of influence in fleet planning that impact both the meaningfulness and the computation times;

- number of different fleets in the portfolio
- number of different aircraft types in the portfolio
- number of OD pairs under consideration
- number of years (or any other discrete measure of time periods) in the planning horizon
- number of realizations of uncertainty per time period under consideration

Research framework The research framework provides a general indication of the steps that need to be taken to achieve the research objective from the gathering of information, to the demarcation of the research scope, formulation and implementation of the proposed solution methodology to observation and analysis of the results and ultimately the drawing of conclusions. A schematic visualization can be found in Figure 3.1.

Programming language and optimization software In this research, conducting experiments is to be viewed from the perspective of a computer programming environment. The proposed methodology and models are translated to computer code using a programming language in order to numerically solve the problem. It is decided to use the Python programming language since its high level abstraction enables relatively quick implementation of algorithms. Moreover, its open source nature results in a large active online community that can be used for rapidly increasing the necessary coding knowledge. For the optimization part it is decided to use Gurobi, which is a commercial software package that is available at no cost to students, it is fast and works intuitively with the Python programming language.

3.3 Scope

To achieve the objective within the available time, space and money it is important to demarcate the research objective. Scoping has already partially taken place in the previous section by detailing how the concepts of meaningful results and reasonable computation times are defined.

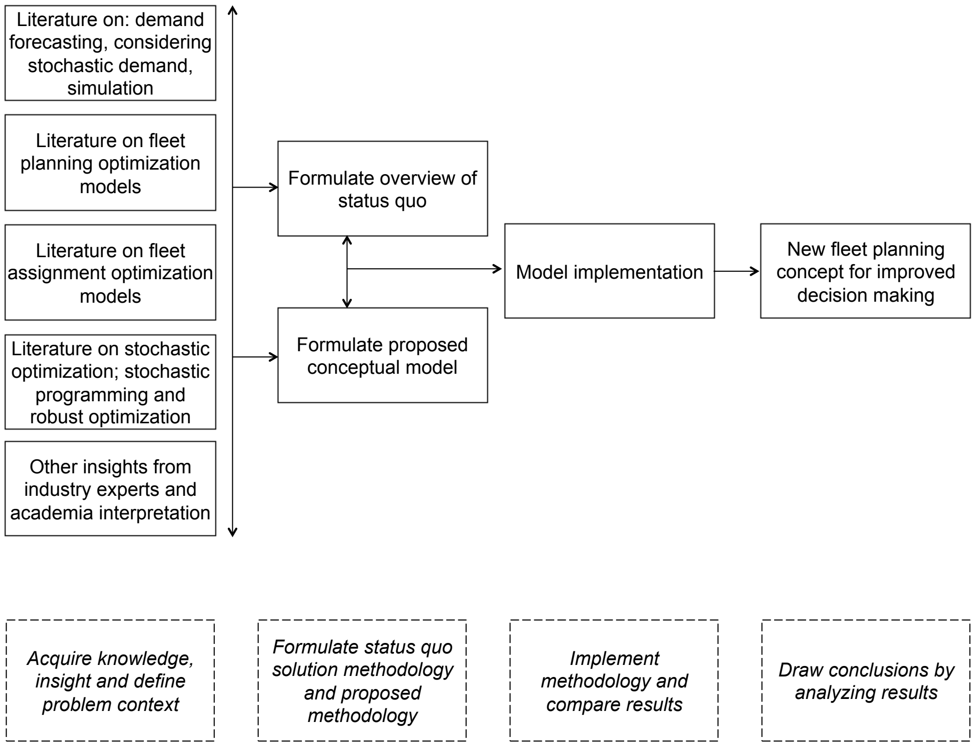


Figure 3.1: Research framework

Essentially, through scoping the assumptions are set that together form the context in which the experiment is going to be performed and as such is also the context in which the results should be evaluated. Below is a list of project demarcations:

- The uncertainty that is associated to fuel price volatility is not considered.
- The goal is not to solve a fleet planning problem for a specific airline. The goal is to demonstrate the proof of concept of a new fleet planning methodology that is generic and can be applied to any specific airline, irrespective of its business model, routing network or fleet planning preferences in terms of size and mix.
- The methodology is based on the premise that the poor airline profitability can be improved by optimizing the asset base towards catering future uncertain demand. As such, the underlying economic factors that drive poor profitability (e.g. competition, bargaining power of suppliers, regulation, etc., see Chapter 1) are considered out of scope and are not part of the solution methodology.
- The overall methodology is set up as a decision analysis approach that aims to provide insight by comparing different fleets through simulation, optimization and scenario generation. It is not a stochastic programming approach that aims to return the *single best* fleet given the distribution of uncertainty across the planning horizon.
- The uncertainty that is associated with the negotiation and deal making process is difficult to capture into a mathematical optimization model (even in game theoretic models) in such a way that it reflects reality. Examples include negotiations on aircraft purchase prices, long-term spare parts contracts as well as long-term exclusivity agreements signed between airlines and aircraft manufacturers. The research objective and proposed methodology do not tackle this problem. However, since the methodology provides information about the profitability of fleets across the planning horizon across stochastic demand, it does allow for better understanding of the trade space when faced with these negotiations.

- It is noted that focusing on computation times inherently includes the scientific field of computer science. Computation times depend on how many computers are used, the type of computer used, working memory, processor, the number of cores in the processor, level of abstraction of the programming language, efficiency of the used optimization algorithm, whether use is made of parallel computing or cloud computing, etc. The elements that from a computer science research field perspective impact computation times are considered out of scope for this research.

- The following important elements of impact to fleet planning do not fall within the scope of this research: the timing of future orders and deliveries, lease/buy mix considerations as a measure to increase flexibility and transferring the risk of mismatching supply with demand under uncertainty, no game theoretic considerations (not on the airline competition side and not on the deal making side), no market share-frequency considerations.

3.4 Impact and contribution

In Figure 3.2 the state of the art (i.e. status quo) and the contribution to that state of the art are displayed both from a scientific perspective (i.e. the contribution to the body of knowledge) as well from the industry perspective (i.e. the industry impact).

	Scientific	Industry
Status quo	▪ Deterministic airline fleet planning model (Bazargan, 2012) ▪ Two-stage stochastic programming approach to fleet planning problem assuming point-to-point network (List, 2003) ▪ Two stage stochastic fleet assignment models – dynamic capacity allocation (Listes & Dekker 2005, Pilla 2012) ▪ Value of fleet commonality (Brüggen & Klose, 2010)	Fleet planning decisions are driven by: ▪ Route-based analysis ▪ Availability of cash ▪ Relatively simplistic demand scenarios Perspective: ▪ “Should we buy this 787 for this specific route based on these three simplistic demand scenarios?”
Contribution of proposed methodology	▪ Portfolio-based approach allows for the explicit comparison (financial and non financial) between different fleet sizes and compositions across the planning horizon across different realizations of stochastic demand ▪ Extensive consideration of stochastic demand per origin-destination pair by modeling it as a mean reverting Ornstein-Uhlenbeck process and considering year-to-year evolution of uncertainty using discrete-time Markov transition probability matrices	Fleet planning decisions are driven by: ▪ Network analysis ▪ Demand scenarios based on an extensive and in-depth consideration of stochastic demand Perspective: ▪ “What is the ideal fleet size and composition to have for the entire network under stochastic demand?”

Figure 3.2: Science and industry: status quo and contribution of the proposed methodology

4

Methodology

4.1 The overarching solution methodology

This section provides a high level introduction to the overarching methodology, which will be alternatively referred to as the modeling framework. In order to achieve the research objective the overarching methodology must satisfy two requirements while ensuring reasonable computation times; consider the stochastic nature of demand and allow for the explicit comparison of both financial and non-financial performance metrics of each fleet from the portfolio. Figure 4.1 presents how these requirements are grounded in the structure of the methodology and result in the formulation of three models.

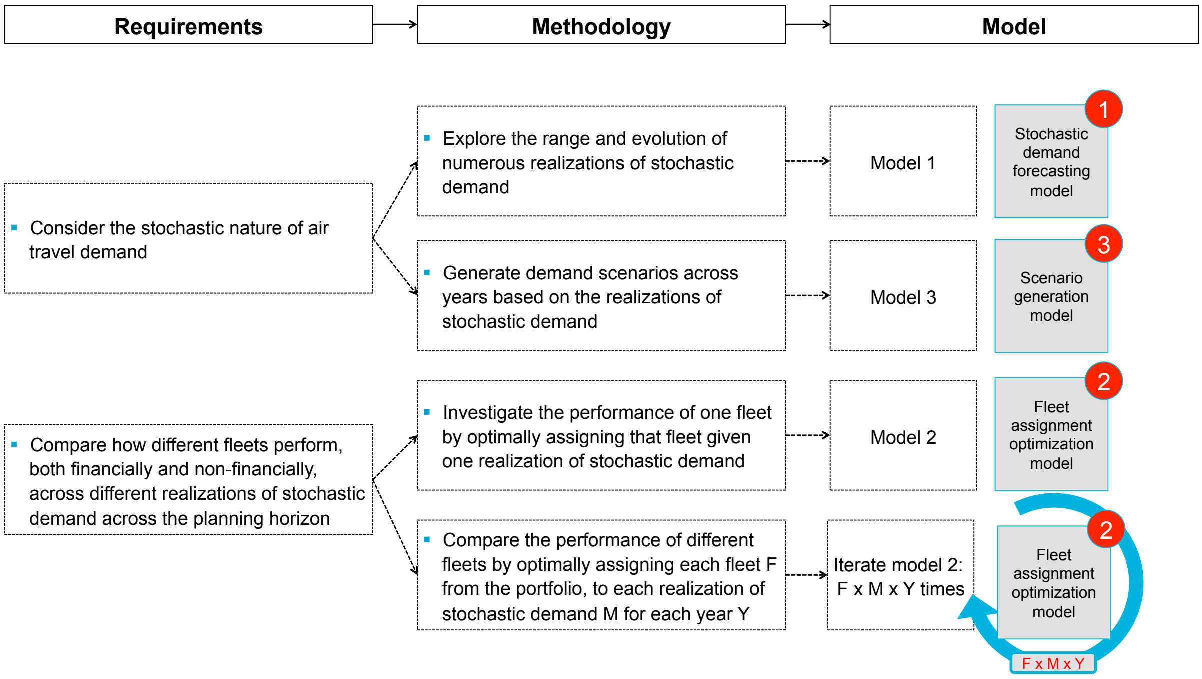


Figure 4.1: The translation from requirements to methodology and models

Table 4.1: Variable notation in the methodology

Notation	Definition
F	# Fleets in portfolio
Y	# Years in planning horizon
D	# Monte Carlo simulations
S	# Sample values per year per OD pair
M	# OD demand matrices per year
N	# Airports under consideration
Z	# OD pairs under consideration
H	# Hubs under consideration
K	# Aircraft types under consideration
B	# Scenarios generated

A brief summary of each model

The variable notation that is used throughout the methodology is presented in Table 4.1. The concept of an origin-destination (OD) demand matrix is used throughout the methodology; an example OD demand matrix is presented in Table 4.2. Sections 4.2, 4.3 and 4.4 respectively present the details of each of the three models that together form the overarching solution methodology. However, to support the high level understanding of the overarching solution methodology, a short summary of the working principles of each model is outlined below. Figure 4.2 presents how these models are connected.

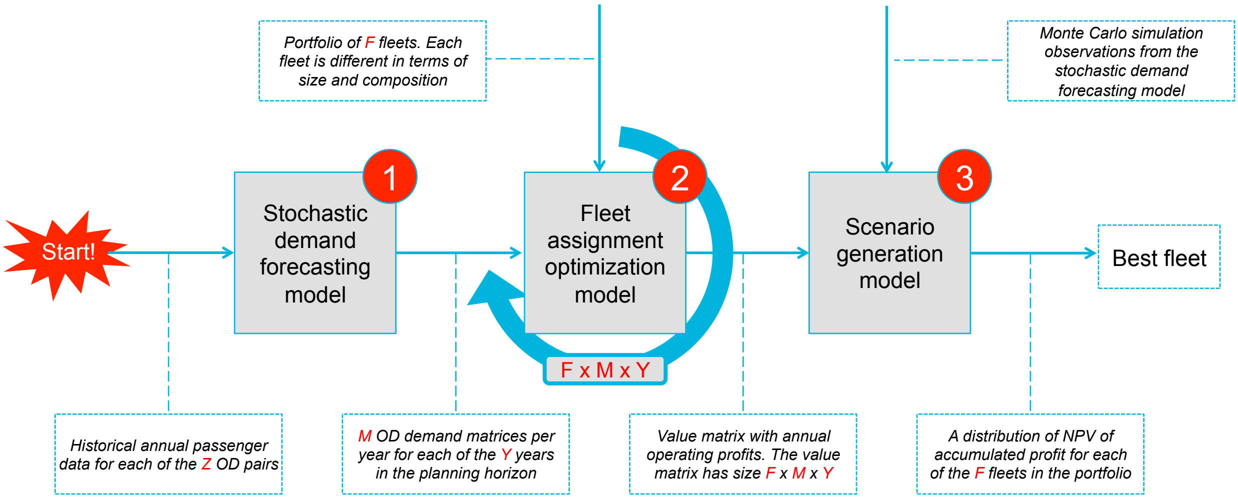


Figure 4.2: The proposed solution methodology consists of three underlying models

Model 1 The stochastic demand forecasting model

- The goal of the stochastic demand forecasting model is to output a set of M OD demand matrices per year for all Y years in the planning horizon, by exploring the evolution of stochastic demand per OD pair. This is achieved by modeling the stochastic nature of air travel demand as a mean reverting Ornstein-Uhlenbeck process and running D Monte Carlo simulation runs across the Y years. The resulting D Monte Carlo simulation observations per OD pair per year are sampled into S representative sample values which are then used to construct M OD demand matrices per year for the Y years in the planning horizon. In total $M \cdot Y$ OD demand matrices are outputted by this model.

Model 2 Fleet assignment optimization model

- The goal of the optimization model is to optimally allocate one fleet in terms of operating profit given one OD demand matrix. This is achieved by mathematically formulating the optimization problem as an Integer Linear Programming (ILP) optimization model based on a weekly flight frequency aircraft type assignment. The mathematical formulation consists of a profit maximizing objective function and a set of demand, capacity, physical and integrality constraints. The formulation is such that it allows for both point-to-point and hub-and-spoke network routing networks. A number of inputs are used for this optimization process: a given OD demand matrix, a given fleet which is characterized by the number of aircraft per aircraft type K, the specific aircraft characteristics of each aircraft type (seats, cruise speed, range, daily utilization, turnaround times, fixed cost, variable cost), as well as yields and distances between airports. The optimization model returns weekly operating profit which is multiplied by 52 to arrive at annual operating profits. Besides financial results, the model also returns non-financial performance metrics such as the average network load factor and aircraft utilization. This optimization process is repeated for each fleet-OD demand matrix combination. Consequently, the optimization model is run $F \cdot M \cdot Y$ times which results in an equal amount of annual operating profits that are stored in a value matrix.

Model 3 Scenario generation model

- The goal of the scenario generation model is to generate numerous paths through the value matrix across the planning horizon. By adopting the discrete-time Markov Chain (DTMC), the D Monte Carlo simulation observations per year per OD pair can be used to construct transition probability matrices which describe the year-to-year transition behavior of Monte Carlo simulation observations. Such a transition probability matrix is constructed for each consecutive year combination for each OD pair, resulting in $(Y-1)\cdot Z$ transition probability matrices. Then, for each consecutive year combination, the OD-pair based transition probability matrices are aggregated so that they essentially describe the transition behavior of an entire OD demand matrix. As result, the number of transition probability matrices is reduced to $(Y-1)$. Using these aggregated transition probability matrices, B paths or scenarios are generated throughout the planning horizon. Each path corresponds to a sequence of Y annual operating profit values per fleet, which are discounted to a single net present value (NPV) of accumulated profits. Subsequently, the output of this model is a distribution of NPVs per fleet that is based on the evolution of stochastic demand across the planning horizon.

Table 4.2: Example of an OD demand matrix

		Destination		
		Airport 1	**Airport 2**	**Airport 3**
Origin	**Airport 1**	0	50	85
	Airport 2	50	0	23
	Airport 3	85	23	0

4.2 Stochastic demand forecasting model

4.2.1 Introduction

This section thoroughly details the working principles of the stochastic demand forecasting model. The goal of the model is to construct M OD demand matrices per year that represent the range of uncertainty within each year, for all the Y years in the planning horizon. Figure 4.3 places the stochastic demand forecasting model in the context of the overarching methodology and thereby highlights that each of the $M \cdot Y$ outputted OD demand matrices is used as input to the fleet assignment optimization model.

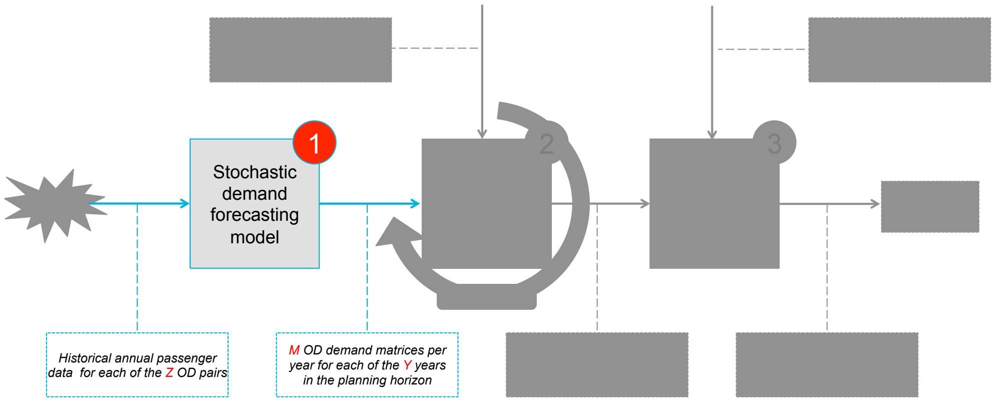


Figure 4.3: The stochastic demand forecasting model in the context of the overarching solution methodology

4.2.2 The mean reverting Ornstein-Uhlenbeck process

In order to explore the evolution of uncertainty into the future, a representation of historical behavior of uncertainty needs to be captured in a mathematical expression. The chosen mathematical expression is the mean reverting Ornstein-Uhlenbeck process. The rationale for choosing this representation of uncertainty, its mathematical formulation and the estimation of its model parameters are elaborated over the next three sections.

Rationale

The mean reverting process is a methodology that can be used to model the stochastic nature of variables that tend to revert about a long run mean value. In this sense the mean reverting process differs from random walk theory in that the latter assumes that variables are equally likely to take on any random value, whereas the former assumes that in the long run it is more likely that variables tend to revert about a certain long-term average value.

The mean reverting process has been successfully applied to model variables that tend to be cyclical. Prime examples include the modeling of stock, commodity and option prices (Bessembinder et al., 1995; Schwartz, 1997). Ultimately these variables tend to correlate to the cyclical behavior of gross domestic product (GDP). In this research the mean reversion concept is not applied to model the stochastic nature of stock prices, but of air travel demand. Although modeling future stock prices is a different activity than modeling future air travel demand, the underlying causes for the variation in these two variables share a common denominator, being the variation in GDP. Since both stock price and air travel demand correlate with GDP and the concept of mean reversion has been effectively applied to forecast future stock prices, it seems promising to apply the same concept to forecast future air travel demand.

An important consideration is to which specific air travel demand variable to apply the mean reverting process: historical passenger data, historical passenger growth data, historical revenue-passenger-mile (RPM) data or historical RPM growth data. All four of these variables have been evaluated and the variable that exhibits the best goodness of fit was chosen; historical passenger growth data. For the validation of this consideration the reader is referred to Section 6.1.1.

Mathematical formulation

The mean reverting process is represented by the following equation;

$$X_{t+1} = X_t + \lambda(\mu - X_t) + \sigma dW_t \tag{4.1}$$

where X_{t+1} is the to be forecasted future air travel demand growth rate between time t and $t = t + 1$, X_t is the air travel demand growth rate between time $t - 1$ and t, λ is the speed of mean reversion, μ is the long-term mean growth rate, σ the standard deviation of the historical estimation error and W_t a random shock with N~(0,1). As can be seen from the $\lambda(\mu - X_t)$ term, the expected corrective movement towards the long-term average growth rate at each point in time depends on the speed of mean reversion λ and the difference between the demand growth rate at that time t, X_t, and the long-term average demand growth rate, μ.

In physical terms, the process displays similar behavior as a spring; the larger the difference between a passenger growth rate at a certain point in time and the average passenger growth rate (i.e. the more a spring is stretched with respect to its equilibrium length), the higher the tendency to revert back to the mean passenger growth rate in the subsequent point in time (i.e. the higher the force with which the spring pushes back).

Furthermore the randomness of future demand is captured in the last term of the equation, σdW_t, which resembles a random error shock with mean 0 and standard deviation equal to the standard deviation of the historical estimation error which is inherited from the estimation of the model parameters.

Estimating the model parameters λ, μ and σ

The model parameters can be estimated by rewriting the mean reversion equation into a form that is suitable for linear least squares regression;

$$y = a + bx + \epsilon \tag{4.2}$$

$$X_{t+1} - X_t = \lambda\mu - \lambda X_t + \sigma dW_t \tag{4.3}$$

So that, $y = X_{t+1} - X_t$, $x = X_t$, $a = \lambda\mu$, $b = -\lambda$. When a linear regression is applied to y and x, the regression coefficients for the intercept, a, and the slope, b, are returned. From these regression coefficients the model parameters λ and μ can be derived in the following fashion;

$$\lambda = -b \tag{4.4}$$

$$\mu = -\frac{a}{b} = \frac{a}{\lambda} \tag{4.5}$$

In order to derive the standard deviation of the historical estimation error σ, the estimated values of y, $\hat{y}$, are compared to the actual historical values of y and taking the root mean square of the historical errors ϵ results in its standard deviation, with N being the number of historical data points used in the regression;

$$\hat{y} = a + bx \tag{4.6}$$

$$y = a + bx + \epsilon \tag{4.7}$$

$$y = \hat{y} + \epsilon \tag{4.8}$$

$$\epsilon = y - \hat{y} \tag{4.9}$$

$$\sigma = \sqrt{\frac{\sum(y - \hat{y})^2}{N - 1}} \tag{4.10}$$

Knowing these three model parameters λ, μ and σ and the last available historical growth rate X_t, the future demand growth rates can be calculated using:

$$X_{t+1} = X_t + \lambda(\mu - X_t) + \sigma dW_t \tag{4.11}$$

The concept of mean reversion is applied to forecast future demand growth rates. However, ultimately the goal is forecast future demand levels, which can be easily calculated using;

$$D_{t+1} = D_t \cdot (1 + X_{t+1}) \tag{4.12}$$

where D_{t+1} is the demand value at time $t+1$, D_t is the demand value at time t and X_{t+1} is the growth rate of demand between t and $t+1$.

4.2.3 Monte Carlo simulation and sampling

Monte Carlo simulation

Once the mean reverting model parameters λ, μ and σ are known, the mean reverting equation is likely to return a different result every time it is run, due a different realization of uncertainty which is brought about by the random error shock term W_t with N∼(0,1). In order to numerically explore the range of possible outcomes, the process is iterated D (e.g. 5000) times. This process of iterating over an equation with different realizations of uncertainty in order to numerically approximate the probability distribution of the stochastic variable is called a Monte Carlo simulation. A visual example of a Monte Carlo simulation is shown in Figure 4.4.

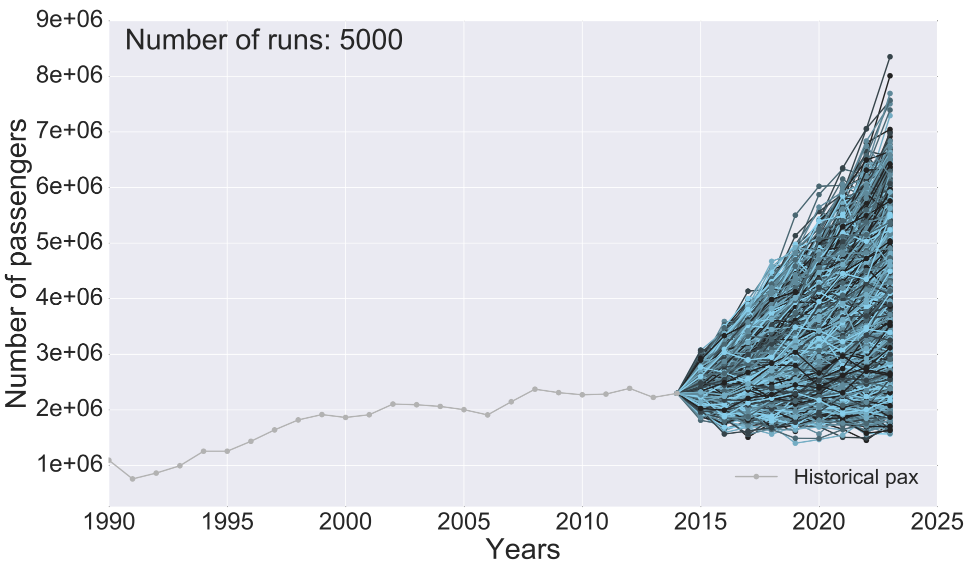


Figure 4.4: Example of a Monte Carlo simulation with D = 5000

Defining the sampling strategy

This section details the relation between the number of realizations of uncertainty per year per OD pair D, the number of sample values per year per OD pair S, the number of OD demand matrices per year M and the overall computation time of the fleet assignment optimization model CT_{model2}. Although the models seem fairly isolated, it will be shown that the sampling process in the stochastic demand forecasting model directly and significantly impacts the overall computation time of the fleet assignment optimization model. Therefore the sampling process is a key component when striving for an acceptable balance between meaningful results and reasonable computation times. The overall computation time of the fleet assignment optimization model is given by;

$$CT_{\text{model 2}} = F \cdot Y \cdot M \cdot CT_O \tag{4.13}$$

where F is the number of fleets in the portfolio, Y is the number of years in the planning horizon, M is the number of OD demand matrices per year and CT_O is the computation time of a single optimization run. It can be observed that the computation time linearly scales with the number of OD demand matrices per year M. Knowing the relation between $CT_{\text{model 2}}$ and M, it is interesting to investigate how M is affected by the sampling process; $M \stackrel{?}{=} f(S, D, Z)$.

As result of the Monte Carlo simulation, D realizations of uncertainty (i.e. or Monte Carlo simulation observations) are obtained per year per OD pair. However, these OD pair based realizations of stochastic demand should ultimately be sampled and consolidated into OD demand matrices that contain all OD pairs. The impact of the sampling strategy on the number of OD demand matrices per year M is investigated through four example sample strategies;

Sampling strategy 1 The D number of realizations of uncertainty per year per OD pair are consolidated into one sample value; essentially adopting a deterministic approach

- $M = S^Z = 1^Z = 1$

Sampling strategy 2 The number of sample values S is equal to the number of realizations of uncertainty D

- $M = S^Z = D^Z$

Sampling strategy 3 The D realizations of uncertainty are represented by 10 sample values

- $M = S^Z = 10^Z$

Sampling strategy 4 The D realizations of uncertainty are represented by 10 sample values, and OD pairs are assumed to be perfectly correlated with a value of $+1$ within a year

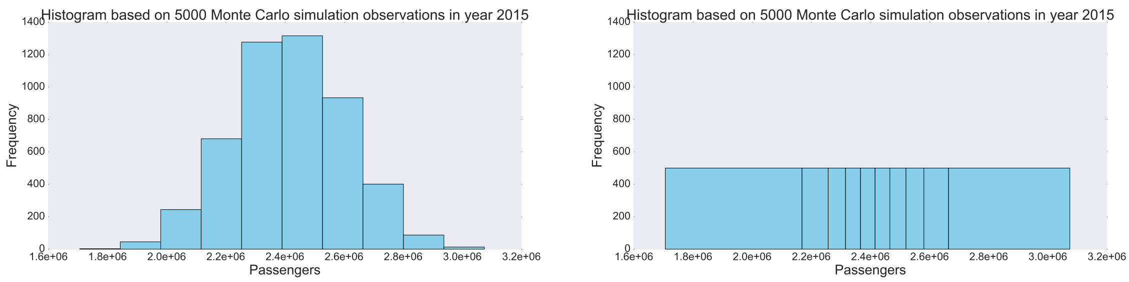
- $M = S = 10$

Of all four cases, sampling strategy 1 yields the lowest number of unique OD demand matrices per year M, but this comes at the cost of loss of information about uncertainty which makes it a deterministic approach. Sampling strategy 2 is the identification of the upper bound on the number of unique OD demand matrices M. It has both a variable in the base as well as in the exponent of the equation which results in a very strong increase in the number of M with increments of D and Z. Although the number of unique OD demand matrices M that result from sampling strategy 3 is significantly lower than in sampling strategy 2, it still scales exponentially with Z. Sample strategy 4 strikes an acceptable balance between meaningful results and reasonable computation times; by assuming correlation between OD pairs within a year the number of unique OD demand matrices per year M greatly reduces and scales one-to-one with the number of sample values S.

Sampling different distributions in the same way

In the previous section it was identified that it is beneficial to sample the D Monte Carlo simulation observations per year per OD pair into 10 (S=10) representative sample values. Reducing the D observations into 10 representative sample values can be done in different ways. It is important to keep in mind that this sampling procedure is not only performed for one set of D observations for one OD pair and one year, but to all years and all OD pairs, that are likely to all have a different distribution of observations.

The sample values should reflect the range and probability of observations in D, irrespective of the specific distribution of D. In other words the goal is to reduce the D observations to 10 sample values, where each sample value is equally like to occur and in essence each sample value represents 1/10 of the probability distribution.



(a) A traditional histogram

(b) An equal probability histogram

Figure 4.5: The difference between a traditional histogram and an equal probability histogram

A straightforward method for sampling would be distribute the D observations across 10 bins and take the average of each bin as a sample value. However, this sampling strategy cannot be performed with a traditional histogram because a non-uniform distribution will result in non-equal bin heights and thus non-equal probabilities. In order to ensure that each bin represents 1/10th of the D dataset, equal probability bin histograms with 10 bins are adopted which set the bin edges at the 0, 10th, 20th, ..., 100th percentiles. Subsequently each bin contains one tenth of the total number of observations D and thus the equal probability property is satisfied. The difference between the concepts of a traditional histogram and an equal probability histogram is visualized in Figure 4.5. Each bin in Figure 4.5b contains 1/10th (i.e. $\frac{D}{S}$) of the observations. These observations are reduced to one sample value by taking the average.

It is noted that by basing the sample value on the average of all the observations in a bin, the information regarding the range of uncertainty of observations within a bin is partially lost. Alternatively, instead of taking the average of each bin it could also be decided to take the median of each bin as sample value. In essence that would be equivalent to taking the sample values equal to the observed values at 5, 15, 25, etc. percentiles. Rather than taking a single observation as a sample value, it is considered a better approach to take the average value of all the observations in a bin as a sample value since that value contains more information regarding the spread an thus the uncertainty of observations.

To summarize, Figure 4.6 provides a visualization of the processes involved in the stochastic demand forecasting model. The evolution of stochastic demand is explored per OD pair for each of the Z OD pairs. Then, per OD pair, per year, S sample values are extracted. By assuming perfect correlation between OD pairs, M OD demand matrices are constructed per year ($M = S$); each OD demand matrix contains all OD pairs with demand sample values from the same bin number.

4.2.4 Assumptions and their implications

- It is assumed that historical passenger data reflects historical demand data;

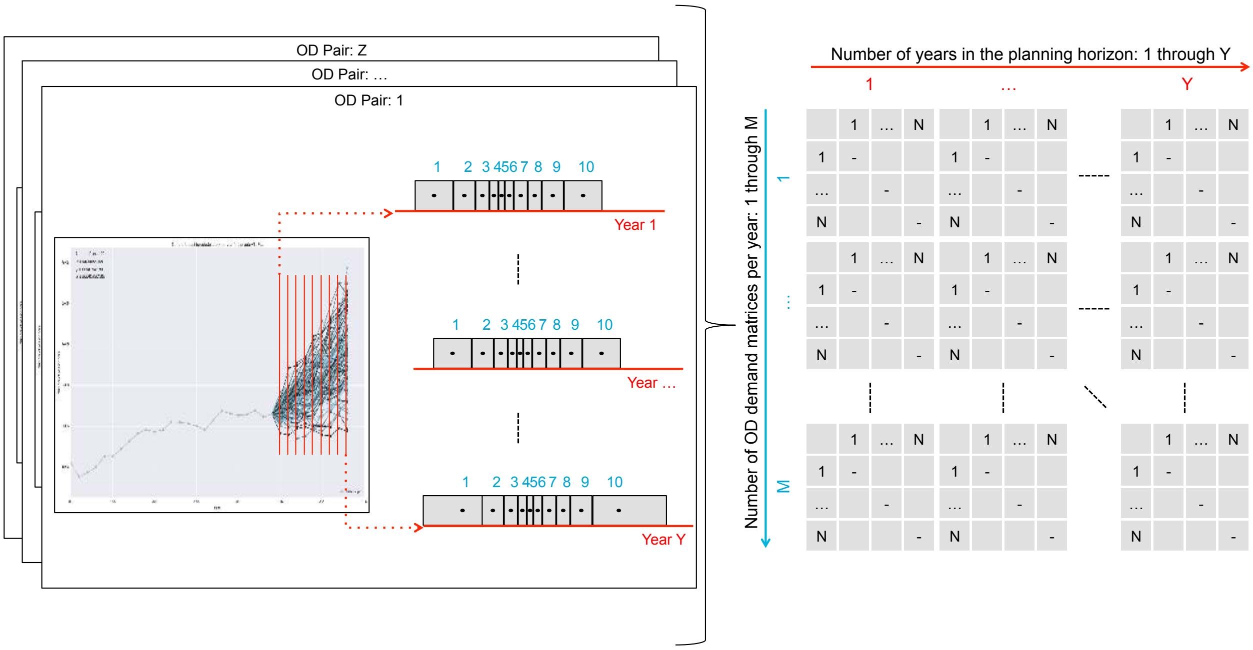


Figure 4.6: A visualization of the stochastic demand forecasting process

  - This essentially neglects the presence of demand-supply interactions at each point in historical time

- The forecasting process is applied to forecast yearly demand;
  - Consequently it does not consider seasonality or trend growth throughout the year
- A sample value is taken as the average of all observations in a bin
  - Although all observations are used to calculate the average, there is some inherent loss of information about the range of uncertainty in a bin when the average is calculated

4.3 Fleet assignment optimization model

4.3.1 Introduction

This section thoroughly details the working principles of the fleet assignment optimization model. The goal of the model is to optimally assign one fleet in terms of operating profit given one OD demand matrix and to iterate this optimization process for each OD demand matrix-fleet combination. With F fleets in the portfolio, M OD demand matrices per year and Y years in the planning horizon this amounts to $F \cdot M \cdot Y$ iterations over the optimization model.

Figure 4.7 places the fleet assignment optimization model in the context of the overarching solution methodology. As input it takes the $M \cdot Y$ OD demand matrices from the stochastic demand forecasting model and the F fleets from the portfolio of fleets. For each OD demand matrix-fleet combination the model outputs both financial (e.g. operating profit, spill) and non-financial performance metrics (e.g. load factor, aircraft utilization). The annual operating profits are stored in a value matrix which is visualized in Figure 4.9. Figure 4.8 provides a visualization of the steps involved in a single optimization run. The schematic picture of the optimization model in the second step serves as a signpost to the math behind the optimization model; it does not have the goal to represent the actual math. The formulation of the decision variables, objective function and constraints is elaborated step by step in Sections 4.3.3, 4.3.4 and 4.3.5

respectively. The mathematical formulation is based on Santos (2013) and can be found in its entirety in Appendix C.1.

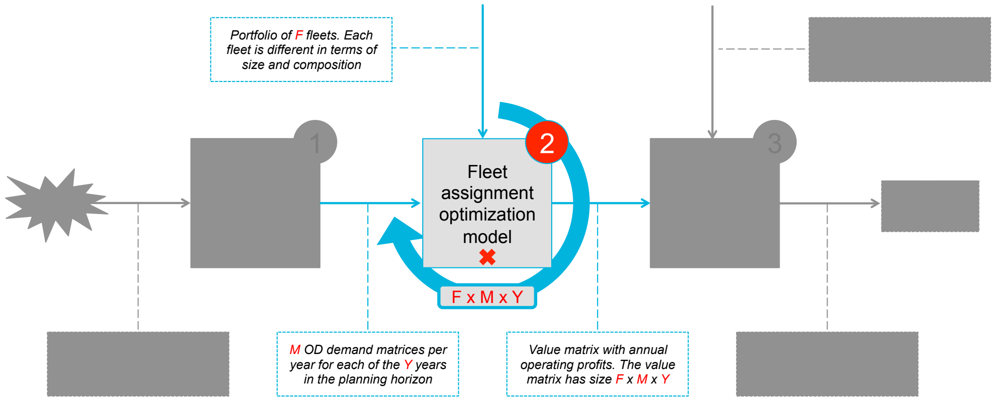


Figure 4.7: The fleet assignment optimization model in the context of the overarching solution methodology

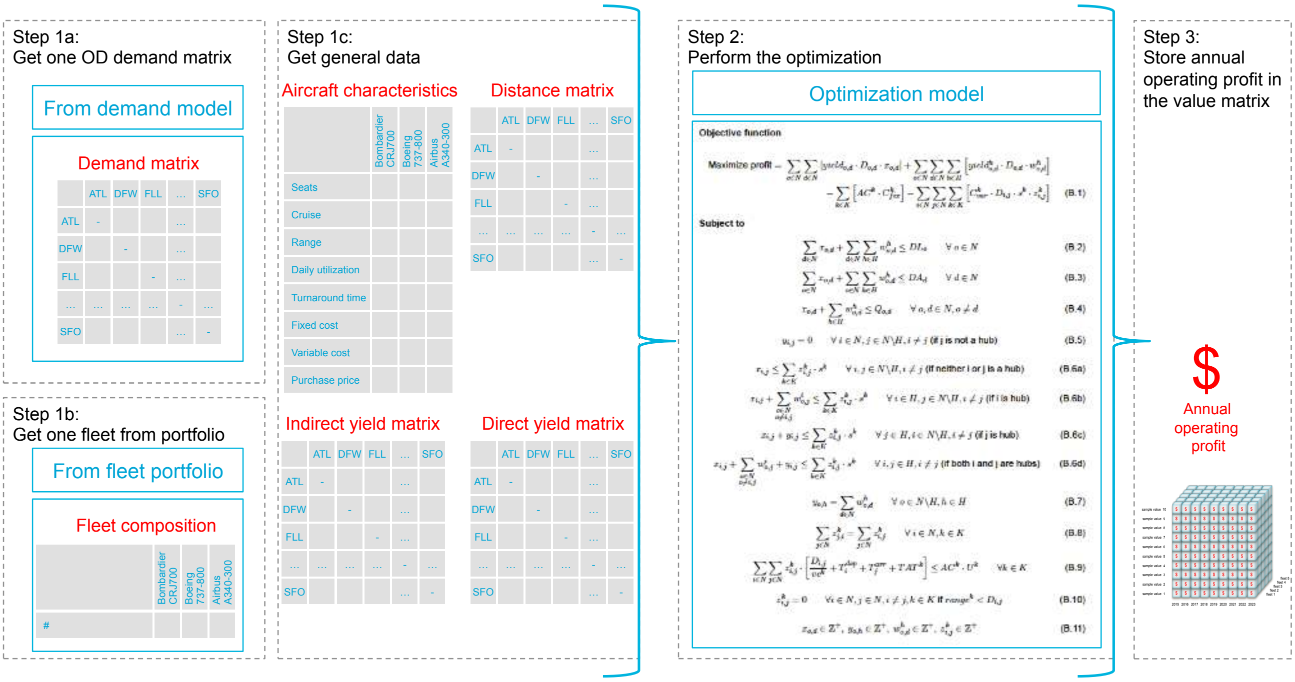


Figure 4.8: A visualization of one run of the fleet assignment optimization model that is run for each OD demand matrix-fleet combination

4.3.2 Problem definition and solution techniques

The fleet assignment optimization model is a typical optimization model in which scarce resources (i.e. the aircraft in the fleet) need to be allocated to competing activities (i.e. the transportation of passengers) in an optimal fashion.

The classical FAM: tail number assignment to a time-space network

A clarification is required with regards to the definition of the fleet assignment model (FAM). In literature, the term FAM is usually adopted for models that allocate *tail numbers* (i.e. specific

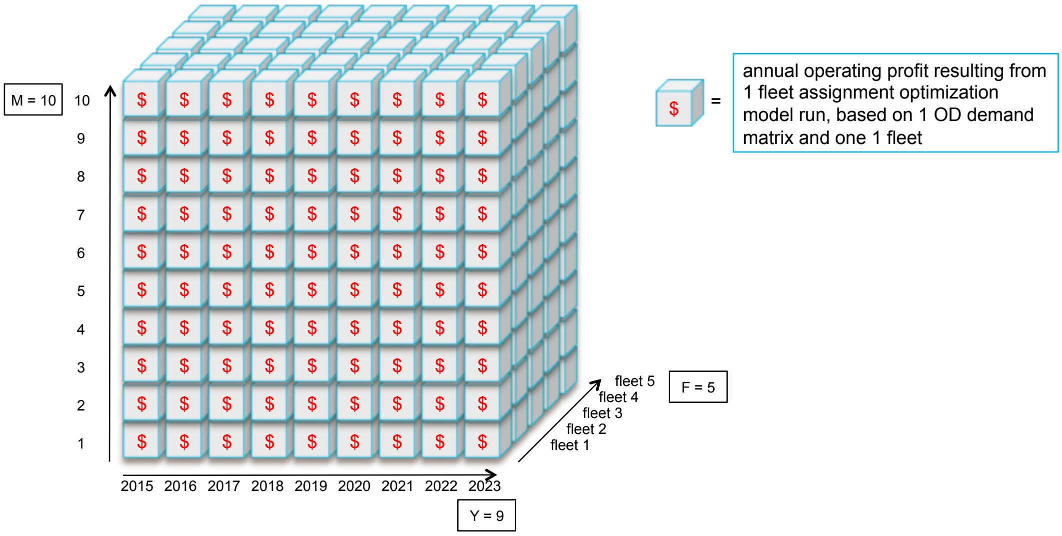


Figure 4.9: An example value matrix with $F = 5$, $M = 10$ and $Y = 2015, ..., 2023$; the value matrix is filled with annual profit values based on assigning each fleet F to each of the OD demand matrices $M \cdot Y$

aircraft) to a time-space network. This means that a tail number is assigned to a specific flight in a schedule, which forms a connection between one point in time-space to another point in time-space. An example is the assignment of a Boeing 787 with tail number *XY-1234* to a flight which departs from ATL 6:30am and arrives in FLL at 7:55am. This type of detailed FAMs entail tremendous problem sizes in terms of the LP-matrix (i.e. the number of decision variables and constraints) and computation times (Abara, 1989; Lohatepanont and Barnhart, 2004). Such a detailed FAM is typically used a couple of weeks or months before the day of operation to optimally assign a fleet to a given schedule.

FAM based on weekly frequency aircraft type assignment

Although the tail number based FAMs are key to optimally assign each tail number in a fleet to a timetable over a short-term planning horizon, the computation times that stem from this detailed FAM are considered unsustainable when used for a multi-year year planning horizon.

Rather than assigning tail numbers to a time-space network, *frequency planning* can be used as a FAM to determine the optimal deployment of a fleet by assigning aircraft types to airport pairs based on a weekly flight frequency. To illustrate, this approach would yield results such as the assignment of a Boeing 787 between ATL and FLL for 10 flights per week.

The advantage of this type of frequency based FAM is that the problem size is smaller which results in lower computation times. However, because aircraft types are used as decision variables instead of aircraft tail numbers, the capacity and utilization of the different tail numbers of the same aircraft type are consolidated, resulting in potential aircraft utilization constraint violations when a transition is made from aircraft type assignment to aircraft tail assignment which is detrimental with regards to the reflection of reality.

Hub-and-spoke versus point-to-point

Another consideration in fleet assignment models is the type of routing network that is considered. Two distinct routing networks are identified, being the point-to-point and hub-and-spoke routing networks. A vast amount of literature exists that investigates the advantages and disadvantages of both routing networks with regards to the impact on service, the ability to capture demand, competition and profitability. Because the overarching solution methodology is focused on providing an approach that is generalizable for any type of airline and thus any type of routing network, the goal here is to set up the mathematical formulation in such a way that both routing networks could be part of a solution.

A real world example

Consider a set of airports between which the demand is supposedly known, i.e. the OD demand matrix is given. Consider a fleet that consists of a set of 3 aircraft types, and 3 specific aircraft of each type totaling at 9 aircraft in the fleet. Each aircraft type has different characteristics in terms of: number of seats, cruise speed, range, daily utilization, turnaround times, fixed cost and variable cost. The optimization model should decide which OD demand to satisfy and how to transport these passengers either nonstop or through a connecting service with which aircraft type while optimizing for profit on an airline level.

Optimization algorithms

A feasible region is defined as the collection of feasible solutions, i.e. solutions for which all the constraints are satisfied (Hillier and Lieberman, 2010). One can image that when a fleet is considered with various aircraft types with different characteristics and a vast amount of OD pairs is considered (e.g. 200) with various levels of demand and different yielding passengers (both nonstop and connecting), a tremendous amount of feasible solutions exist. Finding the optimal solution by first calculating all feasible solutions and then selecting the best feasible (i.e. optimal) solution is not the most efficient optimization methodology and can be troublesome in terms of computation time.

Fortunately, a specific research field is dedicated to these kinds of problems: operations research. Depending on the problem, the search for the best solution can be performed by exploiting the topology of the mathematical formulation. For linear programming (LP) problems that have both a linear objective function as well as linear constraints, the Simplex optimization algorithm can be used to solve the problem. An algorithm is simply defined as a sequence of steps. The Simplex algorithm is a sequence of steps that, based on the structure of the mathematical formulation, searches for an optimal solution using a series of matrix operations that are based on two fundamental insights, being: an optimal solution must be a corner-point feasible solution and, the mathematical formulation can be rewritten into a matrix form that can be used for relatively quick matrix operations to get to a solution using a computer. For more details about this elegant proof the reader is referred to dedicated literature (Bertsimas and Tsitsiklis, 1997; Hillier and Lieberman, 2010).

The problem under consideration has decision variables that preferably can only take integer values. For example, a model that returns that 7 flights per week need to be operated by a Boeing 787 is a more intuitive result than a model that returns that 7.4 flights need to be assigned per week.

Therefore the problem is not an linear program but an integer linear program (ILP). If some but not all decision variables are integers, then the problem is referred to as a Mixed Integer Linear Programming problem (MILP). The Simplex method is used to solve the *linear relaxation*, i.e. the LP-problem without integrality constraints, and then a branch-and-bound method as well as cutting-plane method are used to get to integer solutions. The Gurobi solver automatically employs these optimization algorithms.

4.3.3 Decision variables

The goal of the optimization is to utilize the different aircraft types to transport passengers in such a way that the highest profit is obtained. Consequently, the weekly flight frequency per airport pair per aircraft type as well as the weekly passenger flow per OD pair (both nonstop and connecting flow) need to be determined. These are the decision variables of the problem.

Consequently, four types of decision variables are defined. Three decision variables represent the weekly passenger flow per OD pair and one decision variable represents the weekly flight frequency per aircraft type per airport pair. The concept behind these decision variables is briefly outlined below and illustrated in Figure 4.10.

- x_{od}: Nonstop passenger flow between origin airport o and destination airport d
- y_{oh}: Connecting passenger flow for passengers that are in the segment between the origin airport o and the hub h, irrespective of their final destination airport d
- w^h_{od}: Connecting passenger flow for passengers that originate from airport o and are in the segment between the hub h and the final destination airport d
- z^k_{ij}: Number of flights (i.e. flight frequency) between airport i and airport j operated by aircraft type k

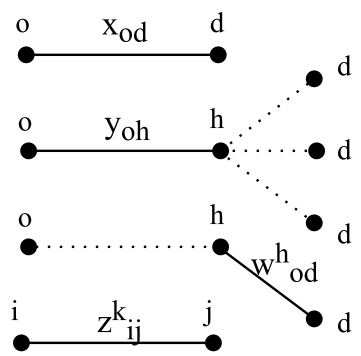


Figure 4.10: An illustration of the four decision variables of the optimization model

4.3.4 Objective function

The basic airline operating profit equation (Belobaba et al., 2009) is given by;

$$\text{Operating profit} = \text{RPM} \cdot \text{yield} - \text{ASM} \cdot \text{unit cost} \tag{4.14}$$

where, RPM are the revenue-passenger-miles (the number of passengers transported times their transported distance), yield is the operating revenue per RPM, ASM are the available-seat-miles (i.e. the number of seats times their transported distance), and the unit cost is the cost per ASM, often referred to as CASM. The CASM is assumed to include all operating cost associated with the operation of an aircraft such a crew, fuel, maintenance, etc. CASMs can be estimated by aggregating historical total operating cost data on an airline level and dividing it by the total offered ASM.

