IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received July 18, 2019, accepted August 14, 2019, date of publication August 20, 2019, date of current version December 23, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2936418

Robust Multi-Frame Joint Frequency Hopping Radar Waveform Parameters Estimation Under Low Signal-Noise-Ratio

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This work was supported in part by the National Natural Science Foundation of China under Grant 61771372 and Grant 61771367, and in part by the National Outstanding Youth Science Fund Project under Grant 61525105.

ABSTRACT Robust estimation of frequency hopping radar signal parameters is the key to achieving stable jam in frequency hopping communication. In order to deal with the poor noise resistance performance of conventional time-frequency analysis based parameter estimation method under low signal-noise-ratio conditions. A robust method is proposed in this paper for frequency hopping radar waveform parameters estimation. The method is performed in time domain. Firstly, the pulse repetition interval of the received radar signal is extracted by matching filtering, which is used as the window length to divide the signal into multiple frames. The coherent integration process is performed on the multi-frame signal to increase the signal-noise-ratio, which effectively ensures the robustness of the algorithm under low signal-noise-ratio conditions. Secondly, the short-time Fourier transform is used to realize the rough estimation of the frequency hopping number, which reduces the range of subsequent hopping counts and reduces the amount of algorithm calculation. Finally, an accurate estimation of the waveform parameters including frequency hopping sequence, hop duration, frequency hopping interval and carrier frequency is realized. Simulation experiments verify the effectiveness of the algorithm and the robustness under various signal-noise-ratio conditions.

INDEX TERMS Hopping frequency waveform, parameter estimation, multiple-frame, short time Fourier transform (STFT).

I. INTRODUCTION

Frequency hopping technology is an effective anti-jamming communication technology, it has wide applications in military communications and other civilian fields [1]–[5]. The carrier frequency of the frequency hopping radar signal is pseudo-randomly hopped with time, so FH radar signal has a high spectrum utilization rate. In addition, the FH radar signal also has obvious advantage in low interception and anti-interference. It is difficult for conventional jammers to perform frequency tracking on this signal. With the complexity of FH radar signal applications, it is of great significance to achieve accurate and robust estimation of the received low signal-noise-ratio (SNR) FH radar signal parameters [6]–[9].

The frequency hopping radar signal is a typical non-stationary signal, which is generally estimated by time-frequency analysis method [10], [11]. The FH radar signal is composed of some single frequency signals, which has equal duration and different carrier frequencies. Therefore, the carrier frequency of the FH radar signal is time-varying. Conventional Fourier transform can only display the frequency domain information of the signal, while the time-frequency analysis can reflect the frequency variation of the signal with time. According to the above property, time-frequency analysis is very suitable for parameter estimation of frequency hopping radar signals. Short-time

The associate editor coordinating the review of this manuscript and approving it for publication was Nagendra Prasad Pathak.

Fourier transform (STFT) [12], Wigner-Ville distribution (WVD) [15] and smoothed pseudo Wigner-Ville distribution (SPWVD) [16] are commonly used time-frequency analysis methods. STFT is a parameter estimation method with small computation complexity. STFT is implemented in [12]–[14] to estimate the FH signal parameters, but the frequency resolution and time resolution of the STFT parameter estimation result cannot meet the high precision requirement at the same time. WVD is proposed by Barbarossa in [15] to estimate parameters such as hop time and hop frequency. This method has good time-frequency aggregation and can achieve better time-frequency resolution, but this method will be affected by severe cross-term when processing FH signals [16]. SPWVD [16]–[18] is an improved method of WVD, which effectively suppresses cross-term interference. But SPWVD adopts time-domain smoothing filtering, which greatly increase the computational complexity of the algorithm. It can be seen that for the single frame parameter estimation method based on time-frequency analysis, when the parameter estimation accuracy is improved, the algorithm computational burden will be significantly increased. Moreover, the performance of time-frequency analysis based method is obviously affected by SNR. Under low SNR conditions, the edge characteristics of signal time-frequency analysis are seriously affected by noise, and the parameter estimation performance of this kind of method will be significantly reduced [16], [19]–[21].

Therefore, non-coherent accumulation is applied in [22] to preprocess the signal, and spectrogram method is then implemented to estimate the parameter of FH signal. Non-coherent accumulation effectively improves signal noise resistance ability. However, the spectrogram method is a computationally intensive algorithm so it is not suitable for rapid parameter estimation. In [23], a maximum likelihood parameter estimation method is proposed, which has lower computational complexity and higher parameter estimation accuracy than time-frequency distribution (TFD) based algorithm. However, the algorithm still performs poorly under low SNR conditions. In addition, sparse linear regression (SLR) is applied in [24], [25] to estimate the FH signals, high-precision estimation with low computational complexity is achieved. However, the impact of low SNR is also a major problem. In [24], it is shown that this method shows good performance when the SNR is greater than or equal to 10dB. And when the SNR is lower than 5dB, the estimation error of the algorithm will increase significantly. Therefore, the motivation of this paper is estimate the parameters of FH signal with high accuracy at low SNR.

In response to these problems, a time domain parameter estimation method is proposed in this paper. Firstly, the pulse repetition interval (PRI) of the radar received signal is extracted by matched filtering. PRI is used to divide the received signal into multi-frame FH radar signals for joint processing. Secondly, the joint multi-frame radar signal is used to implement coherent accumulation, which effectively improves the SNR of the original signal. And multi-frame coherent accumulation guarantees the robustness of the parameter estimation under low SNR conditions. Then, the STFT is performed on the signal after the coherent accumulation to obtain the rough estimation of hop number of FH radar signal. Finally, exhaustive search calculation is performed on the hop number. After that, the frequency hopping sequence, hop duration, frequency hopping interval and signal carrier frequency of FH radar signal can all be estimated. Essentially, the method uses multi-frame signal coherent accumulation to improve the SNR of the signal. Then, the parameter estimation is performed on the accumulated signal, which effectively guarantees the robustness of the algorithm under low SNR conditions. The innovations of this method are summarized as follows:

1) Strong noise resistance ability. The proposal assures a high parameter estimation accuracy under low SNR condition with -10dB;
2) Blind estimation of parameters. Without any prior information, the proposed algorithm can estimate the hop duration, frequency hopping interval, frequency hopping sequence and carrier frequency of FH radar signal blindly.

The content of this paper is as follows: Section II introduces the mathematical model of the frequency hopping radar signal and the characteristics of the signal carrier frequency hopping with time. The problem of conventional single frame time-frequency analysis parameter estimation method is also analyzed. The parameter estimation method of this paper is introduced in Section III. The simulation experiment in Section IV verifies the feasibility and effectiveness of the proposed algorithm. And the proposed algorithm is compared with the conventional single-frame STFT parameter estimation method under various SNR conditions, and the robustness of the proposed algorithm is illustrated.

