

# Robust Global Registration

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## Abstract

We present an algorithm for the automatic alignment of two 3D shapes (data and model), without any assumptions about their initial positions. The algorithm computes for each surface point a descriptor based on local geometry that is robust to noise. A small number of feature points are automatically picked from the data shape according to the uniqueness of the descriptor value at the point. For each feature point on the data, we use the descriptor values of the model to find potential corresponding points. We then develop a fast branch-and-bound algorithm based on distance matrix comparisons to select the optimal correspondence set and bring the two shapes into a coarse alignment. The result of our alignment algorithm is used as the initialization to Iterative Closest Point (ICP) and its variants for fine registration of the data to the model. Our algorithm can be used for matching shapes that overlap only over parts of their extent, for building models from partial range scans, as well as for simple symmetry detection, and for matching shapes undergoing articulated motion.

## Global Registration Problem

**Given**  
Two partially overlapping shapes, data (P) and model (Q), represented by point clouds or surface meshes, in arbitrary initial positions.

**Goal**  
Find a rigid transform that minimizes the distance between the data and the model in the overlap region.



## Method Overview

1. Compute a geometric descriptor for each point of data and model. We use integral volume descriptor.
2. Automatically select a small number of feature points based on uniqueness of descriptor values.
3. Use descriptor values to identify potential corresponding points for each feature.
4. Efficiently explore entire correspondence search space using a branch and bound algorithm to find the optimal set of correspondences.
5. Refine the alignment using Iterated Closest Point (ICP) algorithm.

## Integral Volume Descriptor

**Definition**

$$V_r(p) = \int_{B_r(p) \cap S} dx$$

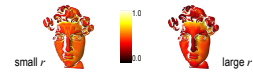


**Properties**

- Related to mean curvature ( $H$ ) as  $V_r(p) = \frac{2\pi}{3}r^3 - \frac{\pi H}{4}r^4 + O(r^5)$ .
- Robust to noise.
- Multi-scale with scale controlled by  $r$ .

**Computation**

Voxelize the interior of the shape onto a grid and convolve with the rasterization of the ball  $B_r$ . Can be efficiently computed using FFT.



## Feature Point Selection

Points with rare descriptor values are candidates for features.  
Rare in the data  $\rightarrow$  rare in the model  $\rightarrow$  few correspondences.

Candidates which are persistent over different scales are chosen as features.

Space points far apart. When a point is picked, exclude all points within  $R_e$ .



**Feature properties**

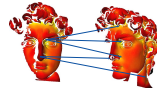
- Sparse
- Robust to noise
- Non-canonical

● large scale feature  
● small scale feature

## Search Space

**Correspondence Selection**

For each feature point  $p_i$  with descriptor  $V(p_i)$ , find all points  $q_j$  in the model such that  $|V(p_i) - V(q_j)| < \epsilon$ . Cluster corresponding points, and pick representatives from each cluster.



**Rigidity Constraint**

Distance between pairs of points and their correspondences should be preserved.



## Correspondence Search

To find the correct set of correspondences for the feature points, we minimize the pairwise distance mismatch:

$$dRMS^2(P, Q) = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n (\|p_i - p_j\| - \|q_i - q_j\|)^2$$

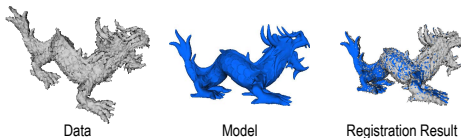
Explore the correspondence search space using a branch and bound algorithm.

- prune if partial correspondence exceeds current best assignment.
- prune if any pairs of points in partial correspondence violates rigidity constraint.

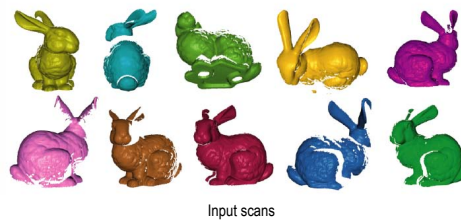
Since we explore the entire search space, we are guaranteed to find the optimal assignment.

## Results

### Whole Object Matching

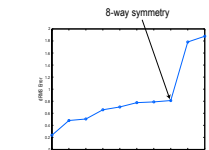


### Automatic Scan Registration



### Applications

#### Symmetry Detection

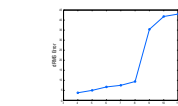


Model

dRMS vs Size of Correspondence Set

### Partial Matching

Maximize the number of feature points that get assigned a valid corresponding point in the model, while still keeping the dRMS error of the correspondence set low.

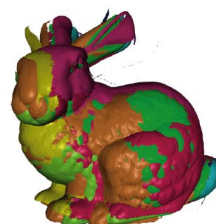


dRMS vs Size of Correspondence Set



Registration using our algorithm

Refinement with ICP

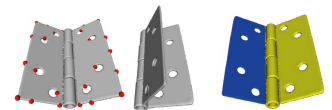


Registration using our algorithm



Refinement with ICP

#### Segmentation and Matching for Simple Articulated Motion



Input Poses

Segmentation