Equation 4.15 is the objective function. It is based on the basic airline operating profit equation and expanded with an ownership cost term. It consists of four terms and aims to maximize weekly operating profit on an airline level. The first term reflects operating revenue that stems from nonstop passengers and is a function of the number of nonstop passengers between each origin and destination $x_{o,d}$, the nonstop yield $yield_{o,d}$ and nonstop distance $D_{o,d}$. Similarly, the second term reflects operating revenue stemming from connecting passengers and is a function of the number of connecting passengers between each origin and destination $w^h_{o,d}$, the yield for connecting passengers $yield^h_{o,d}$ and the nonstop distance $D_{o,d}$. The third term reflects ownership cost and is a function of the number of aircraft per type in the fleet AC^k and the weekly ownership cost per aircraft type C^k_{fix}. It is a simple multiplication of parameters and does not contain any decision variables. The fourth term reflects operating cost and is a function of the number of flights per aircraft type between each airport pair $z^k_{i,j}$, the operating cost per aircraft type C^k_{var}, the distance between two airports $D_{i,j}$ and the number of seats per aircraft type s^k.

$$\text{Maximize profit} = \sum_{o\in N}\sum_{d\in N}\left[yield_{o,d}\cdot D_{o,d}\cdot x_{o,d}\right] + \sum_{o\in N}\sum_{d\in N}\sum_{h\in H}\left[yield^h_{o,d}\cdot D_{o,d}\cdot w^h_{o,d}\right] - \sum_{k\in K}\left[AC^k\cdot C^k_{fix}\right] - \sum_{i\in N}\sum_{j\in N}\sum_{k\in K}\left[C^k_{var}\cdot D_{i,j}\cdot s^k\cdot z^k_{i,j}\right] \quad (4.15)$$

4.3.5 Constraints

Flow-demand constraints

Three constraints ensure that the assigned passenger flows cannot exceed the demand. These constraints are represented by Equations 4.16, 4.17 and 4.18. The sum of all nonstop and connecting passenger flows leaving origin airport o should be smaller than or equal to the demand leaving origin airport o. This constraint must hold for every origin airport $o \in N$, therefore the number of constraints is: $C = N$.

$$\sum_{d\in N} x_{o,d} + \sum_{d\in N}\sum_{h\in H} w^h_{o,d} \le DL_o \qquad \forall\, o \in N \quad (4.16)$$

The sum of all nonstop and connecting passenger flows arriving in destination airport d should be smaller than or equal to the demand arriving in destination airport d. This constraint must hold for every destination airport $d \in N$, therefore the number of constraints is: $C = N$.

$$\sum_{o\in N} x_{o,d} + \sum_{o\in N}\sum_{h\in H} w^h_{o,d} \le DA_d \qquad \forall\, d \in N \quad (4.17)$$

The sum of all nonstop and connecting passenger flows between origin airport o and destination airport d should be smaller than or equal to the demand between origin airport o and destination airport d. This constraint must hold for every origin-destination pair in the network $o, d \in N, o \neq d$, therefore the number of constraints is: $C = N^2 - N$.

$$x_{o,d} + \sum_{h\in H} w^h_{o,d} \le Q_{o,d} \qquad \forall\, o, d \in N, o \neq d \quad (4.18)$$

Hub definition constraint

If airport j is not a hub, then the connecting passenger flow between i and j, y_{ij} must be zero. Although this constraint seems trivial, it is the constraint that ensures there is a distinction between airports that can be used as a hub, and airports that cannot be used as a hub.

While all the other constraints are explicitly implemented, this constraint allows for implicit implementation by simply not initializing y_{ij} decision variables for connections between i and j where j is not a hub. Therefore the number of explicitly implemented constraints is $C = 0$. In terms of the LP-matrix this implicit implementation reduces some rows (by not explicitly implementing the constraint) and some columns (by not initializing the decision variables).

$$y_{i,j} = 0 \qquad \forall\, i \in N, j \in N\backslash H, i \neq j \text{ (if j is not a hub)} \quad (4.19)$$

Passenger flow-capacity constraint

Equations 4.20a, 4.20b, 4.20c, 4.20d ensure that the passenger flow in a certain flight segment between airport i and airport j must be smaller than or equal to the capacity offered between airports i and j.

The capacity in a flight segment is a function of the number of flights per aircraft type between these airports $z_{i,j}^k$ and the number of seats per aircraft type s^k. As such, the capacity is essentially represented by the total number of seats offered between the two airports.

Focusing on a flight segment between two airports i and j, both of these airports can be either a regular airport or act as a hub. Therefore, the passenger flow between the two airports can consist of four different passenger flow mixes; merely nonstop passenger flow if both airports are not a hub, a mix of nonstop and connecting passenger flow where airport i is used as a hub, a mix of nonstop and connecting passenger flow where airport j is used as a hub, or a mix of nonstop passenger flow and connecting passenger where both airport i and j act as a hub. Because these four different passenger flow mixes could occur, the passenger flow-capacity constraint is represented by the following four sub-constraints;

$$x_{i,j} \leq \sum_{k \in K} z_{i,j}^k \cdot s^k \qquad \forall\, i,j \in N \backslash H, i \neq j \text{ (if neither i or j is a hub)} \tag{4.20a}$$

$$x_{i,j} + \sum_{\substack{o \in N \\ o \neq i,j}} w_{o,j}^i \leq \sum_{k \in K} z_{i,j}^k \cdot s^k \qquad \forall\, i \in H, j \in N \backslash H, i \neq j \text{ (if i is hub)} \tag{4.20b}$$

$$x_{i,j} + y_{i,j} \leq \sum_{k \in K} z_{i,j}^k \cdot s^k \qquad \forall\, j \in H, i \in N \backslash H, i \neq j \text{ (if j is hub)} \tag{4.20c}$$

$$x_{i,j} + \sum_{\substack{o \in N \\ o \neq i,j}} w_{o,j}^i + y_{i,j} \leq \sum_{k \in K} z_{i,j}^k \cdot s^k \qquad \forall\, i,j \in H, i \neq j \text{ (if both i and j are hubs)} \tag{4.20d}$$

The number of constraints is a function of the number of hubs H that are adopted and the number of airports N under consideration resulting in the following 4 equations that represent the number of constraints;

- Number of constraints for equation 4.20a: $(N - H)^2 - (N - H)$
- Number of constraints for equation 4.20b: $(N - H) \cdot H$
- Number of constraints for equation 4.20c: $(N - H) \cdot H$
- Number of constraints for equation 4.20d: $H^2 - H$

Consequently, the total number of passenger flow-capacity constraints is given by;

$$C = (N - H)^2 - (N - H) + (N - H) \cdot H + (N - H) \cdot H + H^2 - H \tag{4.21}$$

Connecting passenger flow continuity constraint

Equation 4.22 represents passenger continuity at the hub by ensuring that all connecting passengers that arrive at a hub also leave the hub. This constraint also serves as a balance constraint between the two decision variables that both describe connecting passenger flow before the hub $y_{o,h}$ and after the hub $w_{o,d}^h$. For each hub H, this constraint must hold for all origin airports minus that hub $N - 1$, therefore the number of constraints is: $C = H \cdot (N - 1)$.

$$y_{o,h} = \sum_{d \in N} w_{o,d}^h \qquad \forall\, o \in N \backslash H, h \in H \tag{4.22}$$

Aircraft balance constraint

Equation 4.23 represents aircraft continuity; the total number of inbound flights per aircraft type k that arrive at airport i from all airports j must be equal to the total number of outbound flights per aircraft type k that depart from airport i to all airports j. This constraint must hold for every airport $i \in N$ and for every aircraft type $k \in K$, therefore the number of constraints is: $C = N \cdot K$.

$$\sum_{j \in N} z_{j,i}^{k} = \sum_{j \in N} z_{i,j}^{k} \qquad \forall\, i \in N, k \in K \tag{4.23}$$

Capacity-physical limits constraints

Equation 4.24 ensures per aircraft type that the total weekly operational time does not exceed the weekly aircraft utilization. The aircraft utilization per aircraft type is not based on 24 hour per day availability, rather it reflects the available hours to operation when considering the need for scheduled and unscheduled maintenance.

The total weekly operational time is a function of the number of flights of each aircraft type between each airport pair, the flight time, the taxi time and turnaround time. The flight time is a function of the distance between two airports $D_{i,j}$ and the cruise speed of the aircraft type vc^k; $\frac{D_{i,j}}{vc^k}$. The taxi times are airport dependent and depend on whether the flight is inbound or outbound. The turnaround times range from 30 minutes to one hour and are based on the assumption that larger aircraft have higher turnaround times.

This constraint must hold for every aircraft type $k \in K$, therefore the number of constraints is: $C = K$.

$$\sum_{i \in N} \sum_{j \in N} z_{i,j}^{k} \cdot \left[\frac{D_{i,j}}{vc^k} + T_i^{dep} + T_j^{arr} + TAT^k \right] \leq AC^k \cdot U^k \qquad \forall k \in K \tag{4.24}$$

Aircraft range constraint

Each aircraft type is characterized by its maximum range. Equation 4.25 ensures that a flight between two particular airports i and j can only be operated by a particular aircraft type k if the range of the the respective aircraft type $range^k$ is larger than the distance between two airports $D_{i,j}$. It is impossible to quantify the number of constraints at a higher level of abstraction because it depends on the specific characteristics of the aircraft types and the distances between the airports under consideration.

$$z_{i,j}^{k} = 0 \qquad \forall i \in N, j \in N, i \neq j, k \in K \text{ if } range^k < D_{i,j} \tag{4.25}$$

Integrality and non-negativity constraints

Equation 4.26 ensures that each decision variable can only take positive integer variables, as such it is a combination of non-negativity and integrality constraints. As highlighted in Chapter 2, integrality constraints can have a profound impact on the computation time and can result in intractability of the mathematical formulation (i.e. meaning that it is difficult or impossible to get a solution). Per case study it can be investigated what the computation times are of the LP, MILP or ILP formulation.

$$x_{o,d} \in \mathbb{Z}^+,\ y_{o,h} \in \mathbb{Z}^+,\ w_{o,d}^{h} \in \mathbb{Z}^+,\ z_{i,j}^{k} \in \mathbb{Z}^+ \tag{4.26}$$

4.3.6 Expanding the LP matrix

It is considered interesting to know how the size of the optimization problem scales with the problem size, where the problem size is defined by the number of airports N, hubs H and aircraft types K under consideration.

The size of the optimization problem can be characterized by its LP-matrix, where each additional decision variable adds a column and each constraint adds a row. It is important to note that it is difficult to directly relate the size of the LP matrix to the resulting computation time, especially if some or all of the decision variables or subject to integrality constraints. This is because the computation time depends on the topology of mathematical formulation and how the algorithm exploits that topology. However, knowing the size of the LP matrix, experienced operations researchers can make a rough estimate of the computation time and decide whether or not to solve the problem first as a LP and investigate the ILP at a later stage.

Furthermore it is noted that many ILP and MILP problems in practical situations are NP-hard. Extensive research has been done on NP-hardness, which refers to non-deterministic polynomial time. This topic is a computer science perspective on optimization problems and deals with how computation times scale with the problem size, given the currently available optimization algorithms. In simple terms, polynomial time computation time scales with $O(\text{variable}^{\text{constant}})$ and is therefore faster than exponential time computation time which scales with $O(\text{constant}^{\text{variable}})$.

Equations 4.27 and 4.28 highlight the relation between the number the number of airports N, hubs H and aircraft types K to the number of rows (i.e. constraints) and columns (i.e. decision variables) in the LP-matrix, respectively;

$$\begin{aligned}\text{Rows} &= N + N + N^2 - N + (N-H)^2 - (N-H) + (N-H)\cdot H + (N-H)\cdot H \\ &\quad + H^2 - H + H\cdot(N-1) + N\cdot K + K \\ &= 2N^2 + NH - H + NK + K + x\end{aligned} \tag{4.27}$$

where x is the number of aircraft range constraints.

$$\begin{aligned}\text{Columns} &= N^2 - N + H\cdot(N-1) + H\cdot\left[(N-1)^2 - (N-1)\right] + (N^2-N)\cdot K \\ &= N^2 - N - 2NH + H + N^2H + N^2K - NK\end{aligned} \tag{4.28}$$

4.3.7 Assumptions and their implications

- The optimization is based on a weekly frequency aircraft type assignment which results in a weekly operating profit; it is not based on a (daily) aircraft tail number assignment to a time-space network. This has the following implications;
  - Aircraft utilization per tail number is not considered
  - Maintenance cycles are not considered
  - To derive weekly demand from the forecasted yearly demand, the latter is simply divided by 52 weeks. As such, it is assumed that this average week is representative of the demand throughout the year and thereby neglects seasonality as well as trend growth throughout the year. The optimization returns weekly operating profit, which is multiplied by 52 (i.e. weeks) to arrive at annual profit
- The mathematical formulation allows for both point-to-point and hub-and-spoke solutions
  - Economies of scale at the hub are not considered
  - Hub-and-spoke economics are implemented in a simplistic manner through a yield ratio for nonstop and connecting passengers

  - The methodology does not optimize for the optimal location of the hub; it simply takes hub locations as an input
- All costs are captured by two terms: operating cost and ownership cost
  - A recommendation for future work would be to explicitly breakdown the cost such as maintenance cost, fuel cost, insurance cost, infrastructure cost, ground-handling cost, etc
- There is no recapture of spilled passengers

While the output of the fleet assignment optimization model, i.e. the value matrix, already provides valuable information about the financial performance of different fleets, one additional step can be taken which will yield more fundamental insight in the financial performance of the different fleets across the planning horizon and under the uncertain demand; this additional step is the scenario generation model.

4.4 Scenario generation model

4.4.1 Introduction

The goal of the scenario generation model is to generate paths through the value matrix. The place of the scenario generation model in the context of the overarching methodology is visualized in Figure 4.11. As input it takes the value matrix outputted by the fleet assignment optimization model as well as the Monte Carlo simulation observations from the stochastic demand forecasting model.

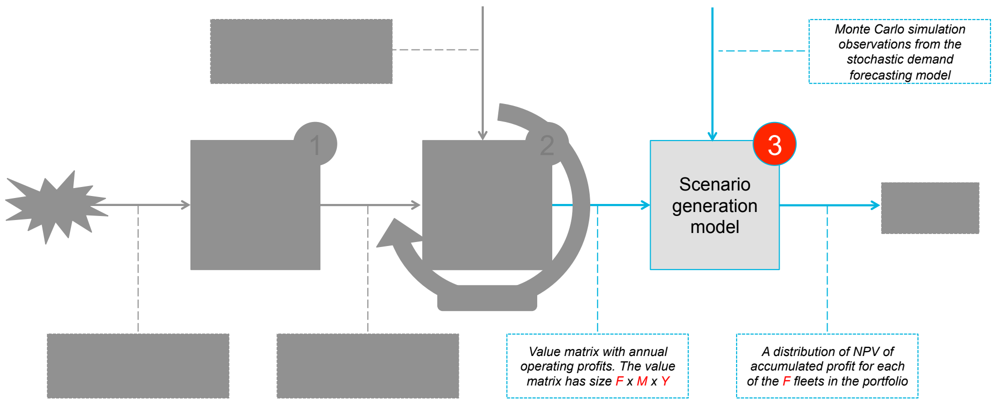


Figure 4.11: The scenario generation model in the context of the overarching solution methodology

This section details two main elements of the model: how the random selection process for scenario generation is based on the adoption of discrete-time Markov Chain (DTMC) transition probability matrices as well as how the simulation of B scenarios is used to construct a distribution of net present profit values per fleet, across the planning horizon across the range of uncertainty. The methodology of the model can be represented by 4 steps;

Step 1 Based on the DTMC property, year-to-year transition probabilities are constructed for each of the Z OD pairs, for each of the consecutive year combinations in the Y year planning horizon. This results in $Z \cdot (Y-1)$ transition probability matrices

Step 2 The $Z \cdot (Y-1)$ OD pair based transition probability matrices are aggregated and reduced to $(Y-1)$ transition probabilities that represent the average behavior of all OD pairs in an

OD demand matrix. Thus, the $(Y-1)$ transition probability matrices describe the transition behavior of OD demand matrices

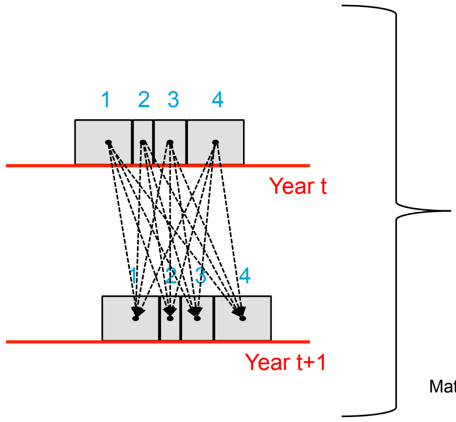
Step 3 B scenarios are generated through the value matrix by making use of the $(Y-1)$ transition probability matrices. Each scenario consists of a sequence of Y annual operating profit values.

Step 4 The sequence of Y annual profit values within one scenario are reduced to one net present value (NPV), resulting in B NPVs per fleet. The distribution of these B NPVs per fleet are visualized in a histogram which allows for comparison between different fleets in terms of cash flow generating capability across the years in the planning horizon across the range of uncertainty

4.4.2 The discrete-time Markov chain transition probability matrix

A discrete-time Markov Chain (DTMC) is a stochastic process that satisfies the Markov property with discrete time steps and a discrete state space. The Markov property describes the memorylessness of the stochastic process: the probability of arriving in a future state only depends on the present state. A transition probability matrix contains the transition probabilities of transitioning from state i at time t to state j at time $t+1$. Based on the Markov property the transition probability should be a square matrix (i.e. the number of states must remain constant over time) and each row should add up to one (i.e. the total probability of arriving in any of the states must be 1).

Translated to the context of this research, a DTMC can be used to model the stochastic process of the evolution of Monte Carlo simulation observations that are outputted by the stochastic demand forecasting model. D Monte Carlo simulation observations per year per OD pair are equally distributed across S bins. These S bins are the discrete states in this research context. Subsequently the transition probability matrix has size $S \times S$.



		Year t+1			
		bin 1	bin 2	bin 3	bin 4
Year t	bin 1	700	300	150	100
	bin 2	200	800	200	50
	bin 3	150	300	500	300
	bin 4	100	250	250	650

Matrix of counted Monte Carlo simulation observation transitions between year t and year t+1

Figure 4.12: An example of the transition process

Step 1: OD pair based transition probability matrices

To illustrate the formation of a transition probability matrix, consider 5000 Monte Carlo simulation observations ($D = 5000$) and 4 bins (S = 4). Figure 4.12 provides a visualization of the transition process; an observation can transition from any of the 4 states at time t to any of the 4 states in $t+1$ resulting in S^2 possible transitions. The figure presents fictitious results of the Monte Carlo simulation observation counting process. For example, 700 Monte Carlo simulation observations transitioned from bin 1 in year t to bin 1 in year $t+1$. Table 4.3 again presents the counted Monte Carlo simulation observation transitions and Table 4.4 presents the derived transition probabilities based on those counts. Note that each row adds up to one, which is effectuated by normalizing each row by the number of observations in that row. Since use is made of equal probability bin histograms to equally distribute the D Monte Carlo simulation observations across S bins, each bin contains the same number of observations: $\frac{D}{S}$. Consequently, rows are normalized by multiplying each element with;

$$\frac{1}{\frac{D}{S}} \tag{4.29}$$

which in this example boils down to $\frac{1}{\frac{D}{S}} = \frac{1}{\frac{5000}{4}} = \frac{1}{1250}$. Such a transition probability matrix can be constructed in an analogues manner for each consecutive year combination $Y-1$ in the Y years of the planning horizon and for each OD pair Z under consideration, resulting in $Z \cdot (Y-1)$ transition probability matrices.

Table 4.3: 5000 Monte Carlo simulation observation counts (D=5000) across 4 bins (S=4)

	1	**2**	**3**	**4**
1	700	300	150	100
2	200	800	200	50
3	150	300	500	300
4	100	250	250	650

Table 4.4: Transition probability matrix based on the Monte Carlo simulation observations counts in Table 4.3

	1	**2**	**3**	**4**
1	0.56	0.24	0.12	0.08
2	0.16	0.64	0.16	0.04
3	0.12	0.24	0.40	0.24
4	0.08	0.20	0.20	0.52

Step 2: OD demand matrix based transition probability matrices

As defined in Step 2 of this model, the OD pair based transition probability matrices must be aggregated and reduced. This is required because the transition probability matrices are used to generate paths through the value matrix, which contains annual operating profit values that each are based on the assignment of a fleet to an OD demand matrix (and not on OD pairs). Therefore there is the need to know the transition behavior of entire OD demand matrices as opposed to the transition behavior of individual OD pairs.

For the construction of OD demand matrix based transition probability matrices the same underlying principles of counting observations and transforming these to probabilities can be used. However, the counting process is iterated for all Z OD pairs and therefore each row contains $Z \cdot \frac{D}{S}$ observations. Consequently, rows are normalized by multiplying each element with;

$$\frac{1}{\frac{D}{S} \cdot Z} \tag{4.30}$$

It is noted that this aggregation is made possible by the decision presented in Section 4.2.3 to set the number of unique OD demand matrices per year equal to the number of sample values ($M = S$). As result of that decision, each OD demand matrix contains demand sample values of different OD pairs that are based on the same bin number. Moreover, this aggregation is only valid if the different OD pairs display similar transition behavior, which is validated in Chapter 6.

4.4.3 Scenario generation and a distribution of net present values

This section elaborates Step 3 and Step 4 of the scenario generation model; the generation of B scenarios and the subsequent formation of a distribution of NPVs, for each fleet F in the portfolio.

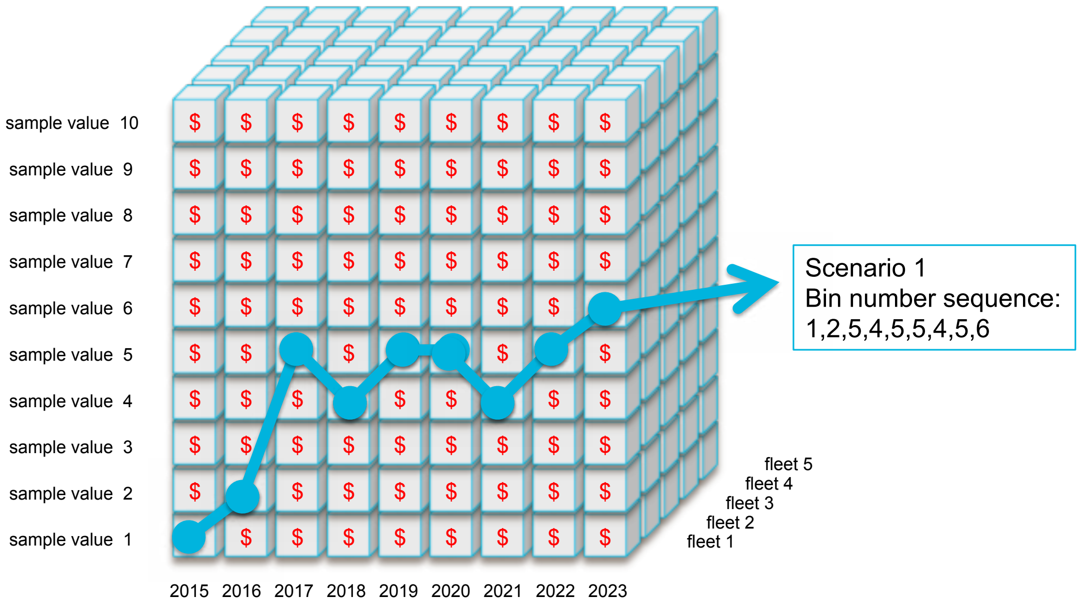


Figure 4.13: An example scenario through the value matrix

Step 3: generating scenarios

Due to the memoryless property of the DTMC, a scenario can be generated throughout the planning horizon Y by utilizing the $Y-1$ OD demand matrix based transition probability matrices. The process of one scenario generation resembles a roulette process which is executed $Y-1$ times in sequence using the known probabilities from the $Y-1$ transition probabilities and acknowledging that the first roulette is defined by a 0.1 probability for each state (i.e. bin number) because use is made of equal probability bins.

A scenario is essentially a sequence of Y bin numbers; i.e. one bin number for each year in the planning horizon. Because each bin number in each year can be related to one specific OD demand matrix in each year that contains OD demand sample values that are based on that bin number, a scenario can be related to a sequence of OD demand matrices. The sequence of OD demand matrices can in turn be related to a sequence of annual operating profit values using the value matrix. This process is visualized in Figure 4.13.

Ultimately, the generation of B scenarios results in B sequences of annual operating profits each of length Y.

Step 4: a distribution of net present profit values

One scenario corresponds to a sequence of Y annual operating profit values. These Y values can be reduced to a single monetary value; the so called net present value (NPV) in the following fashion;

$$\sum_{i=1}^{Y} \frac{\text{annual profit}_i}{(1+r)^i} \tag{4.31}$$

where r is the discount rate and i is the year, with i = 1, ..., Y. When B scenarios are generated the resulting B NPVs can be used to form a distribution of NPVs. Moreover, this procedure is executed for each fleet F in the portfolio so that ultimately F distributions of NPVs are outputted that can be used to compare the profit generating capabilities between fleets across the planning horizon across the range of stochastic demand. It is noted that it is key that each fleet is subject to the same set of scenarios to ensure fair comparison.

4.4.4 Assumptions and their implications

- It is assumed that the aggregated set of OD demand matrix transition probabilities reflect the same transition behavior as the transition probabilities of each OD pair.
  - The validation of the aggregation of OD pair based transition probability matrices into OD demand matrix based transition probability matrices is presented in Chapter 6
  - This assumption only holds because each OD demand matrix in each year contains OD demand sample values that are based on the same bin number
- Transition probabilities are derived by modeling year-to-year behavior of Monte Carlo simulation observations as a Markov process.
  - Therefore, each year and each OD pair must be represented by the same number of sample values.

5

Case study

A case study serves as proof of concept of the proposed methodology. This chapter contains four sections. Section 5.1 defines the characteristics of the case study. The case study specific data and assumptions are listed in Sections 5.2 and 5.3, respectively. Section 5.4 provides a detailed observation and analysis of the results.

5.1 Description and context

The variables that are used to describe the methodology in Chapter 4 are substituted by case study specific values. These values are listed in Table 5.1.

Table 5.1: Case study specific variable values

Notation	Definition	Case study value
F	# Fleets in portfolio	8
Y	# Years in planning horizon	9
D	# Monte Carlo simulations	5000
S	# Sample values	10
M	# OD demand matrices per year	10
N	# Airports under consideration	10
Z	# OD pairs under consideration	10
H	# Hubs under consideration	1
K	# Aircraft types under consideration	3
B	# Scenarios generated	5000

The portfolio of fleets is presented in Table A.1. Each of the 8 fleets in the portfolio ($F = 8$) is characterized by the number of aircraft per aircraft type. Fleets 1-5 have the same composition ratios but a different total fleet size whereas fleets 5-8 have the same total fleet size but different composition ratios.

The forecasting period consists of 9 years ($Y = 9$) with 2014 as the last historical year and the following forecasting years: 2015, 2016, ..., 2023. This time span roughly coincides with one business cycle (Pearce, 2013). It is decided to perform 5000 Monte Carlo simulation runs

Table 5.2: OD pairs

OD pairs
ATL-FLL
ATL-MCO
DFW-LAX
JFK-LAX
JFK-SFO
LAS-LAX
LAX-ORD
LAX-SFO
LGA-ORD
ORD-SFO

Table 5.3: OD demand matrix

	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL			x					x		
DFW						x				
FLL	x									
JFK						x				x
LAS						x				
LAX		x		x	x				x	x
LGA									x	
MCO	x									
ORD						x	x			x
SFO				x		x			x	

($D = 5000$) because, as will be discussed in Section 6.1.3, it provides a well balanced trade-off between the consistency of forecasted demand sample values and the resulting computation time. The 5000 Monte Carlo simulation observations per OD per year are sampled into 10 demand sample values ($S = 10$) and by assuming correlation between OD pairs this results in 10 OD demand matrices per year ($M = 10$). The rationale for these values is grounded in the goal to adopt a meaningful sampling strategy that can be performed in reasonable computation times. This discussion is introduced in Section 4.2.3 by investigating the relationship between the sampling strategy and the number of unique OD demand matrices per year M. Here, the discussion is completed with case study specific data by relating the sampling strategy to the actual total computation time of the fleet assignment optimization model CT_{model2}. If a computation time of $1s$ is assumed for a single optimization run CT_o, and given that $F = 8$ and $Y = 9$, the total computation time of the fleet assignment optimization model reduces to;

$$
\begin{aligned}
CT_{\text{model 2}} &= F \cdot Y \cdot M \cdot CT_O \\
&= 8 \cdot 9 \cdot M \cdot 1 \\
&= 72 \cdot M
\end{aligned} \tag{5.1}
$$

Sample strategy 1 is disregarded because it was identified that it yields a deterministic approach, which is not in line with the research objective. Therefore, the three remaining sampling strategies yield the following computation times;

Sample strategy 2 The number of sample values S is equal to the number of realizations of uncertainty D

- $M = S^Z = D^Z = 5000^{10}$
- $CT_{\text{model 2}} = 72 \cdot M = 72 \cdot 5000^{10} \approx 2.23 \times 10^{31}$ years
- Analysis: a computation time of 2.23×10^{31} years is clearly unacceptable

Sample strategy 3 The D realizations of uncertainty are represented by 10 sample values

- $M = S^Z = 10^Z = 10^{10}$
- $CT_{\text{model 2}} = 72 \cdot M = 72 \cdot 10^{10} \approx 2.28 \times 10^4$ years
- Analysis: a computation time of 2.28×10^4 years is an improvement compared to the computation time of sampling strategy 2, however it is still unacceptable

Sample strategy 4 The D realizations of uncertainty are represented by 10 sample values, and OD pairs are assumed to be perfectly correlated within a year

- $M = S = 10$
- $CT_{\text{model 2}} = 72 \cdot M = 72 \cdot 10 = 720\ s$
- Analysis: assuming perfect correlation between OD pairs which results in a computation time of $720\ s$ is considered an acceptable balance between the meaningfulness of results and the resulting computation times

In this analysis CT_o was assumed to be $1s$. Actual values of CT_o are of the same order of magnitude and are presented in Section 5.4.2. As result of the adoption of the fourth sampling strategy, the total number of generated OD demand matrices amounts to $Y \cdot M = 9 \cdot 10 = 90$, the total number of fleet assignment optimization model runs amounts to $F \cdot Y \cdot M = 8 \cdot 9 \cdot 10 = 720$ and therefore the total computation time of the fleet assignment optimization model is given by;

$$CT_{\text{model 2}} = F \cdot Y \cdot M \cdot CT_O = 8 \cdot 9 \cdot 10 \cdot CT_O = 720 \cdot CT_O \tag{5.2}$$

From a group of 10 different airports ($N = 10$), 10 OD pairs are selected for this case study ($Z = 10$) and are presented in Appendix A. The OD pairs are selected based on 2014 air travel data; they reflect the 10 most high-density OD connections in the total US domestic economic passenger market in 2014. Because $N = 10$ and $Z = 10$, the OD demand matrix has size 10×10 and contains 20 nonzero elements. The OD pairs under consideration and the resulting OD demand matrix structure are presented in Table 5.2 and 5.3, respectively.

The Atlanta airport (ATL) is selected to be a hub ($H = 1$) due to its relatively central location. Consequently constraint 4.20d in the mathematical formulation of the optimization, which is only active if two or more hubs are selected, remains unused. Three different aircraft types are considered ($K = 3$) each with different characteristics; the Airbus A340-300, the Boeing 737-800 and the Bombardier CRJ700. These aircraft are chosen because of their strong differing characteristics in terms of seating capacities, range, purchase price, etc.

Based on Equations 4.27 and 4.28 which identified the relation between the problem size and the size of the LP matrix, the number of decision variables and constraints for this case study are given by;

$$\begin{aligned} \text{\# Constraints} &= 2N^2 + NH - H + NK + K + x \\ &= 2 \cdot 10^2 + 10 \cdot 1 - 1 + 10 \cdot 3 + 3 + 38 \\ &= 280 \end{aligned} \tag{5.3}$$

$$\begin{aligned} \text{\# Decision variables} &= N^2 - N - 2NH + H + N^2H + N^2K - NK \\ &= 10^2 - 10 - 2 \cdot 10 \cdot 1 + 1 + 10^2 \cdot 1 + 10^2 \cdot 3 - 10 \cdot 3 \\ &= 441 \end{aligned} \tag{5.4}$$

where x is the number of aircraft range constraints, which amounts to 38 for this case study.

As presented in Section 4.4, the number of OD pair based transition probability matrices is given by $Z \cdot (Y - 1)$ which amounts to $10 \cdot (9 - 1) = 80$. Likewise, the number of OD demand matrix based transition probabilities is $Y - 1 = 9 - 1 = 8$ in this case study. The number of scenarios generated by the scenario generation model is set at 5000 ($D = 5000$). Consequently, each distribution of NPVs per fleet is based on 5000 data points.

5.2 Data

5.2.1 Demand data

The historical passenger data is extracted from the TranStats database of the Bureau of Transportation Statistics (BTS), which is part of the United States Department of Transportation (US

DOT). The underlying dataset that was used is the T-100 Domestic Market (U.S. Carriers) data table that contains monthly scheduled US domestic passenger data based on a 10 percent ticket sale information dataset, aggregated for all airlines for the period 1990-2014.

In order to be able to use a manageable dataset (e.g. small in size) with only the required information for the specific purpose of this research, the extracted data has been subject to some data mining and cleaning to eliminate unused data and restructure the data in the right format. The dataset is reduced to contain the following: number of passengers per market per year for the period 1990-2014.

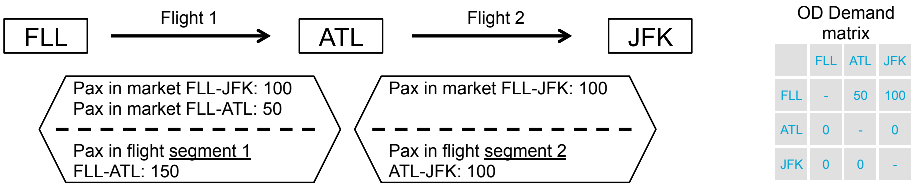


Figure 5.1: A visualization of the difference between market data and segment data

At this point it is important to highlight the difference between market data and segment data. Market data reflects the passenger data between an origin and destination airport (i.e. an OD pair) irrespective of how a connection was made between those two airports. Segment data reflects the number of passengers in a certain flight segment between two airports; however these passengers could have different itineraries and therefore could use this flight segment as part of different origin-destination connections. See Figure 5.1 for a visualization of these two concepts.

5.2.2 Aircraft characteristics

Different aircraft types are differentiated by their characteristics, which are the number of seats, cruise speed, range, daily utilization, turnaround time, variable cost, ownership cost and purchase price. These characteristics are specified in Table A.3. The number of seats s^k, cruise speed vc^k, range $range^k$ and purchase price (i.e. list price) are based on information provided on the internet (AxleGeeks, 2016). Weekly utilization U^k, turnaround times TAT^k and operating cost C_{var}^k are virtual data and are based on the assumption that larger aircraft tend to have a higher daily utilization, higher turnaround time and lower unit operating cost (Belobaba et al., 2009). The yearly ownership cost C_{fix}^k are based on the aircraft purchase price and assuming a 20 year linear depreciation period and residual value of 15% at the end of the depreciation period which is based on an example depreciation scheme provided by Doganis (2002);

$$\text{Yearly ownership cost} = \frac{\text{Purchase price} \cdot (1 - \text{residual value})}{\text{Depreciation term in years}} = \frac{0.85 \cdot \text{Purchase price}}{20} \quad (5.5)$$

5.2.3 Adjusting the OD demand matrices

Each of the 90 outputted OD demand matrices by the stochastic demand forecasting model contains total yearly demand, which needs to be reduced to airline specific weekly demand. This is achieved in two steps. First, each element in the OD demand matrix is multiplied with $\frac{1}{52}$ to reduce the annual data to weekly data. Consequently, it is assumed that the demand matrix reflects an average week of the year and thereby neglects seasonality or trend growth throughout the year. Second, each element in the OD demand matrix is multiplied with a 20% market share to arrive at airline specific demand. This market share reflects the share of total market demand that can be potentially captured by the airline given its competitive characteristics. The 20% market share assumption is adopted for each OD pair.

$$\text{Weekly airline specific demand} = \frac{\text{Annual market demand}}{\text{Number of weeks per year}} \cdot \text{Airline market share} \tag{5.6}$$

$$= \frac{\text{Annual market demand}}{52} \cdot 0.2 \tag{5.7}$$

5.2.4 Yields

Yield is defined as revenue per revenue-passenger-mile in 2014 US Dollar cents. The yields are based on actual average fare data in 2014. This data stems from the BTS US DOT database and the underlying dataset that was used is Table 1a Domestic Airline Airfare Report (2011 - 2014) which contains average fare data per OD pair per quarter for the period 2011-2014 for a large set of OD pairs (but not all OD pairs) in the US. Yield data is calculated by dividing the fares by their origin-destination distance. It is noted that the average fares that form the basis for this dataset are not only averaged for the year 2014, but also reflect average fares as listed by all airlines in the marketplace and irrespective of the offered service (i.e. nonstop or connecting). Unfortunately the fare data was not available for all OD pairs under consideration in the case study. Therefore the missing data has to be estimated. This is achieved by identifying the yield-distance relationship for the OD pairs for which fare data is available through a power fit, and using this relationship together with the known origin-destination distance to estimate fares and subsequently yields for the OD pairs for which fare data was missing. The available fare data and the fitted yield-distance relationship is shown in Figure 5.2. The yields can be found in Table A.7.

The ratio between yields for nonstop and connecting passengers is set at 1.0 in the case study, which results in the nonstop flow being more profitable. In practice, these yield ratios are airline specific and highly depend on how an airline prioritizes nonstop or connecting flow per OD pair in their revenue management models, based on the competitive environment.

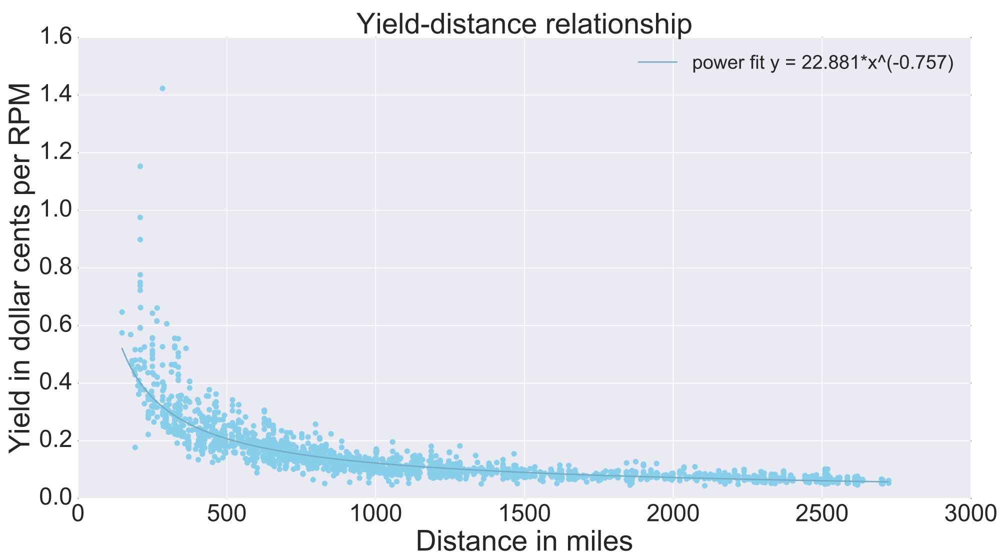


Figure 5.2: The yield-distance relationship

5.2.5 Airport characteristics

The taxi-in and taxi-out times stem from the BTS US DOT database. The underlying dataset that was used is the Airline On-Time Statistics - Origin and Destination Airport dataset that provides 2014 data on taxi-in and taxi-out times in minutes per OD pair averaged for all airlines. The taxi-out times and taxi-in times can be respectively found in Tables A.4 and A.5. The OD distances can be found in Table A.6.

5.2.6 Inflation, discount rate and tax rate

Inflation is considered on the cost side (i.e. operating and ownership cost) and on the revenue side (i.e. nonstop and connecting yields). In the case study the inflation is assumed to be 1.5% per year for all the 9 years in the planning horizon. This number is calculated as the average inflation in the US between 2010 and 2014 which is based on data from the US Bureau of Labor Statistics (US BLS) and specifically the dataset provided in Table 24 Historical Consumer Price Index for all Urban consumers (CPI-U).

For the NPV calculation, the discount rate r is set at the weighted average cost of capital (WACC). The assumed WACC value is 7.4 % which is the average historical WACC for US airlines (both legacy carriers and low cost carriers) between 2004 and 2011 (Pearce, 2013).

The effective corporate tax rate in the US depends on a federal and state component and is assumed to be 39%. Using the tax rate the return on invested capital (ROIC) is calculated as;

$$\begin{aligned}\text{Annual return on invested capital (ROIC)} &= \frac{\text{Annual operating profit} - \text{tax}}{\text{Investment}} \\ &= \frac{\text{Annual operating profit} \cdot (1 - \text{tax rate})}{\text{Investment}}\end{aligned} \quad (5.8)$$

5.3 Assumptions and their implications

In addition to the assumptions that are grounded in the proposed methodology there are some case study specific assumptions which are presented in this section.

Stochastic demand forecasting model

- Merely economy class passenger data is used;
  - It could very well be that business class and first class air travel demand displays different mean reverting properties than economy class air travel demand
- Historical passenger data is airport-to-airport related as such the demand forecast also returns airport-to-airport demand;
  - This might impact results considering that certain cities have multiple airports, and it could be possible that passengers chose a particular airport within a city due to the competitive circumstances (e.g. departure times, air fares) at that point in time
- OD pairs are assumed to be perfectly correlated $(+1)$ within a year;
  - This allows for the construction of 10 OD demand matrices per year instead of 10^{10} OD demand matrices per year. In words this assumption states that in a certain year the demand sample values of each OD pair in an OD demand matrix are taken from the same part of the probability distribution. This simplifying assumption also allows for the generation of OD demand matrix based transition probability matrices.
- The used dataset is altered in that the effect of triangular and one-way connections are eliminated by summing traffic in either way of one OD pair and dividing the sum by two to arrive at directional passenger data

Fleet assignment optimization model

- In the case study only 1 hub is adopted; being airport ATL (Atlanta)
- Yield is OD dependent and the yield ratio between nonstop and connecting passengers is 1
  - Yields in the case study are based on average airline fare data, so it is not airline specific but representative of an average airline
  - Assuming a yield ratio of 1 results in that nonstop flow is more profitable than connecting flow for each OD pair
- A market share model is not adopted. The market share is assumed to be 0.2 for all OD pairs
  - Demand-supply interactions are ignored; there is no consideration of a S-curve relation between the offered weekly flight frequency and captured demand; demand-price elasticity is not considered
  - The impact of this assumption is discussed in Section 5.4.4
- Inflation is 1.5 % percent
- Taxi time depends on the airport and whether the flight is inbound or outbound but is not aircraft type dependent
- A straight line 20 year depreciation term is assumed with a residual value of 15%

Scenario generation model

- The discount rate r is set by the WACC, which is based on the historical 7.4 % WACC for US carriers between 2004-2011 (Pearce, 2013)

5.4 Results

The results of the stochastic demand forecasting model, the fleet assignment optimization model and the scenario generation model are presented in Sections 5.4.1, 5.4.2 and 5.4.3, respectively. Section 5.4.4 presents a synthesis of the results and computation times from the perspective of the overarching solution methodology.

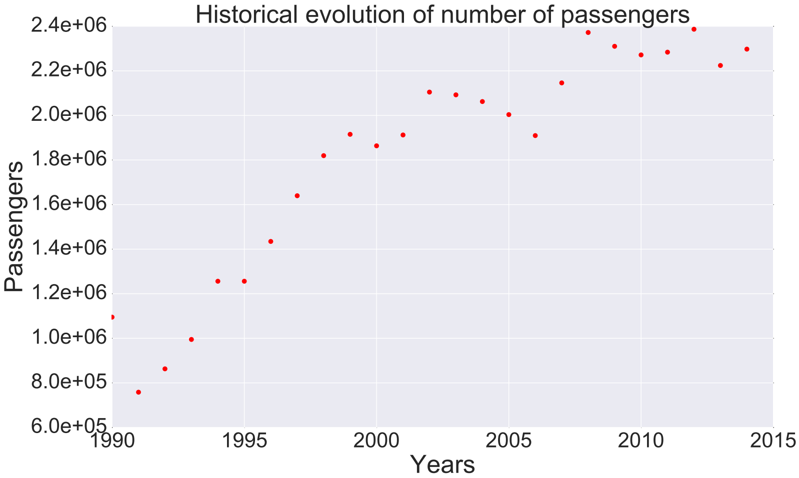
5.4.1 Stochastic demand forecasting model

Mean reverting model parameters for each OD pair

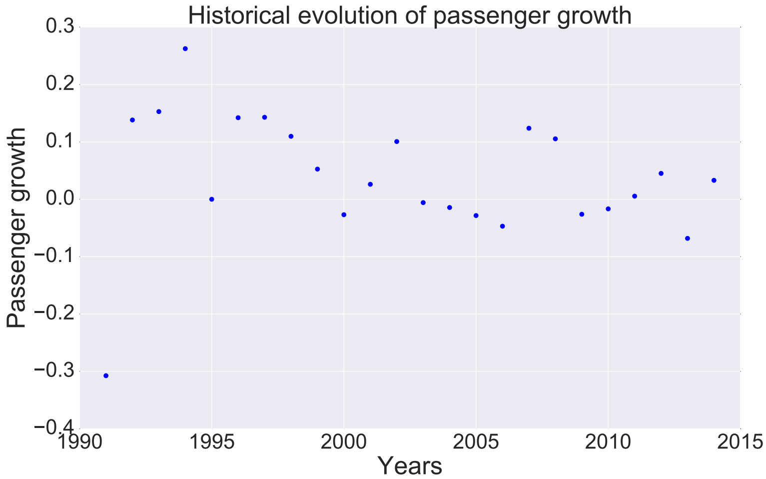
Figure 5.3 shows for one OD pair, ATL-FLL, how the mean reverting model parameters are estimated through linear least squares regression. This procedure is executed for each OD pair and the resulting mean reverting model parameters for the 10 OD pairs that are part of the case study are presented in Table 5.4.

In terms of goodness of fit, it can be observed that for the 10 OD pairs under consideration the R^2 values range from 0.26 to 0.62 and the p-values range from 0.000 to 0.013. A high R^2 together with a low p-value respectively indicate that a lot of the variability is explained and that the predictor variable is significant (i.e. a change in the predictor variable x results in a change in the response variable y).