II. PRELIMINARIES AND PROBLEM ANALYSIS

A. SIGNAL MODEL

The frequency of the FH radar signal is pseudo-randomly hopped according to the frequency hopping pattern. The property of FH signal of frequency varying with time is shown in Fig 1. According to the frequency hopping pattern, the FH radar signal is composed of n single frequency chip signals of the same duration and different carrier frequency and the carrier frequency of each chip signal is related to the code k. Each chip signal is a single frequency signal with a short duration.

The mathematical expression of the FH radar signal model [14] is

$$s(t)=A\cdot\sum_{k=1}^{n} rect\left(\frac{t}{T}\right)\cdot e^{j2\pi(f_c+k\Delta f)(t-kt_k)+j\varphi}+n(t) \tag{1}$$

where, $0 \leq t \leq T$, A is the signal amplitude, T is the pulse repetition time of the FH radar signal, n is the hop number, t_k is the hop duration, f_c is the carrier frequency of FH radar signal, $f_c + k\Delta f$ is the hop frequency, $\{f_c+\Delta f, f_c+2\Delta f, \cdots, f_c+k\Delta f, \cdots, f_c+n\Delta f\}$

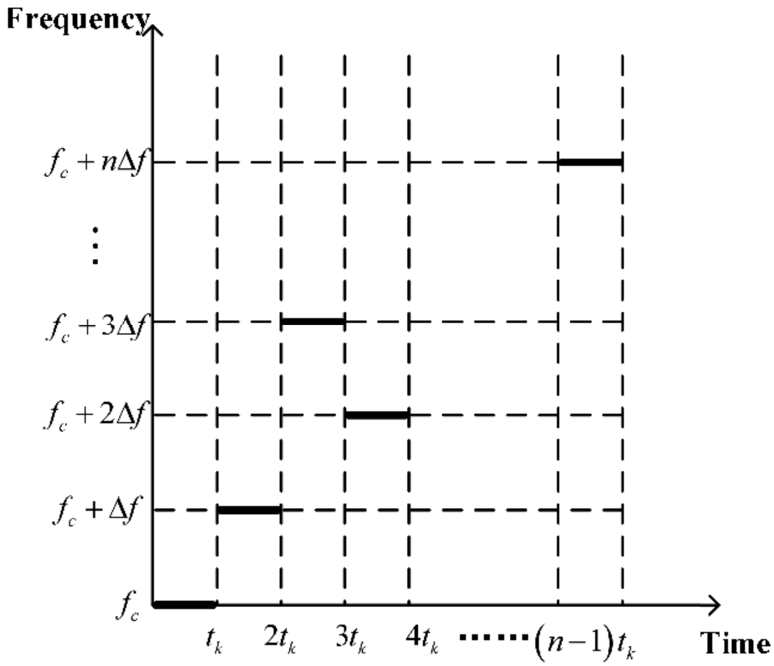


FIGURE 1. Signal hopping pattern.

constitutes the carrier frequency set of FH radar signal, φ is the initial modulation phase, $n(t)$ is a Gaussian white noise signal, and $rect(\cdot)$ is a rectangular window function.

According to the property of the Fourier transform:

$$s(t) = A \cdot e^{j2\pi f_c t} \Leftrightarrow S(\omega) = P \cdot \delta(f - f_c) \tag{2}$$

Then the result of Fourier transforming a single chip signal will obtain a unique peak point at the frequency f_c . In order to ensure the continuity of the signal spectrum, the carrier frequency hopping must be continuous, so the frequency hopping sequence is a linear sequence. With this property, the fine search of hop number can be achieved, and then the frequency hopping sequence, the frequency hopping interval and the carrier frequency can be estimated.

B. STFT BASED PARAMETER ESTIMATION

STFT is commonly used for the analysis of non-stationary signals such as FH signals due to its small computation complexity. The conversion formula of STFT is:

$$STFT(t,f) = \int_{-\infty}^{+\infty} [s(t)\,h(t-\tau)]\, e^{-j2\pi f\tau} d\tau \tag{3}$$

where $s(t)$ is the signal for performing the STFT and $h(t)$ is the fixed-length window function. At time τ, the window function of length L is multiplied by the signal $s(t)$, and then Fourier transform is performed to obtain the STFT result of the signal $s(t)$ at time τ. Move the window function in time order to get the STFT result of the signal over time.

The window function length L is an important parameter. Window length L that directly affects the time resolution and frequency resolution of time-frequency spectrum. The time resolution and the frequency resolution are contradictory to each other, so the window length needs to be determined according to specific requirements.

Select the appropriate window length to get the STFT result time-frequency matrix of FH signal, $TF = STFT(t,f)$. Calculate the maximum value of each column along the time axis for the time-frequency matrix TF, and obtain the time-frequency ridge V of the FH signal. V shows the characteristics of the signal frequency as a function of time. When the SNR condition is better, the shape of V is similar to the frequency hopping pattern of the signal. Differentiate the time-frequency ridge along the time axis, the position where the difference result is not zero is the frequency hop point. The difference between adjacent two hop points is the hop duration, and take the mean of multiple hop durations as an estimate of the parameter. The time-frequency ridge of one chip between adjacent two hop points, taking the frequency mean of the time-frequency ridge as the frequency estimate of the chip. Schematic diagram of STFT using time-frequency ridge to estimate the FH signal parameters is shown in Fig.2. The key to STFT based parameter estimation algorithm is the accurate extraction of the time-frequency ridges.

C. PROBLEM ANALYSIS AT LOW SNR

The actual radar received signal model is:

$$f(t) = s(t) + n(t) \tag{4}$$

where $s(t)$ is a FH radar signal and $n(t)$ is a Gaussian white noise signal. According to the linear nature of STFT,