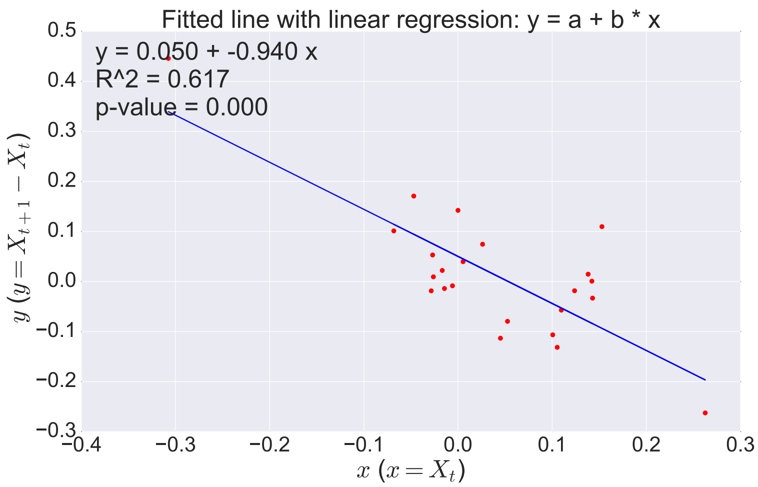
In terms of mean reverting model parameters, it can be observed that the speed of mean reversion λ ranges from 0.83 to 1.02, the long term average growth rates μ range from 0.7%



(a) Historical evolution of number of passengers

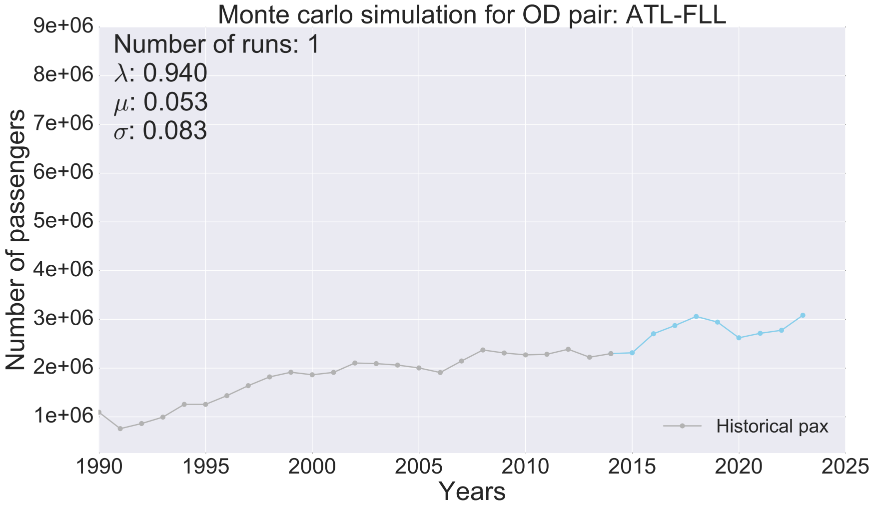


(b) Historical evolution of year-to-year passenger growth rates (X_t)

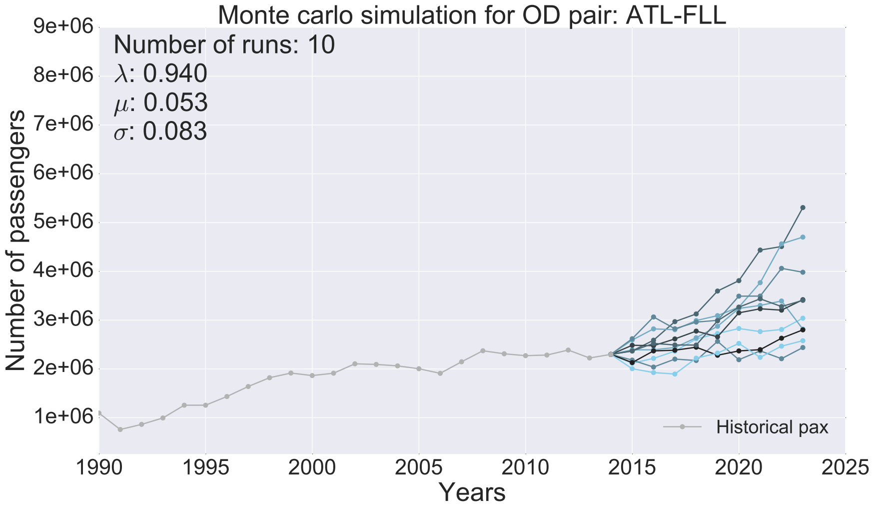


(c) Mean reverting process equation rewritten into a form suitable for linear regression: $y = a + b \cdot x + \epsilon$

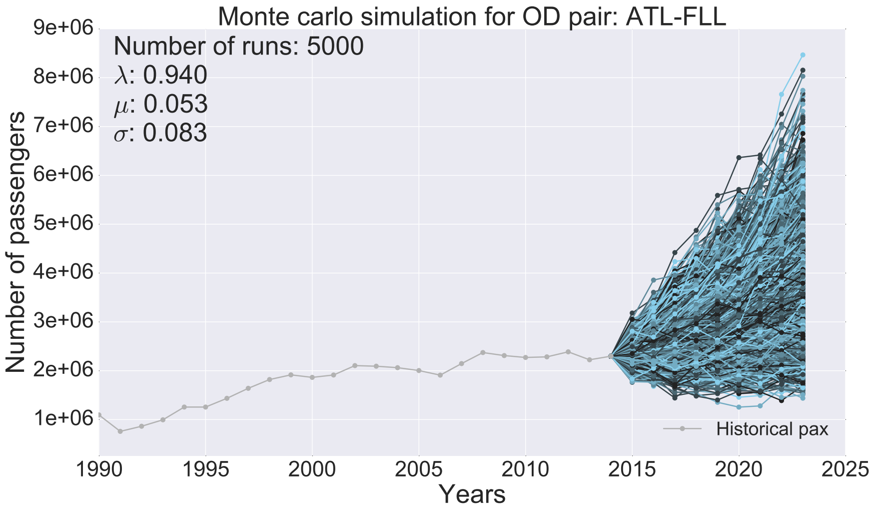
Figure 5.3: Estimating the mean reverting process model parameters through a linear least squares regression



(a) 1 run



(b) 10 runs



(c) 5000 runs

Figure 5.4: Monte Carlo simulation of the mean reverting process

Table 5.4: Mean reverting model parameters for each OD pair

OD pair	a	b	p-value	R^2	λ	μ	σ
ATL-FLL	0.051	-0.963	0.000	0.617	0.963	0.053	0.087
ATL-MCO	0.034	-0.941	0.000	0.656	0.941	0.037	0.070
DFW-LAX	0.024	-1.020	0.000	0.526	1.020	0.024	0.063
JFK-LAX	0.029	-1.011	0.000	0.510	1.011	0.029	0.127
JFK-SFO	0.035	-0.961	0.000	0.485	0.961	0.036	0.144
LAS-LAX	0.018	-0.665	0.004	0.332	0.665	0.027	0.119
LAX-ORD	0.010	-0.953	0.000	0.476	0.953	0.011	0.056
LAX-SFO	0.004	-0.523	0.013	0.259	0.523	0.007	0.107
LGA-ORD	0.023	-0.977	0.000	0.490	0.977	0.023	0.080
ORD-SFO	0.013	-0.831	0.001	0.416	0.831	0.016	0.056

per year to 5.3% per year and the standard deviation of the historical estimation error σ varies between 0.056 and 0.144. Based on these values it can be analyzed that the 10 OD pairs under consideration display strong mean reverting properties because the λ is close to one, they have varying long term average growth rates and also display a different behavior in terms of year-to-year volatility.

It is noted that the case study only focuses on 10 OD pairs and that the evaluation of other OD pairs might yield different results both in terms goodness of fit as well as mean reverting properties.

Monte Carlo simulation of demand

Figure 5.4 displays the Monte Carlo simulation of OD pair ATL-FLL across the 2015-2013 planning horizon, based on 1, 10 and 5000 simulation runs respectively. An overview of the Monte Carlo simulations of all 10 OD pairs can be found in Figure B.1. It can be clearly observed that the uncertainty increases with time. Although the figure gives an interesting insight in the expansion of uncertainty over time, it is noted that this visualization cannot be used to analyze in detail the distribution of the 5000 Monte Carlo simulation observations within a year. This is due to the thickness of the vast amount of plotted lines. In order to analyze the distribution of observations within a year in detail, it is advised to analyze the histograms that are discussed in the next paragraph.

A histogram based on 5000 observations per year per OD pair

Figure 5.5 displays 9 traditional histograms that provide insight into the distribution of 5000 Monte Carlo simulation observations per year for all the 9 forecasting years for the OD pair ATL-FLL. In Section 4.2.3 it is elaborated why and how the data is not binned using traditional histograms but rather using equal probability bin histograms. The latter type of histogram can be found in Figure 5.6 for OD pair ATL-FLL and in Appendix B.2 for all OD pairs.

Five observations can be made. First, the widths of the histograms increase with time, which is in line with expectation because uncertainty increases with time. Second, it can be observed that across years there is a tendency of the weight of the histogram to move to the right; i.e. generally across years there is air travel demand growth. Third, it can be observed that the outer bins (especially bin 1 and bin 10) are much wider than the inner bins. Each bin contains the same number of observations, but apparently the spread between the highest and lowest value of the observation is larger in the outer bins than in the inner bins. This is in line with expectation because when a vast amount of simulations is run, it is likely that some extreme values occur that shift the outer bin edges to the right (for the right bin) and to the left (for the left bin) and therefore increase the bin widths of the outer bins. Fourth, it can be observed that when focusing on the outer bins (bin 1 and bin 10) in general bin 10 is wider than bin 1. This can be attributed to the presence of long-term average growth rates that result in some upward skewness of uncertainty in demand levels. Fifth, it can be observed that generally all OD pairs display the aforementioned four observations. This can be verified through visually inspecting all figures in Appendix B.2.

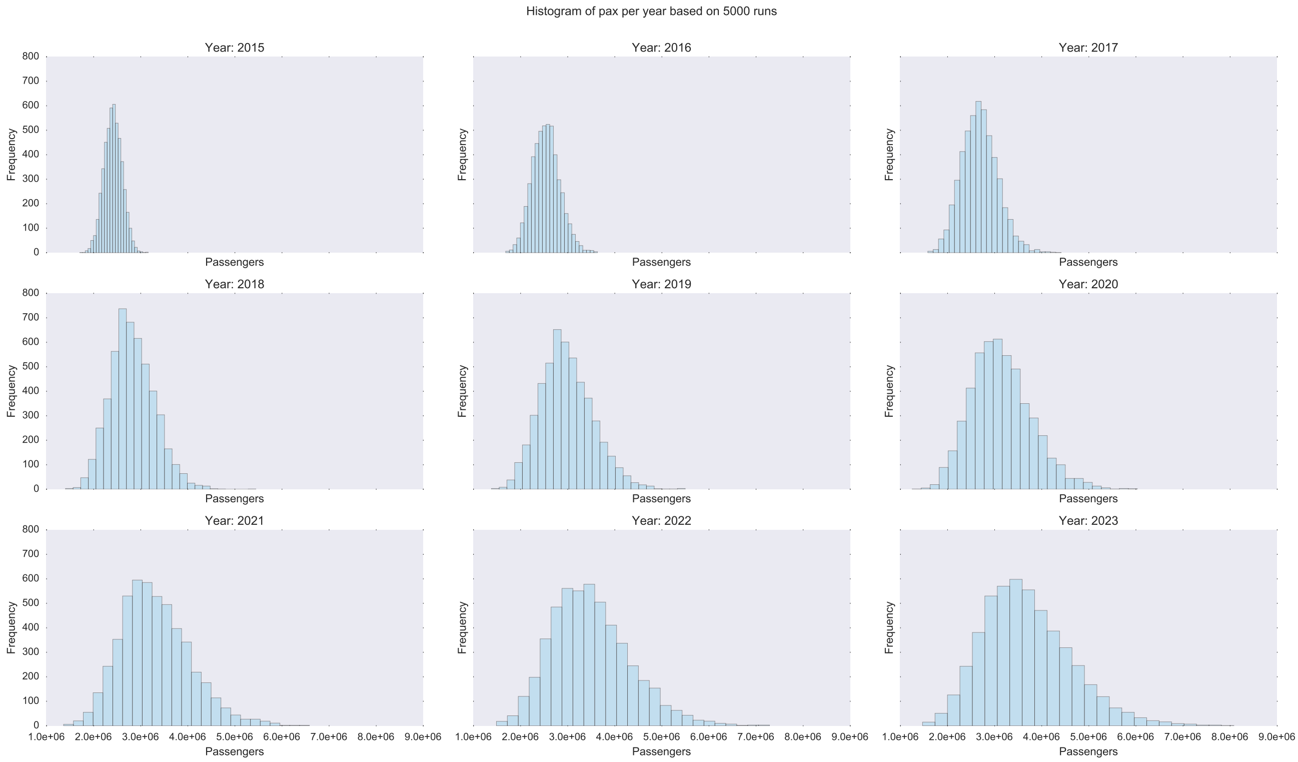


Figure 5.5: Traditional histograms of Monte Carlo simulation observations for the OD pair ATL-FLL for all the years in the planning horizon

90 OD demand matrices

From the perspective of the overarching solution methodology the 90 OD demand matrices are the most important output of the stochastic demand forecasting model. The OD demand matrix based on sample values from bin 1 for the year 2015 is shown in Table 5.5. An overview of all 90 OD demand matrices can be found in Appendix B.3. Three observations are elaborated.

First, focusing on one OD pair and one year, the demand values increase when moving from bin 1 to bin 10. This should always be the case since all the individual values in a bin are larger than the individual values in the bin to the left of it and therefore the average value (i.e. sample value) is also larger.

Second, focusing on one OD pair and one year, the spread between subsequent demand values decreases from bin 1 to 5 and then increases from bin 5 to 10. This is in line with the visual observations; the bin widths decrease from outer to inner bins so probably the averages of those bins also lie closer to each other when moving inward. However, it is noted that this is not necessarily the case because the sample values are averages of all the observations in the bin and not the median of the observations in the bin.

Third, focusing on one OD pair and one bin number, generally the demand sample values decrease from year to year for the bins 1-5 and increase from year to year for bins 6-10. This can

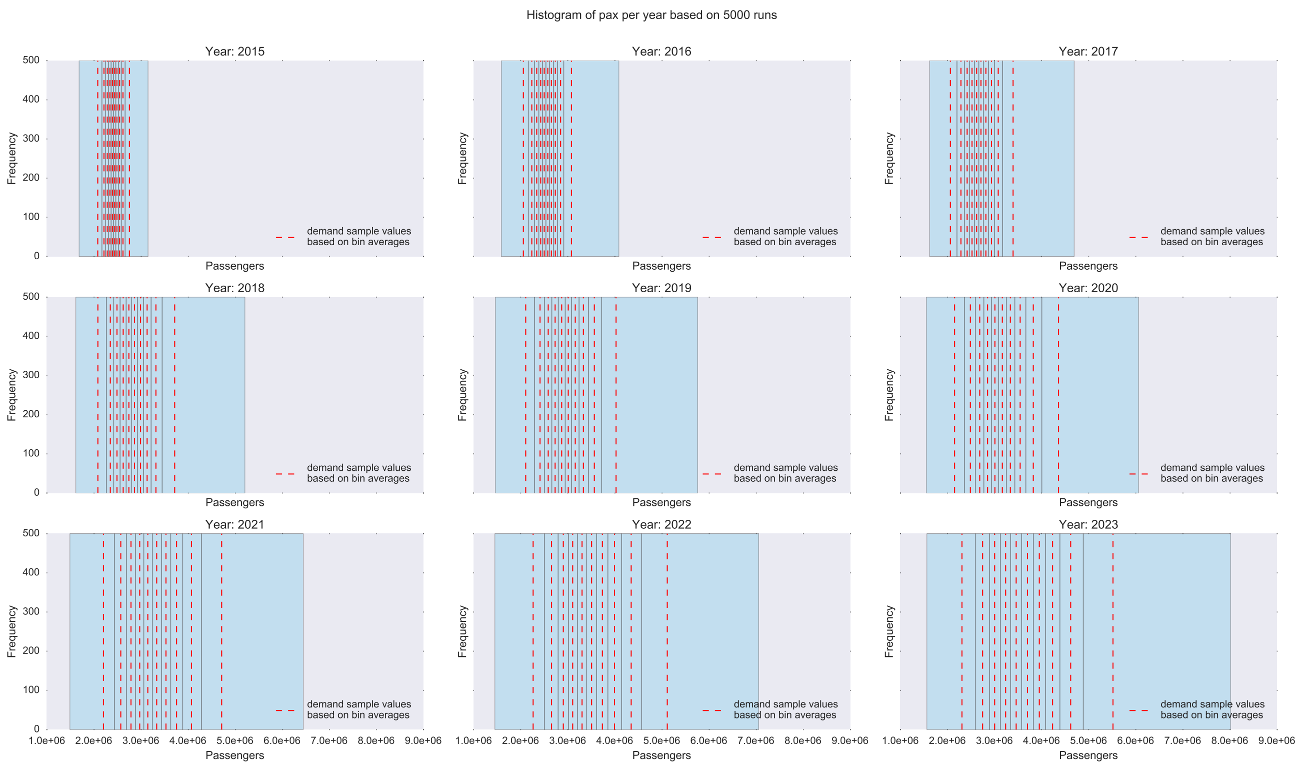


Figure 5.6: Equal probability histograms of Monte Carlo simulation observations for the OD pair ATL-FLL for all the years in the planning horizon

be attributed to the expansion of uncertainty over time, which is both upwards and downwards.

Table 5.5: OD demand matrix based on sample values from bin 1 in 2015

Year 2015 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.03E+06	0	0	0	0	1.13E+06	0	0
DFW	0	0	0	0	0	1.06E+06	0	0	0	0
FLL	1.03E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.29E+06	0	0	0	8.87E+05
LAS	0	0	0	0	0	9.43E+05	0	0	0	0
LAX	0	1.06E+06	0	1.29E+06	9.43E+05	0	0	0	1.08E+06	1.57E+06
LGA	0	0	0	0	0	0	0	0	1.21E+06	0
MCO	1.13E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.08E+06	1.21E+06	0	0	1.01E+06
SFO	0	0	0	8.87E+05	0	1.57E+06	0	0	1.01E+06	0

Computation time

The computation time of the stochastic demand forecasting model approximately scales with $6s$ per OD pair Z, per year Y and can be represented by the following equation;

$$CT_{\text{model 1}} = Y \cdot Z \cdot 6s = 9 \cdot 10 \cdot 6 = 540s \tag{5.9}$$

5.4.2 Fleet assignment optimization model

One optimization model run

Each optimization model run returns the optimal values of the decision variables and objective function. Both financial and non-financial performance metrics are derived from those values. The decision variable and objective function values of one example run are provided in Table 5.6, and the corresponding derived performance metrics are presented in Table 5.7. The results of this example run are based on the assignment of Fleet 6 from the portfolio (which contains 15 Bombardier CRJ700 aircraft) based on the OD demand matrix in 2015 that contains sample values from bin 5. The returned objective function value is $1,310,509.72 (i.e. the weekly operating profit) and the optimality gap is 0.0085% (i.e. the gap between the optimal objective function value with integrality constraints and the optimal value of the linear relaxation). A visualization of the routing network and the weekly frequencies per aircraft type can be found in Figure 5.7.

Table 5.6: Decison variable values

Decision variable	Subscripts	Value
x_{od}	ATL-MCO	4875
x_{od}	MCO-ATL	4875
x_{od}	LAS-LAX	4500
x_{od}	LAX-LAS	4500
x_{od}	LGA-ORD	4875
x_{od}	ORD-LGA	4875
x_{od}	LAX-SFO	7200
x_{od}	SFO-LAX	7200
z_{ij}^k	ATL-CMO-Bombardier CRJ700	65
z_{ij}^k	MCO-ATL-Bombardier CRJ700	65
z_{ij}^k	LAS-LAX-Bombardier CRJ700	60
z_{ij}^k	LAX-LAS-Bombardier CRJ700	60
z_{ij}^k	LGA-ORD-Bombardier CRJ700	65
z_{ij}^k	ORD-LGA-Bombardier CRJ700	65
z_{ij}^k	LAX-SFO-Bombardier CRJ700	96
z_{ij}^k	SFO-LAX-Bombardier CRJ700	96

720 runs: running the optimization model for each fleet-OD demand matrix combination

The optimization model is run for every combination of fleets from the portfolio and OD demand matrix outputted by the stochastic demand forecasting model. The case study encompasses 8 fleets ($F = 8$) in the portfolio, 9 planning years ($Y = 9$) and 10 OD demand matrices per year ($M = 10$), resulting in $F \cdot Y \cdot M = 8 \cdot 9 \cdot 10 = 720$ iterations of the optimization model.

Each iteration returns the same type of financial and non-financial performance metrics as presented in Table 5.7, which are stored in one large data table with 720 rows. This allows for an explicit comparison of both financial and non-financial performance metrics between different fleets across different realizations of stochastic demand across the 9 years in the planning horizon. Only the yearly operating profits are stored in the value matrix which is presented in table form in Appendix C.2. The value matrix is subject to further analysis in the scenario generation model. However, the value matrix can also be plotted per fleet in order to visualize the range of annual operating profits across the planning horizon. An example of such a plot is presented for Fleet 6 in Figure 5.8. The expansion of uncertainty with time on the demand side

Table 5.7: Financial and non-financial performance metrics derived from a single optimization run

Financial perfomance metrics		Non-financial performance metrics	
Metric	**Value**	**Metric**	**Value**
Weekly revenue	$3,597,750.79	Weekly OD pax transported	42900
Weekly operating cost	$1,986,880.50	Weekly seats offered	42900
Weekly ownership cost	$300,360.58	Weekly seats filled	42900
Weekly operating profit	$1,310,509.72	Percentage nonstop flow	100%
Annual operating profit	$68,146,505.33	Percentage OD demand satisfied	42.4%
Operating profit margin	36.43%	Average network load factor	100%
Annual op. profit minus tax	$41,569,368.25	Number of OD pairs served	4
Total investment cost	$367,500,000.00	Weekly utilization per aircraft type	
Annual ROIC	11.31%	Bombardier CRJ700	100%
Spilled revenue percentage	70.49%		

clearly also propagates through the evolution of annual operating profits across the planning horizon.

Computation time

The total computation time of the second model ($CT_{\text{model 2}}$) linearly scales with the computation time of a single optimization run (CT_o), the number of fleets in the portfolio (F), the number of years in the planning horizon (Y) and the number of OD demand matrices per year (M);

$$CT_{\text{model 2}} = F \cdot Y \cdot M \cdot CT_O = 8 \cdot 9 \cdot 10 \cdot 0.72s = 520s \tag{5.10}$$

5.4.3 Scenario generation model

OD pair based transition probability matrix for two consecutive years

Table 5.8 presents the transition probability matrix between 2015 and 2016 for the OD pair ATL-FLL. While it is interesting to now the actual probabilities, it is also interesting to see the pattern of probabilities within a transition probability at one glance. This can be done by visualizing the transition probability matrix as a heat map, where the darker the color the higher the probability. A heat map enables quick visual interpretation of the transition behavior and therefore can be used to analyze one transition probability matrix but also can be used to compare the behavior of different transition probability matrices. Figure 5.9 is the heat map visualization of the transition probability matrix in Table 5.8. Three observations can be made with regards to one transition probability matrix.

First, it can be clearly observed that the probabilities are highest along the diagonal of the transition probability matrix. Essentially this means that it is very likely that from year-to-year an observation falls either in the same bin or in adjacent bins. Moreover, this tendency seems to gradually decrease with an increasing transition gap between bins in consecutive years.

Second, it can clearly be observed that within the diagonal, the highest probability is in the outer bins. Essentially this means that the tendency of an observation to stay in the same bin from year to year is higher for observation that are in the outer bins. This behavior is explained by the usage of equal probability bins and the distribution of uncertainty which results in wider outer bins than inner bins and therefore a smaller likelihood of transitioning from the outer wider bin to another bin.

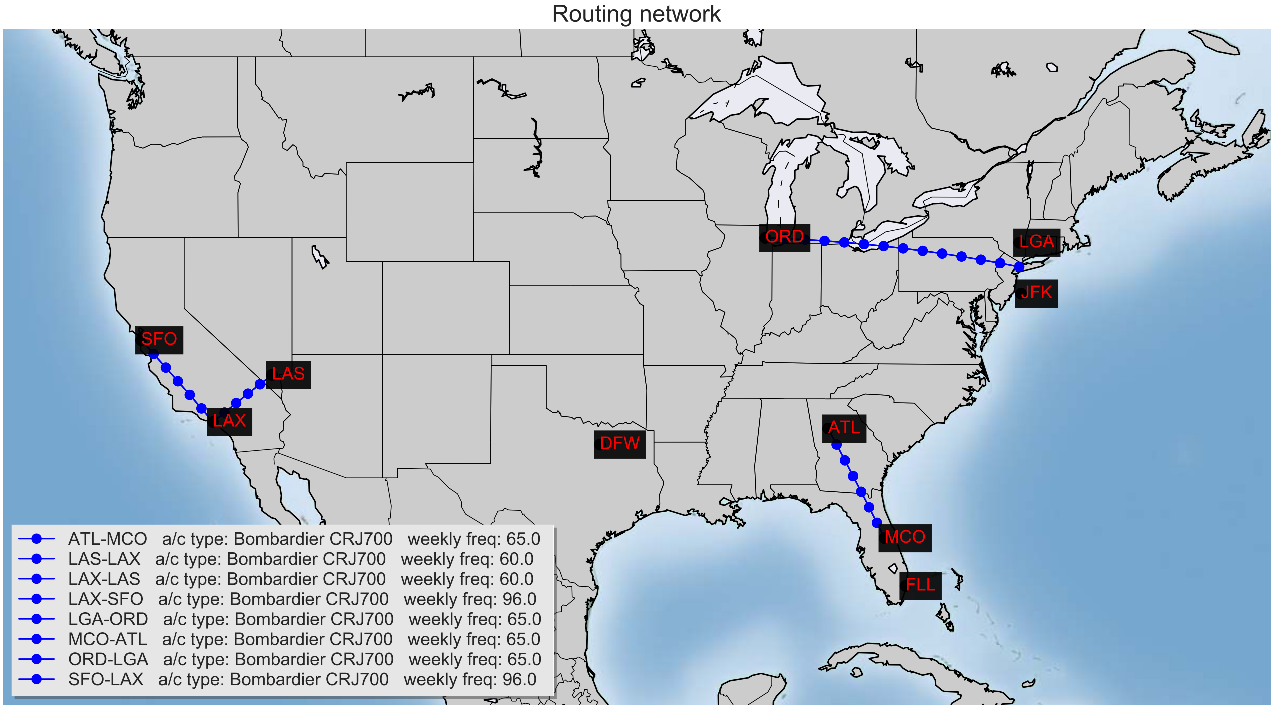


Figure 5.7: Routing network

Third, it is noted that the first two observations hold for all 80 transition probability matrices as can be verified through visual inspection in Appendix D.1.

		Bin number in: 2016									
		1	2	3	4	5	6	7	8	9	10
Bin number in: 2015	1	0.50	0.24	0.14	0.06	0.03	0.02	0.00	0.01	0.00	0.00
	2	0.21	0.25	0.14	0.13	0.14	0.07	0.04	0.02	0.01	0.00
	3	0.11	0.19	0.15	0.19	0.14	0.09	0.08	0.04	0.01	0.00
	4	0.06	0.12	0.17	0.16	0.13	0.12	0.13	0.08	0.03	0.01
	5	0.06	0.08	0.14	0.13	0.14	0.17	0.12	0.09	0.07	0.01
	6	0.04	0.05	0.11	0.13	0.13	0.15	0.13	0.13	0.10	0.04
	7	0.01	0.03	0.07	0.10	0.12	0.13	0.17	0.16	0.15	0.06
	8	0.01	0.02	0.04	0.06	0.09	0.14	0.15	0.18	0.15	0.15
	9	0.01	0.02	0.02	0.04	0.06	0.07	0.10	0.19	0.24	0.25
	10	0.00	0.00	0.02	0.01	0.03	0.04	0.08	0.11	0.23	0.48

Table 5.8: Transition probability matrix for the OD pair ATL-FLL for the consecutive year combination 2015-2016 based on 5000 Monte Carlo simulation observations

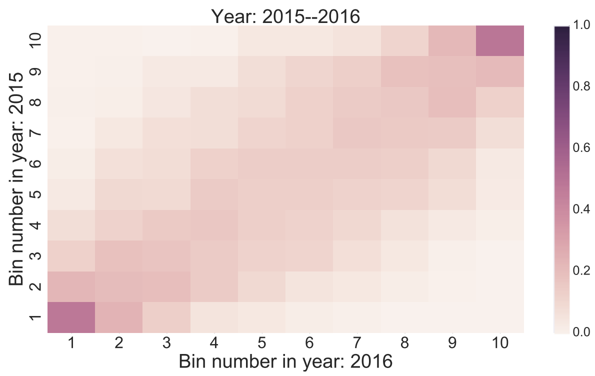


Figure 5.9: Heat map visualization of the transition probability matrix presented in Table 5.8

OD pair based transition probability matrices for all consecutive years

Appendix D.1 contains the heat maps for each consecutive year combination in the planning period 2015-2023 for each of the 10 OD pairs. In addition to the observations that are made about individual transition probability matrices in the previous section, one observation can be made with regards to the evolution of transition probability behavior over time. It can be observed that the probability density across the diagonal increases over time. Essentially this means that the tendency of an observation to fall in the same bin or adjacent bins gets stronger with the evolution of time. This can be explained using the same principle as in the previous section; because uncertainty expands with time, the bin widths also increase and therefore as

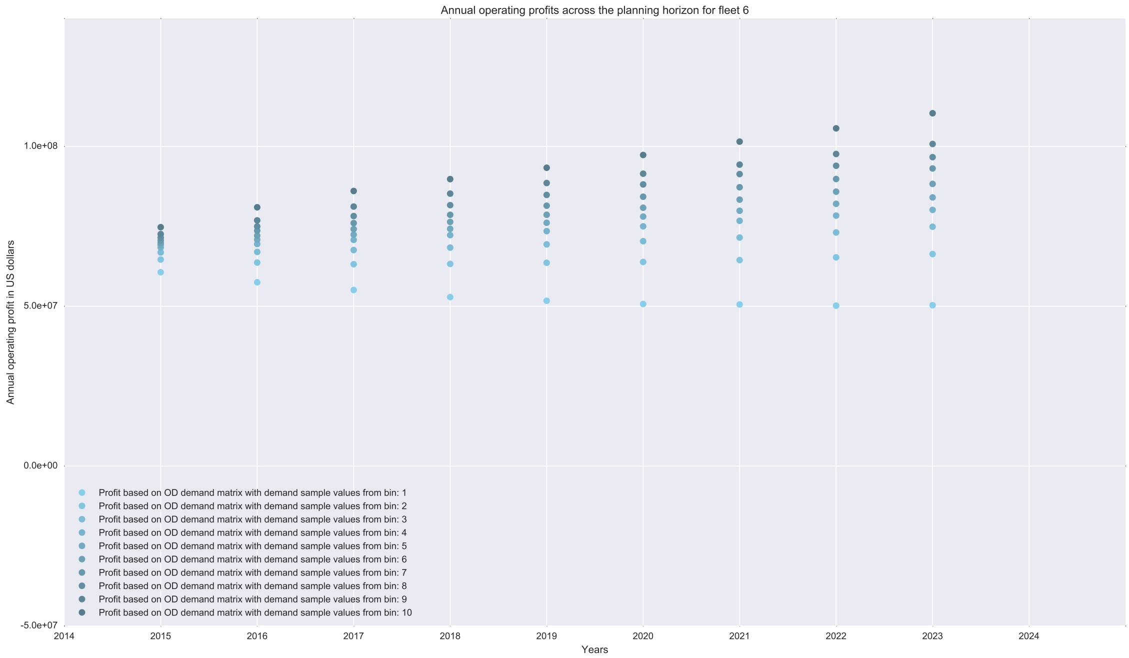


Figure 5.8: A plot of one intersection through the value matrix; annual operating profit of Fleet 6 for each of the 10 OD demand matrices per year for each of the 9 years in the planning horizon

time evolves the likelihood of transferring from one bin to another bin decreases. As result the probability that an observation is transitioned from one bin to another decreases.

OD demand matrix based transition probability matrices for all consecutive years

The tables and figure in Appendix D.2 present the OD demand matrix based transition probability matrices both in table form and as a heat map. These transition probability matrices represent the average year-to-year transition behavior of all OD pairs in an OD demand matrix. It is noted that although the different OD pairs do show similar transition behavior and are of the same order of magnitude, the actual transition probabilities might not be exactly the same. The validation of this aggregation procedure is presented in Chapter 6.3.1.

Distribution of net present profit values for each fleet in the portfolio

Figure 5.10 presents for each fleet in the portfolio the distribution of net present profit values based on the 5.000 scenarios across the planning horizon. Three attributes can be evaluated in order to compare the different fleets from the portfolio.

- The mean of the distribution
  - Does the distribution have a high mean (i.e. distribution to the right) or a low mean (i.e. distribution to the left)? This gives insight in the absolute operating profit generation capability of the fleet across the planning horizon across the range of stochastic demand.
- The spread of the distribution:
  - Is it a wide distribution (i.e. lot of uncertainty) or a narrow distribution (i.e. little uncertainty)? This provides insight in the robustness of the profit generating capability of a fleet to the range of stochastic of demand it is subject to across the planning horizon.

- The location of the distribution with respect to the level of investment required
  - Is the distribution close the investment cost? Is it to the right of it? This observation relates the profit generating capability of a fleet to the magnitude of the investment cost that is required to purchase the fleet. Whereas operating profit is an indicator of how efficient the assets (i.e. the fleet) are deployed, the spread between profitability and investment can be used as an indicator of how efficient the investment is generating a return.

The third observation reveals a key insight in the difference of the profit generating capability of a fleet and the capability to generate returns on invested capital. A fleet could be very profitable in operation in absolute terms (i.e. a distribution with a high mean) but at the same time can be a poor investment because of the disproportionate level of investment that is required to get to that level of absolute operating profits (i.e. the de distribution of operating profits is to the far left of the investment cost such as with Fleet 8 from the portfolio). In short, there can be fleets with high operating margins and low returns. To emphasize that, also the average annual return on invested capital based on the average annual net present profit value in the distribution is shown in the figure.

Together, these attributes define the robustness of a fleet. Based on these three attributes, a fleet planner from industry would probably like to select the fleet with a distribution with a high mean, low spread and preferably close or even to the right of the investment cost.

Computation time

The computation time of the scenario generation model scales with the number of scenarios B and the number of fleets in the portfolio F;

$$CT_{\text{model 3}} = B \cdot F \cdot 0.02754s = 5000 \cdot 8 \cdot 0.0275s = 1100s \tag{5.11}$$

5.4.4 Synthesis of the results and computation times

The three models together produce a vast amount of results, the majority of which is used as intermediate results that are part of the methodology. The most important final results in terms of the overarching methodology are;

- The distribution of net present values of profit across the planning horizon across the range of stochastic demand, per fleet from the portfolio
- The large data table that contains all financial and non-financial performance metrics per fleet per year per OD demand matrix within the year

How should a fleet planner interpret the results?

The information that stems from these two datasets can be used to explicitly compare fleets. First, the distribution of NPVs can be used by fleet planners to get a high level insight in the magnitude and uncertainty of the operating profits across a 9 year planning horizon and how these operating profits relate to the investment necessary to obtain the profits. Second, the vast amount of both financial and non-financial data can be used to unravel the underlying factors that drive the distribution of profitability; what are the aircraft utilizations of the different aircraft types in the fleet? What is the average network load factor? How many passengers are spilled? What is the spilled revenue? How many OD pairs are served? What percentage of the passengers is transported nonstop? What are the weekly operating cost and ownership cost? What is the routing network? This vast amount of detailed information can be used for subsequent detailed analysis.

Based on the distributions presented in Figure 5.10, fleet planner is likely to select fleet 6 due to the relatively narrow distribution of NPVs and the beneficial relation between NPVs and investment cost.

The total computation time

The computation times of each of the three models is presented in Table 5.9.

Table 5.9: An overview of computation times per model

Model	Total computation time (s)	Number of runs	Time per run (s)
Model 1	540	$Y \cdot Z = 9 \cdot 10 = 90$	6
Model 2	520	$F \cdot Y \cdot M = 8 \cdot 9 \cdot 10 = 720$	0.72
Model 3	1100	$B \cdot F = 5.000 \cdot 8 = 40.000$	0.0275

The total computation time is simply calculated as the sum of the computation times of the three models:

$$\begin{aligned} CT_{\text{TOTAL}} &= CT_{\text{model 1}} + CT_{\text{model 2}} + CT_{\text{model 3}} \\ &= Y \cdot Z \cdot 6 + F \cdot Y \cdot M \cdot 0.72 + B \cdot F \cdot 0.0275 \\ &= 9 \cdot 10 \cdot 6 + 8 \cdot 9 \cdot 10 \cdot 0.72 + 5000 \cdot 8 \cdot 0.0275 \\ &= \text{36 minutes} \end{aligned} \tag{5.12}$$

The problem size and computation time of a real world case study

For a larger real world case study with 300 OD pairs under consideration ($Z = 300$), 9 years in the planning horizon ($Y = 9$), 10 OD demand matrices per year ($M = 10$), 5.000 scenarios ($B = 5.000$) and 100 fleet in the portfolio ($F = 100$), the computation would amount to approximately 10 hours, as is calculated in Equation 5.13. However it is noted that this estimation of computation time does neglect the impact of the increased number of OD pairs under consideration on the computation time of a single optimization run CT_o.

$$\begin{aligned} CT_{\text{TOTAL}} &= \\ &= Y \cdot Z \cdot 6 + F \cdot Y \cdot M \cdot 0.72 + B \cdot F \cdot 0.0275 \\ &= 9 \cdot 300 \cdot 6 + 100 \cdot 9 \cdot 10 \cdot 0.72 + 5000 \cdot 100 \cdot 0.0275 \\ &\approx \text{10 hours} \end{aligned} \tag{5.13}$$

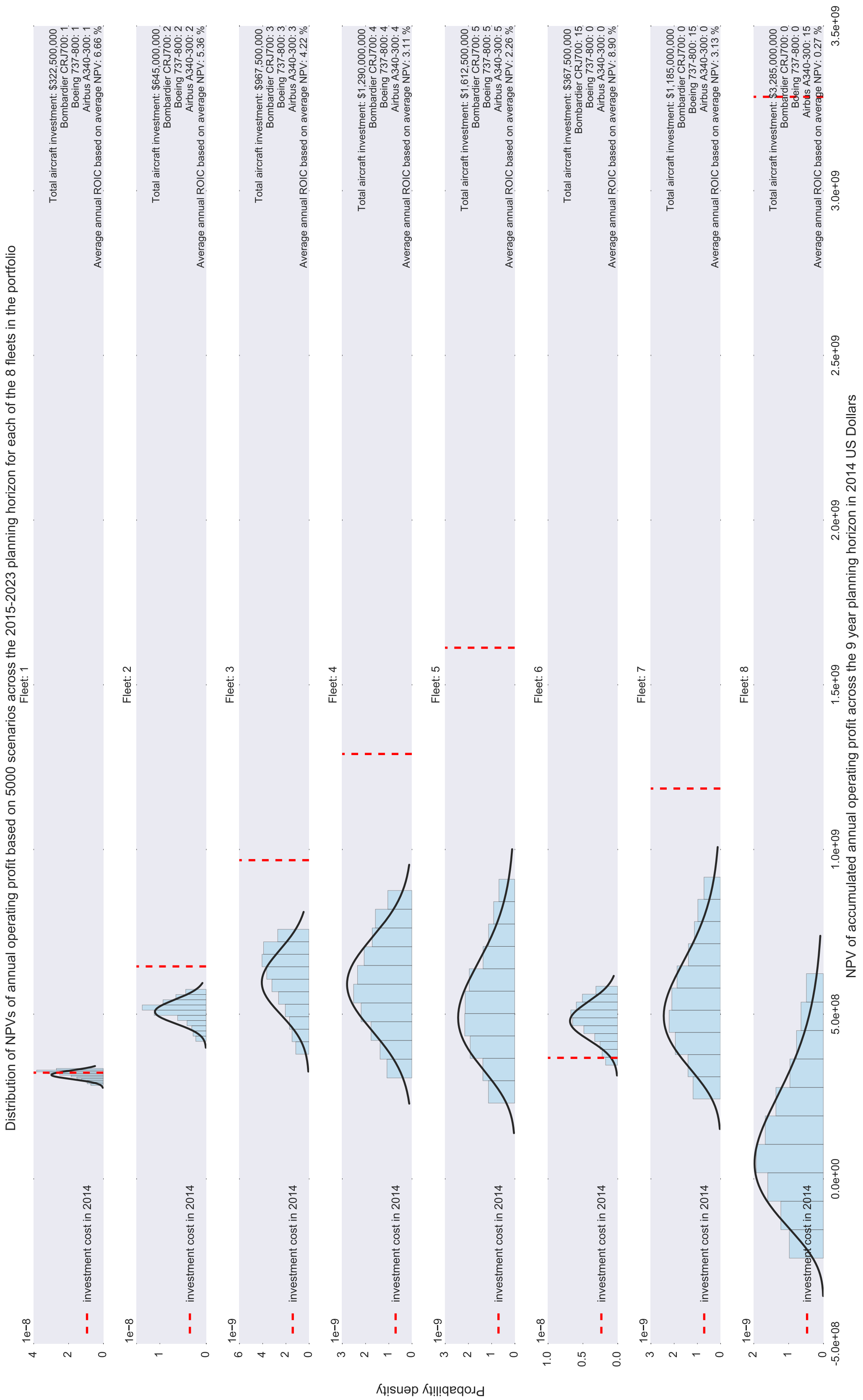


Figure 5.10: A distribution of net present values based on annual operating profits across the planning horizon across numereous realizations of stochastic demand, for each fleet in the portfolio

6

Verification and validation

This chapter details the verification and validation of the models and methodology. It is important to investigate whether the models comply with the specified requirements (i.e. verification) and to what extent the results can be considered to be a reflection of reality (i.e. validation). For each of the three models these verification and validation procedures are detailed in Sections 6.1, 6.2 and 6.3, respectively.

6.1 Stochastic demand forecasting model

6.1.1 Selecting the air travel demand variable

As is briefly highlighted in Section 4.2.2, multiple variables could be used as air travel demand variable: passenger data, passenger growth data, rpm data and rpm growth data. For all 4 variables the total US domestic historical data is used to test the mean reverting model properties and goodness of fit. The results are shown in Table 6.1. Historical passenger data is available for the period 1990-2014, resulting in 25 data points whereas historical rpm data is available for the period 1996-2014, resulting in 19 data points.

Clearly the passenger growth air travel demand variable displays the most desired behavior in terms of goodness of fit with a R^2 of 0.47 and a p-value of 0.000. Moreover, this demand variable shows a strong mean reverting property ($\lambda = 0.9$). Therefore it is decided to use passenger growth data as the mean reverting air travel demand variable in this research.

This decision can be qualitatively corroborated through two notions. On a global level the number of passengers per year steadily increased the past thirty years with an average growth rate of 5% (Belobaba et al., 2009), hinting towards the suitability of using of an air travel demand growth rate (either pax or rpm) as mean reverting air travel demand variable. Moreover it can be argued that passenger data is a better predictor than rpm data because rpm data is even more influenced by historic competition (i.e. which drives how OD passenger are transported over the network and thus the distance term in rpm) than passenger data, which is something that can be influenced in the future.

Table 6.1: Goodness of fit of the four different air travel demand variables (pax, rpm, pax growth, rpm growth)

	# data points	R^2	p-value	λ	μ	σ
pax	25	0.10	0.138	0.08	6.88E+08	2.18E+07
pax growth	25	0.47	0.000	0.90	0.023	0.049
rpm	19	0.12	0.158	0.13	5.97E+11	1.91E+10
rpm growth	19	0.37	0.010	0.73	0.019	0.039

6.1.2 Estimation of the mean reverting model parameters

There is a discrepancy between analytically calculated annual historical passenger growth rates and the μ parameters that stem from the mean reverting model parameter estimation through linear least squares regression. The differences are listed Figure 6.2. For future work it is considered interesting to investigate the impact of other methods to estimate the mean reverting parameters such as the maximum likelihood method. Moreover it can be expected that using more data points, if available, will yield more accurate mean reverting parameter estimates.

Table 6.2: Difference between estimated mean annual growth rate μ and the analytically calculated mean annual growth rate

OD pair	μ	analytical solution
ATL-FLL	0.053	0.038
ATL-MCO	0.037	0.021
DFW-LAX	0.024	0.020
JFK-LAX	0.029	0.025
JFK-SFO	0.036	0.032
LAS-LAX	0.027	0.028
LAX-ORD	0.011	0.010
LAX-SFO	0.007	0.010
LGA-ORD	0.023	0.022
ORD-SFO	0.016	0.017

6.1.3 Implications of the number of Monte Carlo simulations

As discussed in Section 4.2.3 a Monte Carlo simulation is a brute force method to numerically approximate the probability distribution. As such, the accuracy of the approximation depends on the number of simulation runs, which in turns also impacts the computation time. It is strived to set the number of simulation runs at a level for which the resulting sample values display acceptably low inconsistency when the simulation process is performed more than once. Table 6.3 presents for four different quantities of Monte Carlo simulation runs how the model returns sample values across three trials. Inconsistency is defined as the sum of the absolute difference between each sample value and the average sample value of all trials as percentage of the average sample value of all trials. The table shows that the adoption of 5000 runs results in a computation time of 541s and an inconsistency of 0.4% which is considered an acceptable balance. It is noted however that this investigation is performed for the demand sample value based on the average of all Monte Carlo simulation observations in bin 1 for OD pair ATL-FLL in 2015, and might lead to different results when performed for other sample values within the year, other years and other OD pairs. Still, the expectation is that in general terms a higher number of simulation runs will lead to lower inconsistency.

Table 6.3: Balance between the number of Monte Carlo simulation runs, inconsistency of results and computation time

# Runs [#]	Trial 1 [Pax]	Trial 2 [Pax]	Trial 3 [Pax]	Inconsistency [%]	Computation time [s]
500	1031187	1040640	1039374	1.1%	192
1000	1033736	1036129	1027395	1.0%	232
2500	1037334	1026506	1036193	1.3%	354
5000	1031932	1033559	1029341	0.4%	541

6.1.4 Validation using historical data

Order of magnitude and range of uncertainty of forecasted sample values

In order to validate the stochastic demand forecasting model only part of the historical data is used to estimate the model parameters so that the remainder of the historical data can be used to validate the model output. In this example the 1990-2011 years are used to estimate the model parameters and a Monte Carlo simulation is performed for the period 2012-2014. Appendix B.4 shows the validation of the model for all OD pairs. For each OD pair, it presents the three resulting equal probability histograms and the spread of Monte Carlo simulation observations. The red dotted lines represent the actual historical passenger data. Although it is noted that this analysis cannot be used to validate the accuracy of the model since the demand is stochastic, at least it can be observed that the order of magnitude of the outputted forecasts is in line with the actual historical data and the actual data falls within the range of forecasted values.

10 OD demand matrices

In Section 4.2.3 it is decided to adopt 10 OD demand matrices per year where each OD demand matrix contains all OD pairs with demand sample values based on the same bin number. Here it is investigated whether that assumption is a correct reflection of reality. For each of the histograms in Appendix B.4, the actual historical data falls in a particular bin in the forecasted data. The results of this observation for each OD pair and each year are presented in Table 6.4. The assumption of constructing 10 OD demand matrices per year with each OD demand matrix containing OD demand sample values based on the same bin number would be validated if for each year the historical demand data of each OD pair would fall in the same bin number, which is unfortunately not the case. In 2012, 2013 and 2014, respectively, 2, 4 and 4 out of 10 OD pairs displayed historical passenger data that fell in the same bin number. As such the assumption is not a fully correct reflection of reality and a recommendation for future work is to investigate whether this information can be used to improve the construction of a limited set of OD pairs.

6.2 Fleet assignment optimization model

6.2.1 Verification through analytical solution checks

The mathematical optimization model is implemented incrementally by adding constraints one at the time. During this construction phase the optimization model is run and verified by using the resulting decision variable values to analytically calculate both sides of each constraint to check whether the constraints are satisfied and not violated. During the incremental implementation process this verification step has been performed for each constraint and under different circumstances (e.g. yields, number of aircraft per type in the fleet, available daily utilization

Table 6.4: Historical bin numbers per year per OD pair

OD pairs	2012	2013	2014
ATL-FLL	5	2	2
ATL-MCO	4	3	2
DFW-LAX	9	8	8
JFK-LAX	6	5	5
JFK-SFO	7	5	6
LAS-LAX	2	3	4
LAX-ORD	8	6	6
LAX-SFO	7	5	6
LGA-ORD	6	5	5
ORD-SFO	8	6	6

time, etc.). The same checking procedure has also been performed for the verification of the objective function.

One example is provided here to support the understanding of this verification process. Equation 4.20a represented the passenger flow-capacity constraint between two airports that are not a hub. In the case study in Table 5.6, the x_{od} decision variable of passenger flow between ATL-MCO returned a value of 4875 and the z_{ij}^k decision variable of weekly flight frequency between ATL-MCO with aircraft type Bombardier CRJ700 returned a value of 65 and the number of seats of that aircraft type (s^k) is 75. The Bombardier was the only aircraft type under consideration in this example. Consequently the passenger flow-capacity constraint can be verified in the following fashion;

$$
\begin{aligned}
x_{i,j} &\leq \sum_{k \in K} z_{i,j}^k \cdot s^k \\
4875 &\leq 65 \cdot 75 \\
4875 &\leq 4875
\end{aligned}
\tag{6.1}
$$

6.2.2 Validating the observed model behavior

Next to ensuring that the math is implemented correctly, it is also important to investigate and understand the type of behavior the optimization model displays when getting to optimal solutions and check whether that behavior is a correct reflection of reality. This behavior is the result of the mathematical formulation of the decision variables, objective function and constraints as well as the sequence of solution steps adopted by the optimization algorithm.