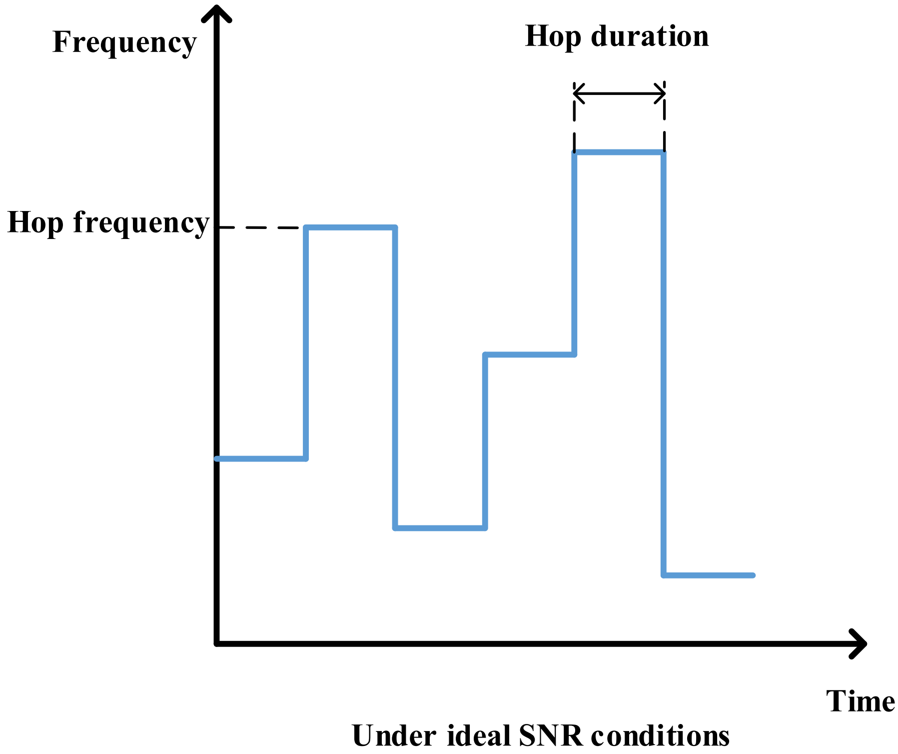


FIGURE 2. Relationship between time-frequency ridges and estimated parameters.

the STFT result of $f(t)$ is equal to the sum of the STFT results of $s(t)$ and $n(t)$. And it can be expressed by:

$$\begin{aligned} STFT(t,f) &= \int_{-\infty}^{+\infty} [f(t)\,h(t-\tau)]\,e^{-j2\pi f\tau}d\tau \\ &= \int_{-\infty}^{+\infty} [s(t)\,h(t-\tau)]\,e^{-j2\pi f\tau}d\tau \\ &\quad + \int_{-\infty}^{+\infty} [n(t)\,h(t-\tau)]\,e^{-j2\pi f\tau}d\tau \end{aligned} \quad (5)$$

From equation (5), the problem of analyzing the effect of SNR on STFT is converted into an analysis of the STFT result for $n(t)$. Gaussian white noise is a power signal with unlimited energy and no Fourier transform. However, the STFT intercepts the signal with a shorter time window and performs a Fourier transform on the intercepted signal. It means that when performing the STFT, $n(t)$ is a Gaussian white noise over a period of time with limited time and limited energy. Therefore, the signal $n(t)$ has a Fourier transform, which can be analyzed by spectrum. According to the Wiener - Khinchin theorem [26], the power spectral density is equal to the Fourier transform of the signal autocorrelation function:

$$PSD_n(\omega) = \int_{-\infty}^{\infty} R_{nn}(\tau)\,e^{-j\omega\tau}d\tau \quad (6)$$

where $PSD_n(\omega)$ is the power spectral density of $n(t)$ and $R_{nn}(\tau)$ is the autocorrelation function of $n(t)$. And because $R_{nn}(\tau)$ has the following properties:

$$\begin{aligned} R_{nn}(\tau) &= n(\tau) * n^*(-\tau) \\ |N(j\omega)|^2 &= N(j\omega)\cdot N^*(j\omega) \\ n(\tau) * n^*(-\tau) &\Leftrightarrow N(j\omega)\cdot N^*(j\omega) \end{aligned} \quad (7)$$

where $*$ is the convolution operator, $(\cdot)^*$ is the conjugate, $N(j\omega)$ is the Fourier transform result of $n(t)$, and $\Leftrightarrow$ represents the Fourier transform relationship. Combining equations (6) and (7), the power spectral density of $n(t)$ is the square of the modulus of the Fourier transform result:

$$PSD_n(\omega) = |N(j\omega)|^2 \quad (8)$$

Just the STFT algorithm also uses the modulus value of the time-frequency matrix to extract the signal time-frequency ridge. Therefore, the power spectral density of $n(t)$ directly affects the extraction of time-frequency ridges in the STFT algorithm. The error extraction of the time-frequency ridge is the most important reason for the error of the STFT parameter estimation algorithm. Therefore, the relationship between the power spectral density $PSD_n(\omega)$ of $n(t)$ and the SNR is analyzed below. It is known that the power spectral density of Gaussian white noise obeys a uniform distribution, and its value is as shown in (9), and the SNR is defined as shown in equation (10).

$$PSD_n(\omega) = \frac{N_0}{2} \quad (9)$$

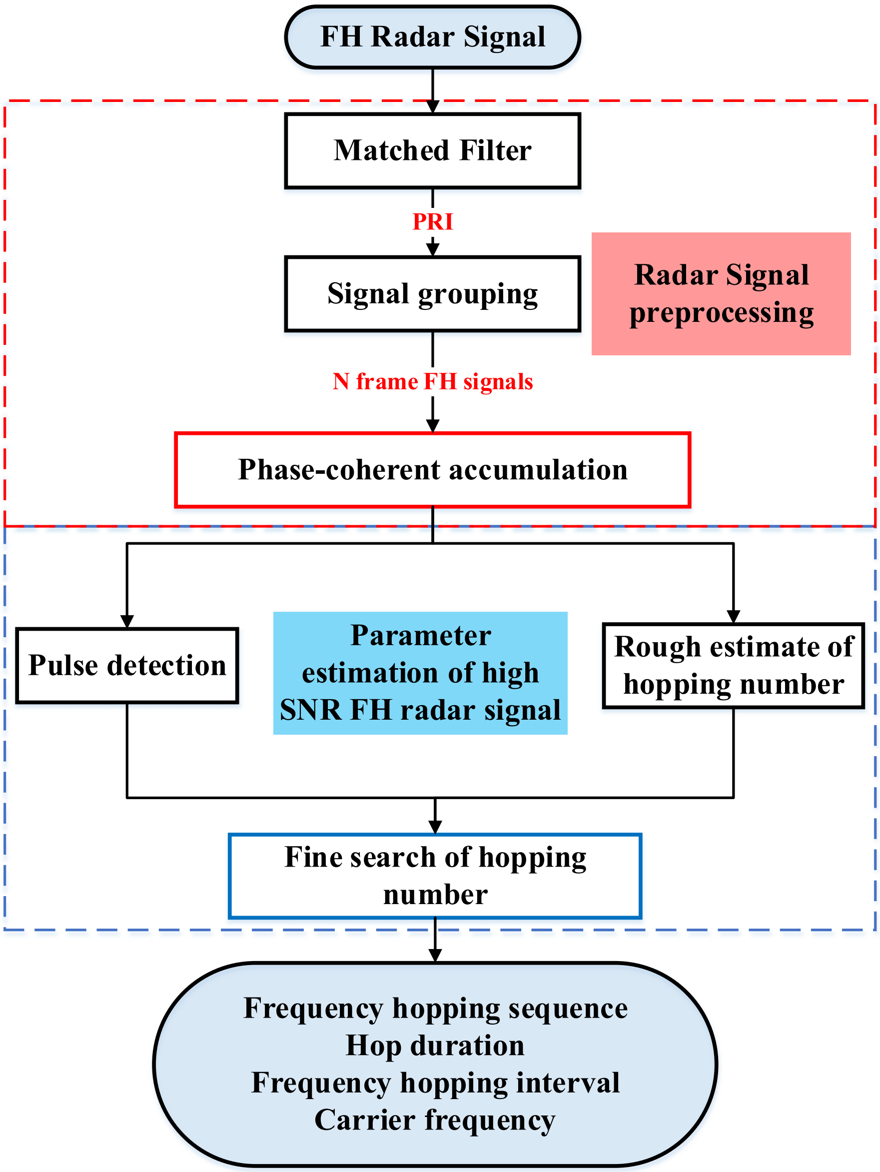


FIGURE 3. Parameter estimation method flow chart.

$$SNR = 10\lg\left(\frac{P_s}{P_n}\right) = 10\lg\left(\frac{P_s}{N_0 \cdot B}\right) \tag{10}$$

where N_0 is a constant, the magnitude of N_0 is related to the variance of Gaussian white noise, P_s is the average power of the signal, P_n is the average power of the noise, and B is the effective bandwidth of the receiver.

It can be seen from (10) that B is generally invariant. If P_s is also assumed to be constant, when the SNR is lowered, it must be caused by an increase in P_n. That is to say, the power spectral density $PSD_n(\omega)$ of the noise is increased. The increase of $PSD_n(\omega)$ means that when the signal $f(t)$ uses the STFT algorithm to extract the time-frequency ridge, the component of the noise may be larger than the component of the signal, thereby detecting the noise as a signal. The falsely detected noise becomes the wrong hop point estimate, which leads to a large error in the STFT algorithm parameter estimation. And the lower the SNR, the more likely this problem will occur. If N_0 is assumed to be constant and P_s is variable, then when the SNR is lowered, the signal energy is reduced. When the SNR drops to a lower level, the noise component may be larger than the signal component, and the reason for the parameter estimation error is similar to the former case.

It can be seen that the SNR has a great influence on the STFT parameter estimation algorithm. Especially in the low SNR condition, the STFT algorithm is easily affected by noise and gets the wrong hop point estimation, which leads to the error estimation of parameter.

III. PROPOSED PARAMETER ESTIMATION ALGORITHM

A. FLOWCHART OF THE PROPOSED ALGORITHM

The algorithm flow of this paper is shown in Fig.3. The algorithm steps are mainly divided into two parts. The first part is the preprocessing of the radar signal, and the second part is the parameter estimation of the high SNR FH radar signal. The steps are described as follows:

a) Radar signal preprocessing. This is the core step of the algorithm to maintain the robustness of parameter estimation under low SNR conditions. Firstly, the received FH radar signal is passed through a matched filter to obtain a PRI estimation value of the pulse signal. Secondly, the original input signal is processed by PRI to obtain a multi-frame FH radar signals. Phase compensation and coherent accumulation are then performed on the multi-frame FH radar signals. Finally, a high SNR FH radar signal is obtained. Parameter estimation of high SNR signals can effectively ensure high accuracy of parameter estimation results.

b) Parameter estimation of high SNR FH radar signal. The accuracy of the proposed algorithm parameter estimation results in this paper mainly depends on the estimation of the hopping number. Firstly, the preprocessed FH radar signal is roughly estimated by the STFT method. Based on the rough estimate of hopping number, the search range is determined to avoid the amount of redundant computation caused by blind search. Secondly, it is known that the frequency hopping sequence is linear and continuous, and as a criterion, the hopping number of FH radar signal is refined. Finally, the frequency hopping sequence, hop duration, frequency hopping interval, and carrier frequency under the best estimation are calculated as the parameter estimation result output.

The algorithm steps are described in detail in Section III-B and III-C. Matched filter and signal grouping in radar signal preprocessing are introduced in Section III-B. The N-frame FH radar signal obtained in Section III-B is used as the input to Section III-C. In III-C, the phase-coherent accumulation is first performed in III-C.1, and then the rough estimation of hopping number is implemented in III-C.2. Finally, the pulse detection and fine search of hopping number are implemented in III-C.3. Through the operation of Section III-B and III-C, an accurate estimation of the parameters of the FH radar signal can be obtained.

B. PRI ESTIMATION AND MULTI-FRAME RADAR SIGNAL ACQUIRING

The radar pulse signal is transmitted repeatedly by the transmitter in a period of PRI. So PRI can be estimated by the signal autocorrelation property. The autocorrelation function expression of the complex signal $s(t)$ with a pulse repetition period of T is:

$$\begin{aligned} R_{ss}(\tau) &= \lim_{\tau\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s(t)\, s^*(t+\tau)\, dt \\ &= \lim_{\tau\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s(t)\, s^*(t+\tau+nT)\, dt \end{aligned} \tag{11}$$

It can be seen from the above equation that the autocorrelation function of the periodic signal is also periodic, and its period is also T. The received radar signal model $f(t)$ contains a pulse signal $s(t)$ and a Gaussian white noise $n(t)$, where $s(t)$ and $n(t)$ are uncorrelated.

$$f(t) = s(t) + n(t) \tag{12}$$

The autocorrelation function of the received signal $f(t)$ is:

$$R_{ff}(\tau) = R_{ss}(\tau) + R_{sn}(\tau) + R_{ns}(\tau) + R_{nn}(\tau) \tag{13}$$

where $R_{ss}(\tau)$ is the autocorrelation function of the pulse signal $s(t)$, $R_{nn}(\tau)$ is the autocorrelation function of the noise signal $n(t)$, and $R_{sn}(\tau)$, $R_{ns}(\tau)$ are the cross-correlation functions of the pulse signal and the noise signal. Since the signal is not correlated with noise, the two values $R_{sn}(\tau)$ and $R_{ns}(\tau)$ are zero. The formula (13) can be written as:

$$R_{ff}(\tau) = R_{ss}(\tau) + R_{nn}(\tau) \tag{14}$$

Also, because the autocorrelation function of Gaussian white noise is an impulse function. Therefore, according to equation (14), it can be approximated that the autocorrelation function of $f(t)$ is equal to the autocorrelation function of $s(t)$. The period of $R_{ss}(\tau)$ is also the period of $R_{ff}(\tau)$. Thus, the estimation of the PRI of the unknown radar signal can be achieved by the following process:

a) The autocorrelation of the signal can be achieved by matched filtering. The PRI of the signal can be extracted by means of matched filtering of the signal. Matched filter is a linear filter that produces the maximum output SNR. The matched filter is the complex conjugate after the time flop of the known pulse signal $s(t)$, then the impulse response of the matched filter is:

$$h(t) = k \cdot s^*(T-t) \tag{15}$$

where k is a constant and T is the signal pulse repetition time.