The impact of objective function coefficients

The objective function aims to maximize profit, which is influenced by four terms; two revenue terms, one constant ownership cost term that does not contain any decision variables and an operating cost term. This set up drives the focus on profitability and as such there are two theoretical results of the optimization process. Either all potential profitable demand is satisfied and capacity has not yet reached its upper bound (i.e. spoilage due to over-capacity), or alternatively the upper bound on capacity is reached but some profitable demand is not satisfied (i.e. spill due to under-capacity). In the first case, the aircraft types with the highest total operating cost (i.e. with the most negative objective function coefficient) will not be deployed whereas in the second case the least revenue generating OD passenger demand (i.e. with the least positive objective function coefficient) will not be satisfied.

This behavior is arguably a partial reflection of reality. It seems sensible that airlines in a real world will try to get a demand share in the most profitable OD pairs. However, it is unlikely that such narrow rationale of prioritizing the allocation of capacity to the most profitable OD pairs is employed in the real world because airlines need to consider also other elements that may impact revenues, cost and profitability on a larger scale. For instance, the total demand an airline can potentially capture in a particular OD pair may possibly also depend on the total number of OD pairs served due to customer loyalty effects. This would shift the focus from allocating as much capacity as possible to a single highly profitable OD pair to distributing capacity across multiple OD pairs in order to cover a larger set of OD pairs. Another example could be that a specific aircraft type is preferably operated from and to a particular maintenance hub with maintenance capabilities for that aircraft type, which might not necessarily coincide to the set of OD pairs that can be served with the highest operational profit with that aircraft type.

Grounding aircraft due to case study specific values

Another behavior can be observed as consequence of the mathematical logic. An OD pair is only served by an aircraft type when the connection results in a positive operating profit. When none of the OD pairs under consideration can be profitably catered by a specific aircraft type, then that aircraft type will be not be deployed. Consequently, situations occur in the case study where aircraft are grounded while the demand in all OD pairs is not fully satisfied and capacity has not yet reached its upper bound. Although this makes sense mathematically, in the real world an airline is not likely to ground aircraft throughout the year because it cannot cater the profitable demand in the limited set of OD pairs that was initially under consideration; it will likely seek new OD pairs in which the aircraft types can be deployed profitably.

Average network load factors

The average network load factors are close to 100 percent across the case study while in reality load factors are typically around 80% (Pearce, 2013). The average network load factor (ANLF) is given by;

$$\text{ANLF} = \frac{\text{Total pax in network}}{\text{Total seats in network}} \tag{6.2}$$

An attempt has been made to incorporate an average network load factor constraint, but this resulted in undesired results. The load factor constraint simply added empty chairs by adding empty flights between airports with the shortest distance in order to satisfy the load factor constraint. Although this might seem a counterintuitive result, it can be shown that this behavior stems from the mathematical logic; from a total operating profit perspective it was less detrimental to add very short flights with empty seats (i.e. between LGA and JFK with a distance of just 10 miles) and thereby increase the magnitude of the denominator in equation 6.2 to enforce the load factor constraint, than to reduce some of the highly profitable flow in the numerator of equation 6.2. It is noted that the latter investigation is a prime example of the verification and validation process; a constraint was implemented, verified but led to an undesired result and therefore was disregarded.

Sensitivity to purchase price

Because depreciation is based on the purchase price, the aircraft purchase price impacts the annual ownership cost and through that impacts the annual operating profit. Moreover it also impacts the total aircraft investment. Consequently, the purchase price impacts the return on invested capital on both sides of the equation: the after tax operating profit and the total invested capital. Because of this impact and because in the real world aircraft are typically sold at a discount against the list price, the sensitivity of the outputted return on invested capital to the inputted purchase price is investigated in Table 6.5. In this calculation the ROIC is the average annual ROIC based on the average annual NPV across the planning horizon across the range of stochastic demand. It can clearly be observed that discounts on the list price have a beneficial impact on the average return on invested capital. In practice such a table can be used for a more thorough understanding of the trade space when negotiating the purchase price with manufacturers.

Table 6.5: Sensitivity of average annual ROIC to aircraft purchase price

	Average annual return on invested capital (ROIC)							
	Fleet 1	Fleet 2	Fleet 3	Fleet 4	Fleet 5	Fleet 6	Fleet 7	Fleet 8
Full list price	6.66%	5.36%	4.22%	3.11%	2.26%	8.90%	3.13%	0.27%
List price - 10%	7.62%	6.18%	4.92%	3.68%	2.74%	10.12%	3.71%	0.53%
List price - 25%	9.54%	7.81%	6.30%	4.81%	3.68%	12.54%	4.85%	1.03%

Sensitivity to yields

Competition can impact airline specific yields, which can in turn impact the airline's profitability. Therefore the sensitivity of average annual return on invested capital to a yield increase and decrease of 10% is evaluated in Table 6.6. It can be observed from this table that the return on invested capital is clearly sensitive to these different levels of yields, with higher yields resulting in a higher ROIC and lower yields resulting in a lower ROIC.

Table 6.6: Sensitivity of average annual ROIC to yields

	Average annual return on invested capital (ROIC)							
	Fleet 1	Fleet 2	Fleet 3	Fleet 4	Fleet 5	Fleet 6	Fleet 7	Fleet 8
Yields - 10%	5.33%	4.12%	3.10%	2.14%	1.41%	6.58%	2.13%	-0.27%
Yields	6.66%	5.36%	4.22%	3.11%	2.26%	8.90%	3.13%	0.27%
Yields + 10%	8.00%	6.64%	5.42%	4.23%	3.33%	11.25%	4.42%	1.05%

Sensitivity to market share

The absence of a market share model and resulting assumption of a fixed market share of 0.2 for each OD pair is a major assumption. Therefore it is considered interesting to investigate how the average return on invested capital relates to changes in the level of market share. The impact of a 0.15 and 0.25 market share (MS) assumption are presented in Table 6.7. It can be observed that a higher market share positively impacts the returns whereas a lower market share negatively impacts returns.

Typically a S-curve relation is assumed between the frequency share of an airline and its market share. Translated to the context of this research this has the following impact on the results presented in Figure 5.10; larger fleets are likely to be capable to offer a higher frequency and therefore capture a higher share of demand. Consequently, the entire distribution of NPVs is expected to shift to the left for small fleets, and to the right for larger fleets, assuming that the more demand that could be captured the higher the profits that could be obtained.

Table 6.7: Sensitivity of average annual ROIC to assumed market share

	Average annual return on invested capital (ROIC)							
	Fleet 1	Fleet 2	Fleet 3	Fleet 4	Fleet 5	Fleet 6	Fleet 7	Fleet 8
MS (0.15)	6.11%	4.63%	3.12%	2.05%	1.33%	7.17%	1.90%	-0.28%
MS (0.20)	6.66%	5.36%	4.22%	3.11%	2.26%	8.90%	3.13%	0.27%
MS (0.25)	6.98%	5.80%	4.96%	4.00%	3.13%	10.05%	4.31%	0.84%

Sensitivity of computation time to integrality constraints

Table 6.8 provides an overview of the computation times. It shows computation times for 1 optimization run CT_o and for all 720 optimization runs $CT_{\text{model 2}}$, as well as how the computation time increases when some or all decision variables are subject to integrality constraints. The LP formulation has no integrality constraints and generates solutions in the lowest computation time. When the flight frequency decision variable $z^k_{i,j}$ is subject to integrality constraints, the LP is transformed to a MILP and this increases computation times roughly by 1.53 times. When all decision variables are subject to integrality constraints the LP is transformed to an ILP and it takes on average generally 1.55 times more time to generate solution with respect to the LP formulation; the difference in computation time compared to the MILP formulation is marginal however. Again, it is noted that this a simple computation time analysis that in reality is affected by a multitude of computer science related factors such as; working memory processor speed, number of cores per processor, etc., as well as the efficiency of the solution techniques of the optimization algorithm.

Table 6.8: Computation times of different optimization model formulations

Formulation	1 iteration CT_O $[s]$	720 iterations $CT_{\text{model 2}}$ $[s]$	multiplier $[\times]$
LP	0.47	335	
MILP	0.71	512	1.53
ILP	0.72	520	1.55

6.3 Scenario generation model

6.3.1 Aggregating the transiton probabilities

Through visually comparing the OD pair based transition probability matrices in Appendix D.1 as well as the aggregated OD demand matrix based transition probability matrices in Appendix D.2 the following observations can be made:

- The same transition behavior patterns can be observed
  - Within each year-to-year transition probability matrix the highest density is across the diagonal and within the diagonal the highest density is in the outer bins
  - Moving across the planning horizon this observed behavior increases for the OD demand matrix based transition probability matrix which is congruous with the behavior of individual OD pairs
- Specific elements in transition probability matrices are of the same order of magnitude but are not exactly the same value

– Although on average the transition behavior tends to be similar, individual transition probabilities can be different. Because the aggregated OD demand matrix based transition probability matrix contains transition behavior that is based on the average transition behavior of all OD pairs, some OD pair based transition probabilities will show a stronger transition behavior and some will show a weaker behavior when compared to the aggregated OD demand matrix based transition probabilities. To illustrate the order of magnitude of these differences, Table 6.9 presents OD pair based and aggregated OD demand matrix based transition probabilities for all years for two example transitions: from bin 1 to bin 1 and from bin 5 to bin 5 in consecutive years. In this table, p refers to the transition probability and Δ refers to the percentage difference between the OD pair based transition probability and the OD demand matrix based transition probability. It can be observed that indeed the Δ terms differ per OD pair with for example OD pair ATL-FLL showing small dissimilarities and OD pair LAX-SFO showing larger dissimilarities.

Table 6.9: Validation of OD pair based transition probabilities to aggregated OD demand matrix based transition probabilities

		OD pair based																				OD demand matrix based
		ATL-FLL		ATL-MCO		DFW-LAX		JFK-LAX		JFK-SFO		LAS-LAX		LAX-ORD		LAX-SFO		LGA-ORD		ORD-SFO		Aggregated
year-to-year	transition	p	Δ	p	Δ	p	Δ	p	Δ	p	Δ	p	Δ	p	Δ	p	Δ	p	Δ	p	Δ	p
2015-2016	1->1	0.49	4%	0.50	3%	0.47	9%	0.48	7%	0.49	4%	0.60	17%	0.51	1%	0.57	12%	0.50	3%	0.52	2%	0.51
2016-2017	1->1	0.60	2%	0.61	0%	0.57	8%	0.56	9%	0.61	0%	0.67	8%	0.61	1%	0.71	15%	0.57	7%	0.63	2%	0.62
2017-2018	1->1	0.66	3%	0.66	3%	0.63	8%	0.64	6%	0.68	1%	0.72	6%	0.67	2%	0.75	10%	0.61	10%	0.66	4%	0.68
2018-2019	1->1	0.70	2%	0.69	4%	0.66	8%	0.68	4%	0.71	0%	0.75	4%	0.69	3%	0.78	9%	0.66	8%	0.73	1%	0.72
2019-2020	1->1	0.72	3%	0.71	4%	0.71	4%	0.71	5%	0.76	2%	0.76	3%	0.75	2%	0.83	12%	0.70	6%	0.77	3%	0.74
2020-2021	1->1	0.74	3%	0.74	3%	0.73	5%	0.74	3%	0.76	0%	0.81	6%	0.77	1%	0.84	10%	0.75	2%	0.77	1%	0.76
2021-2022	1->1	0.78	1%	0.77	1%	0.76	1%	0.76	3%	0.80	3%	0.84	8%	0.77	0%	0.83	7%	0.74	4%	0.80	4%	0.78
2022-2023	1->1	0.76	3%	0.79	1%	0.78	0%	0.76	3%	0.79	1%	0.84	7%	0.78	1%	0.86	10%	0.76	4%	0.80	2%	0.79
2015-2016	5->5	0.14	3%	0.16	13%	0.14	0%	0.17	15%	0.14	4%	0.18	25%	0.16	11%	0.19	32%	0.13	10%	0.17	18%	0.14
2016-2017	5->5	0.20	8%	0.18	5%	0.15	18%	0.19	1%	0.19	1%	0.22	17%	0.18	5%	0.19	4%	0.17	11%	0.20	7%	0.19
2017-2018	5->5	0.22	1%	0.21	4%	0.19	14%	0.19	12%	0.22	0%	0.24	12%	0.19	14%	0.30	37%	0.21	4%	0.26	18%	0.22
2018-2019	5->5	0.23	8%	0.24	5%	0.24	5%	0.21	14%	0.19	23%	0.28	12%	0.24	2%	0.32	30%	0.23	7%	0.23	8%	0.25
2019-2020	5->5	0.27	0%	0.22	19%	0.22	17%	0.26	4%	0.27	1%	0.28	5%	0.25	8%	0.34	28%	0.26	4%	0.28	6%	0.27
2020-2021	5->5	0.30	1%	0.27	9%	0.24	18%	0.26	12%	0.28	5%	0.35	18%	0.29	1%	0.39	31%	0.26	13%	0.31	4%	0.29
2021-2022	5->5	0.31	2%	0.30	6%	0.31	4%	0.26	17%	0.28	13%	0.34	8%	0.29	9%	0.42	32%	0.27	14%	0.30	7%	0.32
2022-2023	5->5	0.32	3%	0.28	16%	0.24	27%	0.29	11%	0.27	17%	0.37	13%	0.28	15%	0.43	30%	0.28	15%	0.35	7%	0.33

6.3.2 Mutually exclusive collectively exhaustive scenario generation

In this research it is decided to run a limited set of scenarios ($B = 5.000$), and to generate the paths based on the DTMC transition probability matrices. Alternatively it could be decided to not run a limited set of scenarios based on the DTMC transition probability matrices, but to enforce the evaluation of all paths. Table 6.10 displays the theoretical number of mutually exclusive collectively exhaustive (MECE) scenarios that could unfold, irrespective of their probability of occurrence, as function of the number of years in the planning horizon and the number of bins per histogram. When considering a 9-year planning horizon and 10 bins per histogram per year the total number of MECE scenarios amounts to 10^9. The benefit of this approach is that it allows for explicitly evaluating all scenarios (i.e. including the most unlikely scenarios), irrespective of their probability of occurrence. The downside is that the information regarding the year-to-year transition probabilities is disregarded.

In the case study it was identified that the runtime of 5.000 scenarios amounted roughly to 20 minutes. Running 1 billion scenarios would therefore take roughly 7 years, which is unacceptable in terms of computation time. Moreover it can be argued that in terms of meaningfulness this approach would not provide a significant contribution because most of the scenarios in the set of MECE scenarios consist of paths that have a near zero probability of occurrence. Still, if a MECE scenario generation approach would be desired while ensuring reasonable computation times, Table 6.10 can be used re-evaluate the decision regarding the number of bins per histogram and the number of years in the planning horizon knowing that the evaluation 5.000 scenarios yields a computation time of approximately 20 minutes.

Table 6.10: Number of alternative mutually exclusive and collectively exhaustive scenarios as function of the number of years in the planning horizon and the number of bins per histogram

		Number of bins per histogram									
		1	2	3	4	5	6	7	8	9	10
Number of years	1	1	2	3	4	5	6	7	8	9	10
	2	1	4	9	16	25	36	49	64	81	100
	3	1	8	27	64	125	216	343	512	729	1000
	4	1	16	81	256	625	1296	2401	4096	6561	10000
	5	1	32	243	1024	3125	7776	16807	32768	59049	1E+05
	6	1	64	729	4096	15625	46656	1E+05	3E+05	5E+05	1E+06
	7	1	128	2187	16384	78125	3E+05	8E+05	2E+06	5E+06	1E+07
	8	1	256	6561	65536	4E+05	2E+06	6E+06	2E+07	4E+07	1E+08
	9	1	512	19683	3E+05	2E+06	1E+07	4E+07	1E+08	4E+08	1E+09

6.3.3 Verification of random selection process

The random selection process of the bin number in a subsequent year given the bin number in the current year is based on the transition probability matrix for these two years. This random selection process is verified in Table 6.11 for two example cases; transitioning from bin 5 in 2015 to any of the bins in 2016 and transitioning from bin 9 in 2022 to any of the bins in 2023, using the corresponding rows in the corresponding transition probability matrices that can be found in Appendix D.2. It can be observed that the frequency of outputted bin numbers by this selection process corresponds with the inputted transition probability of arriving in each bin, which serves as verification of this random selection procedure.

Table 6.11: Verification of the random selection process based on 5000 random draws

2015-2016 bin 5	1	2	3	4	5	6	7	8	9	10
Inputted transition probability	0.04	0.09	0.12	0.14	0.14	0.15	0.13	0.10	0.06	0.02
Random selection occurrence	197	472	608	689	729	772	651	462	318	102
Random selection probability	0.04	0.09	0.12	0.14	0.15	0.15	0.13	0.09	0.06	0.02

2022-2023 bin 9	1	2	3	4	5	6	7	8	9	10
Inputted transition probability	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.23	0.48	0.20
Random selection occurrence	0	0	0	3	19	79	337	1185	2361	1016
Random selection probability	0.00	0.00	0.00	0.00	0.00	0.02	0.07	0.24	0.47	0.20

7
Conclusions and recommendations

A concise overview of conclusions is presented in Section 7.1. The limitations and recommendations for future work are covered in detail in Section 7.2.

7.1 Conclusions

In the introduction the need is identified for airline fleets that are robust in terms of profit generating capability across a long-term planning horizon under stochastic demand. The airline fleet planning problem is an optimization problem under uncertainty, which poses a challenge in terms of generating meaningful results in reasonable computation times. Consequently, the research objective of this research is to develop an innovative fleet planning concept that is capable to generate meaningful results in reasonable computation times.

Meaningfulness is defined as the ability to compare different fleets, on both financial and non-financial performance metrics, across a multi-year planning period across numerous realizations of stochastic demand. A computation time of less than two hours for the case study is considered reasonable and moreover it is considered valuable if the impact of increasing the problem size on the computation time is made explicit.

Meaningful results

The proposed methodology is a three-step modeling framework that harvests insight into the operating profit generating capability of different fleets in terms of size and composition under a multi-year planning horizon under stochastic demand. Two types of results enable the explicit comparison of different fleets in the portfolio.

First, the distribution of net present values of operating profit (NPVs) across the planning horizon across the range of stochastic demand can be used to compare different fleets from the portfolio on three attributes: the absolute magnitude of NPVs, the range of uncertainty of NPVs and the relation between operating profits and return on invested capital.

Second, the vast amount of financial and non-financial performance metrics enables a more detailed analysis and investigation into the underlying drivers of the distribution of profitability by using information such as; the aircraft utilization per aircraft type, average network load factors, the number of origin-destination pairs served, the weekly operating and ownership cost, the spilled revenue, etc.

It can be concluded that the proposed airline fleet planning modeling framework has the potential to identify robust fleet plans through the detailed consideration of stochastic demand per origin-destination pair across a long term planning horizon, and being able to compare both financial and non-financial performance metrics of different fleets across a multi-year planning horizon across multiple realizations of stochastic demand.

Reasonable computation times

The computation time of a small case study, with 8 fleets in the portfolio and 10 origin-destination pairs under consideration and a 9-year planning horizon, amounts to 36 minutes. Explicit information is available with regards to how the computation time scales with input variables such as the number of fleets in the portfolio, the number of origin-destination pairs under consideration, the number of years in the planning horizon, and the number of origin-destination demand matrices per year.

Moreover, it is made explicit how the size of the linear programming matrix (i.e. the number of decision variables and constraints) scales with increasing problem size, through the number of aircraft types, the number of airports and the number of hubs under consideration. Given this information it is estimated that a real world case study, with 100 fleets in the portfolio and 300 origin-destination pairs under consideration and a 9 year planning horizon, would amount to approximately 10 hours of computation time. For a planning decision that covers multiple years it can be argued that it is reasonable if it takes less than one day to generate information that can be used as input to decision making.

It can be concluded that the case study results are generated within 2 hours. Moreover, the relation between increasing the problem size and increasing computation times is made explicit. Furthermore it is noted that the methodology is generic and can be applied to any airline, irrespective of the business model, size, routing network and preference with regards to aircraft types or risk profile.

Contribution to the body of knowledge

- Long-term (multi-year) consideration of stochastic demand per origin-destination pair by modeling it as a mean reverting Ornstein-Uhlenbeck process and using discrete-time Markov chain transition probability matrices to generate scenarios

- Portfolio perspective which allows for explicit comparison of different fleets in terms of size and composition on both financial and non-financial performance metrics. Robust fleets can be selected based on their operating profit generating capability across the long-term planning horizon across the range of stochastic demand

Industry impact

- Detailed and long-term consideration of stochastic demand. The more information is available regarding future demand levels, the better an airline is capable to match supply with demand

- Network analysis of the profitability of the entire fleet as opposed to the traditionally route-based analysis for the acquisition of individual aircraft

- The consideration of return on invested capital provides a strong insight in the relationship between operating profits and returns. Being capable to thoroughly understand the impact of stochastic demand in the long run on the return on invested capital on an airline level might prove to be a key step forward in the improvement of the airlines' financial performance (i.e. generating a return on invested capital at or above weighted average cost of capital). Moreover, this improved information might strengthen the understanding of the trade space that airlines have when negotiating aircraft purchase discounts with aircraft manufacturers

- The modeling framework might be of use to four different groups. First, existing airlines seeking to renew or expand their fleet. Second, start-up airlines that have a clean sheet in terms of deciding which aircraft to have in the fleet but also pose an additional risk to airline investors. Third, aircraft manufacturers could also benefit from this modeling framework by displaying their understanding of the airline's problem in terms of the large capital investment, profitability and returns and try to think with them across the long-term planning horizon on an airline level. Fourth, aircraft leasing companies could use the modeling framework for an analysis of demand development on both a global and regional level in order to forecast the future demand of airlines for specific aircraft types

7.2 Limitations and recommendations for future work

This section provides an extensive discussion of the limitations of the proposed modeling framework. Some limitations are expected to be overcome with minor work; these suggestions are formulated as recommendations. Other limitations require a profound shift of perspective and are formulated as research opportunities for future work.

Limitations

- Implications of the adoption of a portfolio
  - The adoption of a portfolio of fleets is a two-edged sword. On the one hand it provides valuable information by enabling the identification of robust fleets in a portfolio of fleets by being able to explicitly compare the fleets. On the other hand, it does not guarantee that any of the fleets in the portfolio is the optimal fleet. In theory, the collection of fleets in terms of size and composition is of astronomical size. In terms of computation time it is not feasible to evaluate all these fleets. By nature, selecting a small subset of that collection in the portfolio eliminates the possibility to guarantee optimality which changes the perspective to a decision analysis approach. The implication is that the formulation of the portfolio can significantly impact the results, which gives rise to question on how to construct the portfolio.
  - Recommendation: the construction of fleets should be grounded in reality. Input to the construction of the portfolio can be provided by; experienced fleet planners, consideration of fleet commonality and historically financially successful fleets of competitors.
- Number of OD demand matrices per year
  - As part of the methodology it is decided to fill each OD demand matrix with OD pair based demand sample values that are based on averages from the same bin number. This decision allowed for the construction of 10 OD demand matrices per year ($M = S = 10$) as opposed to 10^{10} ($M = S^Z = 10^{10}$), which greatly reduced the computation times from impracticable large (i.e. years) to reasonable (i.e. minutes). However, during the validation it is observed that this construction of 10 OD demand matrices per year is not a fully correct reflection of reality, with between 2-4 out of 10 OD pairs showing actual historical demand values in the same bin number.

  - Research opportunity: there is a need to investigate new ways to construct OD demand matrices per year in a manner that is a correct reflection of reality, while ensuring that the number of OD demand matrices per year remains small considering the impact on the total computation time. Improvements could be made by extending the investigation in Chapter 6.1.4 from 3 years to more years in order to check whether a trend can be identified with respect to the correlation of bin numbers between OD pairs.

    However it is important to keep in mind that the scenario generation model is based on the assumption that OD demand matrix based transition probability matrices reflect the same transition behavior as all the individual OD pairs in the OD demand matrix. When OD demand sample values in an OD demand matrix are taken from different bin numbers this assumption no longer holds and the development of OD demand matrix based transition probability matrices can become troublesome.

- Consideration of competition

  - The model does not consider competitive elements such as service, fares and the relative frequency offered per OD pair. Instead, a 20% market share is assumed for each OD pair when reducing total market demand to airline specific demand. In industry and academia, the impact of competition is typically modeled through the adoption of a S-curve relationship between frequency share of an airline and its market share, or through the adoption of a quality-service index (QSI) that encompasses more differentiating attributes such as service and fares. As result, the main assumption is that a higher frequency offered corresponds to a higher market share. The implication of the absence of a market share model on the results of this research is that the distribution of NPVs is likely to shift to the right for larger fleets considering they can offer a higher flight frequency and vice versa for smaller fleets.
  - Research opportunity: implement a S-curved market share frequency share model or a QSI based market share model in order to capture the competitive elements.

- Consideration of hub-and-spoke economics

  - While the methodology does allow for both point-to-point and hub-and-spoke routing networks, it cannot act as a tool to decide which routing network is more beneficial. This is because the hub-and-spoke economics in the model are not representative of reality. In short this can be attributed to three considerations: a focus on profit instead of revenue, the absence of economies of scale advantages due to cost reduction at the hub and the adoption of a yield ratio. Only yields are a differentiating factor in the model between nonstop and connecting passenger flows. As such, either nonstop or connecting passenger flows can be prioritized through a yield ratio. Consequently, if a hub-and-spoke network is desired, the inputted yield ratio should be such that connecting flow is higher yielding than nonstop flow.

    In reality however, the difference between yields for nonstop and connecting flow is more complex. Typically, a hub-and-spoke network aims to cater the demand in a large number of OD pairs by offering connecting flights through a hub between OD connections that can potentially not be operated profitably with a nonstop connection because of the low demand. However, typically passengers are willing to pay less for a connecting flight than for a nonstop flight, resulting in lower yields for connecting passenger flow. Moreover, it can be argued that the transportation of a passenger through a connection instead of nonstop is more costly than transporting them nonstop because of the additional fuel burnt due to longer stage lengths and additional takeoffs and landings. As result it could be argued that connecting passenger flow tends to generate less revenue (i.e. through lower yields) and more cost (i.e. higher fuel cost) per OD passenger when compared to nonstop flow. On an airline level however, the hub-and-spoke network could generate more revenue because more OD pairs can be catered. In short, a hub-and-spoke network is likely to generate a higher absolute revenue at a lower profit margin. Therefore, it can be concluded that the adoption of a hub-and-spoke network is not necessarily primarily focused on profit but on revenue. A layer of complexity is added when competition is taken into account per OD pair. The impact of this observation is that only a simple yield ratio is not sufficiently representative of reality to let the model decide on whether to adopt a hub-and-spoke or a point-to-point routing network.

  – Research opportunity: implement the impact of economies of scale at the hub and research whether an improved representation of reality can be achieved in the implementation of hub-and-spoke economics, by taking into account revenues, profits and the consideration of competition on the level of OD pairs.

- Absence of dynamic aspect of fleet planning
  – Each fleet in the portfolio has a fixed size and composition across the planning horizon. Consequently, the consideration of aircraft replacement and fleet expansion is neglected. Assuming a fixed fleet over a long-term planning horizon is not realistic and inherently causes decreasing market shares as passenger levels grow consistently.
  – Research opportunity: include the dynamic aspect of fleet planning by changing the number of aircraft per fleet AC^k from a parameter to a decision variable and adding a budget constraint for annual aircraft purchases. Also, a fleet evolution constraint is to be implemented that ensures that the aircraft that are in the fleet in the previous year and are not sold, still remain in the fleet. This approach does however eliminate the possibility to explicitly compare different fixed fleets across the planning horizon. Rather, the perspective is shifted towards comparing different fleet evolutions under the evolution of stochastic demand. Moreover, this new perspective could support the construction of the initial portfolio of fixed fleets. In a more detailed analysis, the consideration of timing of orders and deliveries (i.e. purchase slots and lead times) would contribute to a better reflection of reality since usually the purchase of an aircraft does not lead to ad hoc availability to deployment of that aircraft.

Recommendations

- Expand the consideration of demand
  – On the demand side three recommendations can be made. First, using weekly demand forecasts instead of annual demand forecasts would allow for the consideration of seasonality and trend growth throughout the year. This does multiply the number of time steps by a factor of 12 however, which could greatly influence computation times. Second, it is recommended to research whether business and first class demand can also be appropriately modeled as a mean reverting process. Third, the consideration of city-to-city demand instead of airport-to-airport demand allows for a better reflection of reality.
- Break down the simplistic cost terms
  – The operating cost is assumed to capture all costs from maintenance cost, to fuel cost and crew cost, whereas the ownership cost is solely based on depreciation.
  – Research opportunity: implement a more detailed breakdown of ownership and operating cost by explicitly considering maintenance cost, crew cost (per aircraft type),

fuel cost, insurance, etc. Moreover, when ordering aircraft in the real world, there is typically a choice in engines from different engine manufacturers. These different engines could yield different payload-range combinations, fuel efficiency, noise patterns, operating cost and investment cost. Consequently, it seems promising to incorporate the consideration of engines in the investment decision.

- Consider fuel price volatility
  - Consider the uncertainty associated with future fuel price volatility and its impact on the fleet planning decision.
- Include lease/buy mix consideration
  - It seems promising to research whether the uncertainty of NPVs outputted by the modeling framework can be used to investigate the value of flexibility. This information could possibly be used to make informed decisions about the value of flexibility that can be bought at a leasing company compared to the risk of purchasing the aircraft, given the uncertainty in NPVs.
- Reducing computation times
  - Adopt a computer science perspective in an attempt to reduce computation times. Examples of potential improvements include the consideration of parallel computing and cloud computing. Other measures to reduce computation times can be broken down in two categories. The number of inputs can be reduced (e.g. the number of fleets in the portfolio, number of OD pairs under consideration) or the model can be simplified (e.g. reduce the number of sample values per year per OD pairs from 10 to 3; drop the integrality constraints in the mathematical formulation of the optimization model).
- Implement realistic average network load factor constraint
  - On the optimization side, it is recommended to implement an average network load factor constraint in order to achieve average network load factors that are a better reflection of reality.
- Investigate the potential of adopting the Markowitz portfolio theory
  - It is considered interesting to research whether Markowitz portfolio theory could be implemented as extension to the proposed methodology. Given the distribution of NPVs of operating profits, a distribution of returns on invested capital could be generated. With the different means and standard deviations of ROIC of each fleet, each fleet can be seen as an investment opportunity. Using Markowitz portfolio theory, the means and standard deviations of expected return on invested capital of these different fleets in the portfolio can be plotted and a Pareto front (i.e. or efficient frontier) can be drawn which allows for the identification of the optimal fleet given the risk profile the airline is seeking to pursue.

A

Input data

A.1 Portfolio of fleets

Table A.1: Portfolio of fleets

Fleet	Bombardier CRJ700	Boeing 737-800	Airbus A340-300	Total
Fleet 1	1	1	1	3
Fleet 2	2	2	2	6
Fleet 3	3	3	3	9
Fleet 4	4	4	4	12
Fleet 5	5	5	5	15
Fleet 6	15	0	0	15
Fleet 7	0	15	0	15
Fleet 8	0	0	15	15

A.2 OD pairs

Table A.2: OD pairs

OD pairs	Airport specification
ATL-FLL	Atlanta Hartsfield-Jackson - Fort Lauderdale
ATL-MCO	Atlanta Hartsfield-Jackson - Orlando
DFW-LAX	Dallas/Ft Worth - Los Angeles
JFK-LAX	New York John F Kennedy - Los Angeles
JFK-SFO	New York John F Kennedy - San Francisco
LAS-LAX	Las Vegas McCarran - Los Angeles
LAX-ORD	Los Angeles - Chicago O'Hare
LAX-SFO	Los Angeles - San Francisco
LGA-ORD	New York LaGuardia - Chicago O'Hare
ORD-SFO	Chicago O'Hare - San Francisco

A.3 Aircraft data

Table A.3: Aircraft characteristics per aircraft type: number of seats (s^k), cruise speed (vc^k), range ($range^k$), daily utilization (U^k), turnaround time (TAT^k), ownership cost (C^k_{fix}), variable cost (C^k_{var}) and purchase price (PP^k)

Attributes	s^k	vc^k	$range^k$	U^k	TAT^k	C^k_{fix}	C^k_{var}	PP^k
Units	#	miles/hour	miles	hours	hours	USD	USD	USD
Bombardier CRJ700	75	514	1401	11	0.75	1.23E+06	0.09	2.45E+07
Boeing 737-800	162	543	3582	12	1.00	3.95E+06	0.10	7.90E+07
Airbus A340-300	295	555	8510	14	1.50	1.10E+07	0.13	2.19E+08

A.4 Airport data

Table A.4: Taxi out times in minutes per OD pair (T_i^{dep}), for the 10 OD pairs under consideration

	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0.00	0.00	16.96	0.00	0.00	0.00	0.00	16.37	0.00	0.00
DFW	0.00	0.00	0.00	0.00	0.00	15.84	0.00	0.00	0.00	0.00
FLL	16.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JFK	0.00	0.00	0.00	0.00	0.00	25.58	0.00	0.00	0.00	27.50
LAS	0.00	0.00	0.00	0.00	0.00	17.18	0.00	0.00	0.00	0.00
LAX	0.00	17.08	0.00	17.70	15.50	0.00	0.00	0.00	17.00	16.56
LGA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	25.89	0.00
MCO	17.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ORD	0.00	0.00	0.00	0.00	0.00	17.33	19.26	0.00	0.00	19.13
SFO	0.00	0.00	0.00	19.60	0.00	17.74	0.00	0.00	18.79	0.00

Table A.5: Taxi in times in minutes per OD pair (T_j^{arr}), for the 10 OD pairs under consideration

	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0.00	0.00	4.52	0.00	0.00	0.00	0.00	8.05	0.00	0.00
DFW	0.00	0.00	0.00	0.00	0.00	10.67	0.00	0.00	0.00	0.00
FLL	10.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JFK	0.00	0.00	0.00	0.00	0.00	11.42	0.00	0.00	0.00	7.32
LAS	0.00	0.00	0.00	0.00	0.00	9.98	0.00	0.00	0.00	0.00
LAX	0.00	9.80	0.00	10.29	7.56	0.00	0.00	0.00	11.57	5.75
LGA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	12.43	0.00
MCO	10.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ORD	0.00	0.00	0.00	0.00	0.00	10.99	5.76	0.00	0.00	6.98
SFO	0.00	0.00	0.00	10.08	0.00	11.19	0.00	0.00	12.23	0.00

A.5 Distance data

Table A.6: Origin-destination distance matrix in miles ($D_{i,j}$)

	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	732	581	760	1747	1946	761	404	606	2139
DFW	732	0	1119	1391	1055	1235	1389	985	802	1464
FLL	581	1119	0	1069	2174	2343	1075	177	1182	2584
JFK	760	1391	1069	0	2248	2475	10	944	740	2586
LAS	1747	1055	2174	2248	0	236	2242	2039	1514	414
LAX	1946	1235	2343	2475	236	0	2469	2218	1744	337
LGA	761	1389	1075	10	2242	2469	0	950	733	2580
MCO	404	985	177	944	2039	2218	950	0	1005	2446
ORD	606	802	1182	740	1514	1744	733	1005	0	1846
SFO	2139	1464	2584	2586	414	337	2580	2446	1846	0

A.6 Yield data

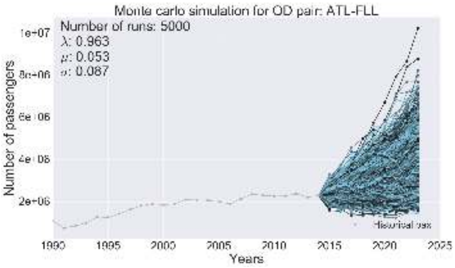
Table A.7: Origin-destination yield matrix with yields for nonstop passengers in 2014 USD cents ($yield_{o,d}$)

	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0.00	0.15	0.13	0.16	0.08	0.08	0.15	0.24	0.20	0.08
DFW	0.15	0.00	0.09	0.11	0.09	0.09	0.11	0.11	0.14	0.09
FLL	0.13	0.09	0.00	0.09	0.05	0.05	0.08	0.19	0.09	0.05
JFK	0.16	0.11	0.09	0.00	0.07	0.08	4.00	0.10	0.13	0.08
LAS	0.08	0.09	0.05	0.07	0.00	0.28	0.06	0.07	0.08	0.18
LAX	0.08	0.09	0.05	0.08	0.28	0.00	0.07	0.07	0.08	0.20
LGA	0.15	0.11	0.08	4.00	0.06	0.07	0.00	0.10	0.15	0.07
MCO	0.24	0.11	0.19	0.10	0.07	0.07	0.10	0.00	0.10	0.06
ORD	0.20	0.14	0.09	0.13	0.08	0.08	0.15	0.10	0.00	0.08
SFO	0.08	0.09	0.05	0.08	0.18	0.20	0.07	0.06	0.08	0.00