b) The received signal $f(t)$ is convolved with the matched filter impulse response $h(t)$ to obtain a matched filtered output:

$$\begin{aligned} y(t) &= \int_{-\infty}^{+\infty} f(\tau)\, h(t-\tau)\, d\tau \\ &= \int_{-\infty}^{+\infty} [s(\tau)+n(\tau)]\, h(t-\tau)\, d\tau \\ &= \int_{-\infty}^{+\infty} s(\tau)\, h(t-\tau)\, d\tau \\ &\quad + \int_{-\infty}^{+\infty} n(\tau)\, h(t-\tau)\, d\tau \end{aligned} \tag{16}$$

The noise signal $n(t)$ is uncorrelated with the matched filter $h(t)$, so the second term of the above equation is zero. Equation (16) can be rewritten as follows:

$$y(t) = \int_{-\infty}^{+\infty} s(\tau)\, h(t-\tau)\, d\tau \tag{17}$$

TABLE 1. The main steps of multi-frame signal generation.

Algorithm 1 The multi-frame signal generation method
Input: $f(t)$, Radar receiving signal, $h(t)$, Matched filter impulse response
Step1: Pass the signal $f(t)$ through the filter $h(t)$ and the matched filter result $y(t)$ is obtained by (18).
Step2: Calculate the PRI estimate based on the matched filter result $y(t)$ by (19).
Step3: Decompose $f(t)$ and get multi-frame radar signal $s_n(t)$ through (20).
Output: The multi-frame FH signals $s_1(t)$,$s_2(t)$, $\cdots$ $s_n(t)$,$\cdots$ $s_N(t)$.

c) Substituting equation (15) into equation (17), let $k = 1$, we can get the following formula:

$$\begin{aligned} y(t) &= \int_{-\infty}^{+\infty} s(\tau)\, s^*(T-t+\tau)\, d\tau \\ &= R_{ss}(T-t) \\ &= R_{ss}(t) \end{aligned} \tag{18}$$

According to the equation (18), the matched filtering result obtains a peak at time $t = 0$. And due to the periodicity of the autocorrelation function, when $t = mT$, $m = 1, 2, 3, \cdots$, the matched filtering result can also obtain a peak value. The median of the peak position interval of the matched filtering result is taken as the estimated value of the PRI:

$$PRI_{est} = mid\,(diff\,(T_n)) \tag{19}$$

where T_n represents the peak position of the matched filtering result, $diff\,(\cdot)$ represents the difference operation, and $mid\,(\cdot)$ represents the median operation.

d) After obtaining the PRI estimation value, the received FH radar signal is decomposed to obtain the N frame FH radar signal. Each frame of the signal can be expressed as:

$$\begin{cases} s_n(t) = f(t_n) \\ (n-1)\,PRI \le t_n \le nPRI \end{cases} \tag{20}$$

Extracting the PRI of a complete FH radar signal and decomposing it into multi-frame signals for joint processing is an important prerequisite for the parameter estimation algorithm. The advantage of the proposed algorithm is that even if the SNR of the input FH radar signal is low, the higher parameter estimation accuracy can still be achieved. This is because the proposed algorithm uses coherent accumulation to improve the SNR of the signal to be processed. Therefore, the robustness of the algorithm under low SNR conditions is effectively guaranteed. To achieve the coherent accumulation, it is necessary to have multiple sets of signals with the same waveform parameters and phase coherence. Therefore, the extraction of multi-frame signals in this section is a necessary precondition for the subsequent parameter estimation algorithm.

C. FH PARAMETER ESTIMATION

The proposed algorithm for estimating the FH radar signal parameters is introduced in this section. The algorithm combines the multi-frame FH radar signals, so that the algorithm can maintain high parameter estimation accuracy under low SNR conditions. And the estimation of hop duration, frequency hopping interval, frequency hopping sequence and carrier frequency is realized.

1) MULTI-FRAME JOINT PHASE-COHERENT ACCUMULATION

Using the multi-frame signal joint accumulation to improve the SNR of the signal to be processed is the key to the parameter estimation algorithm. Combining the multi-frame radar signals obtained in Section III-A, the coherent integration method with better SNR improvement is used to improve the SNR of the FH radar signal. Since each frame of the FH radar signal has the same modulation mode, waveform parameters and mathematical expression, it can be known from equation (1) that the multi-frame FH radar signal obtained in Section III-A can be expressed as:

$$\begin{cases} s_1(t) = A \sum_{k=1}^{n} rect\left(\frac{t}{T}\right) e^{j2\pi(f_c+k\Delta f)(t-kt_k)+j\varphi_1} + n(t) \\ s_2(t) = A \sum_{k=1}^{n} rect\left(\frac{t}{T}\right) e^{j2\pi(f_c+k\Delta f)(t-kt_k)+j\varphi_2} + n(t) \\ \vdots \\ s_N(t) = A \sum_{k=1}^{n} rect\left(\frac{t}{T}\right) e^{j2\pi(f_c+k\Delta f)(t-kt_k)+j\varphi_N} + n(t) \end{cases} \tag{21}$$

where, $0 \le t \le T$, A is the signal amplitude, T is the pulse repetition time of the FH radar signal, n is the hop number, t_k is the hop duration, f_c is the carrier frequency of FH radar signal, $f_c + k\Delta f$ is the hop frequency, $\{f_c + \Delta f, f_c + 2\Delta f, \cdots, f_c + k\Delta f, \cdots, f_c + n\Delta f\}$ constitutes the carrier frequency set of FH radar signal, φ_n is the initial modulation phase, $n(t)$ is a Gaussian white noise signal, and $rect\,(\cdot)$ is a rectangular window function.

It can be seen from equation (21) that there is different initial phase modulation between any two frame of signals, and other modulation methods are the same. Therefore, the phase compensation of the signal is required before the coherent accumulation is performed, so that any two signals become coherent. Suppose that $s_1(t)$ is the reference signal, φ_1 is the reference phase, and $s_n(t)$ is the signal to be compensated, then the phase difference between the signal $s_n(t)$ and the reference signal $s_1(t)$ is $\Delta\varphi = \varphi_1 - \varphi_n$. Phase compensation for signal $s_n(t)$ is :

$$\begin{aligned} s_n'(t) &= s_n(t) \cdot \exp(j\Delta\varphi) \\ &= A \cdot \sum_{k=1}^{n} rect\left(\frac{t}{T}\right) e^{j2\pi(f_c+k\Delta f)(t-kt_k)+j\varphi_1} + n(t) \end{aligned} \tag{22}$$

It can be seen from equation (22) that the phase-compensated signal $s_n'(t)$ is in phase with the reference signal $s_1(t)$. $s_n'(t)$ and $s_1(t)$ can be used directly for coherent accumulation to improve SNR. Let $n = 1, 2, 3, \cdots, N$, for each frame signal, phase compensation processing is performed with reference signal $s_1(t)$, and a set of coherent multi-frame FH radar signals are obtained:

$$\begin{cases} s_1(t) = A \cdot \sum_{k=1}^{n} rect\left(\frac{t}{T}\right) e^{j2\pi(f_c+k\Delta f)(t-kt_k)+j\varphi_1} + n(t) \\ s_2'(t) = A \cdot \sum_{k=1}^{n} rect\left(\frac{t}{T}\right) e^{j2\pi(f_c+k\Delta f)(t-kt_k)+j\varphi_1} + n(t) \\ \vdots \\ s_N'(t) = A \cdot \sum_{k=1}^{n} rect\left(\frac{t}{T}\right) e^{j2\pi(f_c+k\Delta f)(t-kt_k)+j\varphi_1} + n(t) \end{cases} \tag{23}$$

Accumulate the coherent signals in (23) to obtain the accumulated high SNR signal:

$$\begin{aligned} S(t) &= s_1(t) + \sum_{n=2}^{N} s_n'(t) \\ &= N \cdot A \cdot \sum_{k=1}^{n} rect\left(\frac{t}{T}\right) e^{j2\pi(f_c+k\Delta f)(t-kt_k)+j\varphi_1} + n(t) \end{aligned} \tag{24}$$

Comparing (24) with (22), the signal level after the coherent accumulation is significantly improved, and the noise still maintains the original level. It is indicated that SNR of the original signal is effectively improved. Under ideal conditions, after coherent accumulation, the SNR can be increased by N times (N is the number of coherent accumulation signals). After coherent accumulation, parameter estimation of the high SNR FH radar signal $S(t)$ can effectively reduce the impact of noise. Coherent accumulation improves the noise resistance performance of the algorithm, and ensure the robustness of the algorithm under low SNR conditions.

2) HOPPING NUMBER

If the hopping number of FH radar signal is exhaustively searched directly, it is necessary to ensure that the search range is large enough. As a result, a lot of unnecessary calculation are generated during the search process. It can be seen from Section III-C.1 that the SNR of the FH radar signal is significantly improved after multi-frame accumulation, so the STFT with the smallest amount of computation in the time-frequency analysis method can be used to estimate the hopping number. That is to say, the search of hopping number is preprocessed first, so that the search range is narrowed and the unnecessary calculation amount of the algorithm is reduced. The STFT formula is calculated as follows:

$$TF = STFT(t, f) = \int_{-\infty}^{+\infty} [s(t)\, h(t-\tau)]\, e^{-j2\pi f\tau} d\tau \tag{25}$$

TABLE 2. The main steps of hopping number estimation.

Algorithm 2 The hopping number estimation method
Input: $S(t)$,Multi-frame signal coherent accumulation result
Step1: Perform a STFT on $S(t)$, and obtain the time-frequency matrix TF of the FH radar signal according to (25).
Step2: Extract time-frequency ridges by (26).
Step3: Calculate the number of frequency hopping points by (27) (28), and the hopping number is $X = x + 1$.
Output: The hopping number X.

The STFT of the high SNR radar signal $S(t)$ can obtain the time-frequency matrix TF of FH radar signal. Calculating the maximum value of each column along the time axis for the time-frequency matrix TF and obtaining the time-frequency ridge V of the FH radar signal:

$$V(t) = \max_f [STFT(t, f)] = \max_f [TF(t)] \tag{26}$$

V shows the characteristics of signal frequency with time. When the SNR is better, its shape is similar to the signal hopping pattern. The time-frequency ridge V is differentiated along the time axis:

$$d = diff[V(t)] \tag{27}$$
$$x = numel(d > 0) \tag{28}$$

The position where the difference result is not zero is the frequency hopping point. Assuming that the number of frequency hopping points is x, then the rough estimate of hop number is $X = x + 1$.

3) FREQUENCY HOPPING RADAR SIGNAL PARAMETERS

A high SNR FH radar signal $S(t)$ is obtained by coherent accumulation in Section III-C.1. And the pulse signal of $S(t)$ is clearly distinguished from the noise, so that the pulse signal can be separated from the noise by the threshold detection method. Then, based on X, the pulse signal is exhaustively cut within a certain range to realize the fine search of the FH radar signal hopping number. Thereby, the frequency hopping sequence, hop duration, frequency hopping interval and carrier frequency estimation result of the FH radar signal are obtained.

Setting a noise threshold T_h, detecting a pulse signal $p(t)$ in the signal $S(t)$ that is higher than T_h. Extracting the pulse width L_T of the signal $p(t)$, and according to the sampling rate F_s, the pulse time width estimation of radar signal $S(t)$ is obtained by the following formula:

$$T_p = \frac{L_T}{F_s} \tag{29}$$

Based on X, the pulse signal $p(t)$ is cut into $m = X \pm I$ equal parts (I is a positive integer). Each part is one chip signal $C_i(t), i = 1, 2, 3, \cdots m$. Fourier transform is performed on the signal $C_i(t)$ to obtain the frequency domain result

$C_i(\omega) = FFT[C_i(t)]$. The frequency corresponding to each signal $C_i(t)$ is:

$$\begin{cases} [m, pos] = \max[C_i(\omega)] \\ F_i = \dfrac{pos}{length[C_i(\omega)]} \cdot F_s \end{cases}, \quad i = 1, 2, 3, \cdots, m \tag{30}$$