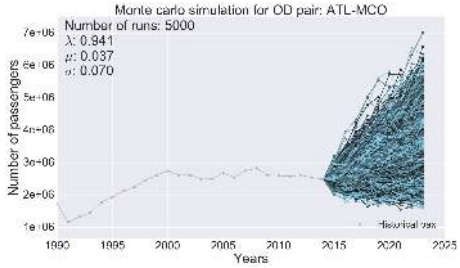
B

Stochastic demand forecasting model results

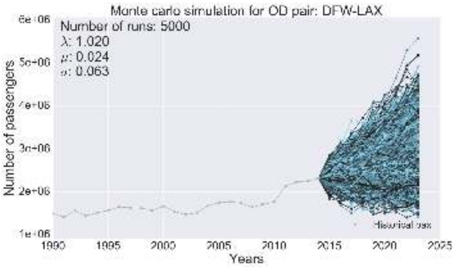
B.1 Monte Carlo simulations



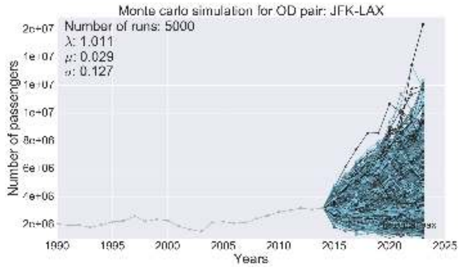
(a) ATL-FLL



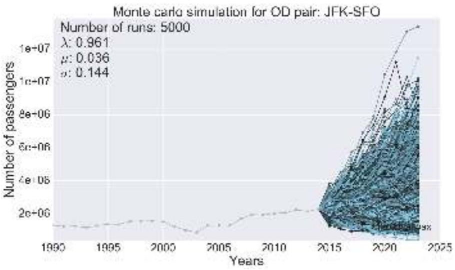
(b) ATL-MCO



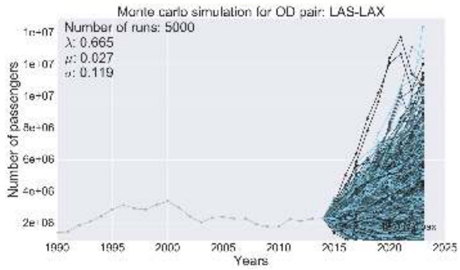
(c) DFW-LAX



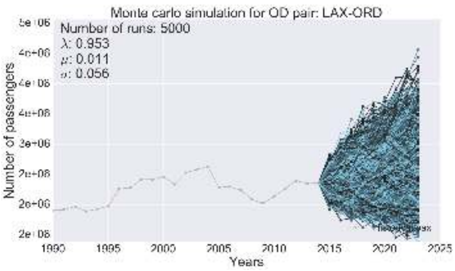
(d) JFK-LAX



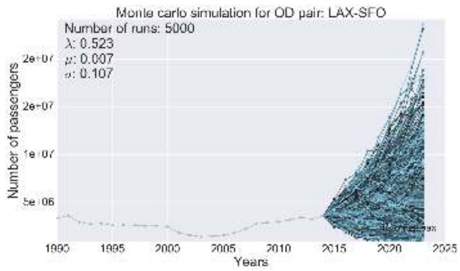
(e) JFK-SFO



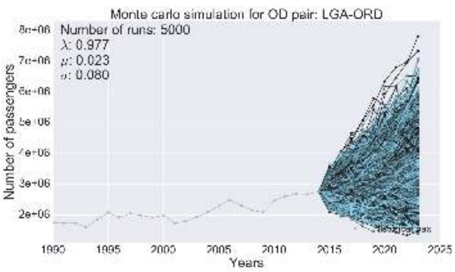
(f) LAS-LAX



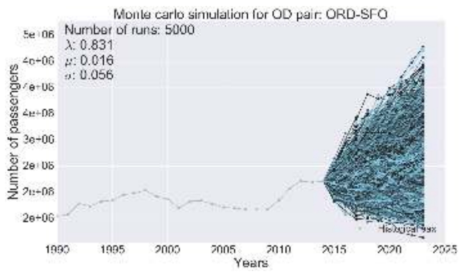
(g) LAX-ORD



(h) LAX-SFO



(i) LGA-ORD



(j) ORD-SFO

Figure B.1: Monte Carlo simulations for all 10 OD pairs

B.2 Equal probability histograms

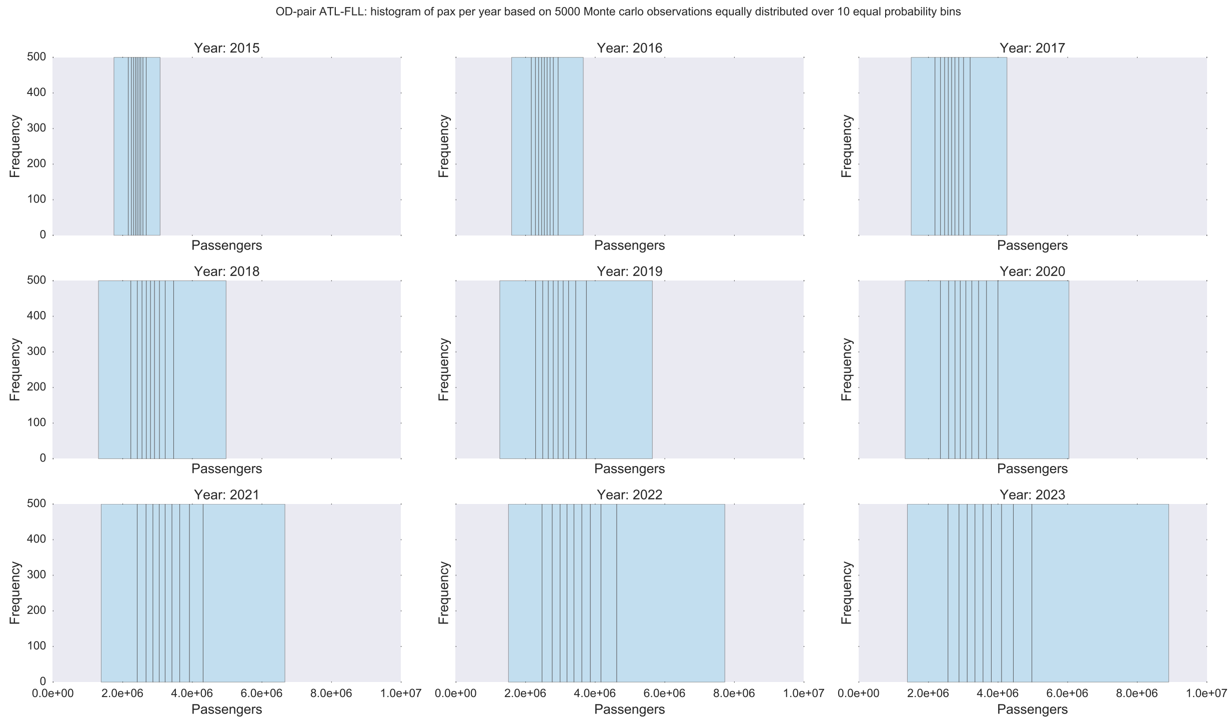


Figure B.2: Histograms of Monte Carlo simulation observations - OD pair: ATL-FLL

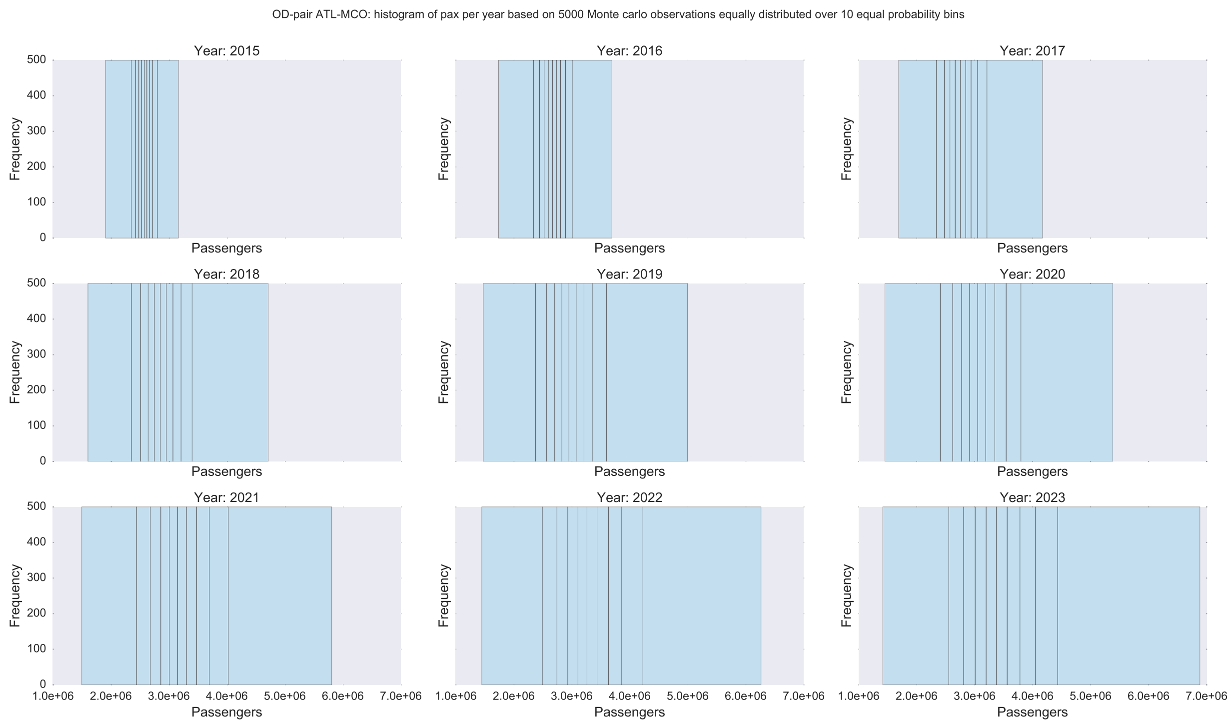


Figure B.3: Histograms of Monte Carlo simulation observations - OD pair: ATL-MCO

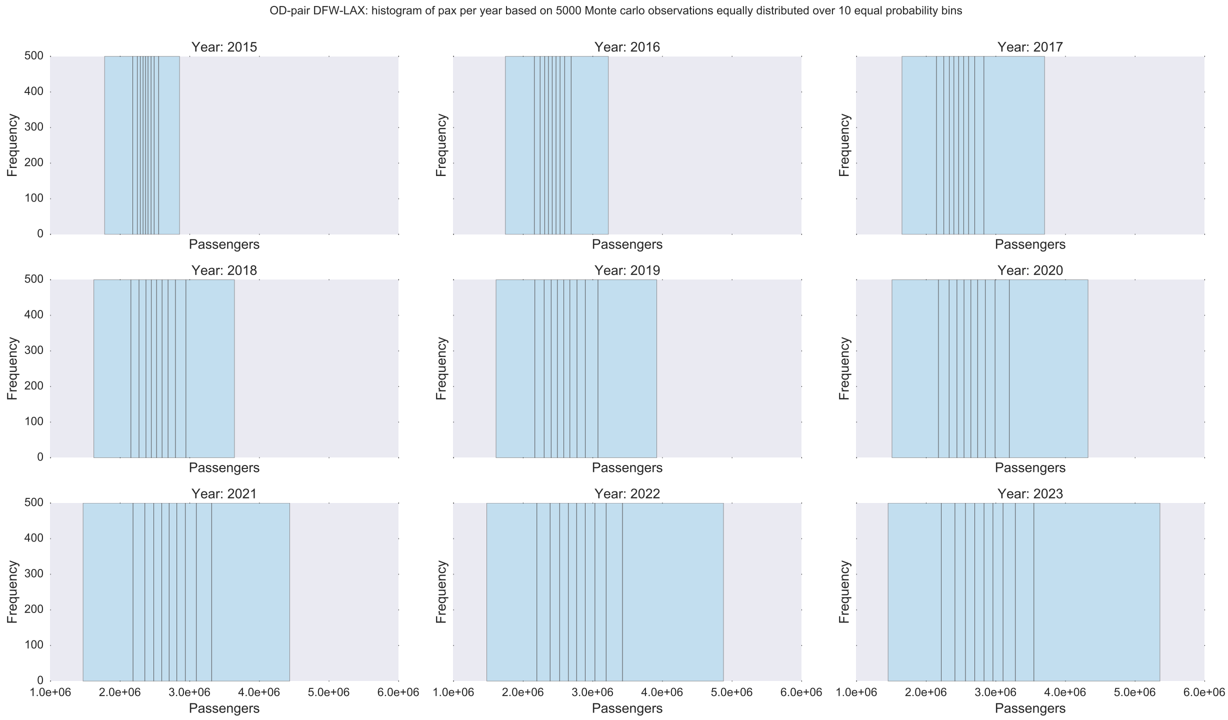


Figure B.4: Histograms of Monte Carlo simulation observations - OD pair: DFW-LAX

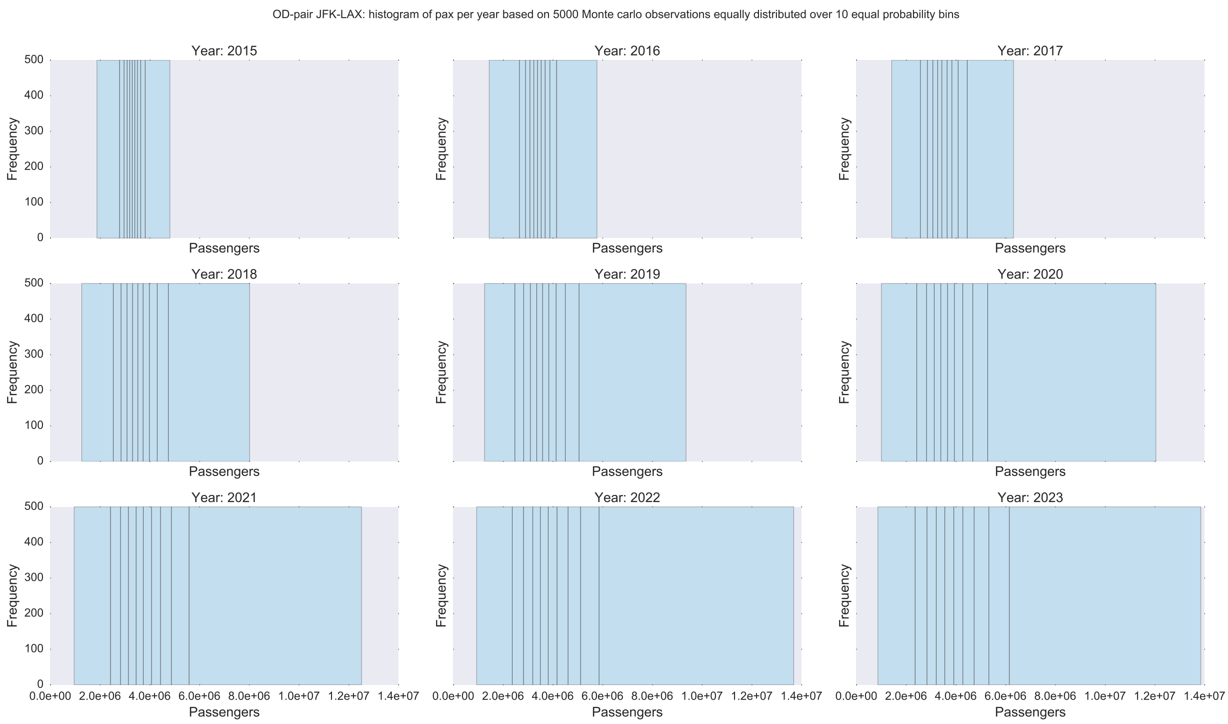


Figure B.5: Histograms of Monte Carlo simulation observations - OD pair: JFK-LAX

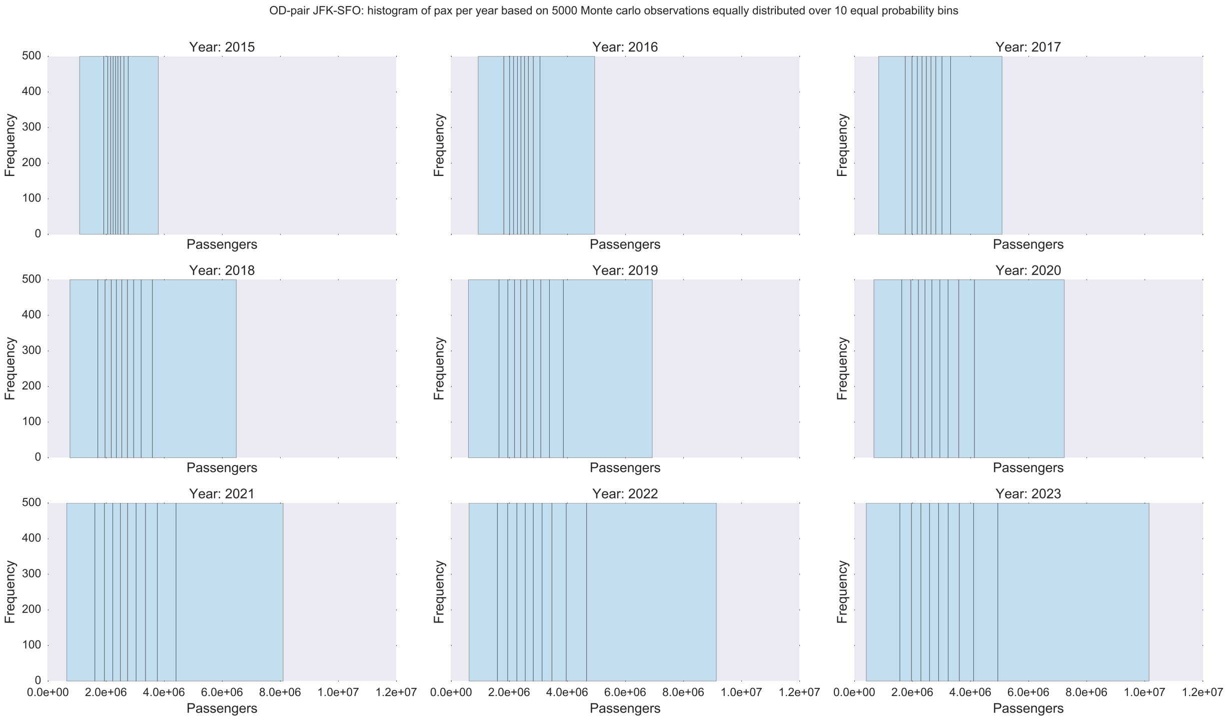


Figure B.6: Histograms of Monte Carlo simulation observations - OD pair: JFK-SFO

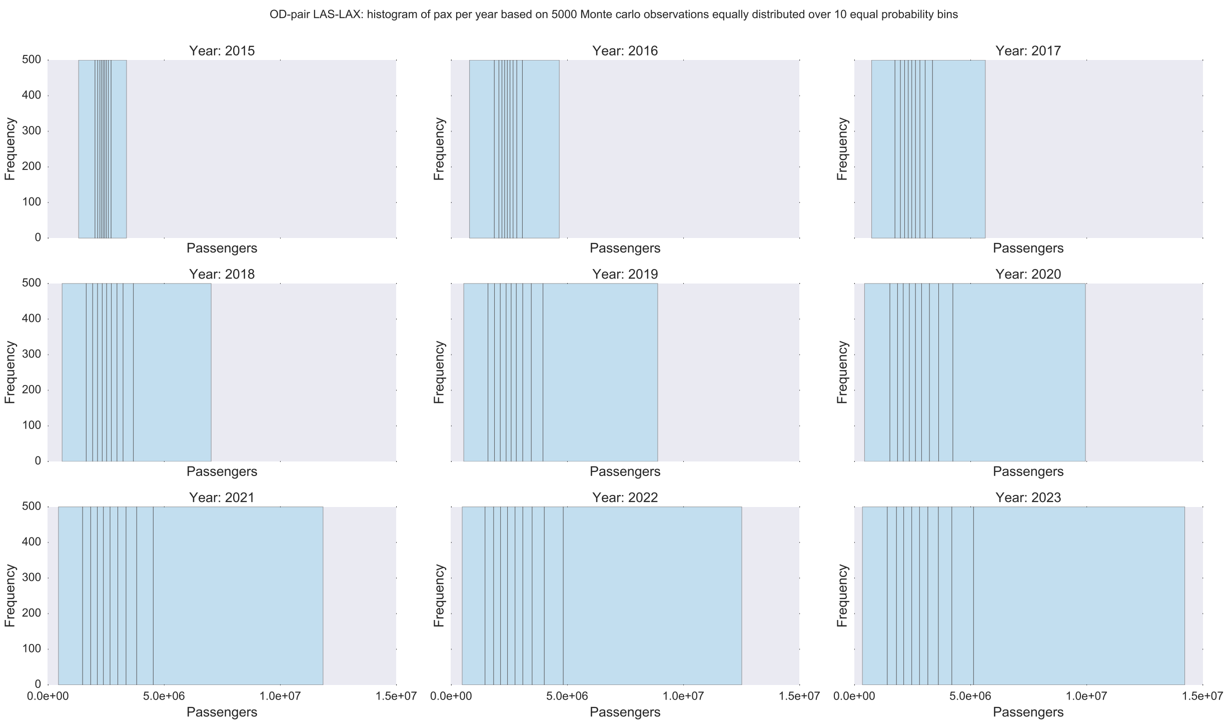


Figure B.7: Histograms of Monte Carlo simulation observations - OD pair: LAS-LAX

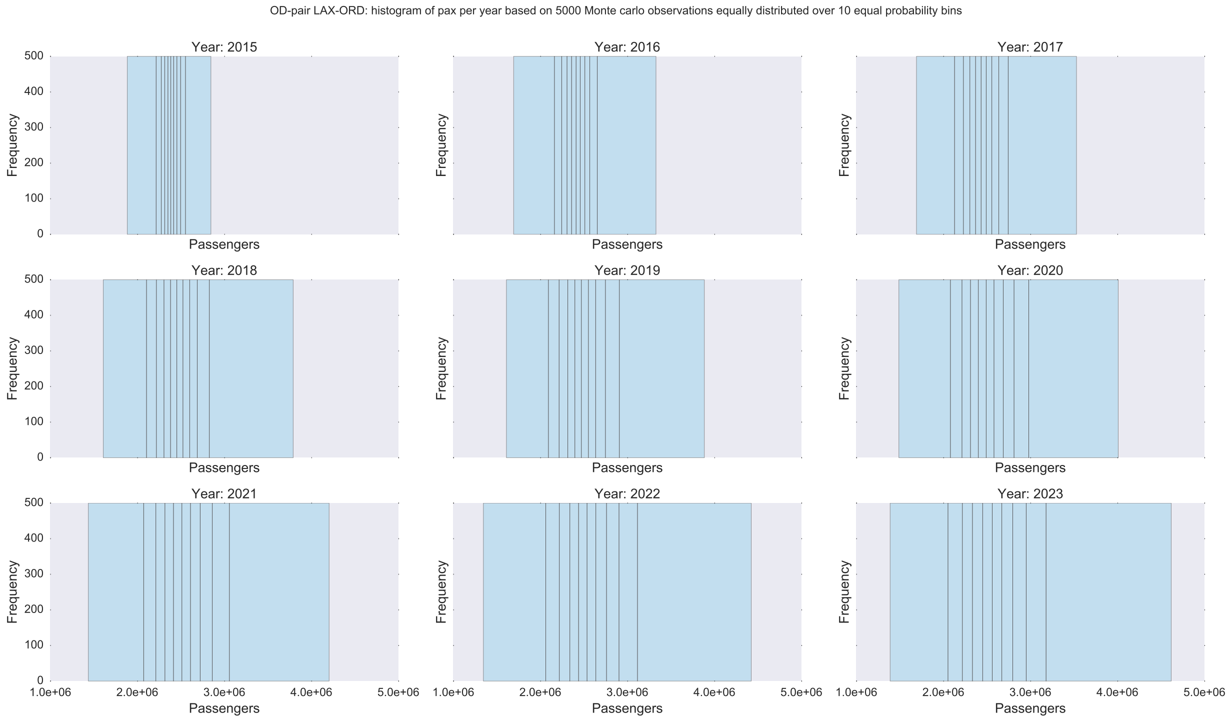


Figure B.8: Histograms of Monte Carlo simulation observations - OD pair: LAX-ORD

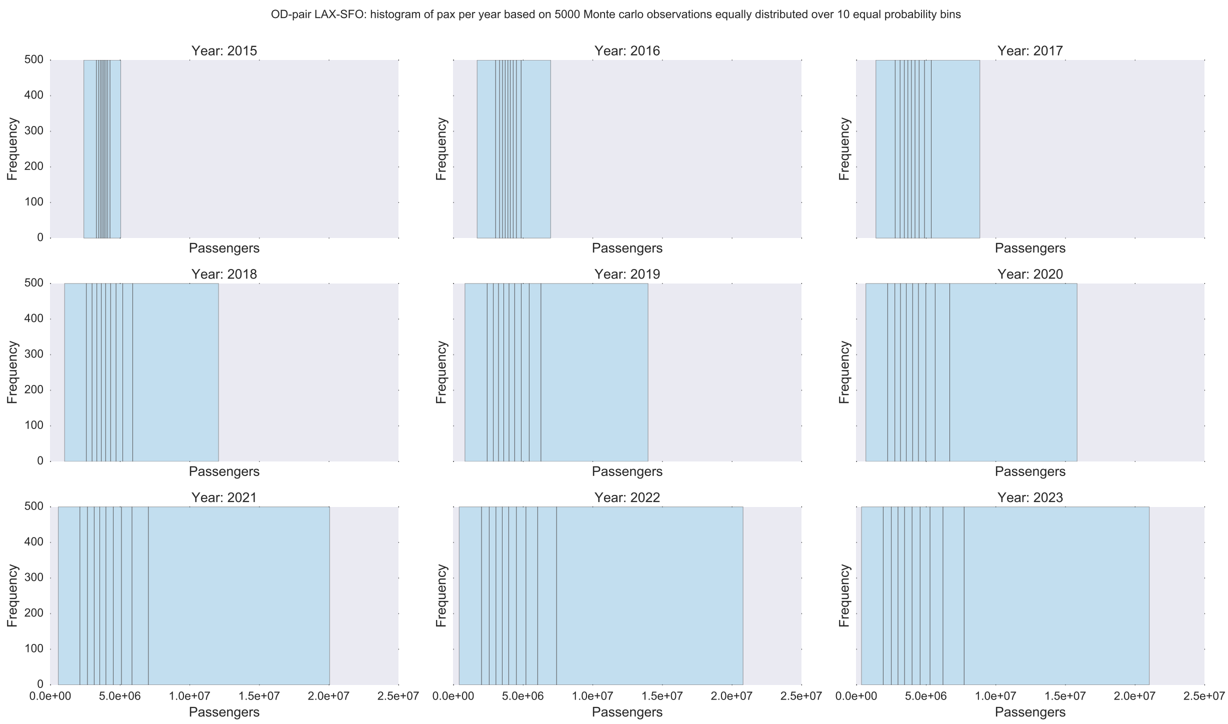


Figure B.9: Histograms of Monte Carlo simulation observations - OD pair: LAX-SFO

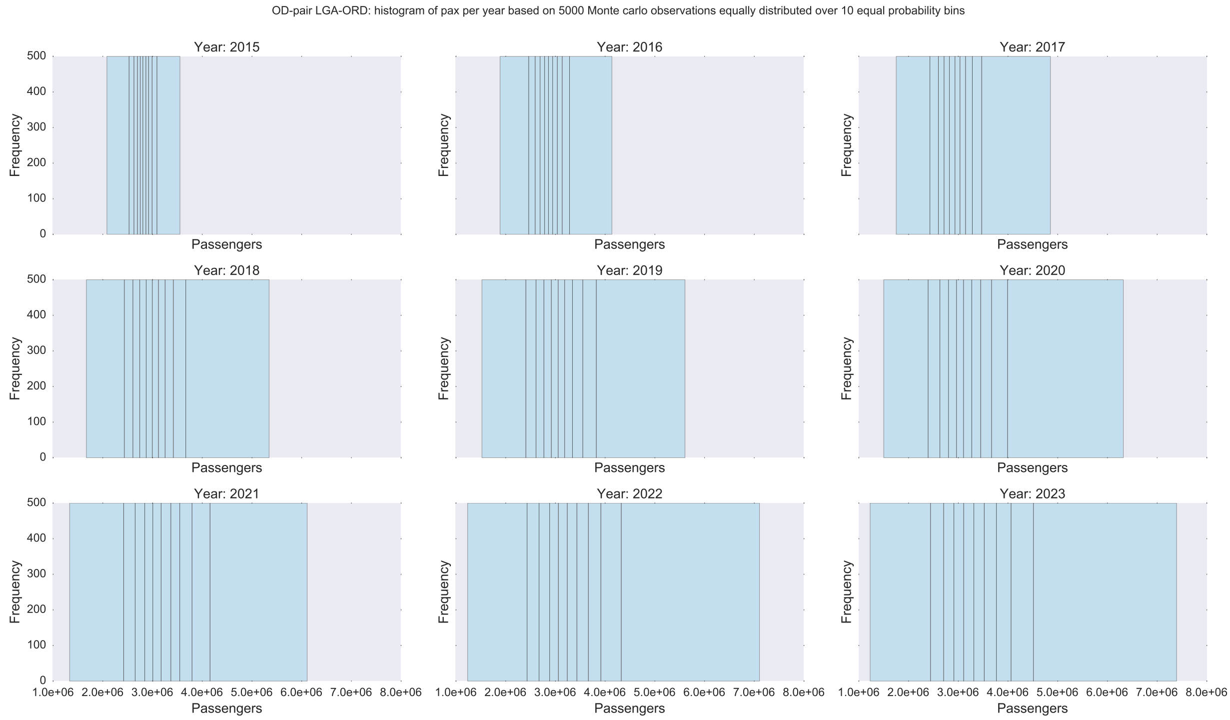


Figure B.10: Histograms of Monte Carlo simulation observations - OD pair: LGA-ORD

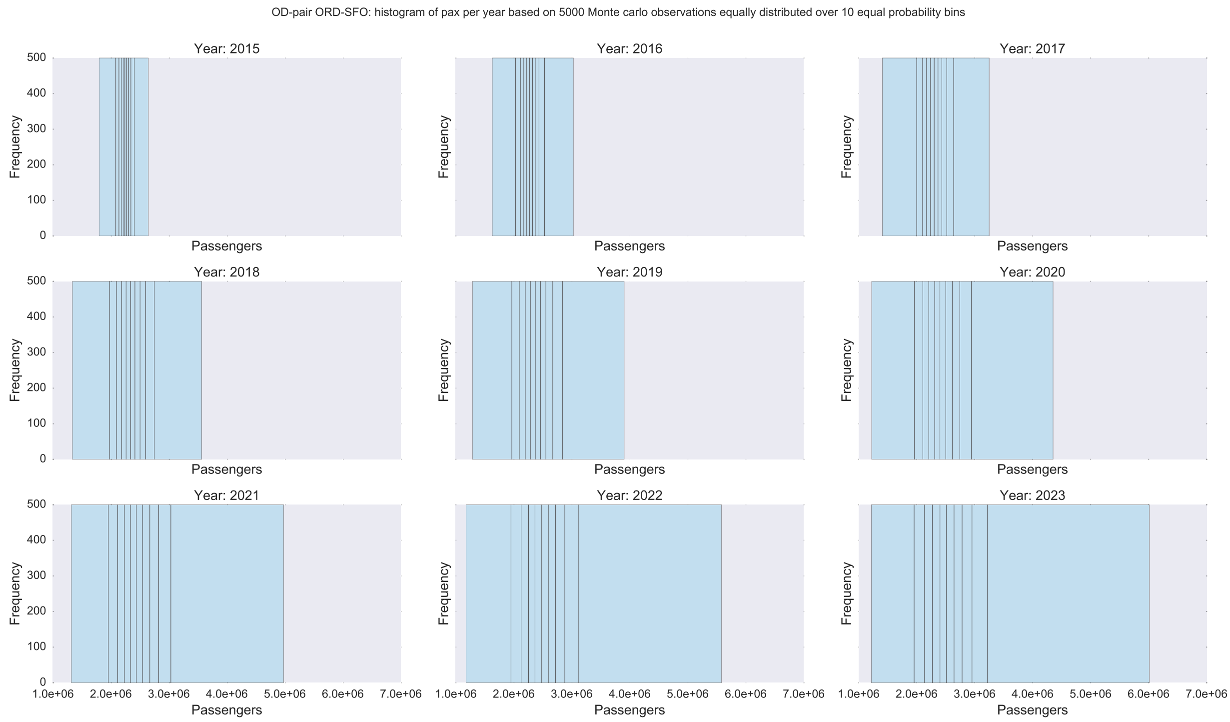


Figure B.11: Histograms of Monte Carlo simulation observations - OD pair: ORD-SFO

B.3 OD demand matrices

Year 2015 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.03E+06	0	0	0	0	1.13E+06	0	0
DFW	0	0	0	0	0	1.06E+06	0	0	0	0
FLL	1.03E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.29E+06	0	0	0	8.87E+05
LAS	0	0	0	0	0	9.43E+05	0	0	0	0
LAX	0	1.06E+06	0	1.29E+06	9.43E+05	0	0	0	1.08E+06	1.57E+06
LGA	0	0	0	0	0	0	0	0	1.21E+06	0
MCO	1.13E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.08E+06	1.21E+06	0	0	1.01E+06
SFO	0	0	0	8.87E+05	0	1.57E+06	0	0	1.01E+06	0

Year 2015 bin 2	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.10E+06	0	0	0	0	1.19E+06	0	0
DFW	0	0	0	0	0	1.11E+06	0	0	0	0
FLL	1.10E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.43E+06	0	0	0	9.99E+05
LAS	0	0	0	0	0	1.04E+06	0	0	0	0
LAX	0	1.11E+06	0	1.43E+06	1.04E+06	0	0	0	1.12E+06	1.70E+06
LGA	0	0	0	0	0	0	0	0	1.29E+06	0
MCO	1.19E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.12E+06	1.29E+06	0	0	1.06E+06
SFO	0	0	0	9.99E+05	0	1.70E+06	0	0	1.06E+06	0

Year 2015 bin 3	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.14E+06	0	0	0	0	1.23E+06	0	0
DFW	0	0	0	0	0	1.13E+06	0	0	0	0
FLL	1.14E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.51E+06	0	0	0	1.06E+06
LAS	0	0	0	0	0	1.09E+06	0	0	0	0
LAX	0	1.13E+06	0	1.51E+06	1.09E+06	0	0	0	1.15E+06	1.77E+06
LGA	0	0	0	0	0	0	0	0	1.33E+06	0
MCO	1.23E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.15E+06	1.33E+06	0	0	1.08E+06
SFO	0	0	0	1.06E+06	0	1.77E+06	0	0	1.08E+06	0

Year 2015 bin 4	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.17E+06	0	0	0	0	1.25E+06	0	0
DFW	0	0	0	0	0	1.16E+06	0	0	0	0
FLL	1.17E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.57E+06	0	0	0	1.11E+06
LAS	0	0	0	0	0	1.13E+06	0	0	0	0
LAX	0	1.16E+06	0	1.57E+06	1.13E+06	0	0	0	1.17E+06	1.83E+06
LGA	0	0	0	0	0	0	0	0	1.36E+06	0
MCO	1.25E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.17E+06	1.36E+06	0	0	1.10E+06
SFO	0	0	0	1.11E+06	0	1.83E+06	0	0	1.10E+06	0

Year 2015 bin 5	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.19E+06	0	0	0	0	1.27E+06	0	0
DFW	0	0	0	0	0	1.17E+06	0	0	0	0
FLL	1.19E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.62E+06	0	0	0	1.16E+06
LAS	0	0	0	0	0	1.17E+06	0	0	0	0
LAX	0	1.17E+06	0	1.62E+06	1.17E+06	0	0	0	1.18E+06	1.88E+06
LGA	0	0	0	0	0	0	0	0	1.39E+06	0
MCO	1.27E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.18E+06	1.39E+06	0	0	1.11E+06
SFO	0	0	0	1.16E+06	0	1.88E+06	0	0	1.11E+06	0

Year 2015 bin 6	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.22E+06	0	0	0	0	1.30E+06	0	0
DFW	0	0	0	0	0	1.19E+06	0	0	0	0
FLL	1.22E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.67E+06	0	0	0	1.20E+06
LAS	0	0	0	0	0	1.21E+06	0	0	0	0
LAX	0	1.19E+06	0	1.67E+06	1.21E+06	0	0	0	1.20E+06	1.92E+06
LGA	0	0	0	0	0	0	0	0	1.42E+06	0
MCO	1.30E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.20E+06	1.42E+06	0	0	1.13E+06
SFO	0	0	0	1.20E+06	0	1.92E+06	0	0	1.13E+06	0

Year 2015 bin 7	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.25E+06	0	0	0	0	1.32E+06	0	0
DFW	0	0	0	0	0	1.21E+06	0	0	0	0
FLL	1.25E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.72E+06	0	0	0	1.24E+06
LAS	0	0	0	0	0	1.24E+06	0	0	0	0
LAX	0	1.21E+06	0	1.72E+06	1.24E+06	0	0	0	1.22E+06	1.98E+06
LGA	0	0	0	0	0	0	0	0	1.45E+06	0
MCO	1.32E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.22E+06	1.45E+06	0	0	1.15E+06
SFO	0	0	0	1.24E+06	0	1.98E+06	0	0	1.15E+06	0

Year 2015 bin 8	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.28E+06	0	0	0	0	1.35E+06	0	0
DFW	0	0	0	0	0	1.23E+06	0	0	0	0
FLL	1.28E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.78E+06	0	0	0	1.28E+06
LAS	0	0	0	0	0	1.28E+06	0	0	0	0
LAX	0	1.23E+06	0	1.78E+06	1.28E+06	0	0	0	1.24E+06	2.04E+06
LGA	0	0	0	0	0	0	0	0	1.48E+06	0
MCO	1.35E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.24E+06	1.48E+06	0	0	1.16E+06
SFO	0	0	0	1.28E+06	0	2.04E+06	0	0	1.16E+06	0

Year 2015 bin 9	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.31E+06	0	0	0	0	1.38E+06	0	0
DFW	0	0	0	0	0	1.26E+06	0	0	0	0
FLL	1.31E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.86E+06	0	0	0	1.34E+06
LAS	0	0	0	0	0	1.33E+06	0	0	0	0
LAX	0	1.26E+06	0	1.86E+06	1.33E+06	0	0	0	1.26E+06	2.11E+06
LGA	0	0	0	0	0	0	0	0	1.52E+06	0
MCO	1.38E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.26E+06	1.52E+06	0	0	1.19E+06
SFO	0	0	0	1.34E+06	0	2.11E+06	0	0	1.19E+06	0

Year 2015 bin 10	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.38E+06	0	0	0	0	1.44E+06	0	0
DFW	0	0	0	0	0	1.32E+06	0	0	0	0
FLL	1.38E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.01E+06	0	0	0	1.46E+06
LAS	0	0	0	0	0	1.43E+06	0	0	0	0
LAX	0	1.32E+06	0	2.01E+06	1.43E+06	0	0	0	1.31E+06	2.24E+06
LGA	0	0	0	0	0	0	0	0	1.60E+06	0
MCO	1.44E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.31E+06	1.60E+06	0	0	1.23E+06
SFO	0	0	0	1.46E+06	0	2.24E+06	0	0	1.23E+06	0

Year 2016 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.02E+06	0	0	0	0	1.11E+06	0	0
DFW	0	0	0	0	0	1.04E+06	0	0	0	0
FLL	1.02E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.21E+06	0	0	0	8.23E+05
LAS	0	0	0	0	0	8.28E+05	0	0	0	0
LAX	0	1.04E+06	0	1.21E+06	8.28E+05	0	0	0	1.04E+06	1.37E+06
LGA	0	0	0	0	0	0	0	0	1.17E+06	0
MCO	1.11E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.04E+06	1.17E+06	0	0	9.70E+05
SFO	0	0	0	8.23E+05	0	1.37E+06	0	0	9.70E+05	0

Year 2016 bin 2	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.12E+06	0	0	0	0	1.19E+06	0	0
DFW	0	0	0	0	0	1.10E+06	0	0	0	0
FLL	1.12E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.39E+06	0	0	0	9.68E+05
LAS	0	0	0	0	0	9.77E+05	0	0	0	0
LAX	0	1.10E+06	0	1.39E+06	9.77E+05	0	0	0	1.10E+06	1.59E+06
LGA	0	0	0	0	0	0	0	0	1.27E+06	0
MCO	1.19E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.10E+06	1.27E+06	0	0	1.04E+06
SFO	0	0	0	9.68E+05	0	1.59E+06	0	0	1.04E+06	0

Year 2016 bin 3	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.17E+06	0	0	0	0	1.24E+06	0	0
DFW	0	0	0	0	0	1.14E+06	0	0	0	0
FLL	1.17E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.49E+06	0	0	0	1.05E+06
LAS	0	0	0	0	0	1.06E+06	0	0	0	0
LAX	0	1.14E+06	0	1.49E+06	1.06E+06	0	0	0	1.14E+06	1.71E+06
LGA	0	0	0	0	0	0	0	0	1.33E+06	0
MCO	1.24E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.14E+06	1.33E+06	0 H	0	1.07E+06
SFO	0	0	0	1.05E+06	0	1.71E+06	0	0	1.07E+06	0

Year 2016 bin 4	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.21E+06	0	0	0	0	1.28E+06	0	0
DFW	0	0	0	0	0	1.17E+06	0	0	0	0
FLL	1.21E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.57E+06	0	0	0	1.12E+06
LAS	0	0	0	0	0	1.13E+06	0	0	0	0
LAX	0	1.17E+06	0	1.57E+06	1.13E+06	0	0	0	1.17E+06	1.81E+06
LGA	0	0	0	0	0	0	0	0	1.37E+06	0
MCO	1.28E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.17E+06	1.37E+06	0	0	1.10E+06
SFO	0	0	0	1.12E+06	0	1.81E+06	0	0	1.10E+06	0

Year 2016 bin 5	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.25E+06	0	0	0	0	1.31E+06	0	0
DFW	0	0	0	0	0	1.20E+06	0	0	0	0
FLL	1.25E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.64E+06	0	0	0	1.18E+06
LAS	0	0	0	0	0	1.19E+06	0	0	0	0
LAX	0	1.20E+06	0	1.64E+06	1.19E+06	0	0	0	1.19E+06	1.90E+06
LGA	0	0	0	0	0	0	0	0	1.41E+06	0
MCO	1.31E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.19E+06	1.41E+06	0	0	1.12E+06
SFO	0	0	0	1.18E+06	0	1.90E+06	0	0	1.12E+06	0

Year 2016 bin 6	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.29E+06	0	0	0	0	1.35E+06	0	0
DFW	0	0	0	0	0	1.22E+06	0	0	0	0
FLL	1.29E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.71E+06	0	0	0	1.24E+06
LAS	0	0	0	0	0	1.25E+06	0	0	0	0
LAX	0	1.22E+06	0	1.71E+06	1.25E+06	0	0	0	1.21E+06	2.00E+06
LGA	0	0	0	0	0	0	0	0	1.45E+06	0
MCO	1.35E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.21E+06	1.45E+06	0	0	1.15E+06
SFO	0	0	0	1.24E+06	0	2.00E+06	0	0	1.15E+06	0

Year 2016 bin 7	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.33E+06	0	0	0	0	1.38E+06	0	0
DFW	0	0	0	0	0	1.25E+06	0	0	0	0
FLL	1.33E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.79E+06	0	0	0	1.31E+06
LAS	0	0	0	0	0	1.31E+06	0	0	0	0
LAX	0	1.25E+06	0	1.79E+06	1.31E+06	0	0	0	1.24E+06	2.09E+06
LGA	0	0	0	0	0	0	0	0	1.50E+06	0
MCO	1.38E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.24E+06	1.50E+06	0	0	1.17E+06
SFO	0	0	0	1.31E+06	0	2.09E+06	0	0	1.17E+06	0

Year 2016 bin 8	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.37E+06	0	0	0	0	1.42E+06	0	0
DFW	0	0	0	0	0	1.28E+06	0	0	0	0
FLL	1.37E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.88E+06	0	0	0	1.38E+06
LAS	0	0	0	0	0	1.38E+06	0	0	0	0
LAX	0	1.28E+06	0	1.88E+06	1.38E+06	0	0	0	1.27E+06	2.21E+06
LGA	0	0	0	0	0	0	0	0	1.55E+06	0
MCO	1.42E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.27E+06	1.55E+06	0	0	1.20E+06
SFO	0	0	0	1.38E+06	0	2.21E+06	0	0	1.20E+06	0

Year 2016 bin 9	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.43E+06	0	0	0	0	1.47E+06	0	0
DFW	0	0	0	0	0	1.32E+06	0	0	0	0
FLL	1.43E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.00E+06	0	0	0	1.47E+06
LAS	0	0	0	0	0	1.48E+06	0	0	0	0
LAX	0	1.32E+06	0	2.00E+06	1.48E+06	0	0	0	1.31E+06	2.36E+06
LGA	0	0	0	0	0	0	0	0	1.61E+06	0
MCO	1.47E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.31E+06	1.61E+06	0	0	1.24E+06
SFO	0	0	0	1.47E+06	0	2.36E+06	0	0	1.24E+06	0

Year 2016 bin 10	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.56E+06	0	0	0	0	1.58E+06	0	0
DFW	0	0	0	0	0	1.40E+06	0	0	0	0
FLL	1.56E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.24E+06	0	0	0	1.67E+06
LAS	0	0	0	0	0	1.67E+06	0	0	0	0
LAX	0	1.40E+06	0	2.24E+06	1.67E+06	0	0	0	1.38E+06	2.64E+06
LGA	0	0	0	0	0	0	0	0	1.73E+06	0
MCO	1.58E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.38E+06	1.73E+06	0	0	1.31E+06
SFO	0	0	0	1.67E+06	0	2.64E+06	0	0	1.31E+06	0

Year 2017 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.02E+06	0	0	0	0	1.10E+06	0	0
DFW	0	0	0	0	0	1.01E+06	0	0	0	0
FLL	1.02E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.15E+06	0	0	0	7.76E+05
LAS	0	0	0	0	0	7.49E+05	0	0	0	0
LAX	0	1.01E+06	0	1.15E+06	7.49E+05	0	0	0	1.02E+06	1.22E+06
LGA	0	0	0	0	0	0	0	0	1.14E+06	0
MCO	1.10E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.02E+06	1.14E+06	0	0	9.48E+05
SFO	0	0	0	7.76E+05	0	1.22E+06	0	0	9.48E+05	0

Year 2017 bin 2	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.13E+06	0	0	0	0	1.20E+06	0	0
DFW	0	0	0	0	0	1.10E+06	0	0	0	0
FLL	1.13E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.36E+06	0	0	0	9.47E+05
LAS	0	0	0	0	0	9.28E+05	0	0	0	0
LAX	0	1.10E+06	0	1.36E+06	9.28E+05	0	0	0	1.09E+06	1.48E+06
LGA	0	0	0	0	0	0	0	0	1.26E+06	0
MCO	1.20E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.09E+06	1.26E+06	0	0	1.02E+06
SFO	0	0	0	9.47E+05	0	1.48E+06	0	0	1.02E+06	0

Year 2017 bin 3	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.20E+06	0	0	0	0	1.26E+06	0	0
DFW	0	0	0	0	0	1.15E+06	0	0	0	0
FLL	1.20E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.48E+06	0	0	0	1.04E+06
LAS	0	0	0	0	0	1.03E+06	0	0	0	0
LAX	0	1.15E+06	0	1.48E+06	1.03E+06	0	0	0	1.14E+06	1.65E+06
LGA	0	0	0	0	0	0	0	0	1.33E+06	0
MCO	1.26E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.14E+06	1.33E+06	0	0	1.07E+06
SFO	0	0	0	1.04E+06	0	1.65E+06	0	0	1.07E+06	0

Year 2017 bin 4	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.25E+06	0	0	0	0	1.31E+06	0	0
DFW	0	0	0	0	0	1.18E+06	0	0	0	0
FLL	1.25E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.57E+06	0	0	0	1.13E+06
LAS	0	0	0	0	0	1.12E+06	0	0	0	0
LAX	0	1.18E+06	0	1.57E+06	1.12E+06	0	0	0	1.17E+06	1.79E+06
LGA	0	0	0	0	0	0	0	0	1.39E+06	0
MCO	1.31E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.17E+06	1.39E+06	0	0	1.11E+06
SFO	0	0	0	1.13E+06	0	1.79E+06	0	0	1.11E+06	0

Year 2017 bin 5	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.30E+06	0	0	0	0	1.36E+06	0	0
DFW	0	0	0	0	0	1.22E+06	0	0	0	0
FLL	1.30E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.67E+06	0	0	0	1.21E+06
LAS	0	0	0	0	0	1.20E+06	0	0	0	0
LAX	0	1.22E+06	0	1.67E+06	1.20E+06	0	0	0	1.20E+06	1.91E+06
LGA	0	0	0	0	0	0	0	0	1.44E+06	0
MCO	1.36E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.20E+06	1.44E+06	0	0	1.14E+06
SFO	0	0	0	1.21E+06	0	1.91E+06	0	0	1.14E+06	0

Year 2017 bin 6	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.35E+06	0	0	0	0	1.40E+06	0	0
DFW	0	0	0	0	0	1.25E+06	0	0	0	0
FLL	1.35E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.76E+06	0	0	0	1.28E+06
LAS	0	0	0	0	0	1.29E+06	0	0	0	0
LAX	0	1.25E+06	0	1.76E+06	1.29E+06	0	0	0	1.23E+06	2.04E+06
LGA	0	0	0	0	0	0	0	0	1.49E+06	0
MCO	1.40E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.23E+06	1.49E+06	0	0	1.17E+06
SFO	0	0	0	1.28E+06	0	2.04E+06	0	0	1.17E+06	0

Year 2017 bin 7	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.41E+06	0	0	0	0	1.44E+06	0	0
DFW	0	0	0	0	0	1.29E+06	0	0	0	0
FLL	1.41E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.86E+06	0	0	0	1.37E+06
LAS	0	0	0	0	0	1.38E+06	0	0	0	0
LAX	0	1.29E+06	0	1.86E+06	1.38E+06	0	0	0	1.26E+06	2.18E+06
LGA	0	0	0	0	0	0	0	0	1.54E+06	0
MCO	1.44E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.26E+06	1.54E+06	0	0	1.20E+06
SFO	0	0	0	1.37E+06	0	2.18E+06	0	0	1.20E+06	0

Year 2017 bin 8	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.47E+06	0	0	0	0	1.49E+06	0	0
DFW	0	0	0	0	0	1.32E+06	0	0	0	0
FLL	1.47E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.98E+06	0	0	0	1.46E+06
LAS	0	0	0	0	0	1.47E+06	0	0	0	0
LAX	0	1.32E+06	0	1.98E+06	1.47E+06	0	0	0	1.30E+06	2.35E+06
LGA	0	0	0	0	0	0	0	0	1.60E+06	0
MCO	1.49E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.30E+06	1.60E+06	0	0	1.24E+06
SFO	0	0	0	1.46E+06	0	2.35E+06	0	0	1.24E+06	0

Year 2017 bin 9	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.55E+06	0	0	0	0	1.56E+06	0	0
DFW	0	0	0	0	0	1.38E+06	0	0	0	0
FLL	1.55E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.13E+06	0	0	0	1.59E+06
LAS	0	0	0	0	0	1.60E+06	0	0	0	0
LAX	0	1.38E+06	0	2.13E+06	1.60E+06	0	0	0	1.35E+06	2.56E+06
LGA	0	0	0	0	0	0	0	0	1.69E+06	0
MCO	1.56E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.35E+06	1.69E+06	0	0	1.29E+06
SFO	0	0	0	1.59E+06	0	2.56E+06	0	0	1.29E+06	0

Year 2017 bin 10	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.71E+06	0	0	0	0	1.69E+06	0	0
DFW	0	0	0	0	0	1.48E+06	0	0	0	0
FLL	1.71E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.46E+06	0	0	0	1.86E+06
LAS	0	0	0	0	0	1.90E+06	0	0	0	0
LAX	0	1.48E+06	0	2.46E+06	1.90E+06	0	0	0	1.44E+06	3.03E+06
LGA	0	0	0	0	0	0	0	0	1.85E+06	0
MCO	1.69E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.44E+06	1.85E+06	0	0	1.38E+06
SFO	0	0	0	1.86E+06	0	3.03E+06	0	0	1.38E+06	0

Year 2018 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.02E+06	0	0	0	0	1.10E+06	0	0
DFW	0	0	0	0	0	1.01E+06	0	0	0	0
FLL	1.02E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.11E+06	0	0	0	7.46E+05
LAS	0	0	0	0	0	7.00E+05	0	0	0	0
LAX	0	1.01E+06	0	1.11E+06	7.00E+05	0	0	0	1.00E+06	1.10E+06
LGA	0	0	0	0	0	0	0	0	1.12E+06	0
MCO	1.10E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.00E+06	1.12E+06	0	0	9.30E+05
SFO	0	0	0	7.46E+05	0	1.10E+06	0	0	9.30E+05	0

Year 2018 bin 2	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.16E+06	0	0	0	0	1.22E+06	0	0
DFW	0	0	0	0	0	1.11E+06	0	0	0	0
FLL	1.16E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.34E+06	0	0	0	9.25E+05
LAS	0	0	0	0	0	8.97E+05	0	0	0	0
LAX	0	1.11E+06	0	1.34E+06	8.97E+05	0	0	0	1.09E+06	1.40E+06
LGA	0	0	0	0	0	0	0	0	1.26E+06	0
MCO	1.22E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.09E+06	1.26E+06	0	0	1.02E+06
SFO	0	0	0	9.25E+05	0	1.40E+06	0	0	1.02E+06	0

Year 2018 bin 3	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.24E+06	0	0	0	0	1.29E+06	0	0
DFW	0	0	0	0	0	1.16E+06	0	0	0	0
FLL	1.24E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.48E+06	0	0	0	1.03E+06
LAS	0	0	0	0	0	1.02E+06	0	0	0	0
LAX	0	1.16E+06	0	1.48E+06	1.02E+06	0	0	0	1.13E+06	1.59E+06
LGA	0	0	0	0	0	0	0	0	1.34E+06	0
MCO	1.29E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.13E+06	1.34E+06	0	0	1.07E+06
SFO	0	0	0	1.03E+06	0	1.59E+06	0	0	1.07E+06	0

Year 2018 bin 4	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.30E+06	0	0	0	0	1.34E+06	0	0
DFW	0	0	0	0	0	1.20E+06	0	0	0	0
FLL	1.30E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.59E+06	0	0	0	1.13E+06
LAS	0	0	0	0	0	1.12E+06	0	0	0	0
LAX	0	1.20E+06	0	1.59E+06	1.12E+06	0	0	0	1.17E+06	1.76E+06
LGA	0	0	0	0	0	0	0	0	1.41E+06	0
MCO	1.34E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.17E+06	1.41E+06	0	0	1.11E+06
SFO	0	0	0	1.13E+06	0	1.76E+06	0	0	1.11E+06	0

Year 2018 bin 5	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.36E+06	0	0	0	0	1.40E+06	0	0
DFW	0	0	0	0	0	1.24E+06	0	0	0	0
FLL	1.36E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.70E+06	0	0	0	1.22E+06
LAS	0	0	0	0	0	1.21E+06	0	0	0	0
LAX	0	1.24E+06	0	1.70E+06	1.21E+06	0	0	0	1.21E+06	1.92E+06
LGA	0	0	0	0	0	0	0	0	1.47E+06	0
MCO	1.40E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.21E+06	1.47E+06	0	0	1.15E+06
SFO	0	0	0	1.22E+06	0	1.92E+06	0	0	1.15E+06	0

Year 2018 bin 6	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.42E+06	0	0	0	0	1.45E+06	0	0
DFW	0	0	0	0	0	1.28E+06	0	0	0	0
FLL	1.42E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.80E+06	0	0	0	1.32E+06
LAS	0	0	0	0	0	1.31E+06	0	0	0	0
LAX	0	1.28E+06	0	1.80E+06	1.31E+06	0	0	0	1.24E+06	2.08E+06
LGA	0	0	0	0	0	0	0	0	1.53E+06	0
MCO	1.45E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.24E+06	1.53E+06	0	0	1.19E+06
SFO	0	0	0	1.32E+06	0	2.08E+06	0	0	1.19E+06	0

Year 2018 bin 7	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.49E+06	0	0	0	0	1.50E+06	0	0
DFW	0	0	0	0	0	1.32E+06	0	0	0	0
FLL	1.49E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.93E+06	0	0	0	1.42E+06
LAS	0	0	0	0	0	1.43E+06	0	0	0	0
LAX	0	1.32E+06	0	1.93E+06	1.43E+06	0	0	0	1.28E+06	2.26E+06
LGA	0	0	0	0	0	0	0	0	1.59E+06	0
MCO	1.50E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.28E+06	1.59E+06	0	0	1.23E+06
SFO	0	0	0	1.42E+06	0	2.26E+06	0	0	1.23E+06	0

Year 2018 bin 8	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.56E+06	0	0	0	0	1.56E+06	0	0
DFW	0	0	0	0	0	1.37E+06	0	0	0	0
FLL	1.56E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.06E+06	0	0	0	1.54E+06
LAS	0	0	0	0	0	1.56E+06	0	0	0	0
LAX	0	1.37E+06	0	2.06E+06	1.56E+06	0	0	0	1.32E+06	2.47E+06
LGA	0	0	0	0	0	0	0	0	1.66E+06	0
MCO	1.56E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.32E+06	1.66E+06	0	0	1.27E+06
SFO	0	0	0	1.54E+06	0	2.47E+06	0	0	1.27E+06	0

Year 2018 bin 9	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.66E+06	0	0	0	0	1.65E+06	0	0
DFW	0	0	0	0	0	1.43E+06	0	0	0	0
FLL	1.66E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.24E+06	0	0	0	1.71E+06
LAS	0	0	0	0	0	1.73E+06	0	0	0	0
LAX	0	1.43E+06	0	2.24E+06	1.73E+06	0	0	0	1.38E+06	2.75E+06
LGA	0	0	0	0	0	0	0	0	1.76E+06	0
MCO	1.65E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.38E+06	1.76E+06	0	0	1.33E+06
SFO	0	0	0	1.71E+06	0	2.75E+06	0	0	1.33E+06	0

Year 2018 bin 10	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.87E+06	0	0	0	0	1.82E+06	0	0
DFW	0	0	0	0	0	1.56E+06	0	0	0	0
FLL	1.87E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.64E+06	0	0	0	2.06E+06
LAS	0	0	0	0	0	2.12E+06	0	0	0	0
LAX	0	1.56E+06	0	2.64E+06	2.12E+06	0	0	0	1.50E+06	3.39E+06
LGA	0	0	0	0	0	0	0	0	1.96E+06	0
MCO	1.82E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.50E+06	1.96E+06	0	0	1.45E+06
SFO	0	0	0	2.06E+06	0	3.39E+06	0	0	1.45E+06	0

Year 2019 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.03E+06	0	0	0	0	1.10E+06	0	0
DFW	0	0	0	0	0	1.00E+06	0	0	0	0
FLL	1.03E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.07E+06	0	0	0	7.20E+05
LAS	0	0	0	0	0	6.62E+05	0	0	0	0
LAX	0	1.00E+06	0	1.07E+06	6.62E+05	0	0	0	9.84E+05	1.00E+06
LGA	0	0	0	0	0	0	0	0	1.11E+06	0
MCO	1.10E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	9.84E+05	1.11E+06	0	0	9.17E+05
SFO	0	0	0	7.20E+05	0	1.00E+06	0	0	9.17E+05	0