F_i is arranged from small to large and the result is differentially processed. Taking the median of the difference result as the reference value of frequency hopping interval:

$$\Delta f_{ref} = mid[diff[sort(F_i)]] \tag{31}$$

where $mid(\cdot)$ represents the median operation and $diff(\cdot)$ represents the difference operation. The frequency hopping sequence estimation result under this cutting condition can be obtained by (32) and (33):

$$P = \frac{F_i - F_1}{\Delta f_{ref}} \tag{32}$$

$$Code_m = round(-(P - \max(P) - 1)) \tag{33}$$

where $round(\cdot)$ is rounded off. Taking the difference between the maximum value and the minimum value of $Code_m$, the sequence coding interval under the cutting condition is:

$$\Delta Code_m = \frac{\Delta C_m}{m} \tag{34}$$

Constructing a linear frequency hopping sequences of FH radar signal by the following formula:

$$U_m = [1, 2, 3, \cdots i, \cdots m] \cdot \Delta Code_m \tag{35}$$

where i is a positive integer. Sequence $Code_m$ is arranged from small to large to obtain sequence Q_m. Then, the estimation error of the frequency hopping sequence under this cutting condition is $D_m = Q_m - U_m$. The mean square error of D_m can be expressed as:

$$\sigma_m = \sqrt{\frac{1}{m} \cdot \sum_{i=1}^{m} (D_m(i) - mean(D_m))^2} \tag{36}$$

where $D_m(i)$ represents the i th value of D_m, $i = 1, 2, \cdots m$, $mean(D_m)$ represents the mean of D_m. The mean square error σ_m represents the degree of deviation between the estimated frequency hopping sequence and the linear sequence under the cutting condition.

Change the value of I, repeat the above steps to obtain the mean square error σ_m under different cutting conditions. When σ_m takes the minimum value, it is indicated that the estimated frequency hopping sequence at this time is the sequence closest to the linear arrangement. Therefore, the frequency hopping sequence $Code_l$ corresponding to $\sigma_l = \sigma_{\min}$ is taken as the frequency hopping sequence estimation of the FH radar signal, and l is the result of the fine search of the frequency hopping number.

TABLE 3. The main steps of parameters estimation.

Algorithm 3 The FH radar signal parameters estimation method
Input: $S(t)$,Multi-frame signal coherent accumulation result, X, Hopping number
Step1: Pulse detection, set a noise threshold T_h to obtain the pulse signal $p(t) = [S(t) > T_h]$, and get pulse width by (29).
Step2: Cut $p(t)$ into m equal parts and each part is $C_i(t), i = 1, 2, 3 \cdots m$, Get the frequency hopping sequence $Code_m$ by (33). And get the mean square error σ_m by (34).
Step3: Chang the value of m , and repeat Step 2.
Step4: Compare all σ_m and take the minimum value as the best estimation. If $\sigma_l = \sigma_{\min}$, then the estimate of the frequency hopping sequence is $Code_l$ and the hop duration is $t_l = \frac{T_p}{l}$.
Step5: Cut $p(t)$ into l equal parts and each part is $C_i(t), i = 1, 2, 3 \cdots l$ and get the frequency F_i by (30). Thus, the estimated carrier frequency is $f_c = \min(F_i)$ and the estimated frequency hopping interval is $\Delta f = mean[diff(sort(F_i))]$.
Output: Frequency hopping sequence $Code_l$, Hop duration t_l, Carrier frequency f_c, and Frequency hopping interval Δf.

D. ALGORITHM PERFORMANCE DISCUSSION

The implementation steps of the proposed parameter estimation algorithm are given in Section III-B and III-C. However, the following aspects should be pay attention to when applying the proposed algorithm:

a) The multi-frame signal accumulation is needed in the proposed algorithm, which improves the SNR to ensure the robustness of the parameter estimation. In other words, it is necessary to ensure that the parameters of the radar signal to be processed do not be change within a certain pulse group. Therefore, the algorithm is not suitable for agile radar signals.

b) Due to the characteristics of the parameter estimation algorithm, the algorithm is mainly for the FH radar signal with fixed hop duration and fixed frequency hopping interval. For the FH radar signal with unfixed hop duration or unfixed frequency hopping interval, the parameter estimation cannot be realized by the proposed algorithm.

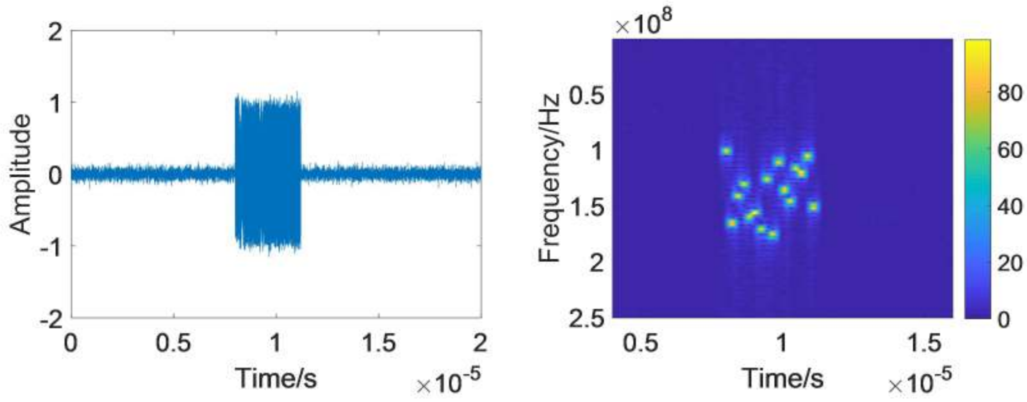
IV. EXPERIMENTS

A. HIGH SNR EXPERIMENT

The simulation experiment firstly verifies the feasibility of the proposed algorithm under the condition of $SNR = 15dB$. The Gaussian white noise is added to the signal, and the SNR is defined as shown in equation (37). The FH radar signal with a frequency hopping number of 16 is designed for parameter estimation experiments. The signal parameters are shown in Table 4. The signal waveform obtained by the simulation and its STFT time-frequency diagram are shown in Fig.4.

$$SNR = 10\lg\left(\frac{E_s}{E_n}\right) \tag{37}$$

where E_s is the received signal energy and E_n is the noise energy.



(a) (b)

FIGURE 4. Simulated FH radar signal. (a) FH radar signal waveform (b) the STFT result of FH radar signal.

TABLE 4. FH radar signal parameters.

Parameter	Value
Hopping number	16
Hop duration	$0.2\mu s$
Frequency hopping interval	5MHz
Carrier frequency	100MHz
Pulse repetition frequency	50KHz
Sampling rate	500MHz
Frequency hopping sequence	1 14 9 7 13 12 15 6 16 3 8 10 4 5 2 11

The parameters of the above simulation signal are estimated by the algorithm steps in Section 3. Considering that the radar received signal may change waveforms in the form of a pulse group, 6-frame signal is used for accumulation in this paper. And the conventional single-frame STFT parameter estimation method is used to compare with the proposed algorithm. The estimation results of the parameters are shown in Table 5.