Year 2019 bin 2	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.20E+06	0	0	0	0	1.23E+06	0	0
DFW	0	0	0	0	0	1.11E+06	0	0	0	0
FLL	1.20E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.32E+06	0	0	0	9.15E+05
LAS	0	0	0	0	0	8.72E+05	0	0	0	0
LAX	0	1.11E+06	0	1.32E+06	8.72E+05	0	0	0	1.08E+06	1.33E+06
LGA	0	0	0	0	0	0	0	0	1.26E+06	0
MCO	1.23E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.08E+06	1.26E+06	0	0	1.01E+06
SFO	0	0	0	9.15E+05	0	1.33E+06	0	0	1.01E+06	0

Year 2019 bin 3	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.28E+06	0	0	0	0	1.31E+06	0	0
DFW	0	0	0	0	0	1.17E+06	0	0	0	0
FLL	1.28E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.47E+06	0	0	0	1.04E+06
LAS	0	0	0	0	0	1.00E+06	0	0	0	0
LAX	0	1.17E+06	0	1.47E+06	1.00E+06	0	0	0	1.13E+06	1.54E+06
LGA	0	0	0	0	0	0	0	0	1.35E+06	0
MCO	1.31E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.13E+06	1.35E+06	0	0	1.07E+06
SFO	0	0	0	1.04E+06	0	1.54E+06	0	0	1.07E+06	0

Year 2019 bin 4	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.36E+06	0	0	0	0	1.38E+06	0	0
DFW	0	0	0	0	0	1.22E+06	0	0	0	0
FLL	1.36E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.59E+06	0	0	0	1.15E+06
LAS	0	0	0	0	0	1.12E+06	0	0	0	0
LAX	0	1.22E+06	0	1.59E+06	1.12E+06	0	0	0	1.18E+06	1.73E+06
LGA	0	0	0	0	0	0	0	0	1.42E+06	0
MCO	1.38E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.18E+06	1.42E+06	0	0	1.12E+06
SFO	0	0	0	1.15E+06	0	1.73E+06	0	0	1.12E+06	0

Year 2019 bin 5	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.43E+06	0	0	0	0	1.44E+06	0	0
DFW	0	0	0	0	0	1.26E+06	0	0	0	0
FLL	1.43E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.72E+06	0	0	0	1.25E+06
LAS	0	0	0	0	0	1.23E+06	0	0	0	0
LAX	0	1.26E+06	0	1.72E+06	1.23E+06	0	0	0	1.22E+06	1.91E+06
LGA	0	0	0	0	0	0	0	0	1.49E+06	0
MCO	1.44E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.22E+06	1.49E+06	0	0	1.16E+06
SFO	0	0	0	1.25E+06	0	1.91E+06	0	0	1.16E+06	0

Year 2019 bin 6	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.50E+06	0	0	0	0	1.50E+06	0	0
DFW	0	0	0	0	0	1.31E+06	0	0	0	0
FLL	1.50E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.84E+06	0	0	0	1.36E+06
LAS	0	0	0	0	0	1.35E+06	0	0	0	0
LAX	0	1.31E+06	0	1.84E+06	1.35E+06	0	0	0	1.26E+06	2.11E+06
LGA	0	0	0	0	0	0	0	0	1.56E+06	0
MCO	1.50E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.26E+06	1.56E+06	0	0	1.21E+06
SFO	0	0	0	1.36E+06	0	2.11E+06	0	0	1.21E+06	0

Year 2019 bin 7	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.58E+06	0	0	0	0	1.56E+06	0	0
DFW	0	0	0	0	0	1.36E+06	0	0	0	0
FLL	1.58E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.98E+06	0	0	0	1.48E+06
LAS	0	0	0	0	0	1.48E+06	0	0	0	0
LAX	0	1.36E+06	0	1.98E+06	1.48E+06	0	0	0	1.30E+06	2.32E+06
LGA	0	0	0	0	0	0	0	0	1.63E+06	0
MCO	1.56E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.30E+06	1.63E+06	0	0	1.25E+06
SFO	0	0	0	1.48E+06	0	2.32E+06	0	0	1.25E+06	0

Year 2019 bin 8	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.66E+06	0	0	0	0	1.64E+06	0	0
DFW	0	0	0	0	0	1.42E+06	0	0	0	0
FLL	1.66E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.15E+06	0	0	0	1.63E+06
LAS	0	0	0	0	0	1.63E+06	0	0	0	0
LAX	0	1.42E+06	0	2.15E+06	1.63E+06	0	0	0	1.34E+06	2.57E+06
LGA	0	0	0	0	0	0	0	0	1.72E+06	0
MCO	1.64E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.34E+06	1.72E+06	0	0	1.30E+06
SFO	0	0	0	1.63E+06	0	2.57E+06	0	0	1.30E+06	0

Year 2019 bin 9	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.78E+06	0	0	0	0	1.73E+06	0	0
DFW	0	0	0	0	0	1.48E+06	0	0	0	0
FLL	1.78E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.37E+06	0	0	0	1.82E+06
LAS	0	0	0	0	0	1.85E+06	0	0	0	0
LAX	0	1.48E+06	0	2.37E+06	1.85E+06	0	0	0	1.42E+06	2.92E+06
LGA	0	0	0	0	0	0	0	0	1.82E+06	0
MCO	1.73E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.42E+06	1.82E+06	0	0	1.37E+06
SFO	0	0	0	1.82E+06	0	2.92E+06	0	0	1.37E+06	0

Year 2019 bin 10	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	2.03E+06	0	0	0	0	1.94E+06	0	0
DFW	0	0	0	0	0	1.63E+06	0	0	0	0
FLL	2.03E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.83E+06	0	0	0	2.26E+06
LAS	0	0	0	0	0	2.35E+06	0	0	0	0
LAX	0	1.63E+06	0	2.83E+06	2.35E+06	0	0	0	1.56E+06	3.74E+06
LGA	0	0	0	0	0	0	0	0	2.06E+06	0
MCO	1.94E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.56E+06	2.06E+06	0	0	1.51E+06
SFO	0	0	0	2.26E+06	0	3.74E+06	0	0	1.51E+06	0

Year 2020 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.05E+06	0	0	0	0	1.11E+06	0	0
DFW	0	0	0	0	0	1.00E+06	0	0	0	0
FLL	1.05E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.05E+06	0	0	0	6.97E+05
LAS	0	0	0	0	0	6.27E+05	0	0	0	0
LAX	0	1.00E+06	0	1.05E+06	6.27E+05	0	0	0	9.69E+05	9.21E+05
LGA	0	0	0	0	0	0	0	0	1.10E+06	0
MCO	1.11E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	9.69E+05	1.10E+06	0	0	9.04E+05
SFO	0	0	0	6.97E+05	0	9.21E+05	0	0	9.04E+05	0

Year 2020 bin 2	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.23E+06	0	0	0	0	1.26E+06	0	0
DFW	0	0	0	0	0	1.12E+06	0	0	0	0
FLL	1.23E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.31E+06	0	0	0	9.12E+05
LAS	0	0	0	0	0	8.46E+05	0	0	0	0
LAX	0	1.12E+06	0	1.31E+06	8.46E+05	0	0	0	1.08E+06	1.26E+06
LGA	0	0	0	0	0	0	0	0	1.26E+06	0
MCO	1.26E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.08E+06	1.26E+06	0	0	1.01E+06
SFO	0	0	0	9.12E+05	0	1.26E+06	0	0	1.01E+06	0

Year 2020 bin 3	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.33E+06	0	0	0	0	1.34E+06	0	0
DFW	0	0	0	0	0	1.19E+06	0	0	0	0
FLL	1.33E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.47E+06	0	0	0	1.04E+06
LAS	0	0	0	0	0	9.92E+05	0	0	0	0
LAX	0	1.19E+06	0	1.47E+06	9.92E+05	0	0	0	1.13E+06	1.51E+06
LGA	0	0	0	0	0	0	0	0	1.36E+06	0
MCO	1.34E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.13E+06	1.36E+06	0	0	1.07E+06
SFO	0	0	0	1.04E+06	0	1.51E+06	0	0	1.07E+06	0

Year 2020 bin 4	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.42E+06	0	0	0	0	1.42E+06	0	0
DFW	0	0	0	0	0	1.24E+06	0	0	0	0
FLL	1.42E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.60E+06	0	0	0	1.16E+06
LAS	0	0	0	0	0	1.12E+06	0	0	0	0
LAX	0	1.24E+06	0	1.60E+06	1.12E+06	0	0	0	1.18E+06	1.72E+06
LGA	0	0	0	0	0	0	0	0	1.44E+06	0
MCO	1.42E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.18E+06	1.44E+06	0	0	1.13E+06
SFO	0	0	0	1.16E+06	0	1.72E+06	0	0	1.13E+06	0

Year 2020 bin 5	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.50E+06	0	0	0	0	1.49E+06	0	0
DFW	0	0	0	0	0	1.29E+06	0	0	0	0
FLL	1.50E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.74E+06	0	0	0	1.28E+06
LAS	0	0	0	0	0	1.25E+06	0	0	0	0
LAX	0	1.29E+06	0	1.74E+06	1.25E+06	0	0	0	1.22E+06	1.92E+06
LGA	0	0	0	0	0	0	0	0	1.52E+06	0
MCO	1.49E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.22E+06	1.52E+06	0	0	1.18E+06
SFO	0	0	0	1.28E+06	0	1.92E+06	0	0	1.18E+06	0

Year 2020 bin 6	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.58E+06	0	0	0	0	1.56E+06	0	0
DFW	0	0	0	0	0	1.34E+06	0	0	0	0
FLL	1.58E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.89E+06	0	0	0	1.40E+06
LAS	0	0	0	0	0	1.39E+06	0	0	0	0
LAX	0	1.34E+06	0	1.89E+06	1.39E+06	0	0	0	1.27E+06	2.14E+06
LGA	0	0	0	0	0	0	0	0	1.60E+06	0
MCO	1.56E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.27E+06	1.60E+06	0	0	1.23E+06
SFO	0	0	0	1.40E+06	0	2.14E+06	0	0	1.23E+06	0

Year 2020 bin 7	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.67E+06	0	0	0	0	1.63E+06	0	0
DFW	0	0	0	0	0	1.39E+06	0	0	0	0
FLL	1.67E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.05E+06	0	0	0	1.54E+06
LAS	0	0	0	0	0	1.54E+06	0	0	0	0
LAX	0	1.39E+06	0	2.05E+06	1.54E+06	0	0	0	1.32E+06	2.36E+06
LGA	0	0	0	0	0	0	0	0	1.68E+06	0
MCO	1.63E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.32E+06	1.68E+06	0	0	1.28E+06
SFO	0	0	0	1.54E+06	0	2.36E+06	0	0	1.28E+06	0

Year 2020 bin 8	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.77E+06	0	0	0	0	1.71E+06	0	0
DFW	0	0	0	0	0	1.45E+06	0	0	0	0
FLL	1.77E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.24E+06	0	0	0	1.71E+06
LAS	0	0	0	0	0	1.71E+06	0	0	0	0
LAX	0	1.45E+06	0	2.24E+06	1.71E+06	0	0	0	1.37E+06	2.66E+06
LGA	0	0	0	0	0	0	0	0	1.76E+06	0
MCO	1.71E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.37E+06	1.76E+06	0	0	1.34E+06
SFO	0	0	0	1.71E+06	0	2.66E+06	0	0	1.34E+06	0

Year 2020 bin 9	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.91E+06	0	0	0	0	1.82E+06	0	0
DFW	0	0	0	0	0	1.53E+06	0	0	0	0
FLL	1.91E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.48E+06	0	0	0	1.93E+06
LAS	0	0	0	0	0	1.96E+06	0	0	0	0
LAX	0	1.53E+06	0	2.48E+06	1.96E+06	0	0	0	1.45E+06	3.08E+06
LGA	0	0	0	0	0	0	0	0	1.89E+06	0
MCO	1.82E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.45E+06	1.89E+06	0	0	1.41E+06
SFO	0	0	0	1.93E+06	0	3.08E+06	0	0	1.41E+06	0

Year 2020 bin 10	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	2.21E+06	0	0	0	0	2.08E+06	0	0
DFW	0	0	0	0	0	1.70E+06	0	0	0	0
FLL	2.21E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	3.02E+06	0	0	0	2.46E+06
LAS	0	0	0	0	0	2.60E+06	0	0	0	0
LAX	0	1.70E+06	0	3.02E+06	2.60E+06	0	0	0	1.60E+06	4.07E+06
LGA	0	0	0	0	0	0	0	0	2.16E+06	0
MCO	2.08E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.60E+06	2.16E+06	0	0	1.58E+06
SFO	0	0	0	2.46E+06	0	4.07E+06	0	0	1.58E+06	0

Year 2021 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.07E+06	0	0	0	0	1.13E+06	0	0
DFW	0	0	0	0	0	1.01E+06	0	0	0	0
FLL	1.07E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.03E+06	0	0	0	6.81E+05
LAS	0	0	0	0	0	6.00E+05	0	0	0	0
LAX	0	1.01E+06	0	1.03E+06	6.00E+05	0	0	0	9.53E+05	8.51E+05
LGA	0	0	0	0	0	0	0	0	1.09E+06	0
MCO	1.13E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	9.53E+05	1.09E+06	0	0	8.99E+05
SFO	0	0	0	6.81E+05	0	8.51E+05	0	0	8.99E+05	0

Year 2021 bin 2	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.27E+06	0	0	0	0	1.29E+06	0	0
DFW	0	0	0	0	0	1.14E+06	0	0	0	0
FLL	1.27E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.30E+06	0	0	0	9.11E+05
LAS	0	0	0	0	0	8.32E+05	0	0	0	0
LAX	0	1.14E+06	0	1.30E+06	8.32E+05	0	0	0	1.07E+06	1.21E+06
LGA	0	0	0	0	0	0	0	0	1.26E+06	0
MCO	1.29E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.07E+06	1.26E+06	0	0	1.01E+06
SFO	0	0	0	9.11E+05	0	1.21E+06	0	0	1.01E+06	0

Year 2021 bin 3	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.39E+06	0	0	0	0	1.38E+06	0	0
DFW	0	0	0	0	0	1.21E+06	0	0	0	0
FLL	1.39E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.46E+06	0	0	0	1.05E+06
LAS	0	0	0	0	0	9.81E+05	0	0	0	0
LAX	0	1.21E+06	0	1.46E+06	9.81E+05	0	0	0	1.13E+06	1.47E+06
LGA	0	0	0	0	0	0	0	0	1.37E+06	0
MCO	1.38E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.13E+06	1.37E+06	0	0	1.08E+06
SFO	0	0	0	1.05E+06	0	1.47E+06	0	0	1.08E+06	0

Year 2021 bin 4	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.48E+06	0	0	0	0	1.46E+06	0	0
DFW	0	0	0	0	0	1.26E+06	0	0	0	0
FLL	1.48E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.62E+06	0	0	0	1.18E+06
LAS	0	0	0	0	0	1.12E+06	0	0	0	0
LAX	0	1.26E+06	0	1.62E+06	1.12E+06	0	0	0	1.19E+06	1.69E+06
LGA	0	0	0	0	0	0	0	0	1.46E+06	0
MCO	1.46E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.19E+06	1.46E+06	0	0	1.14E+06
SFO	0	0	0	1.18E+06	0	1.69E+06	0	0	1.14E+06	0

Year 2021 bin 5	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.57E+06	0	0	0	0	1.53E+06	0	0
DFW	0	0	0	0	0	1.32E+06	0	0	0	0
FLL	1.57E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.78E+06	0	0	0	1.31E+06
LAS	0	0	0	0	0	1.27E+06	0	0	0	0
LAX	0	1.32E+06	0	1.78E+06	1.27E+06	0	0	0	1.23E+06	1.91E+06
LGA	0	0	0	0	0	0	0	0	1.54E+06	0
MCO	1.53E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.23E+06	1.54E+06	0	0	1.19E+06
SFO	0	0	0	1.31E+06	0	1.91E+06	0	0	1.19E+06	0

Year 2021 bin 6	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.66E+06	0	0	0	0	1.61E+06	0	0
DFW	0	0	0	0	0	1.37E+06	0	0	0	0
FLL	1.66E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.94E+06	0	0	0	1.45E+06
LAS	0	0	0	0	0	1.42E+06	0	0	0	0
LAX	0	1.37E+06	0	1.94E+06	1.42E+06	0	0	0	1.28E+06	2.15E+06
LGA	0	0	0	0	0	0	0	0	1.63E+06	0
MCO	1.61E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.28E+06	1.63E+06	0	0	1.25E+06
SFO	0	0	0	1.45E+06	0	2.15E+06	0	0	1.25E+06	0

Year 2021 bin 7	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.76E+06	0	0	0	0	1.69E+06	0	0
DFW	0	0	0	0	0	1.43E+06	0	0	0	0
FLL	1.76E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.11E+06	0	0	0	1.60E+06
LAS	0	0	0	0	0	1.60E+06	0	0	0	0
LAX	0	1.43E+06	0	2.11E+06	1.60E+06	0	0	0	1.33E+06	2.41E+06
LGA	0	0	0	0	0	0	0	0	1.71E+06	0
MCO	1.69E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.33E+06	1.71E+06	0	0	1.30E+06
SFO	0	0	0	1.60E+06	0	2.41E+06	0	0	1.30E+06	0

Year 2021 bin 8	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.88E+06	0	0	0	0	1.79E+06	0	0
DFW	0	0	0	0	0	1.49E+06	0	0	0	0
FLL	1.88E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.31E+06	0	0	0	1.78E+06
LAS	0	0	0	0	0	1.80E+06	0	0	0	0
LAX	0	1.49E+06	0	2.31E+06	1.80E+06	0	0	0	1.39E+06	2.75E+06
LGA	0	0	0	0	0	0	0	0	1.82E+06	0
MCO	1.79E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.39E+06	1.82E+06	0	0	1.37E+06
SFO	0	0	0	1.78E+06	0	2.75E+06	0	0	1.37E+06	0

Year 2021 bin 9	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	2.03E+06	0	0	0	0	1.92E+06	0	0
DFW	0	0	0	0	0	1.58E+06	0	0	0	0
FLL	2.03E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.60E+06	0	0	0	2.04E+06
LAS	0	0	0	0	0	2.09E+06	0	0	0	0
LAX	0	1.58E+06	0	2.60E+06	2.09E+06	0	0	0	1.48E+06	3.25E+06
LGA	0	0	0	0	0	0	0	0	1.95E+06	0
MCO	1.92E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.48E+06	1.95E+06	0	0	1.45E+06
SFO	0	0	0	2.04E+06	0	3.25E+06	0	0	1.45E+06	0

Year 2021 bin 10	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	2.40E+06	0	0	0	0	2.20E+06	0	0
DFW	0	0	0	0	0	1.77E+06	0	0	0	0
FLL	2.40E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	3.24E+06	0	0	0	2.67E+06
LAS	0	0	0	0	0	2.82E+06	0	0	0	0
LAX	0	1.77E+06	0	3.24E+06	2.82E+06	0	0	0	1.65E+06	4.41E+06
LGA	0	0	0	0	0	0	0	0	2.26E+06	0
MCO	2.20E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.65E+06	2.26E+06	0	0	1.64E+06
SFO	0	0	0	2.67E+06	0	4.41E+06	0	0	1.64E+06	0

Year 2022 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.09E+06	0	0	0	0	1.14E+06	0	0
DFW	0	0	0	0	0	1.01E+06	0	0	0	0
FLL	1.09E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.02E+06	0	0	0	6.66E+05
LAS	0	0	0	0	0	5.77E+05	0	0	0	0
LAX	0	1.01E+06	0	1.02E+06	5.77E+05	0	0	0	9.44E+05	7.91E+05
LGA	0	0	0	0	0	0	0	0	1.08E+06	0
MCO	1.14E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	9.44E+05	1.08E+06	0	0	8.90E+05
SFO	0	0	0	6.66E+05	0	7.91E+05	0	0	8.90E+05	0

Year 2022 bin 2	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.30E+06	0	0	0	0	1.31E+06	0	0
DFW	0	0	0	0	0	1.15E+06	0	0	0	0
FLL	1.30E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.31E+06	0	0	0	9.16E+05
LAS	0	0	0	0	0	8.19E+05	0	0	0	0
LAX	0	1.15E+06	0	1.31E+06	8.19E+05	0	0	0	1.07E+06	1.17E+06
LGA	0	0	0	0	0	0	0	0	1.27E+06	0
MCO	1.31E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.07E+06	1.27E+06	0	0	1.02E+06
SFO	0	0	0	9.16E+05	0	1.17E+06	0	0	1.02E+06	0

Year 2022 bin 3	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.43E+06	0	0	0	0	1.41E+06	0	0
DFW	0	0	0	0	0	1.22E+06	0	0	0	0
FLL	1.43E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.49E+06	0	0	0	1.07E+06
LAS	0	0	0	0	0	9.75E+05	0	0	0	0
LAX	0	1.22E+06	0	1.49E+06	9.75E+05	0	0	0	1.14E+06	1.43E+06
LGA	0	0	0	0	0	0	0	0	1.38E+06	0
MCO	1.41E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.14E+06	1.38E+06	0	0	1.09E+06
SFO	0	0	0	1.07E+06	0	1.43E+06	0	0	1.09E+06	0

Year 2022 bin 4	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.55E+06	0	0	0	0	1.50E+06	0	0
DFW	0	0	0	0	0	1.28E+06	0	0	0	0
FLL	1.55E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.66E+06	0	0	0	1.20E+06
LAS	0	0	0	0	0	1.13E+06	0	0	0	0
LAX	0	1.28E+06	0	1.66E+06	1.13E+06	0	0	0	1.19E+06	1.67E+06
LGA	0	0	0	0	0	0	0	0	1.48E+06	0
MCO	1.50E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.19E+06	1.48E+06	0	0	1.15E+06
SFO	0	0	0	1.20E+06	0	1.67E+06	0	0	1.15E+06	0

Year 2022 bin 5	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.64E+06	0	0	0	0	1.59E+06	0	0
DFW	0	0	0	0	0	1.35E+06	0	0	0	0
FLL	1.64E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.82E+06	0	0	0	1.35E+06
LAS	0	0	0	0	0	1.28E+06	0	0	0	0
LAX	0	1.35E+06	0	1.82E+06	1.28E+06	0	0	0	1.24E+06	1.91E+06
LGA	0	0	0	0	0	0	0	0	1.57E+06	0
MCO	1.59E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.24E+06	1.57E+06	0	0	1.21E+06
SFO	0	0	0	1.35E+06	0	1.91E+06	0	0	1.21E+06	0

Year 2022 bin 6	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.74E+06	0	0	0	0	1.67E+06	0	0
DFW	0	0	0	0	0	1.40E+06	0	0	0	0
FLL	1.74E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.98E+06	0	0	0	1.49E+06
LAS	0	0	0	0	0	1.45E+06	0	0	0	0
LAX	0	1.40E+06	0	1.98E+06	1.45E+06	0	0	0	1.29E+06	2.16E+06
LGA	0	0	0	0	0	0	0	0	1.66E+06	0
MCO	1.67E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.29E+06	1.66E+06	0	0	1.26E+06
SFO	0	0	0	1.49E+06	0	2.16E+06	0	0	1.26E+06	0

Year 2022 bin 7	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.85E+06	0	0	0	0	1.76E+06	0	0
DFW	0	0	0	0	0	1.47E+06	0	0	0	0
FLL	1.85E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.17E+06	0	0	0	1.66E+06
LAS	0	0	0	0	0	1.63E+06	0	0	0	0
LAX	0	1.47E+06	0	2.17E+06	1.63E+06	0	0	0	1.35E+06	2.46E+06
LGA	0	0	0	0	0	0	0	0	1.76E+06	0
MCO	1.76E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.35E+06	1.76E+06	0	0	1.33E+06
SFO	0	0	0	1.66E+06	0	2.46E+06	0	0	1.33E+06	0

Year 2022 bin 8	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.98E+06	0	0	0	0	1.87E+06	0	0
DFW	0	0	0	0	0	1.54E+06	0	0	0	0
FLL	1.98E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.38E+06	0	0	0	1.87E+06
LAS	0	0	0	0	0	1.88E+06	0	0	0	0
LAX	0	1.54E+06	0	2.38E+06	1.88E+06	0	0	0	1.42E+06	2.83E+06
LGA	0	0	0	0	0	0	0	0	1.87E+06	0
MCO	1.87E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.42E+06	1.87E+06	0	0	1.40E+06
SFO	0	0	0	1.87E+06	0	2.83E+06	0	0	1.40E+06	0

Year 2022 bin 9	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	2.16E+06	0	0	0	0	2.01E+06	0	0
DFW	0	0	0	0	0	1.64E+06	0	0	0	0
FLL	2.16E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.71E+06	0	0	0	2.16E+06
LAS	0	0	0	0	0	2.21E+06	0	0	0	0
LAX	0	1.64E+06	0	2.71E+06	2.21E+06	0	0	0	1.51E+06	3.40E+06
LGA	0	0	0	0	0	0	0	0	2.03E+06	0
MCO	2.01E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.51E+06	2.03E+06	0	0	1.49E+06
SFO	0	0	0	2.16E+06	0	3.40E+06	0	0	1.49E+06	0

Year 2022 bin 10	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	2.58E+06	0	0	0	0	2.33E+06	0	0
DFW	0	0	0	0	0	1.84E+06	0	0	0	0
FLL	2.58E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	3.45E+06	0	0	0	2.88E+06
LAS	0	0	0	0	0	3.09E+06	0	0	0	0
LAX	0	1.84E+06	0	3.45E+06	3.09E+06	0	0	0	1.69E+06	4.74E+06
LGA	0	0	0	0	0	0	0	0	2.37E+06	0
MCO	2.33E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.69E+06	2.37E+06	0	0	1.70E+06
SFO	0	0	0	2.88E+06	0	4.74E+06	0	0	1.70E+06	0

Year 2023 bin 1	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.12E+06	0	0	0	0	1.16E+06	0	0
DFW	0	0	0	0	0	1.01E+06	0	0	0	0
FLL	1.12E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.00E+06	0	0	0	6.57E+05
LAS	0	0	0	0	0	5.53E+05	0	0	0	0
LAX	0	1.01E+06	0	1.00E+06	5.53E+05	0	0	0	9.36E+05	7.40E+05
LGA	0	0	0	0	0	0	0	0	1.08E+06	0
MCO	1.16E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	9.36E+05	1.08E+06	0	0	8.85E+05
SFO	0	0	0	6.57E+05	0	7.40E+05	0	0	8.85E+05	0

Year 2023 bin 2	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.35E+06	0	0	0	0	1.34E+06	0	0
DFW	0	0	0	0	0	1.16E+06	0	0	0	0
FLL	1.35E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.31E+06	0	0	0	9.19E+05
LAS	0	0	0	0	0	8.08E+05	0	0	0	0
LAX	0	1.16E+06	0	1.31E+06	8.08E+05	0	0	0	1.06E+06	1.13E+06
LGA	0	0	0	0	0	0	0	0	1.28E+06	0
MCO	1.34E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.06E+06	1.28E+06	0	0	1.02E+06
SFO	0	0	0	9.19E+05	0	1.13E+06	0	0	1.02E+06	0

Year 2023 bin 3	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.49E+06	0	0	0	0	1.45E+06	0	0
DFW	0	0	0	0	0	1.24E+06	0	0	0	0
FLL	1.49E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.50E+06	0	0	0	1.08E+06
LAS	0	0	0	0	0	9.77E+05	0	0	0	0
LAX	0	1.24E+06	0	1.50E+06	9.77E+05	0	0	0	1.14E+06	1.39E+06
LGA	0	0	0	0	0	0	0	0	1.40E+06	0
MCO	1.45E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.14E+06	1.40E+06	0	0	1.10E+06
SFO	0	0	0	1.08E+06	0	1.39E+06	0	0	1.10E+06	0

Year 2023 bin 4	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.61E+06	0	0	0	0	1.55E+06	0	0
DFW	0	0	0	0	0	1.30E+06	0	0	0	0
FLL	1.61E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.68E+06	0	0	0	1.23E+06
LAS	0	0	0	0	0	1.14E+06	0	0	0	0
LAX	0	1.30E+06	0	1.68E+06	1.14E+06	0	0	0	1.19E+06	1.65E+06
LGA	0	0	0	0	0	0	0	0	1.51E+06	0
MCO	1.55E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.19E+06	1.51E+06	0	0	1.16E+06
SFO	0	0	0	1.23E+06	0	1.65E+06	0	0	1.16E+06	0

Year 2023 bin 5	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.72E+06	0	0	0	0	1.63E+06	0	0
DFW	0	0	0	0	0	1.37E+06	0	0	0	0
FLL	1.72E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	1.85E+06	0	0	0	1.37E+06
LAS	0	0	0	0	0	1.31E+06	0	0	0	0
LAX	0	1.37E+06	0	1.85E+06	1.31E+06	0	0	0	1.25E+06	1.91E+06
LGA	0	0	0	0	0	0	0	0	1.61E+06	0
MCO	1.63E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.25E+06	1.61E+06	0	0	1.22E+06
SFO	0	0	0	1.37E+06	0	1.91E+06	0	0	1.22E+06	0

Year 2023 bin 6	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.83E+06	0	0	0	0	1.73E+06	0	0
DFW	0	0	0	0	0	1.43E+06	0	0	0	0
FLL	1.83E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.03E+06	0	0	0	1.53E+06
LAS	0	0	0	0	0	1.48E+06	0	0	0	0
LAX	0	1.43E+06	0	2.03E+06	1.48E+06	0	0	0	1.30E+06	2.18E+06
LGA	0	0	0	0	0	0	0	0	1.71E+06	0
MCO	1.73E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.30E+06	1.71E+06	0	0	1.28E+06
SFO	0	0	0	1.53E+06	0	2.18E+06	0	0	1.28E+06	0

Year 2023 bin 7	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	1.95E+06	0	0	0	0	1.83E+06	0	0
DFW	0	0	0	0	0	1.50E+06	0	0	0	0
FLL	1.95E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.23E+06	0	0	0	1.72E+06
LAS	0	0	0	0	0	1.68E+06	0	0	0	0
LAX	0	1.50E+06	0	2.23E+06	1.68E+06	0	0	0	1.37E+06	2.49E+06
LGA	0	0	0	0	0	0	0	0	1.81E+06	0
MCO	1.83E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.37E+06	1.81E+06	0	0	1.35E+06
SFO	0	0	0	1.72E+06	0	2.49E+06	0	0	1.35E+06	0

Year 2023 bin 8	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	2.10E+06	0	0	0	0	1.95E+06	0	0
DFW	0	0	0	0	0	1.58E+06	0	0	0	0
FLL	2.10E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.47E+06	0	0	0	1.96E+06
LAS	0	0	0	0	0	1.94E+06	0	0	0	0
LAX	0	1.58E+06	0	2.47E+06	1.94E+06	0	0	0	1.44E+06	2.89E+06
LGA	0	0	0	0	0	0	0	0	1.94E+06	0
MCO	1.95E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.44E+06	1.94E+06	0	0	1.43E+06
SFO	0	0	0	1.96E+06	0	2.89E+06	0	0	1.43E+06	0

Year 2023 bin 9	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	2.31E+06	0	0	0	0	2.11E+06	0	0
DFW	0	0	0	0	0	1.69E+06	0	0	0	0
FLL	2.31E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	2.82E+06	0	0	0	2.30E+06
LAS	0	0	0	0	0	2.32E+06	0	0	0	0
LAX	0	1.69E+06	0	2.82E+06	2.32E+06	0	0	0	1.54E+06	3.51E+06
LGA	0	0	0	0	0	0	0	0	2.10E+06	0
MCO	2.11E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.54E+06	2.10E+06	0	0	1.53E+06
SFO	0	0	0	2.30E+06	0	3.51E+06	0	0	1.53E+06	0

Year 2023 bin 10	ATL	DFW	FLL	JFK	LAS	LAX	LGA	MCO	ORD	SFO
ATL	0	0	2.78E+06	0	0	0	0	2.46E+06	0	0
DFW	0	0	0	0	0	1.91E+06	0	0	0	0
FLL	2.78E+06	0	0	0	0	0	0	0	0	0
JFK	0	0	0	0	0	3.68E+06	0	0	0	3.09E+06
LAS	0	0	0	0	0	3.34E+06	0	0	0	0
LAX	0	1.91E+06	0	3.68E+06	3.34E+06	0	0	0	1.75E+06	5.08E+06
LGA	0	0	0	0	0	0	0	0	2.48E+06	0
MCO	2.46E+06	0	0	0	0	0	0	0	0	0
ORD	0	0	0	0	0	1.75E+06	2.48E+06	0	0	1.76E+06
SFO	0	0	0	3.09E+06	0	5.08E+06	0	0	1.76E+06	0

B.4 Validation

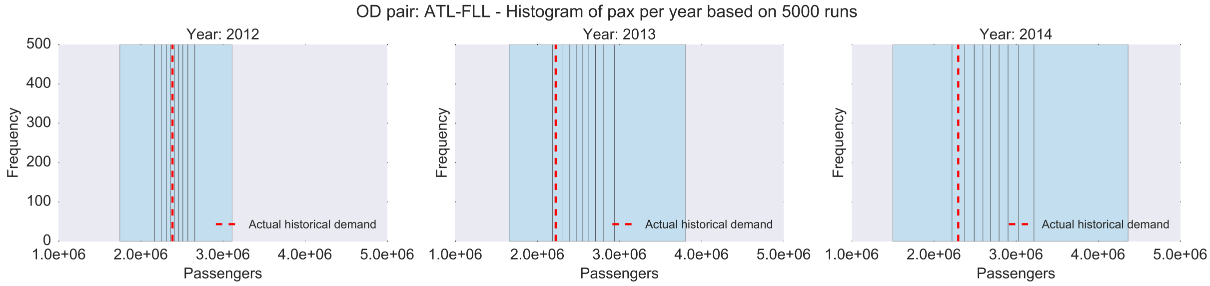


Figure B.12: Validation of the mean reverting process using historical data - OD pair: ATL-FLL

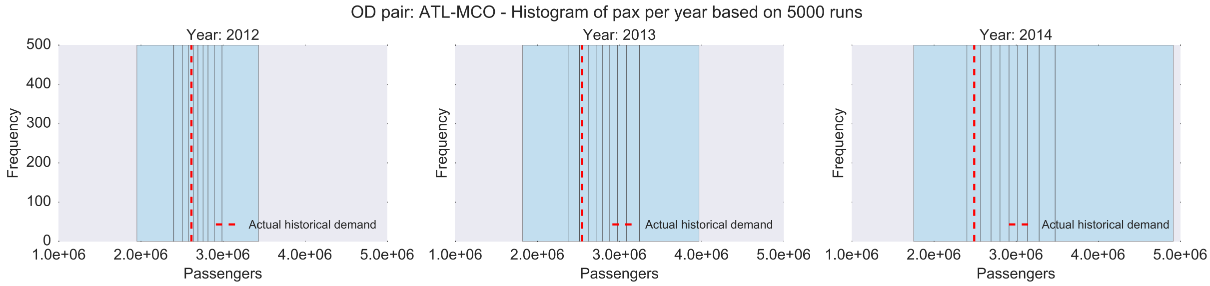


Figure B.13: Validation of the mean reverting process using historical data - OD pair: ATL-MCO

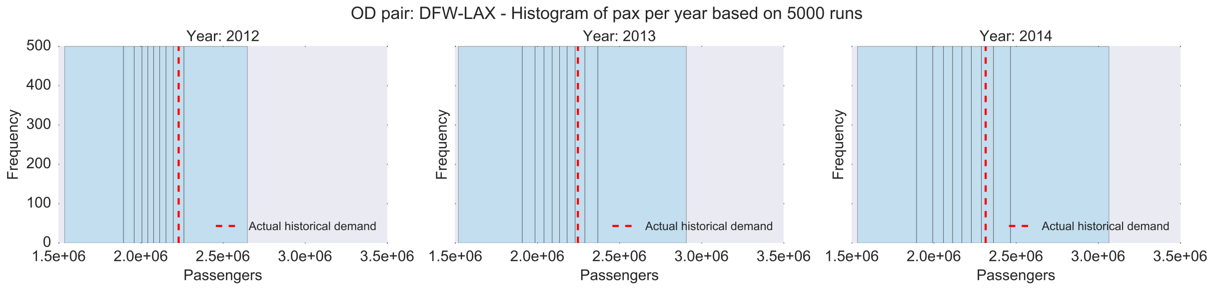


Figure B.14: Validation of the mean reverting process using historical data - OD pair: DFW-LAX

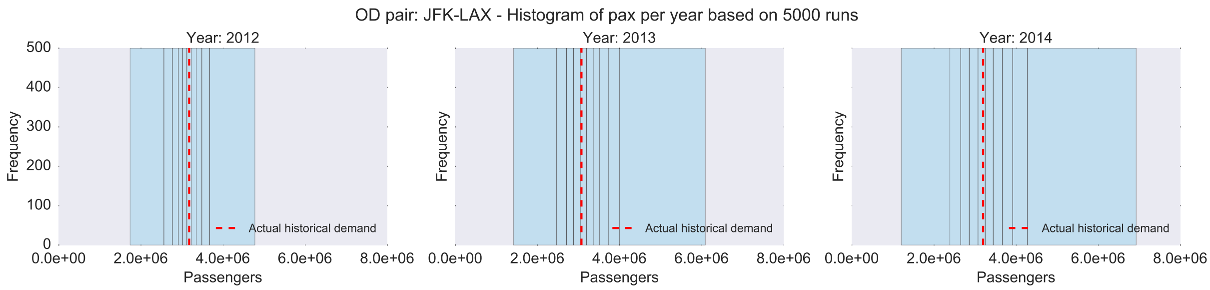


Figure B.15: Validation of the mean reverting process using historical data - OD pair: JFK-LAX

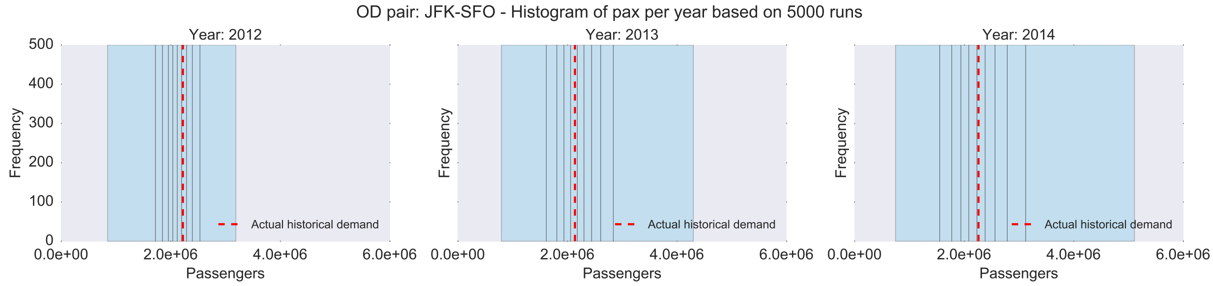


Figure B.16: Validation of the mean reverting process using historical data - OD pair: JFK-SFO

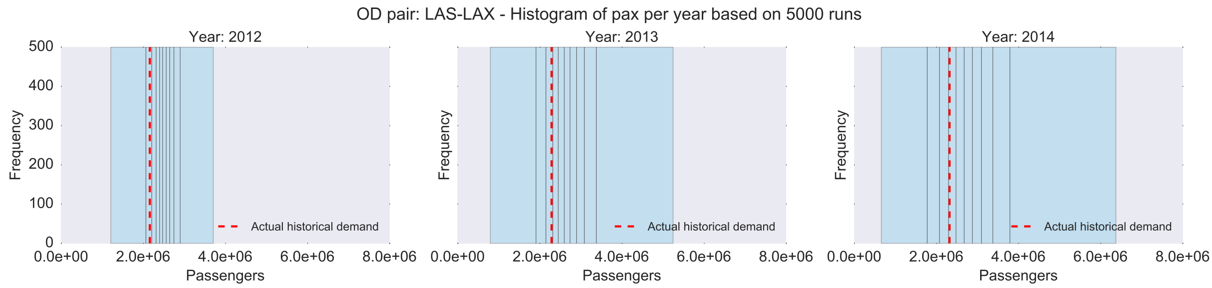


Figure B.17: Validation of the mean reverting process using historical data - OD pair: LAS-LAX

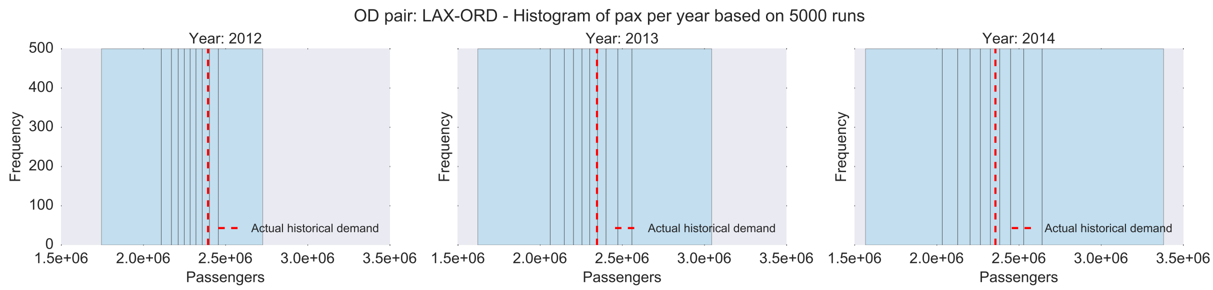


Figure B.18: Validation of the mean reverting process using historical data - OD pair: LAX-ORD

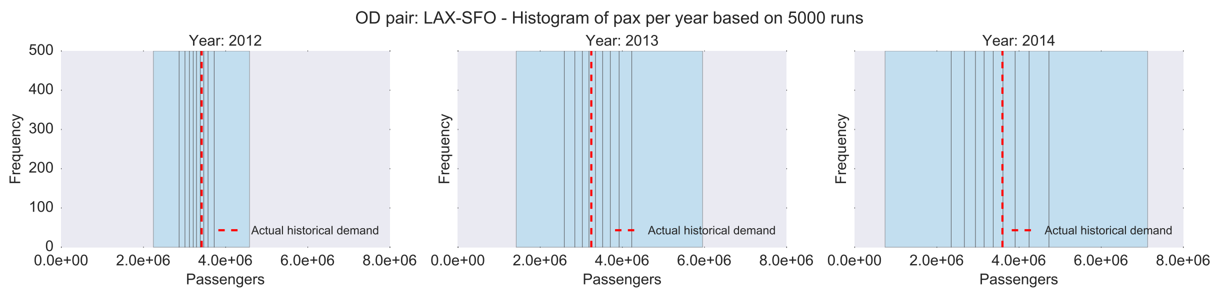


Figure B.19: Validation of the mean reverting process using historical data - OD pair: LAX-SFO

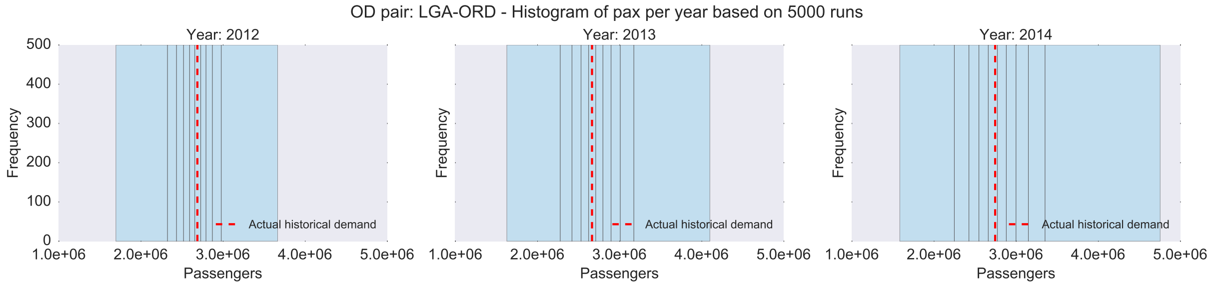


Figure B.20: Validation of the mean reverting process using historical data - OD pair: LGA-ORD

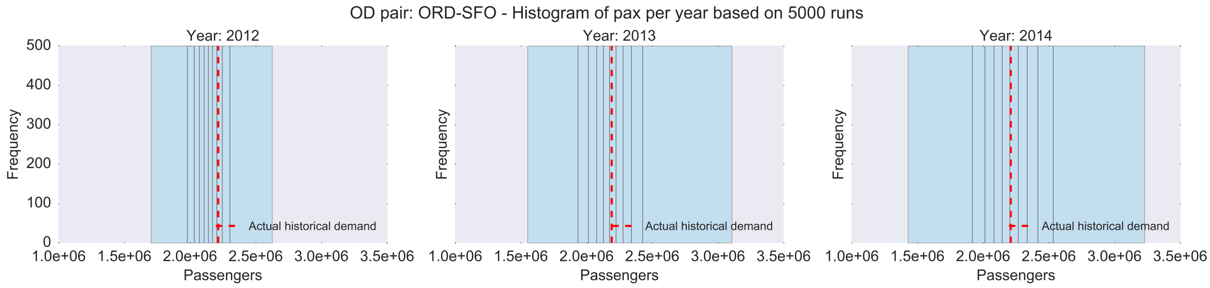


Figure B.21: Validation of the mean reverting process using historical data - OD pair: ORD-SFO

C

Fleet assignment optimization model results

C.1 The mathematical formulation of the optimization model at a glance

Sets

N	Set of airports
K	Set of aircraft types
H	Set of hubs

Index

o,d	Indices used for origin and destination airports (for passengers)
i,j	Indices used for departure and arrival airports (for aircraft)
h	Hub index
k	Aircraft type index

Parameters

$Q_{o,d}$	demand between airports o and d
DL_o	demand leaving from origin airport o
DA_d	demand arriving in destination airport d
$D_{o,d}$	distance between airports o and d
$yield_{o,d}$	yield per route for nonstop connections
$yield_{o,d}^h$	yield per route for connections through hub h
AC^k	number of aircraft of aircraft type k in the fleet
U^k	aircraft utilization per aircraft type k
C_{fix}^k	aircraft ownership cost per aircraft type k
C_{var}^k	aircraft operating cost per aircraft type k (i.e. CASM)
s^k	number of seats per aircraft type k
vc^k	cruise speed per aircraft type k
T_i^{dep}	and T_j^{arr} taxi time per departure and arrival airport, respectively
$range^k$	range per a/c type k

Decision variables

$x_{o,d}$	Nonstop passenger flow between origin airport o and destination airport d
$w_{o,d}^h$	Connecting passenger flow for passengers that are in the segment between the origin airport o and the hub h irrespective of their final destination airport d
$y_{o,h}$	Connecting passenger flow for passengers that originate from airport o and are in the segment between the hub h and the final destination airport d
$z_{i,j}^k$	Number of flights (i.e. flight frequency) between airport i and airport j operated by aircraft type k operated by aircraft type k

Objective function

$$\text{Maximize profit} = \sum_{o\in N}\sum_{d\in N}\left[yield_{o,d}\cdot D_{o,d}\cdot x_{o,d}\right] + \sum_{o\in N}\sum_{d\in N}\sum_{h\in H}\left[yield^h_{o,d}\cdot D_{o,d}\cdot w^h_{o,d}\right] - \sum_{k\in K}\left[AC^k\cdot C^k_{fix}\right] - \sum_{i\in N}\sum_{j\in N}\sum_{k\in K}\left[C^k_{var}\cdot D_{i,j}\cdot s^k\cdot z^k_{i,j}\right] \quad \text{(C.1)}$$