It can be seen from Table 5 that under the condition of $SNR = 15dB$, both the proposed algorithm and the conventional STFT algorithm can effectively estimate the parameters of the FH radar signal. The estimation accuracy of the two algorithms is similar in the hop duration, but the proposed algorithm has a higher precision in the estimation of the frequency hopping interval and the carrier frequency. This is because the time and frequency resolution of the conventional STFT algorithm are mutually limited. When the accuracy of the time parameter estimation is high, the error of the frequency parameter estimation result increases. However, the proposed algorithm does not have such a limitation, and the time and frequency parameter estimation results can achieve high precision at the same time.

B. LOW SNR EXPERIMENT

In order to further verify the robustness of the proposed algorithm under low SNR conditions, keep the signal parameters unchanged and a low SNR environment with $SNR = -8dB$ is selected. The proposed algorithm and the conventional single-frame STFT algorithm are used to estimate the parameter of the FH radar signal, and the parameter estimation results are compared and analyzed. The parameter estimation results are shown in Table 6.

It can be seen from Table 6 that under the condition of low SNR, the conventional STFT algorithm has been difficult to accurately estimate the parameters of the FH radar signal. However, the proposed algorithm in this paper can still estimate the FH radar signal parameters effectively and accurately, and the accuracy of its parameter estimation result is similar to the parameter estimation result under $SNR = 15dB$.

C. DIFFERENT SNR STATISTICAL COMPARISON EXPERIMENTS

In order to verify the performance of the proposed algorithm under different SNR conditions. This section firstly adjust the noise environment and set the SNR condition from -10dB to 18dB. And then 500 times statistical independent experiments were carried out on the proposed algorithm and STFT algorithm under the above-mentioned SNR conditions. Finally, the relative error curve of the hop duration and the frequency hopping interval is obtained, as shown in Fig.5. An error threshold $E_{th} = 1\%$ is set for the parameter estimation result. It is considered that the parameter estimation result whose error is less than the threshold E_{th} is valid, and whose error is larger than the threshold E_{th} is inaccurate. The relative error rate is calculated by:

$$\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\frac{P_i - P}{P} \times 100\% \tag{38}$$

where N is the number of statistical independent experiments, P is the true value of the parameter, and P_i is the parameter estimation result for each experiment.

It can be seen from Fig.5 that in the ideal SNR environment, the parameter estimation accuracy of the proposed algorithm and the conventional single-frame STFT

TABLE 5. Comparison of estimation results of two algorithms under SNR=15dB.

Parameter name	Proposed algorithm	Relative error rate	STFT	Relative error rate
Hop duration(μs)	0.1999	0.05%	0.2001	0.05%
Frequency hopping interval(MHz)	5.0000	0.00%	5.0250	0.5%
Carrier frequency(MHz)	100	0.00%	101.56	0.05%
Hopping number	16		16	
Frequency hopping sequence	1 14 9 7 13 12 15 6 16 3 8 10 4 5 2 11		1 14 9 7 13 12 15 6 16 3 8 10 4 5 2 11	

TABLE 6. Comparison of estimation results of two algorithms under SNR=-8dB.

Parameter name	Proposed algorithm	Relative error rate	STFT	Relative error rate
Hop duration(μs)	0.1986	0.70%	0.2152	7.60%
Frequency hopping interval(MHz)	4.9975	0.05%	5.2322	4.64%
Carrier frequency(MHz)	100.08	0.08%	102.03	2.03%
Hop number	16		15	
Frequency hopping sequence	1 14 9 7 13 12 15 6 16 3 8 10 4 5 2 11		1 13 8 6 12 11 14 6 14 3 7 9 4 2 10	

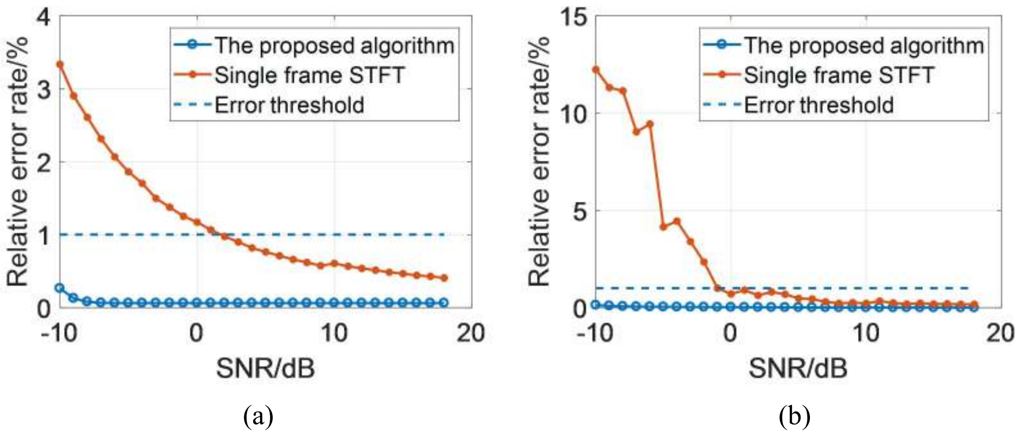


FIGURE 5. Algorithm error curve under different SNR conditions. (a) Hop duration error curve (b) frequency hopping interval error curve.

algorithm are both stable in a low range. However, as the SNR decreases, the estimation accuracy of the STFT algorithm becomes worse and worse. This is because under low SNR conditions, the extraction of time-frequency ridges is more susceptible to noise, resulting in lower accuracy of parameter estimation. However, the proposed algorithm in this paper improves the SNR by coherent accumulation of multi-frame signal, which effectively reduces the influence of noise on parameter estimation. Therefore, the proposed algorithm can still maintain high parameter estimation accuracy under low SNR conditions.

According to the error threshold E_{th} in Fig.5, when the SNR is lower than 0dB, the parameter estimation result of the STFT algorithm becomes not accurate any more. However, even under the SNR condition is as low as -10dB, the parameter estimation result of the proposed algorithm is still accurate. It is shown that the proposed algorithm effectively enhances the noise resistance performance of parameter estimation and has good robustness.

V. CONCLUSION

In order to deal with the poor noise resistance performance of conventional parameter estimation method under low SNR conditions. A robust method is proposed in this paper for multi-frame joint frequency hopping radar waveform parameters estimation. It is shown in both theoretical analysis and simulation results that the proposed algorithm can effectively estimate the hop duration, frequency hopping interval, frequency hopping sequence and carrier frequency of the FH radar signal. Compared with the conventional single-frame STFT algorithm, the noise resistance performance of parameter estimation has been significantly improved in the proposed algorithm, which maintains a high parameter estimation accuracy under the condition of SNR as low as -10dB.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their valuable comments to improve the paper quality.

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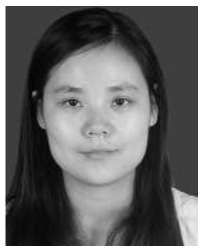

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