Subject to

$$\sum_{d\in N} x_{o,d} + \sum_{d\in N}\sum_{h\in H} w^h_{o,d} \leq DL_o \qquad \forall\ o\in N \quad \text{(C.2)}$$

$$\sum_{o\in N} x_{o,d} + \sum_{o\in N}\sum_{h\in H} w^h_{o,d} \leq DA_d \qquad \forall\ d\in N \quad \text{(C.3)}$$

$$x_{o,d} + \sum_{h\in H} w^h_{o,d} \leq Q_{o,d} \qquad \forall\ o,d\in N, o\neq d \quad \text{(C.4)}$$

$$y_{i,j} = 0 \qquad \forall\ i\in N, j\in N\backslash H, i\neq j \text{ (if j is not a hub)} \quad \text{(C.5)}$$

$$x_{i,j} \leq \sum_{k\in K} z^k_{i,j}\cdot s^k \qquad \forall\ i,j\in N\backslash H, i\neq j \text{ (if neither i or j is a hub)} \quad \text{(C.6a)}$$

$$x_{i,j} + \sum_{\substack{o\in N\\ o\neq i,j}} w^i_{o,j} \leq \sum_{k\in K} z^k_{i,j}\cdot s^k \qquad \forall\ i\in H, j\in N\backslash H, i\neq j \text{ (if i is hub)} \quad \text{(C.6b)}$$

$$x_{i,j} + y_{i,j} \leq \sum_{k\in K} z^k_{i,j}\cdot s^k \qquad \forall\ j\in H, i\in N\backslash H, i\neq j \text{ (if j is hub)} \quad \text{(C.6c)}$$

$$x_{i,j} + \sum_{\substack{o\in N\\ o\neq i,j}} w^i_{o,j} + y_{i,j} \leq \sum_{k\in K} z^k_{i,j}\cdot s^k \qquad \forall\ i,j\in H, i\neq j \text{ (if both i and j are hubs)} \quad \text{(C.6d)}$$

$$y_{o,h} = \sum_{d\in N} w^h_{o,d} \qquad \forall\ o\in N\backslash H, h\in H \quad \text{(C.7)}$$

$$\sum_{j\in N} z^k_{j,i} = \sum_{j\in N} z^k_{i,j} \qquad \forall\ i\in N, k\in K \quad \text{(C.8)}$$

$$\sum_{i\in N}\sum_{j\in N} z^k_{i,j}\cdot\left[\frac{D_{i,j}}{vc^k} + T^{dep}_i + T^{arr}_j + TAT^k\right] \leq AC^k\cdot U^k \qquad \forall k\in K \quad \text{(C.9)}$$

$$z^k_{i,j} = 0 \qquad \forall i\in N, j\in N, i\neq j, k\in K \text{ if } range^k < D_{i,j} \quad \text{(C.10)}$$

$$x_{o,d}\in\mathbb{Z}^+,\ y_{o,h}\in\mathbb{Z}^+,\ w^h_{o,d}\in\mathbb{Z}^+,\ z^k_{i,j}\in\mathbb{Z}^+ \quad \text{(C.11)}$$

C.2 The value matrix

Table C.1: The value matrix with annual operating profits for each of the 8 fleets in the portfolio for each of the 9 years in the planning horizon for each of the 10 OD demand matrices per year

Run	Fleet	Bin	Year	Annual operating profit	Run	Fleet	Bin	Year	Annual operating profit	Run	Fleet	Bin	Year	Annual operating profit
1	1	1	2015	$43,363,869	**81**	1	9	2023	$56,840,445	**161**	2	8	2022	$92,105,000
2	1	1	2016	$43,196,298	**82**	1	10	2015	$48,050,047	**162**	2	8	2023	$94,815,265
3	1	1	2017	$43,322,341	**83**	1	10	2016	$49,306,059	**163**	2	9	2015	$75,817,889
4	1	1	2018	$43,708,668	**84**	1	10	2017	$50,536,331	**164**	2	9	2016	$78,508,443
5	1	1	2019	$44,132,722	**85**	1	10	2018	$51,832,530	**165**	2	9	2017	$81,307,660
6	1	1	2020	$44,566,023	**86**	1	10	2019	$52,932,723	**166**	2	9	2018	$83,989,269
7	1	1	2021	$45,335,146	**87**	1	10	2020	$54,120,695	**167**	2	9	2019	$86,955,030
8	1	1	2022	$45,993,932	**88**	1	10	2021	$55,306,909	**168**	2	9	2020	$89,635,706
9	1	1	2023	$46,794,355	**89**	1	10	2022	$56,459,857	**169**	2	9	2021	$92,382,074
10	1	2	2015	$44,452,620	**90**	1	10	2023	$57,706,787	**170**	2	9	2022	$95,457,679
11	1	2	2016	$44,786,350	**91**	2	1	2015	$69,231,354	**171**	2	9	2023	$98,613,812
12	1	2	2017	$45,274,171	**92**	2	1	2016	$66,986,372	**172**	2	10	2015	$76,856,708
13	1	2	2018	$45,941,563	**93**	2	1	2017	$65,605,865	**173**	2	10	2016	$80,538,428
14	1	2	2019	$46,657,094	**94**	2	1	2018	$64,782,888	**174**	2	10	2017	$84,224,935
15	1	2	2020	$47,467,387	**95**	2	1	2019	$63,929,348	**175**	2	10	2018	$87,751,852
16	1	2	2021	$48,453,353	**96**	2	1	2020	$63,270,652	**176**	2	10	2019	$91,157,739
17	1	2	2022	$49,403,836	**97**	2	1	2021	$63,195,998	**177**	2	10	2020	$95,020,631
18	1	2	2023	$50,221,407	**98**	2	1	2022	$63,216,005	**178**	2	10	2021	$99,016,182
19	1	3	2015	$44,994,844	**99**	2	1	2023	$63,473,166	**179**	2	10	2022	$102,784,509
20	1	3	2016	$45,609,715	**100**	2	2	2015	$72,493,786	**180**	2	10	2023	$106,810,562
21	1	3	2017	$46,407,083	**101**	2	2	2016	$72,452,365	**181**	3	1	2015	$68,043,959
22	1	3	2018	$47,240,731	**102**	2	2	2017	$72,093,669	**182**	3	1	2016	$63,206,657
23	1	3	2019	$48,144,237	**103**	2	2	2018	$72,501,525	**183**	3	1	2017	$59,885,183
24	1	3	2020	$49,177,627	**104**	2	2	2019	$73,283,243	**184**	3	1	2018	$57,671,728
25	1	3	2021	$50,117,295	**105**	2	2	2020	$74,009,458	**185**	3	1	2019	$56,400,908
26	1	3	2022	$51,293,122	**106**	2	2	2021	$74,985,164	**186**	3	1	2020	$55,254,217
27	1	3	2023	$52,494,559	**107**	2	2	2022	$76,057,268	**187**	3	1	2021	$55,047,564
28	1	4	2015	$45,422,414	**108**	2	2	2023	$77,181,165	**188**	3	1	2022	$54,743,407
29	1	4	2016	$46,355,307	**109**	2	3	2015	$73,219,971	**189**	3	1	2023	$54,921,387
30	1	4	2017	$47,297,324	**110**	2	3	2016	$74,116,641	**190**	3	2	2015	$74,794,911
31	1	4	2018	$48,382,946	**111**	2	3	2017	$75,310,986	**191**	3	2	2016	$73,203,090
32	1	4	2019	$49,423,085	**112**	2	3	2018	$76,578,845	**192**	3	2	2017	$72,242,057
33	1	4	2020	$50,559,686	**113**	2	3	2019	$77,814,316	**193**	3	2	2018	$72,102,072
34	1	4	2021	$51,852,664	**114**	2	3	2020	$79,223,109	**194**	3	2	2019	$72,641,974
35	1	4	2022	$52,779,065	**115**	2	3	2021	$80,682,759	**195**	3	2	2020	$73,183,294
36	1	4	2023	$54,007,376	**116**	2	3	2022	$82,348,576	**196**	3	2	2021	$74,014,217
37	1	5	2015	$45,838,252	**117**	2	3	2023	$83,936,884	**197**	3	2	2022	$75,108,706
38	1	5	2016	$47,004,702	**118**	2	4	2015	$73,500,055	**198**	3	2	2023	$76,454,899
39	1	5	2017	$48,004,435	**119**	2	4	2016	$74,937,154	**199**	3	3	2015	$78,169,606
40	1	5	2018	$49,274,255	**120**	2	4	2017	$76,231,937	**200**	3	3	2016	$78,609,374
41	1	5	2019	$50,548,970	**121**	2	4	2018	$77,761,332	**201**	3	3	2017	$79,277,682
42	1	5	2020	$51,683,840	**122**	2	4	2019	$79,284,358	**202**	3	3	2018	$80,504,808
43	1	5	2021	$52,876,289	**123**	2	4	2020	$80,909,162	**203**	3	3	2019	$81,919,962
44	1	5	2022	$53,913,379	**124**	2	4	2021	$82,604,176	**204**	3	3	2020	$83,319,420
45	1	5	2023	$54,845,882	**125**	2	4	2022	$84,213,193	**205**	3	3	2021	$85,659,578
46	1	6	2015	$46,421,748	**126**	2	4	2023	$86,046,326	**206**	3	3	2022	$87,757,276
47	1	6	2016	$47,596,951	**127**	2	5	2015	$74,013,713	**207**	3	3	2023	$90,193,087
48	1	6	2017	$48,762,196	**128**	2	5	2016	$75,601,625	**208**	3	4	2015	$80,894,179
49	1	6	2018	$50,091,290	**129**	2	5	2017	$77,068,060	**209**	3	4	2016	$82,917,466
50	1	6	2019	$51,428,150	**130**	2	5	2018	$78,808,233	**210**	3	4	2017	$84,878,582
51	1	6	2020	$52,317,967	**131**	2	5	2019	$80,614,120	**211**	3	4	2018	$87,392,974
52	1	6	2021	$53,236,799	**132**	2	5	2020	$82,242,044	**212**	3	4	2019	$89,863,184
53	1	6	2022	$54,239,343	**133**	2	5	2021	$84,085,488	**213**	3	4	2020	$92,293,353
54	1	6	2023	$55,466,948	**134**	2	5	2022	$85,896,344	**214**	3	4	2021	$95,361,093
55	1	7	2015	$46,762,746	**135**	2	5	2023	$87,777,030	**215**	3	4	2022	$98,549,051
56	1	7	2016	$48,050,663	**136**	2	6	2015	$74,511,458	**216**	3	4	2023	$101,917,909
57	1	7	2017	$49,502,360	**137**	2	6	2016	$76,251,020	**217**	3	5	2015	$83,499,656
58	1	7	2018	$50,668,128	**138**	2	6	2017	$78,065,955	**218**	3	5	2016	$86,711,619
59	1	7	2019	$51,544,795	**139**	2	6	2018	$79,847,108	**219**	3	5	2017	$90,007,633
60	1	7	2020	$52,450,049	**140**	2	6	2019	$81,724,899	**220**	3	5	2018	$93,619,203
61	1	7	2021	$53,637,420	**141**	2	6	2020	$83,734,683	**221**	3	5	2019	$97,046,203
62	1	7	2022	$54,891,185	**142**	2	6	2021	$85,753,308	**222**	3	5	2020	$100,547,904
63	1	7	2023	$55,838,355	**143**	2	6	2022	$87,828,705	**223**	3	5	2021	$103,366,301
64	1	8	2015	$47,040,443	**144**	2	6	2023	$89,796,603	**224**	3	5	2022	$105,988,213
65	1	8	2016	$48,652,177	**145**	2	7	2015	$74,810,124	**225**	3	5	2023	$108,908,915
66	1	8	2017	$49,919,338	**146**	2	7	2016	$76,887,098	**226**	3	6	2015	$85,691,867
67	1	8	2018	$50,783,049	**147**	2	7	2017	$78,823,716	**227**	3	6	2016	$90,748,419
68	1	8	2019	$51,674,925	**148**	2	7	2018	$80,849,028	**228**	3	6	2017	$95,092,187
69	1	8	2020	$53,043,985	**149**	2	7	2019	$83,013,673	**229**	3	6	2018	$98,778,646
70	1	8	2021	$54,200,156	**150**	2	7	2020	$85,157,271	**230**	3	6	2019	$101,936,580
71	1	8	2022	$55,149,232	**151**	2	7	2021	$87,543,340	**231**	3	6	2020	$105,408,679
72	1	8	2023	$56,269,576	**152**	2	7	2022	$89,821,672	**232**	3	6	2021	$109,065,415
73	1	9	2015	$47,325,725	**153**	2	7	2023	$92,172,258	**233**	3	6	2022	$112,402,910
74	1	9	2016	$49,070,065	**154**	2	8	2015	$75,281,982	**234**	3	6	2023	$115,947,027
75	1	9	2017	$50,032,561	**155**	2	8	2016	$77,605,316	**235**	3	7	2015	$88,061,018
76	1	9	2018	$50,986,550	**156**	2	8	2017	$79,828,192	**236**	3	7	2016	$94,681,168
77	1	9	2019	$52,376,729	**157**	2	8	2018	$82,254,525	**237**	3	7	2017	$98,681,544
78	1	9	2020	$53,399,168	**158**	2	8	2019	$84,615,471	**238**	3	7	2018	$102,559,115
79	1	9	2021	$54,412,444	**159**	2	8	2020	$87,060,916	**239**	3	7	2019	$106,689,892
80	1	9	2022	$55,510,300	**160**	2	8	2021	$89,558,423	**240**	3	7	2020	$110,821,715

Run	Fleet	Bin	Year	Annual operating profit	Run	Fleet	Bin	Year	Annual operating profit	Run	Fleet	Bin	Year	Annual operating profit
241	3	7	2021	$115,288,282	**321**	4	6	2020	$102,574,868	**401**	5	5	2019	$79,263,090
242	3	7	2022	$118,995,570	**322**	4	6	2021	$107,945,327	**402**	5	5	2020	$83,219,398
243	3	7	2023	$123,257,184	**323**	4	6	2022	$113,132,015	**403**	5	5	2021	$87,551,195
244	3	8	2015	$90,774,859	**324**	4	6	2023	$118,606,786	**404**	5	5	2022	$91,852,054
245	3	8	2016	$97,628,482	**325**	4	7	2015	$80,988,134	**405**	5	5	2023	$96,284,242
246	3	8	2017	$102,451,704	**326**	4	7	2016	$87,634,551	**406**	5	6	2015	$67,861,936
247	3	8	2018	$107,198,827	**327**	4	7	2017	$93,723,678	**407**	5	6	2016	$73,066,677
248	3	8	2019	$112,141,947	**328**	4	7	2018	$99,689,903	**408**	5	6	2017	$77,932,507
249	3	8	2020	$116,975,874	**329**	4	7	2019	$105,904,780	**409**	5	6	2018	$82,738,078
250	3	8	2021	$120,228,245	**330**	4	7	2020	$112,587,381	**410**	5	6	2019	$88,052,075
251	3	8	2022	$123,151,588	**331**	4	7	2021	$119,356,966	**411**	5	6	2020	$93,495,020
252	3	8	2023	$126,514,905	**332**	4	7	2022	$125,397,673	**412**	5	6	2021	$99,225,506
253	3	9	2015	$93,912,057	**333**	4	7	2023	$132,606,845	**413**	5	6	2022	$104,893,936
254	3	9	2016	$100,782,346	**334**	4	8	2015	$83,759,019	**414**	5	6	2023	$110,412,566
255	3	9	2017	$106,899,215	**335**	4	8	2016	$92,553,097	**415**	5	7	2015	$70,486,685
256	3	9	2018	$113,085,689	**336**	4	8	2017	$100,457,881	**416**	5	7	2016	$77,910,345
257	3	9	2019	$116,365,952	**337**	4	8	2018	$108,365,102	**417**	5	7	2017	$84,335,970
258	3	9	2020	$119,718,643	**338**	4	8	2019	$116,172,674	**418**	5	7	2018	$90,902,942
259	3	9	2021	$123,083,272	**339**	4	8	2020	$124,093,714	**419**	5	7	2019	$97,609,734
260	3	9	2022	$126,615,442	**340**	4	8	2021	$132,611,123	**420**	5	7	2020	$104,715,832
261	3	9	2023	$130,292,064	**341**	4	8	2022	$139,543,036	**421**	5	7	2021	$111,535,239
262	3	10	2015	$97,591,215	**342**	4	8	2023	$145,016,438	**422**	5	7	2022	$117,439,219
263	3	10	2016	$107,457,920	**343**	4	9	2015	$87,202,583	**423**	5	7	2023	$124,717,508
264	3	10	2017	$112,956,394	**344**	4	9	2016	$98,639,524	**424**	5	8	2015	$73,758,759
265	3	10	2018	$117,133,576	**345**	4	9	2017	$109,198,756	**425**	5	8	2016	$83,342,717
266	3	10	2019	$121,198,234	**346**	4	9	2018	$119,680,202	**426**	5	8	2017	$91,994,739
267	3	10	2020	$125,369,176	**347**	4	9	2019	$130,374,671	**427**	5	8	2018	$100,793,525
268	3	10	2021	$129,713,985	**348**	4	9	2020	$137,306,594	**428**	5	8	2019	$108,513,413
269	3	10	2022	$134,446,317	**349**	4	9	2021	$143,965,406	**429**	5	8	2020	$116,595,037
270	3	10	2023	$139,350,545	**350**	4	9	2022	$151,117,202	**430**	5	8	2021	$125,036,345
271	4	1	2015	$58,632,805	**351**	4	9	2023	$158,340,463	**431**	5	8	2022	$133,608,102
272	4	1	2016	$52,986,417	**352**	4	10	2015	$93,987,203	**432**	5	8	2023	$141,992,335
273	4	1	2017	$49,014,995	**353**	4	10	2016	$111,149,269	**433**	5	9	2015	$77,661,264
274	4	1	2018	$46,280,747	**354**	4	10	2017	$127,504,498	**434**	5	9	2016	$90,317,391
275	4	1	2019	$44,663,787	**355**	4	10	2018	$137,911,912	**435**	5	9	2017	$101,668,408
276	4	1	2020	$43,204,303	**356**	4	10	2019	$147,278,521	**436**	5	9	2018	$112,438,241
277	4	1	2021	$42,775,111	**357**	4	10	2020	$155,429,198	**437**	5	9	2019	$122,941,694
278	4	1	2022	$42,359,370	**358**	4	10	2021	$160,348,441	**438**	5	9	2020	$133,056,287
279	4	1	2023	$42,315,915	**359**	4	10	2022	$165,432,152	**439**	5	9	2021	$143,518,269
280	4	2	2015	$66,212,074	**360**	4	10	2023	$171,101,594	**440**	5	9	2022	$155,230,839
281	4	2	2016	$64,314,165	**361**	5	1	2015	$47,474,663	**441**	5	9	2023	$167,237,143
282	4	2	2017	$63,033,896	**362**	5	1	2016	$41,575,170	**442**	5	10	2015	$85,353,883
283	4	2	2018	$62,677,661	**363**	5	1	2017	$37,370,731	**443**	5	10	2016	$103,830,753
284	4	2	2019	$63,057,350	**364**	5	1	2018	$34,443,264	**444**	5	10	2017	$120,300,130
285	4	2	2020	$63,184,910	**365**	5	1	2019	$32,645,057	**445**	5	10	2018	$136,755,164
286	4	2	2021	$63,919,388	**366**	5	1	2020	$30,934,010	**446**	5	10	2019	$152,835,755
287	4	2	2022	$65,247,017	**367**	5	1	2021	$30,417,723	**447**	5	10	2020	$169,123,768
288	4	2	2023	$66,493,480	**368**	5	1	2022	$29,748,168	**448**	5	10	2021	$179,615,988
289	4	3	2015	$70,203,914	**369**	5	1	2023	$29,527,826	**449**	5	10	2022	$190,697,755
290	4	3	2016	$70,309,220	**370**	5	2	2015	$55,180,471	**450**	5	10	2023	$202,317,857
291	4	3	2017	$70,953,497	**371**	5	2	2016	$53,008,389	**451**	6	1	2015	$60,637,096
292	4	3	2018	$72,135,570	**372**	5	2	2017	$51,478,217	**452**	6	1	2016	$57,507,552
293	4	3	2019	$73,539,885	**373**	5	2	2018	$50,952,791	**453**	6	1	2017	$55,079,165
294	4	3	2020	$74,811,303	**374**	5	2	2019	$51,185,514	**454**	6	1	2018	$52,873,878
295	4	3	2021	$77,134,619	**375**	5	2	2020	$51,129,444	**455**	6	1	2019	$51,717,033
296	4	3	2022	$79,377,320	**376**	5	2	2021	$51,739,043	**456**	6	1	2020	$50,707,471
297	4	3	2023	$81,830,588	**377**	5	2	2022	$52,742,899	**457**	6	1	2021	$50,552,411
298	4	4	2015	$73,383,590	**378**	5	2	2023	$53,921,995	**458**	6	1	2022	$50,194,672
299	4	4	2016	$75,521,211	**379**	5	3	2015	$59,072,296	**459**	6	1	2023	$50,335,597
300	4	4	2017	$77,289,953	**380**	5	3	2016	$59,124,630	**460**	6	2	2015	$64,618,752
301	4	4	2018	$79,826,097	**381**	5	3	2017	$59,641,667	**461**	6	2	2016	$63,668,693
302	4	4	2019	$82,111,494	**382**	5	3	2018	$60,526,639	**462**	6	2	2017	$63,158,149
303	4	4	2020	$84,574,961	**383**	5	3	2019	$61,781,874	**463**	6	2	2018	$63,245,906
304	4	4	2021	$87,738,276	**384**	5	3	2020	$62,927,685	**464**	6	2	2019	$63,608,204
305	4	4	2022	$90,863,597	**385**	5	3	2021	$64,972,518	**465**	6	2	2020	$63,859,904
306	4	4	2023	$94,020,678	**386**	5	3	2022	$67,089,975	**466**	6	2	2021	$64,444,741
307	4	5	2015	$76,303,371	**387**	5	3	2023	$69,291,316	**467**	6	2	2022	$65,302,717
308	4	5	2016	$79,408,436	**388**	5	4	2015	$62,486,366	**468**	6	2	2023	$66,330,075
309	4	5	2017	$82,739,532	**389**	5	4	2016	$64,222,619	**469**	6	3	2015	$66,843,534
310	4	5	2018	$86,301,608	**390**	5	4	2017	$65,938,031	**470**	6	3	2016	$66,997,584
311	4	5	2019	$89,647,144	**391**	5	4	2018	$68,415,030	**471**	6	3	2017	$67,586,421
312	4	5	2020	$93,495,171	**392**	5	4	2019	$70,919,873	**472**	6	3	2018	$68,352,412
313	4	5	2021	$97,366,851	**393**	5	4	2020	$73,367,195	**473**	6	3	2019	$69,358,893
314	4	5	2022	$101,624,331	**394**	5	4	2021	$76,402,264	**474**	6	3	2020	$70,359,891
315	4	5	2023	$105,851,198	**395**	5	4	2022	$79,611,206	**475**	6	3	2021	$71,497,918
316	4	6	2015	$78,481,207	**396**	5	4	2023	$83,235,891	**476**	6	3	2022	$73,091,795
317	4	6	2016	$83,514,336	**397**	5	5	2015	$65,224,448	**477**	6	3	2023	$74,878,667
318	4	6	2017	$87,982,698	**398**	5	5	2016	$68,507,130	**478**	6	4	2015	$68,386,411
319	4	6	2018	$92,525,429	**399**	5	5	2017	$72,017,189	**479**	6	4	2016	$69,485,024
320	4	6	2019	$97,350,706	**400**	5	5	2018	$75,808,578	**480**	6	4	2017	$70,768,182

Run	Fleet	Bin	Year	Annual operating profit	Run	Fleet	Bin	Year	Annual operating profit	Run	Fleet	Bin	Year	Annual operating profit
481	6	4	2018	$72,261,069	**561**	7	3	2017	$60,870,930	**641**	8	2	2016	$-15,112,553
482	6	4	2019	$73,499,642	**562**	7	3	2018	$61,761,013	**642**	8	2	2017	$-18,077,837
483	6	4	2020	$75,001,176	**563**	7	3	2019	$63,232,940	**643**	8	2	2018	$-19,326,087
484	6	4	2021	$76,733,928	**564**	7	3	2020	$64,455,025	**644**	8	2	2019	$-20,959,561
485	6	4	2022	$78,362,607	**565**	7	3	2021	$66,094,990	**645**	8	2	2020	$-21,864,572
486	6	4	2023	$80,146,003	**566**	7	3	2022	$68,483,189	**646**	8	2	2021	$-22,293,516
487	6	5	2015	$69,168,703	**567**	7	3	2023	$70,163,467	**647**	8	2	2022	$-21,461,673
488	6	5	2016	$70,808,995	**568**	7	4	2015	$63,730,538	**648**	8	2	2023	$-22,627,686
489	6	5	2017	$72,416,676	**569**	7	4	2016	$65,137,400	**649**	8	3	2015	$-6,143,015
490	6	5	2018	$74,222,615	**570**	7	4	2017	$67,623,031	**650**	8	3	2016	$-7,424,913
491	6	5	2019	$76,120,795	**571**	7	4	2018	$69,763,126	**651**	8	3	2017	$-8,604,449
492	6	5	2020	$78,055,244	**572**	7	4	2019	$71,867,337	**652**	8	3	2018	$-8,012,604
493	6	5	2021	$79,879,948	**573**	7	4	2020	$74,466,754	**653**	8	3	2019	$-7,648,921
494	6	5	2022	$82,050,257	**574**	7	4	2021	$77,309,392	**654**	8	3	2020	$-7,482,678
495	6	5	2023	$84,049,274	**575**	7	4	2022	$80,628,134	**655**	8	3	2021	$-5,033,747
496	6	6	2015	$69,836,819	**576**	7	4	2023	$84,237,377	**656**	8	3	2022	$-4,692,604
497	6	6	2016	$72,095,974	**577**	7	5	2015	$66,362,043	**657**	8	3	2023	$-3,173,964
498	6	6	2017	$74,134,744	**578**	7	5	2016	$69,591,788	**658**	8	4	2015	$-2,299,397
499	6	6	2018	$76,382,220	**579**	7	5	2017	$73,537,322	**659**	8	4	2016	$-1,051,354
500	6	6	2019	$78,627,878	**580**	7	5	2018	$76,908,876	**660**	8	4	2017	$203,498
501	6	6	2020	$80,822,218	**581**	7	5	2019	$80,519,941	**661**	8	4	2018	$1,309,988
502	6	6	2021	$83,353,801	**582**	7	5	2020	$84,372,284	**662**	8	4	2019	$3,289,589
503	6	6	2022	$85,849,658	**583**	7	5	2021	$88,006,529	**663**	8	4	2020	$4,757,416
504	6	6	2023	$88,273,893	**584**	7	5	2022	$92,478,638	**664**	8	4	2021	$8,093,612
505	6	7	2015	$70,621,853	**585**	7	5	2023	$97,492,750	**665**	8	4	2022	$9,438,139
506	6	7	2016	$73,647,945	**586**	7	6	2015	$69,195,336	**666**	8	4	2023	$13,772,616
507	6	7	2017	$76,020,321	**587**	7	6	2016	$74,225,488	**667**	8	5	2015	$697,166
508	6	7	2018	$78,610,028	**588**	7	6	2017	$79,200,909	**668**	8	5	2016	$3,536,133
509	6	7	2019	$81,463,829	**589**	7	6	2018	$83,886,801	**669**	8	5	2017	$6,900,745
510	6	7	2020	$84,263,241	**590**	7	6	2019	$88,850,909	**670**	8	5	2018	$10,838,005
511	6	7	2021	$87,226,610	**591**	7	6	2020	$94,586,903	**671**	8	5	2019	$12,927,967
512	6	7	2022	$89,810,583	**592**	7	6	2021	$99,777,646	**672**	8	5	2020	$15,799,987
513	6	7	2023	$93,111,358	**593**	7	6	2022	$105,233,713	**673**	8	5	2021	$20,854,294
514	6	8	2015	$71,493,027	**594**	7	6	2023	$111,583,537	**674**	8	5	2022	$23,569,872
515	6	8	2016	$74,980,139	**595**	7	7	2015	$71,426,605	**675**	8	5	2023	$28,742,562
516	6	8	2017	$78,219,562	**596**	7	7	2016	$78,927,472	**676**	8	6	2015	$4,406,694
517	6	8	2018	$81,639,538	**597**	7	7	2017	$85,079,823	**677**	8	6	2016	$9,832,731
518	6	8	2019	$84,829,998	**598**	7	7	2018	$91,589,606	**678**	8	6	2017	$13,380,180
519	6	8	2020	$88,119,875	**599**	7	7	2019	$98,524,515	**679**	8	6	2018	$18,257,965
520	6	8	2021	$91,346,656	**600**	7	7	2020	$105,418,650	**680**	8	6	2019	$23,589,024
521	6	8	2022	$93,950,557	**601**	7	7	2021	$113,527,391	**681**	8	6	2020	$28,378,758
522	6	8	2023	$96,651,516	**602**	7	7	2022	$119,483,687	**682**	8	6	2021	$34,817,460
523	6	9	2015	$72,581,839	**603**	7	7	2023	$127,657,322	**683**	8	6	2022	$40,073,535
524	6	9	2016	$76,866,799	**604**	7	8	2015	$75,144,810	**684**	8	6	2023	$45,698,968
525	6	9	2017	$81,185,789	**605**	7	8	2016	$84,298,095	**685**	8	7	2015	$7,511,411
526	6	9	2018	$85,233,954	**606**	7	8	2017	$93,221,965	**686**	8	7	2016	$15,160,322
527	6	9	2019	$88,551,965	**607**	7	8	2018	$101,772,783	**687**	8	7	2017	$22,076,062
528	6	9	2020	$91,467,619	**608**	7	8	2019	$110,098,017	**688**	8	7	2018	$27,916,977
529	6	9	2021	$94,304,339	**609**	7	8	2020	$118,698,372	**689**	8	7	2019	$34,363,685
530	6	9	2022	$97,639,552	**610**	7	8	2021	$127,426,210	**690**	8	7	2020	$42,612,441
531	6	9	2023	$100,764,029	**611**	7	8	2022	$137,097,678	**691**	8	7	2021	$50,009,935
532	6	10	2015	$74,738,349	**612**	7	8	2023	$146,503,685	**692**	8	7	2022	$57,287,987
533	6	10	2016	$80,966,850	**613**	7	9	2015	$79,085,696	**693**	8	7	2023	$64,505,907
534	6	10	2017	$86,058,658	**614**	7	9	2016	$91,661,965	**694**	8	8	2015	$11,008,752
535	6	10	2018	$89,774,575	**615**	7	9	2017	$102,998,782	**695**	8	8	2016	$21,988,935
536	6	10	2019	$93,333,387	**616**	7	9	2018	$114,438,346	**696**	8	8	2017	$30,637,944
537	6	10	2020	$97,323,600	**617**	7	9	2019	$125,745,416	**697**	8	8	2018	$38,984,212
538	6	10	2021	$101,504,367	**618**	7	9	2020	$136,845,438	**698**	8	8	2019	$48,710,946
539	6	10	2022	$105,657,211	**619**	7	9	2021	$148,658,199	**699**	8	8	2020	$58,463,331
540	6	10	2023	$110,380,496	**620**	7	9	2022	$161,409,106	**700**	8	8	2021	$67,879,094
541	7	1	2015	$49,276,589	**621**	7	9	2023	$168,862,768	**701**	8	8	2022	$77,734,473
542	7	1	2016	$43,197,122	**622**	7	10	2015	$86,552,323	**702**	8	8	2023	$87,650,513
543	7	1	2017	$39,813,616	**623**	7	10	2016	$105,365,513	**703**	8	9	2015	$16,534,730
544	7	1	2018	$36,119,231	**624**	7	10	2017	$123,308,584	**704**	8	9	2016	$29,745,236
545	7	1	2019	$34,464,889	**625**	7	10	2018	$141,085,783	**705**	8	9	2017	$42,842,868
546	7	1	2020	$33,479,725	**626**	7	10	2019	$157,263,371	**706**	8	9	2018	$55,114,146
547	7	1	2021	$32,266,667	**627**	7	10	2020	$168,075,427	**707**	8	9	2019	$68,344,407
548	7	1	2022	$31,639,852	**628**	7	10	2021	$179,122,694	**708**	8	9	2020	$80,548,559
549	7	1	2023	$31,615,724	**629**	7	10	2022	$189,463,706	**709**	8	9	2021	$93,185,391
550	7	2	2015	$56,446,561	**630**	7	10	2023	$196,339,581	**710**	8	9	2022	$107,606,592
551	7	2	2016	$54,249,298	**631**	8	1	2015	$-20,667,224	**711**	8	9	2023	$119,132,488
552	7	2	2017	$53,290,046	**632**	8	1	2016	$-28,009,820	**712**	8	10	2015	$26,446,101
553	7	2	2018	$52,541,597	**633**	8	1	2017	$-33,631,053	**713**	8	10	2016	$46,768,937
554	7	2	2019	$52,495,878	**634**	8	1	2018	$-38,769,016	**714**	8	10	2017	$68,083,449
555	7	2	2020	$52,801,761	**635**	8	1	2019	$-42,696,003	**715**	8	10	2018	$87,871,048
556	7	2	2021	$53,170,066	**636**	8	1	2020	$-45,201,031	**716**	8	10	2019	$107,659,531
557	7	2	2022	$54,293,014	**637**	8	1	2021	$-46,434,716	**717**	8	10	2020	$125,584,861
558	7	2	2023	$55,360,758	**638**	8	1	2022	$-48,292,866	**718**	8	10	2021	$145,358,982
559	7	3	2015	$60,686,556	**639**	8	1	2023	$-49,588,366	**719**	8	10	2022	$164,912,456
560	7	3	2016	$60,057,358	**640**	8	2	2015	$-10,933,335	**720**	8	10	2023	$187,058,250

D

Scenario generation model results

D.1 OD pair based transition probability matrices

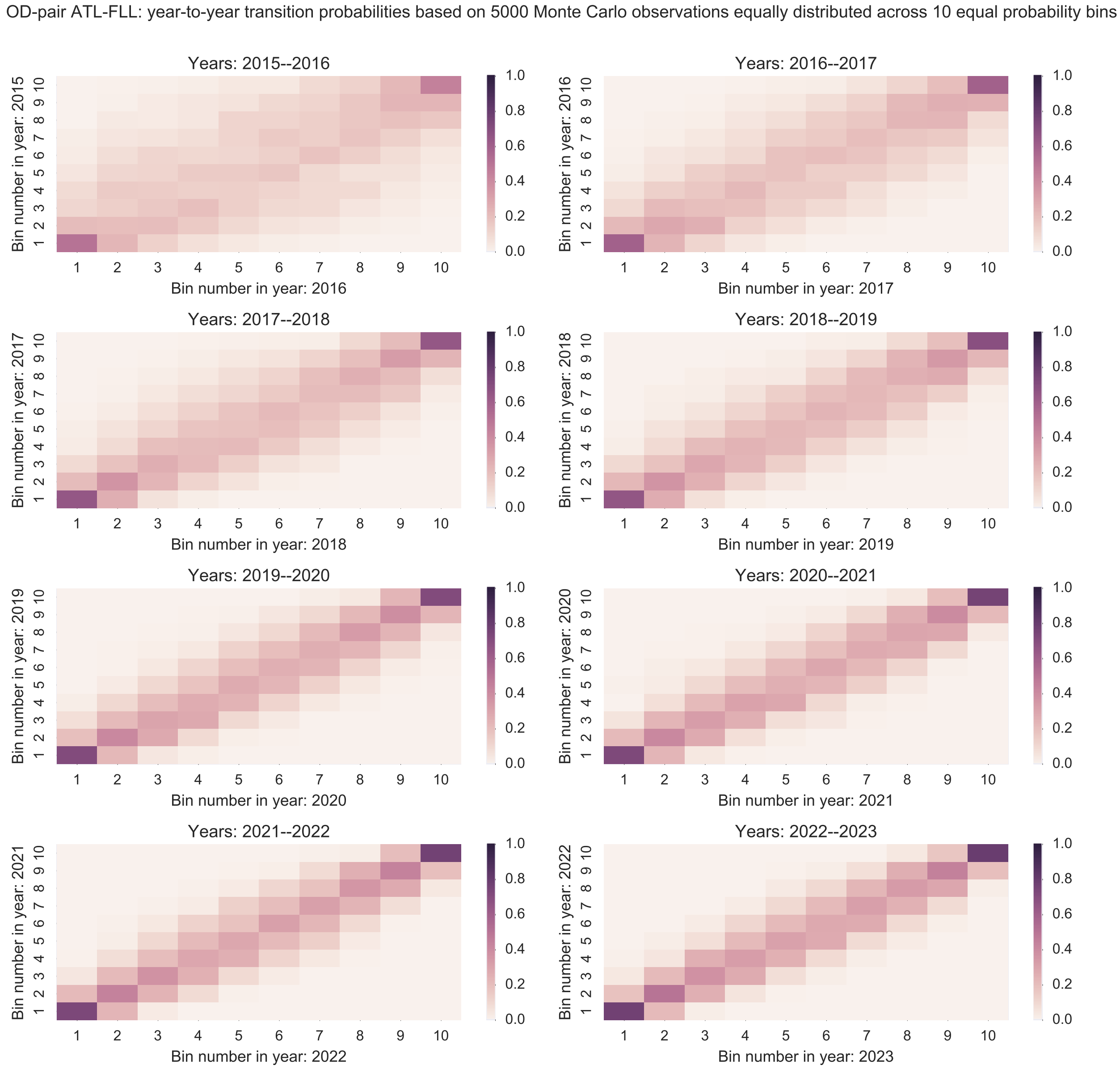


Figure D.1: Heat maps of OD pair based transition probability matrices - OD pair: ATL-FLL

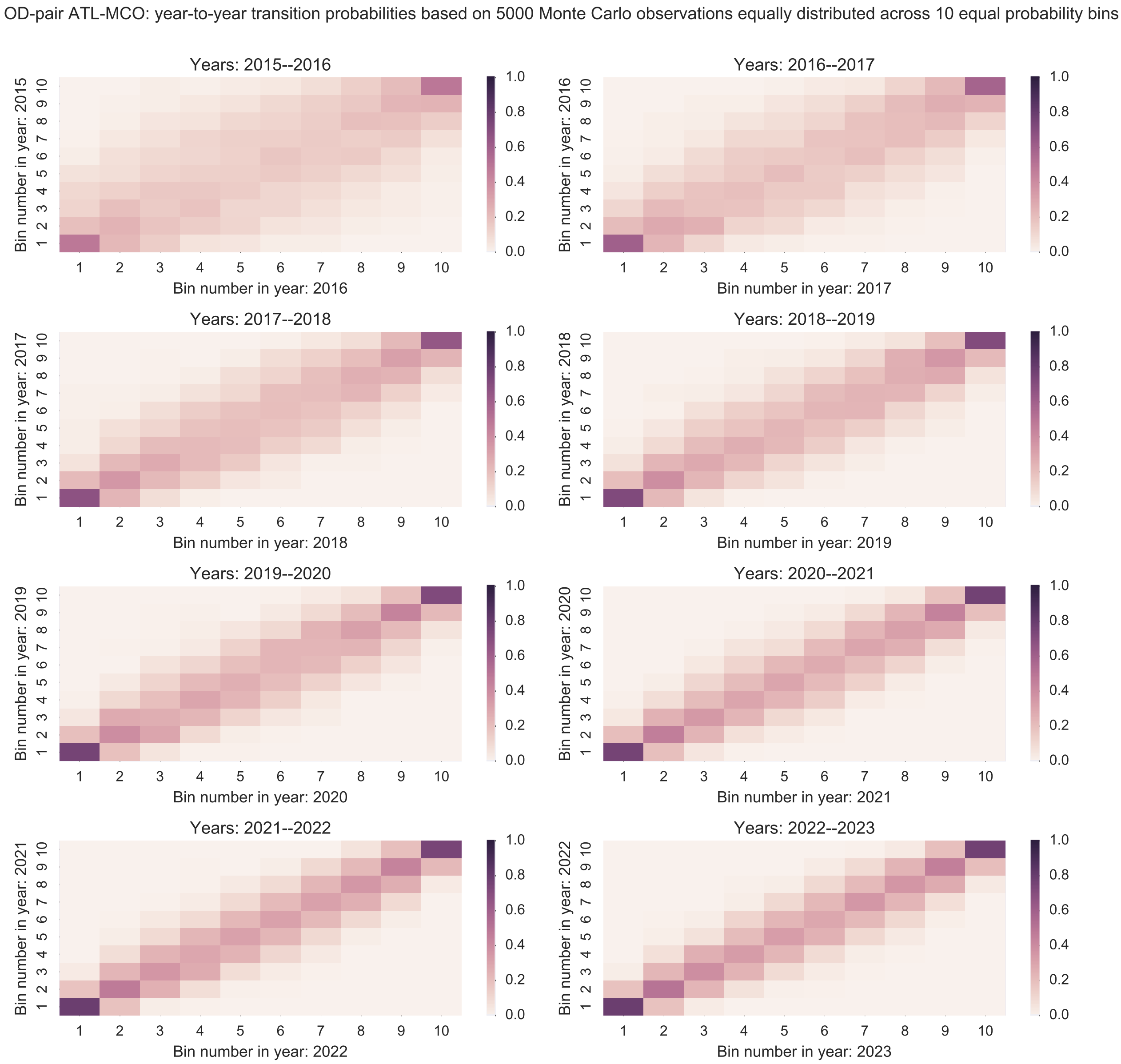


Figure D.2: Heat maps of OD pair based transition probability matrices - OD pair: ATL-MCO

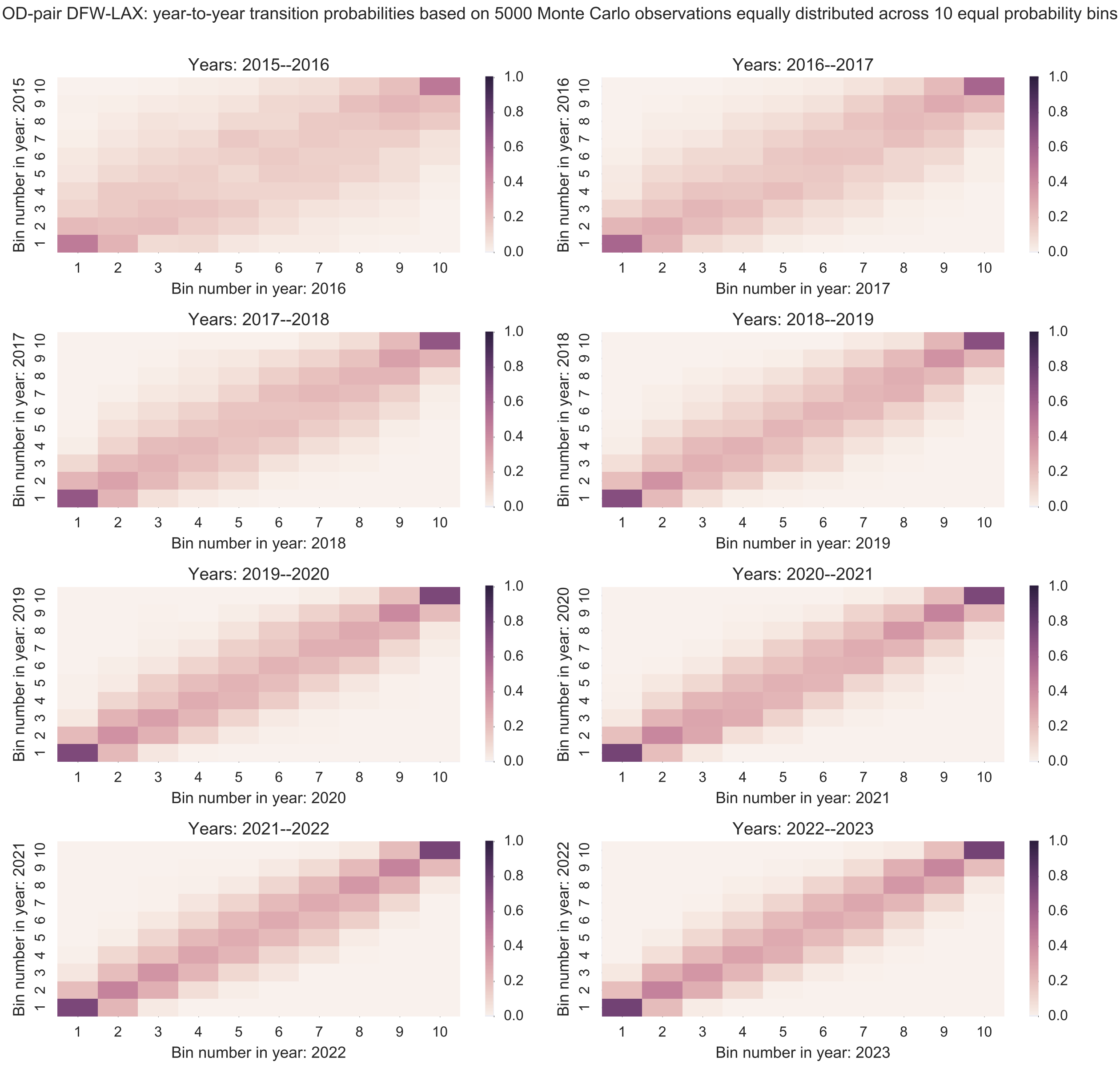


Figure D.3: Heat maps of OD pair based transition probability matrices - OD pair: DFW-LAX

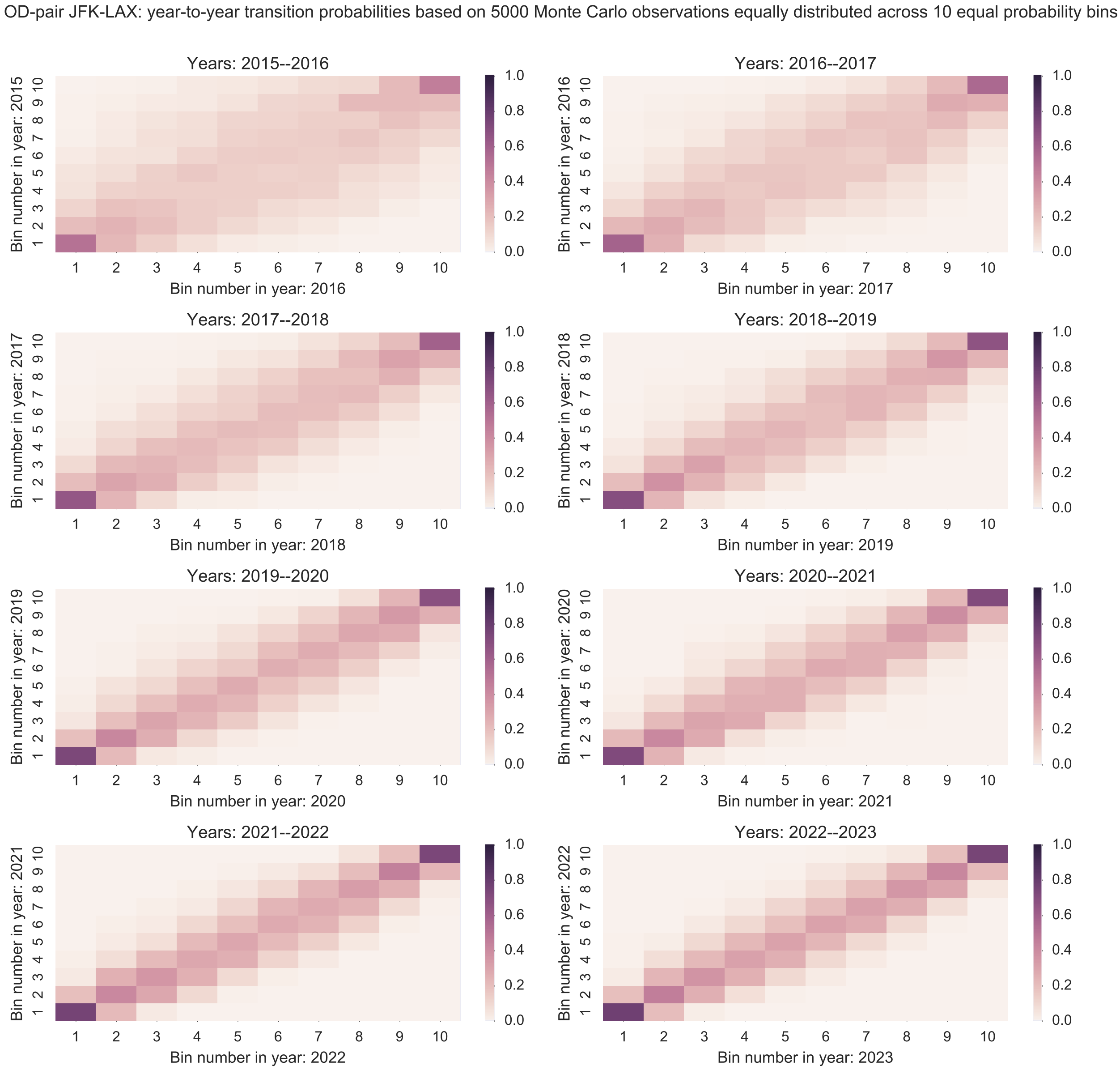


Figure D.4: Heat maps of OD pair based transition probability matrices - OD pair: JFK-LAX

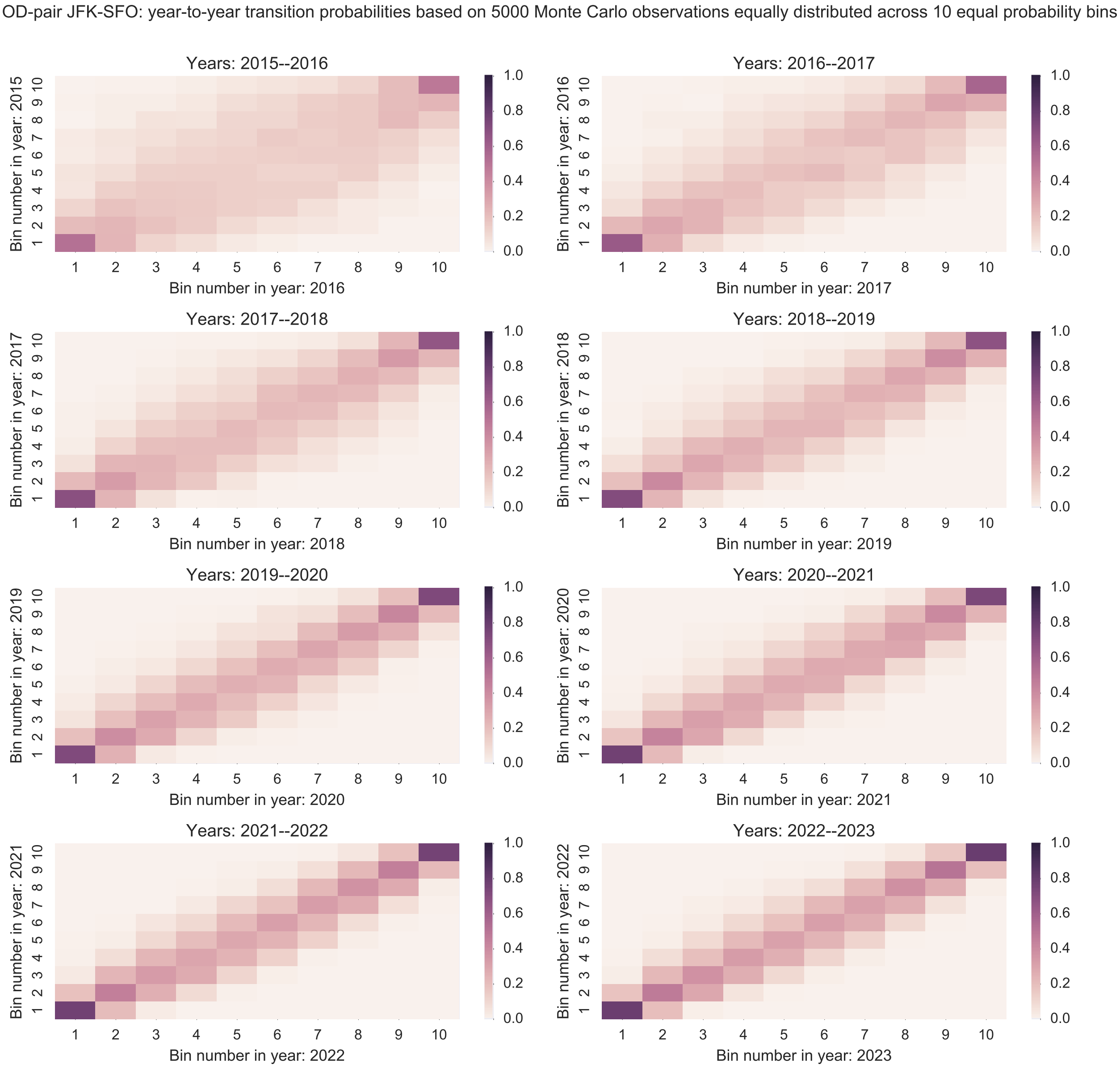


Figure D.5: Heat maps of OD pair based transition probability matrices - OD pair: JFK-SFO

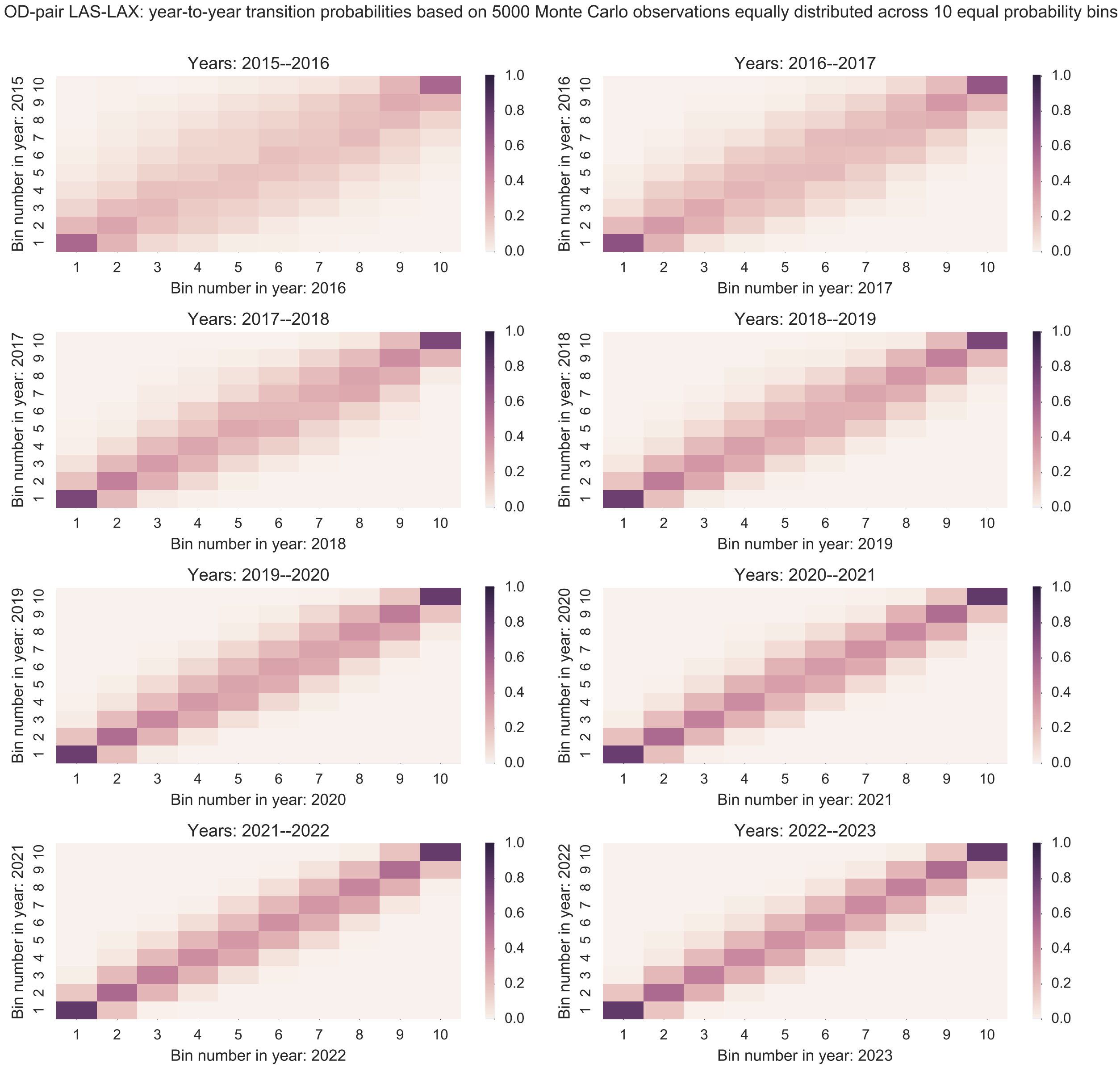


Figure D.6: Heat maps of OD pair based transition probability matrices - OD pair: LAS-LAX

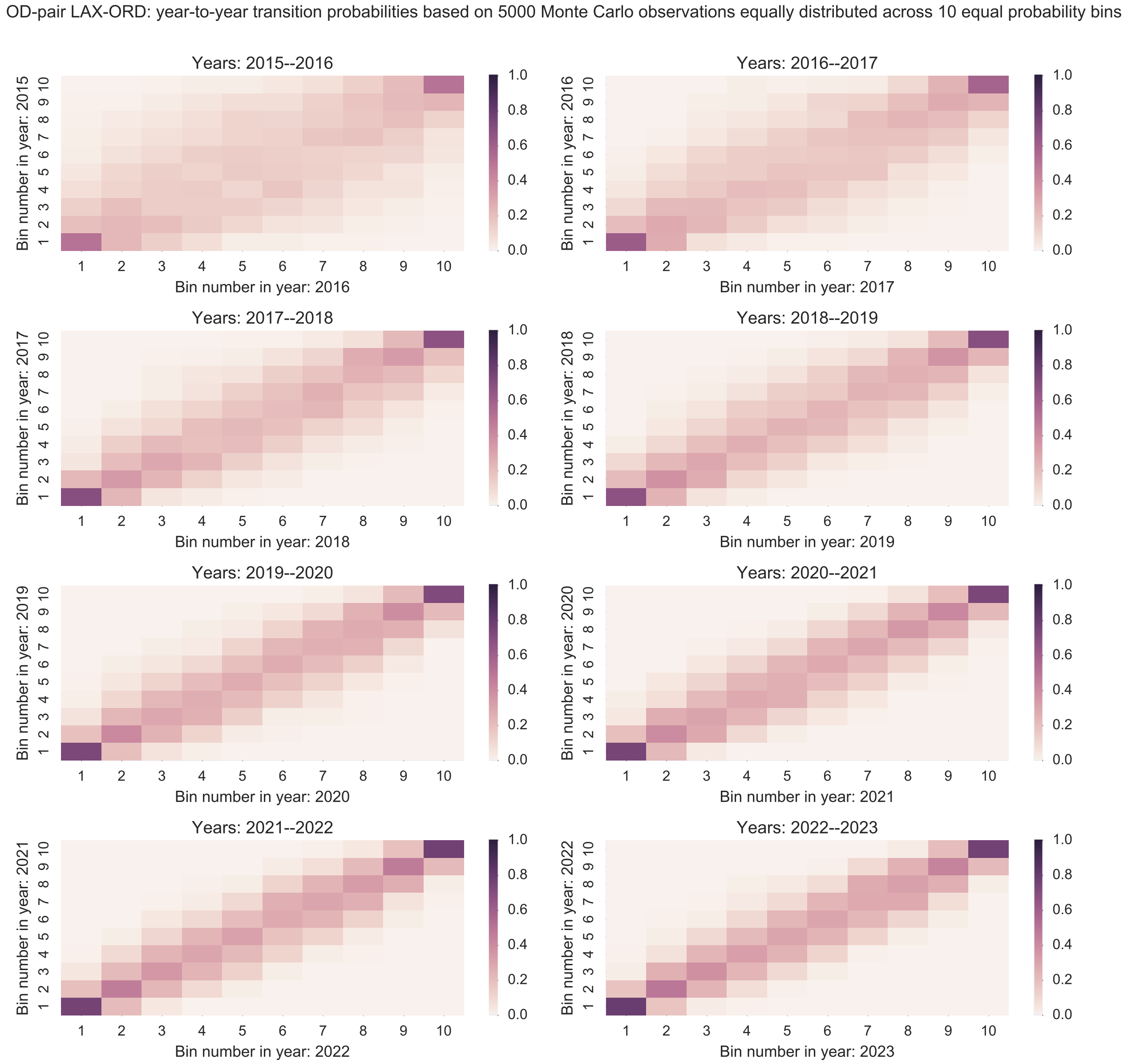


Figure D.7: Heat maps of OD pair based transition probability matrices - OD pair: LAX-ORD

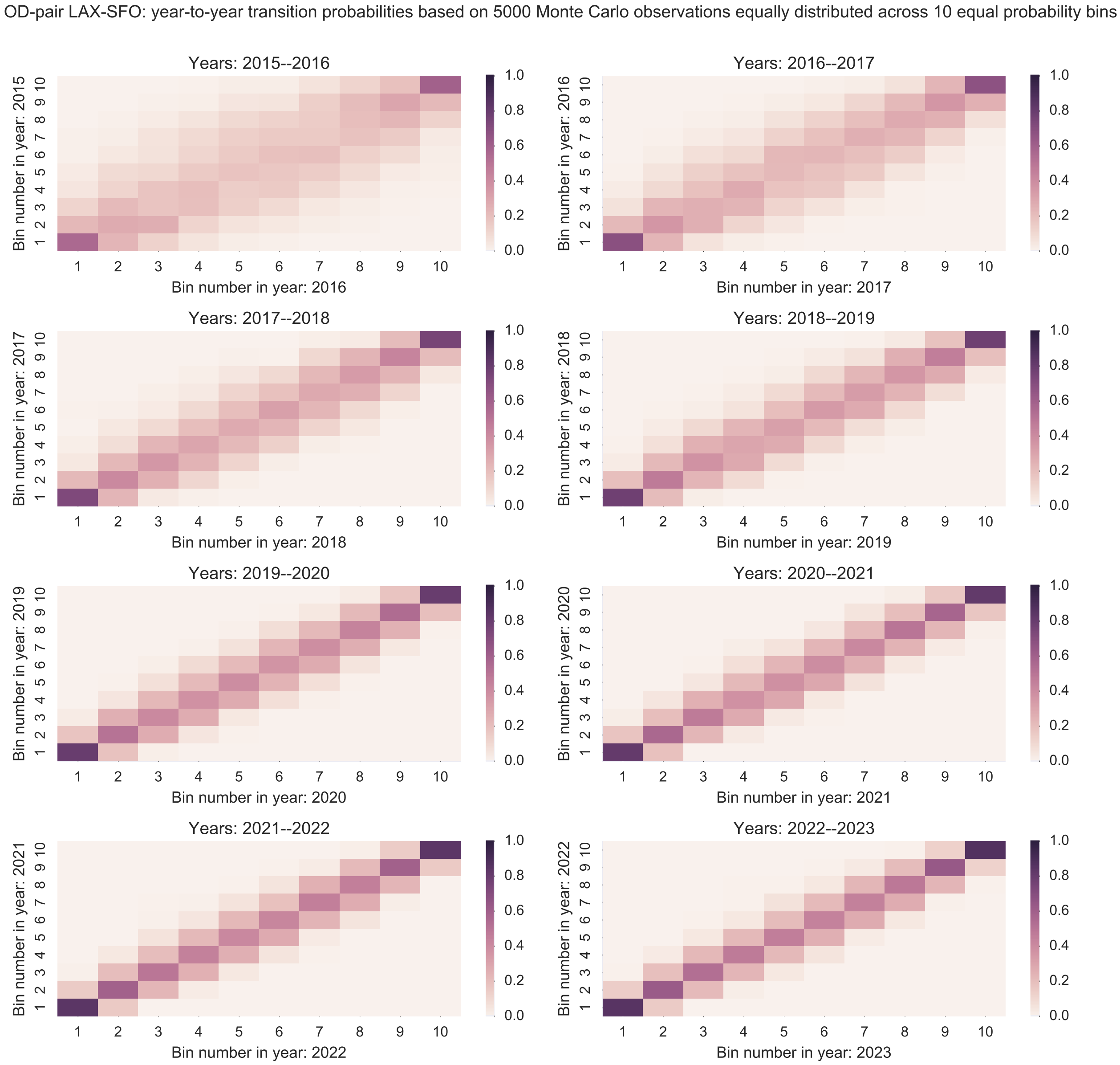


Figure D.8: Heat maps of OD pair based transition probability matrices - OD pair: LAX-SFO

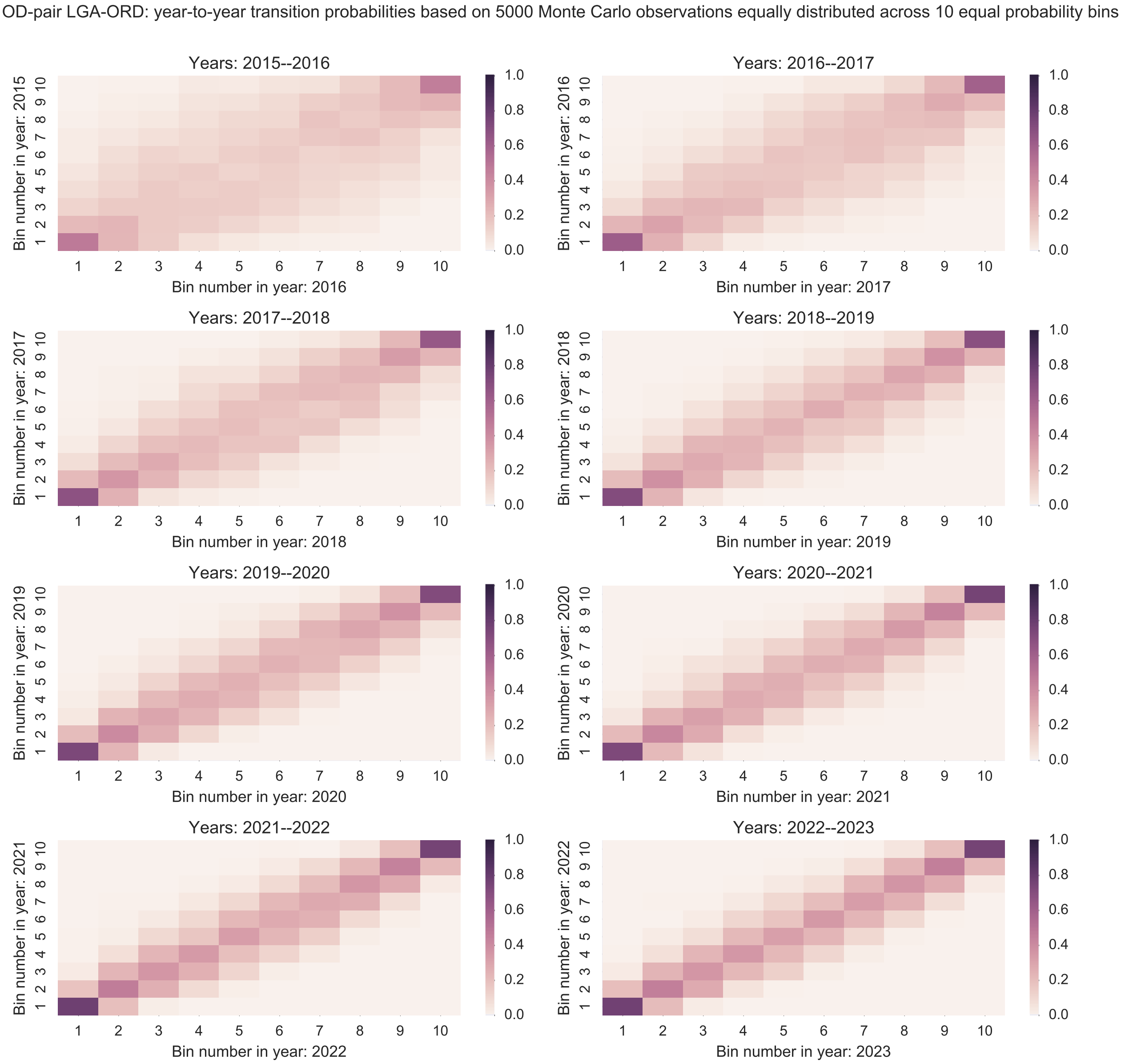


Figure D.9: Heat maps of OD pair based transition probability matrices - OD pair: LGA-ORD

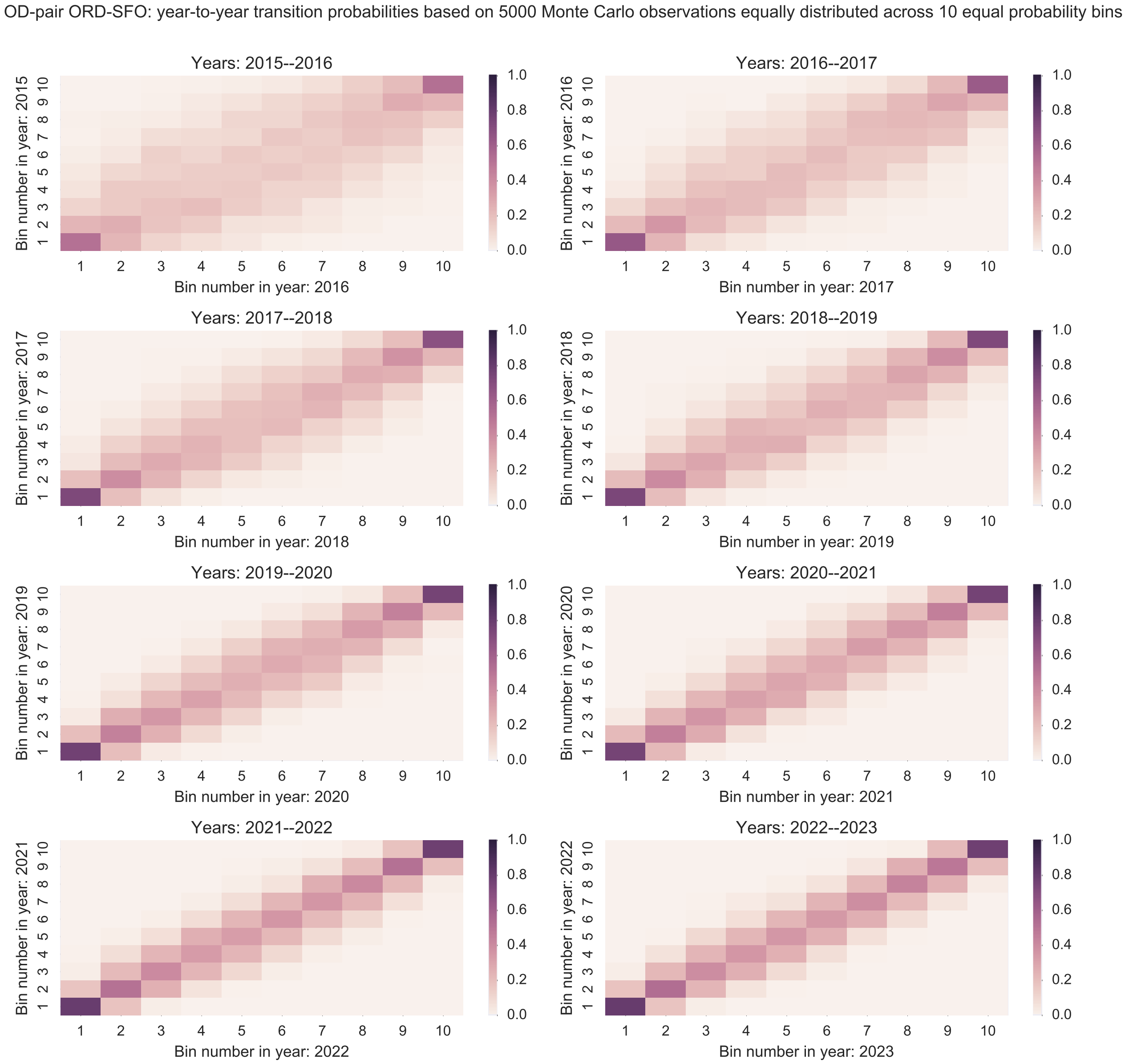


Figure D.10: Heat maps of OD pair based transition probability matrices - OD pair: ORD-SFO

D.2 OD demand matrix based transition probability matrices

2015-2016	1	2	3	4	5	6	7	8	9	10
1	0.513	0.235	0.125	0.068	0.037	0.015	0.004	0.002	0.000	0.000
2	0.213	0.233	0.193	0.138	0.102	0.065	0.038	0.016	0.003	0.000
3	0.113	0.178	0.180	0.172	0.126	0.097	0.070	0.039	0.021	0.003
4	0.066	0.127	0.147	0.161	0.157	0.135	0.105	0.063	0.033	0.007
5	0.042	0.091	0.122	0.141	0.144	0.147	0.130	0.099	0.063	0.021
6	0.026	0.055	0.092	0.115	0.141	0.155	0.149	0.128	0.099	0.039
7	0.013	0.041	0.068	0.095	0.111	0.136	0.161	0.167	0.140	0.069
8	0.009	0.022	0.044	0.062	0.095	0.126	0.152	0.175	0.193	0.122
9	0.004	0.014	0.021	0.033	0.062	0.085	0.128	0.183	0.233	0.238
10	0.001	0.004	0.007	0.016	0.025	0.040	0.063	0.127	0.216	0.502

2016-2017	1	2	3	4	5	6	7	8	9	10
1	0.617	0.240	0.094	0.033	0.009	0.005	0.001	0.000	0.000	0.000
2	0.223	0.308	0.223	0.133	0.071	0.026	0.011	0.003	0.001	0.000
3	0.089	0.196	0.238	0.194	0.140	0.085	0.039	0.015	0.004	0.000
4	0.039	0.127	0.184	0.196	0.182	0.144	0.082	0.036	0.010	0.000
5	0.018	0.064	0.118	0.176	0.187	0.170	0.148	0.081	0.034	0.004
6	0.009	0.035	0.075	0.130	0.165	0.180	0.180	0.138	0.073	0.016
7	0.003	0.018	0.041	0.074	0.123	0.179	0.200	0.193	0.138	0.032
8	0.002	0.007	0.019	0.039	0.078	0.118	0.188	0.236	0.219	0.095
9	0.000	0.003	0.007	0.020	0.035	0.073	0.117	0.200	0.296	0.249
10	0.000	0.000	0.001	0.005	0.010	0.021	0.035	0.098	0.226	0.604

2017-2018	1	2	3	4	5	6	7	8	9	10
1	0.683	0.229	0.066	0.017	0.005	0.001	0.000	0.000	0.000	0.000
2	0.214	0.361	0.240	0.124	0.044	0.015	0.004	0.000	0.000	0.000
3	0.065	0.217	0.265	0.218	0.137	0.069	0.026	0.004	0.000	0.000
4	0.026	0.106	0.208	0.232	0.194	0.140	0.069	0.022	0.003	0.000
5	0.007	0.053	0.117	0.184	0.218	0.199	0.144	0.060	0.017	0.001
6	0.004	0.021	0.062	0.123	0.192	0.214	0.205	0.130	0.046	0.004
7	0.001	0.010	0.028	0.063	0.116	0.182	0.222	0.231	0.127	0.020
8	0.000	0.003	0.011	0.030	0.065	0.123	0.193	0.257	0.246	0.072
9	0.000	0.001	0.003	0.008	0.025	0.051	0.113	0.221	0.341	0.236
10	0.000	0.000	0.001	0.001	0.004	0.008	0.026	0.075	0.219	0.667

2018-2019	1	2	3	4	5	6	7	8	9	10
1	0.716	0.223	0.050	0.010	0.001	0.000	0.000	0.000	0.000	0.000
2	0.200	0.393	0.248	0.117	0.032	0.008	0.002	0.000	0.000	0.000
3	0.059	0.229	0.294	0.234	0.122	0.048	0.012	0.002	0.000	0.000
4	0.017	0.097	0.210	0.250	0.222	0.138	0.055	0.010	0.002	0.000
5	0.007	0.042	0.118	0.194	0.248	0.210	0.129	0.044	0.008	0.000
6	0.001	0.012	0.052	0.121	0.192	0.247	0.211	0.128	0.035	0.001
7	0.001	0.004	0.019	0.053	0.114	0.191	0.271	0.232	0.107	0.009
8	0.000	0.001	0.007	0.019	0.053	0.109	0.205	0.299	0.252	0.054
9	0.000	0.000	0.001	0.004	0.014	0.043	0.100	0.228	0.382	0.227
10	0.000	0.000	0.000	0.000	0.001	0.005	0.015	0.056	0.214	0.709

2019-2020	1	2	3	4	5	6	7	8	9	10
1	0.741	0.209	0.041	0.008	0.001	0.000	0.000	0.000	0.000	0.000
2	0.196	0.415	0.263	0.095	0.028	0.003	0.000	0.000	0.000	0.000
3	0.050	0.240	0.319	0.238	0.112	0.034	0.006	0.001	0.000	0.000
4	0.009	0.093	0.214	0.287	0.232	0.120	0.038	0.006	0.001	0.000
5	0.003	0.029	0.107	0.202	0.268	0.236	0.115	0.037	0.003	0.000
6	0.001	0.010	0.039	0.107	0.198	0.273	0.229	0.119	0.023	0.000
7	0.000	0.003	0.014	0.047	0.106	0.201	0.284	0.244	0.094	0.007
8	0.000	0.001	0.003	0.013	0.042	0.103	0.224	0.321	0.254	0.038
9	0.000	0.000	0.000	0.002	0.011	0.027	0.091	0.223	0.418	0.227
10	0.000	0.000	0.000	0.001	0.001	0.003	0.012	0.049	0.206	0.728

2020-2021	1	2	3	4	5	6	7	8	9	10
1	0.763	0.203	0.031	0.003	0.000	0.000	0.000	0.000	0.000	0.000
2	0.188	0.455	0.256	0.077	0.020	0.003	0.000	0.000	0.000	0.000
3	0.038	0.231	0.349	0.256	0.097	0.024	0.006	0.000	0.000	0.000
4	0.009	0.078	0.220	0.313	0.231	0.116	0.031	0.003	0.000	0.000
5	0.003	0.025	0.098	0.200	0.294	0.235	0.115	0.027	0.003	0.000
6	0.000	0.007	0.036	0.099	0.217	0.288	0.242	0.096	0.016	0.000
7	0.000	0.001	0.008	0.040	0.098	0.212	0.309	0.241	0.087	0.003
8	0.000	0.001	0.002	0.010	0.036	0.095	0.213	0.355	0.255	0.034
9	0.000	0.000	0.000	0.002	0.006	0.024	0.076	0.235	0.450	0.206
10	0.000	0.000	0.000	0.000	0.001	0.003	0.009	0.043	0.188	0.756

2021-2022	1	2	3	4	5	6	7	8	9	10
1	0.776	0.202	0.020	0.002	0.000	0.000	0.000	0.000	0.000	0.000
2	0.189	0.479	0.252	0.070	0.010	0.000	0.000	0.000	0.000	0.000
3	0.027	0.226	0.375	0.255	0.095	0.021	0.002	0.000	0.000	0.000
4	0.007	0.068	0.235	0.321	0.245	0.100	0.022	0.002	0.000	0.000
5	0.001	0.021	0.083	0.217	0.317	0.236	0.099	0.024	0.001	0.000
6	0.000	0.004	0.026	0.097	0.203	0.311	0.248	0.100	0.011	0.000
7	0.000	0.001	0.008	0.030	0.097	0.220	0.326	0.243	0.072	0.003
8	0.000	0.000	0.001	0.007	0.027	0.088	0.222	0.366	0.267	0.022
9	0.000	0.000	0.000	0.001	0.005	0.023	0.076	0.231	0.467	0.197
10	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.034	0.182	0.778

2022-2023	1	2	3	4	5	6	7	8	9	10
1	0.785	0.197	0.017	0.001	0.000	0.000	0.000	0.000	0.000	0.000
2	0.185	0.494	0.258	0.054	0.008	0.001	0.000	0.000	0.000	0.000
3	0.028	0.227	0.382	0.256	0.090	0.017	0.001	0.000	0.000	0.000
4	0.002	0.063	0.233	0.354	0.240	0.092	0.014	0.002	0.000	0.000
5	0.000	0.016	0.084	0.215	0.328	0.241	0.099	0.014	0.002	0.000
6	0.000	0.003	0.020	0.089	0.216	0.323	0.252	0.091	0.007	0.000
7	0.000	0.000	0.005	0.025	0.093	0.221	0.344	0.250	0.059	0.002
8	0.000	0.000	0.001	0.005	0.021	0.088	0.222	0.384	0.259	0.020
9	0.000	0.000	0.000	0.001	0.004	0.015	0.064	0.230	0.483	0.202
10	0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.028	0.190	0.776

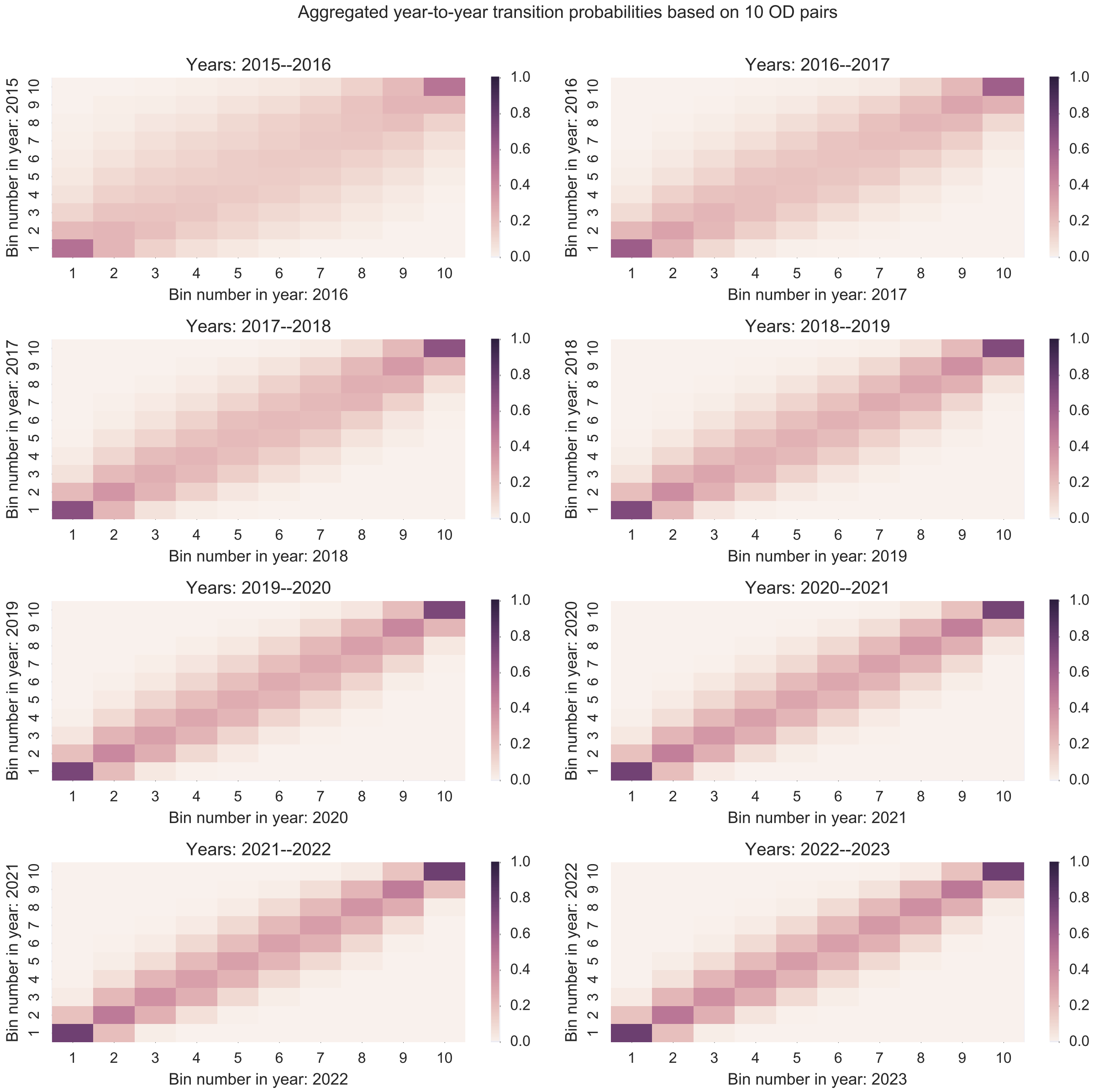


Figure D.11: Heat maps of aggregated OD demand matrix based transition probability matrices

Bibliography

Abara, J. (1989), ‘Applying Integer Linear Programming to the Fleet Assignment Problem’, *Interfaces* **19**(4), 20–28.

AxleGeeks (2016), ‘AxleGeeks - Planes [Accessed on: 2015/07/28]’.
URL: *http://planes.axlegeeks.com/*

Barnhart, C. (2003), ‘1.206J Airline Schedule Planning, Spring 2003 (Massachusetts Institute of Technology: MIT OpenCourseWare), http://ocw.mit.edu [Accessed on: 2016/04/15]’.

Barnhart, C., Boland, N. L., Clarke, L. W., Johnson, E. L., Nemhauser, G. L. and Shenoi, R. G. (1998), ‘Flight String Models for Aircraft Fleeting and Routing’, *Transportation Science* **32**(3), 208–220.

Barnhart, C. and Cohn, A. (2004), ‘Airline Schedule Planning: Accomplishments and Opportunities’, *Manufacturing & Service Operations Management* **6**(1), 3–22.

Barnhart, C., Hane, C. a., Johnson, E. L. and Sigismondi, G. (1994), ‘A column generation and partitioning approach for multi-commodity flow problems’, *Telecommunication Systems* **3**(3), 239–258.

Barnhart, C., Johnson, E. L., Nemhauser, G. L., Savelsbergh, M. W. P. and Vance, P. H. (1998), ‘Branch-and-Price: Column Generation for Solving Huge Integer Programs’, *Operations Research* **46**(3), 316–329.

Barnhart, C., Kniker, T. S. and Lohatepanont, M. (2002), ‘Itinerary-Based Airline Fleet Assignment’, *Transportation Science* **36**(2), 199–217.

Bazargan, M. (2004), *Airline operations and scheduling*, Ashgate Publishing, Ltd.

Bazargan, M. and Hartman, J. (2012), ‘Aircraft replacement strategy: Model and analysis’, *Journal of Air Transport Management* **25**, 26–29.

Belobaba, P., Odoni, A. and Barnhart, C. (2009), *The global airline industry*, John Wiley & Sons.

Berge, M. E. and Hopperstad, C. A. (1993), ‘Demand Driven Dispatch: A Method for Dynamic Aircraft Capacity Assignment, Models and Algorithms’, *Operations Research* **41**(1), 153–168.

Bertsimas, D. and Tsitsiklis, J. N. (1997), *Introduction to linear optimization*, Vol. 6, Athena Scientific Belmont, MA.

Bessembinder, H., Coughenour, J. F., Seguin, P. J. and Smoller, M. M. (1995), ‘Mean-reversion in equilibrium asset prices: Evidence from the futures term structure’, *Journal of Finance* **50**(1), 361–375.

Bojović, N. J. (2002), ‘A general system theory approach to rail freight car fleet sizing’, *European Journal of Operational Research* **136**, 136–172.

Carter, D. a., Rogers, D. a. and Simkins, B. J. (2006), 'Hedging and value in the U.S. airline industry', *Journal of Applied Corporate Finance* **18**(4), 21–33.

Clark, P. (2007), *Buying the big jets: fleet planning for airlines*, Ashgate Publishing, Ltd.

Clarke, J.-P. (2004), 'The Case for Robust Schedules - MIT International Center for Air Transportation [Accessed on: 2015/07/28]'.
URL: *https://www.researchgate.net/publication/266576377_M_I_T_I_The_Case_for_Robust_Schedules*

Clarke, L. W., Hane, C. a., Johnson, E. L. and Nemhauser, G. L. (1996), 'Maintenance and Crew Considerations in Fleet Assignment', *Transportation Science* **30**(3), 249–260.

Doganis, R. (2002), *Flying off course: The economics of international airlines*, Psychology Press.

Dožić, S. Z. and Kalić, M. D. (2013), Two-stage airline fleet planning model, *in* '1st Logistics International Conference, November 2013, Belgrade, Serbia', pp. 60–65.

Dožić, S. Z. and Kalić, M. D. (2015), 'Three-stage airline fleet planning model', *Journal of Air Transport Management* **46**, 30–39.

Ferguson, A. R. and Dantzig, G. B. (1956), 'The Allocation of Aircraft to Routes - An Example of Linear Programming Under Uncertain Demand', *Management Science* **3**(1), 45–73.

Gabrel, V., Murat, C. and Thiele, A. (2014), 'Recent advances in robust optimization: An Overview', *European Journal of Operational Research* **235**(3), 471–483.

Gao, C., Johnson, E. and Smith, B. (2009), 'Integrated Airline Fleet and Crew Robust Planning', *Transportation Science* **43**(1), 2–16.

Gibson, W. E. (2010), Aircraft investment planning and uncertainty, PhD thesis, Cranfield University.

Gu, Z., Johnson, E. L., Nemhauser, G. L. and Yinhua, W. (1994), 'Some properties of the fleet assignment problem', *Operations Research Letters* **15**(2), 59–71.

Hane, C. a., Barnhart, C., Johnson, E. L., Marsten, R. E., Nemhauser, G. L. and Sigismondi, G. (1995), 'The fleet assignment problem: Solving a large-scale integer program', *Mathematical Programming* **70**(1-3), 211–232.

Hillier, F. S. and Lieberman, G. J. (2010), *Introduction to Operations Research*, McGraw-Hill.

Hsu, C.-I., Chao, C.-C. and Huang, P.-S. (2011), 'Fleet dry/wet lease planning of airlines on strategic alliance', *Transportmetrica A: Transport Science* **9**(7), 603–628.

Hsu, C.-I., Li, H.-C., Liu, S.-M. and Chao, C.-C. (2011), 'Aircraft replacement scheduling: A dynamic programming approach', *Transportation Research Part E: Logistics and Transportation Review* **47**(1), 41–60.

Jacobs, T. L., Johnson, E. L. and Smith, B. C. (1999), O&D FAM: Incorporating passenger flows into the fleeting process, *in* '39th AGIFORS Symposium Proceedings', New Orleans, LA, USA.

Jarrah, A. I., Goodstein, J. and Narasimhan, R. (2000), 'An Efficient Airline Re-Fleeting Model for the Incremental Modification of Planned Fleet Assignments', *Transportation Science* **34**(4), 349–363.

Jin, D. and Kite-Powell, H. L. (2000), 'Optimal fleet utilization and replacement', *Transportation Research Part E: Logistics and Transportation Review* **36**(1), 3–20.

Justin, C., Garcia, E. and Mavris, D. (2010), Aircraft valuation: a network approach to the evaluation of aircraft for fleet planning and strategic decision making, *in* '10th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference, Fort Worth, Texas, September 2010', pp. 1–25.

Kanafani, A. and Ghobrial, A. (1982), 'Aircraft Evaluation in Air Network Planning', *Transportation Engineering Journal* **108**(3), 282–300.

Khoo, H. L. and Teoh, L. E. (2014), 'A bi-objective dynamic programming approach for airline green fleet planning', *Transportation Research Part D: Transport and Environment* **33**, 166–185.

Khoo, H. and Teoh, L. (2013), 'An optimal aircraft fleet management decision model under uncertainty', *Journal of Advanced Transportation* **48**(7), 798–820.

List, G. F., Wood, B., Nozick, L. K., Turnquist, M. a., Jones, D. a., Kjeldgaard, E. a. and Lawton, C. R. (2003), 'Robust optimization for fleet planning under uncertainty', *Transportation Research Part E: Logistics and Transportation Review* **39**(3), 209–227.

List, G. F., Wood, B., Turnquist, M. a., Nozick, L. K., Jones, D. a. and Lawton, C. R. (2006), 'Logistics planning under uncertainty for disposition of radioactive wastes', *Computers and Operations Research* **33**(3), 701–723.

Listes, O. and Dekker, R. (2005), 'A Scenario Aggregation-Based Approach for Determining a Robust Airline Fleet Composition for Dynamic Capacity Allocation', *Transportation Science* **39**(3), 367–382.

Lohatepanont, M. and Barnhart, C. (2004), 'Airline Schedule Planning: Integrated Models and Algorithms for Schedule Design and Fleet Assignment', *Transportation Science* **38**(1), 19–32.

Maggioni, F., Potra, F. and Bertocchi, M. (2014), Stochastic versus Robust Optimization for a Transportation Problem, *in* 'Optimization-Online.Org, November 2014', pp. 1–35.

Milenković, M. and Bojović, N. (2013), 'A fuzzy random model for rail freight car fleet sizing problem', *Transportation Research Part C: Emerging Technologies* **33**, 107–133.

Oum, T. H., Zhang, A. and Zhang, Y. (2000), 'Optimal demand for operating lease of aircraft', *Transportation Research Part B: Methodological* **34**(1), 17–29.

Pearce, B. (2013), Profitability and the air transport value chain - IATA Economics Briefings, Technical report.

Pilla, V. L., Rosenberger, J. M., Chen, V. C. P. and Smith, B. (2008), 'A statistical computer experiments approach to airline fleet assignment', *IIE Transactions* **40**(5), 524–537.

Pilla, V. L., Rosenberger, J. M., Chen, V., Engsuwan, N. and Siddappa, S. (2012), 'A multivariate adaptive regression splines cutting plane approach for solving a two-stage stochastic programming fleet assignment model', *European Journal of Operational Research* **216**(1), 162–171.

Porter, M. E. (1979), 'How competitive forces shape stategy', *Harvard Business Review* pp. 137–141.

Powell, W. B. and Carvalho, T. A. (1997), 'Dynamic control of multicommodity fleet management problems', *European Journal of Operational Research* **98**(3), 522–541.

Rosenberger, J. M., Johnson, E. L. and Nemhauser, G. L. (2004), 'A Robust Fleet-Assignment Model with Hub Isolation and Short Cycles', *Transportation Science* **38**(3), 357–368.

Rosskopf, M., Lehner, S. and Gollnick, V. (2014), 'Economic-environmental trade-offs in long-term airline fleet planning', *Journal of Air Transport Management* **34**, 109–115.

Rosskopf, M. and Luetjens, K. (2012), Balancing Economic and Environmental Goals in Airline Fleet Planning: a Multi-Objective Optimization Approach, *in* '16th Air Transport Research Society (ATRS) World Conference Tainan, Taiwan, June 27-30, 2012'.

Rushmeier, R. a. and Kontogiorgis, S. a. (1997), 'Advances in the Optimization of Airline Fleet Assignment', *Transportation Science* **31**(2), 159–169.

Santos, B. (2013), 'Lecture Notes - Network and Fleet Planning - Delft University of Technology [Accessed on: 2015/07/28]'.
URL: *https://blackboard.tudelft.nl*

Sayarshad, H. R. and Ghoseiri, K. (2009), 'A simulated annealing approach for the multi-periodic rail-car fleet sizing problem', *Computers and Operations Research* **36**(6), 1789–1799.

Sayarshad, H. R. and Tavakkoli-Moghaddam, R. (2010), 'Solving a multi periodic stochastic model of the rail-car fleet sizing by two-stage optimization formulation', *Applied Mathematical Modelling* **34**(5), 1164–1174.

Schick, G. J. and Stroup, J. W. (1981), 'Experience with a multi-year fleet planning model', *Omega - The International Journal of Management Science* **9**(4), 389–396.

Schwartz, E. S. (1997), 'The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging', *The Journal of Finance* **52**(3), 923–973.

Sherali, H. D., Bish, E. K. and Zhu, X. (2006), 'Airline fleet assignment concepts, models, and algorithms', *European Journal of Operational Research* **172**(1), 1–30.

Sherali, H. D. and Zhu, X. (2008), 'Two-Stage Fleet Assignment Model Considering Stochastic Passenger Demands', *Operations Research* **56**(2), 383–399.

Shube, D. and Stroup, J. (1975), Fleet Planning Model, *in* 'Winter Computer Simulation Conference Proceedings', Society for Computer Simulation, pp. 45–50.

Stonier, J. E. (1999), What Is an Aircraft Purchase Option Worth ? Quantifying Asset Flexibility Created trough Manufacturer Lead-Time Reductions and Product Commonality, *in* 'Handbook of Airline Finance', Aviation Week Group (McGraw-Hill).

Subramanian, R., Scheff, R. P., Quillinan, J. D., Wiper, D. S. and Marsten, R. E. (1994), 'Coldstart: Fleet Assignment at Delta Air Lines', *Interfaces* **24**(1), 104–120.

Swan, W. M. (2002), 'Airline demand distributions: Passenger revenue management and spill', *Transportation Research Part E: Logistics and Transportation Review* **38**(3-4), 253–263.

Talluri, K. T. (1996), 'Swapping Applications in a Daily Airline Fleet Assignment', *Transportation Science* **30**(3), 237–248.

Teodorović, D. and Krčmar-Nožić, E. (1989), 'Multicriteria Model to Determine Flight Frequencies on an Airline Network under Competitive Conditions', *Transportation Science* **23**(1), 14–25.

Wang, T., Meng, Q. and Wang, S. (2012), ‘Robust Optimization Model for Liner Ship Fleet Planning with Container Transshipment and Uncertain Demand’, *Transportation Research Record: Journal of the Transportation Research Board* **2273**(1), 18–28.

Wang, Y., Sun, H., Zhu, J. and Zhu, B. (2015), ‘Optimization Model and Algorithm Design for Airline Fleet Planning in a Multiairline Competitive Environment’, *Mathematical Problems in Engineering* pp. Article ID 783917, in press.

Wei, W. and Hansen, M. (2007), ‘Airlines’ competition in aircraft size and service frequency in duopoly markets’, *Transportation Research Part E: Logistics and Transportation Review* **43**(4), 409–424.

Zhang, B. (2015), ‘Cover page picture [Accessed on: 2015/08/04]’.
URL: *http://static.businessinsider.com/image/55303a816bb3f7c76ee8a4d7-1200/image.jpg*