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ROBUST MVDR BEAMFORMER FOR NULLING LEVEL CONTROL VIA MULTI-PARAMETRIC QUADRATIC PROGRAMMING

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Abstract—MVDR beamformer is one of the well-known adaptive beamforming techniques that offers the ability to resolve signals that are separated by a fraction of an antenna beamwidth. In an ideal scenario, the MVDR beamformer can not only minimize the array output power but also maintain a distortionless mainlobe response toward the desired signal. Unfortunately, the MVDR beamformer may have unacceptably low nulling level, which may lead to significant performance degradation in the case of unexpected interfering signals. A new robust MVDR beamforming is presented to control the nulling level of adaptive antenna array. In this proposed approach, the beamforming optimization problem is formulated as a multi-parametric quadratic programming (mp-QP) problem such that the optimal weight vector can be easily obtained by real-valued computation. The presented method can guarantee that the nulling level are strictly below the prescribed threshold. Simulation results are presented to verify the efficiency of the proposed method.

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1. INTRODUCTION

Adaptive beamforming is a technique for receiving the signal of interest from specific directions while suppressing the interfering signals adaptively other directions using an array of sensors. This technique is able to automatically optimize the array pattern by adjusting the elemental control weights until a prescribed objective function is satisfied. That is, it provides a means for separating a desired signal from interfering signals. It has found numerous applications to radar, sonar, wireless communications, seismology, and microphone arrays.

One of the most popular approaches to adaptive beamforming is proposed by Capon [1]. His algorithm leads to an adaptive beamformer with a minimum-variance distortionless response (MVDR). Since some constraints, such as, the antenna gain maintained constant in the desired look direction, are used to ensure that the desired signals are not filtered out along with the unwanted signals. The MVDR beamformer can not only minimize the array output power but also maintain a distortionless mainlobe response toward the desired signal. Unfortunately, the MVDR beamformer may have unacceptably low nulling level, which may lead to significant performance degradation in the case of unexpected interfering signals. Specially, the performance of MVDR degrades in rapidly moving jammer environments. This degradation occurs due to the jammer motion that may bring the jammers out of the sharp notches of the adapted pattern. In order to achieve high interference suppression and signal-of-interest (SOI) enhancement, an adaptive array must introduce deep and widened nulls in the directions of arrival (DOAs) of strong interferences, while keeping the desired signal distortionless. Thus, the issue of nulling level control is especially important for both deterministic and adaptive arrays. A large number of approaches [2–11] have been presented in the array processing literature in order to widen nulls in the DOAs of interference signal sources. For example, the covariance matrix tapers and derivative constraints in the directions of jammers are proposed for broadening the null in adaptive processing [2, 3]. An important class of adapted pattern modification techniques are realized by the application of a conformal matrix "pater" to the original sample covariance matrix. From the Schur product theorem and Kolmogorv's existence theorem, this method established that CMT's are, in fact, the solution to a minimum variance optimum beamformer associated with an auxiliary stochastic process that is related to the original by a Hadamard product. Reference [4] utilizes the space-time averaging techniques and rotation techniques of the steering vectors to improve the nulling level control performances. This method can provide increased robustness against the mismatch problem as well as control over the sidelobe level. When the antenna platform vibrates or interference moves quickly, it is possible that the mismatching occurs between adaptive weight and data due to the perturbation of the interference location. To solve these problems above, a robust beamforming control method based on semidefinite programming is presented to widen nulls of adaptive antenna array [5]. The presented method can provide an improved robustness against the interference angle shaking and suppress the interference signals and make the mean output array signal-to-interference-and-noise ratio (SINR). In [6], the beamforming optimization problem is formulated as a second-order cone programming problem. This method can guarantee that the sidelobes are strictly below some given (prescribed) threshold. The advantage of the proposed method is that it can not only guarantee that the sidelobes are strictly below some given (prescribed) threshold value but also improve the robustness of the supergain beamformer agianst random errors such as amplitude and phase errors in sensor chanels and imprecise positioning of sensors. In the reference [7], a zero-space based transmitting beamforming algorithm suitable for combining with MIMO techniques is proposed to widen the nulls. An adaptive beamforming null widening technology is presented by utilizing the numerical method to correlated variance matrix via the rotation of the steering vector [8, 9]. Reference [10] gives a new method based on amplitude-only perturbations for the pattern synthesis of linear antenna array with prescribed nulls. Reference [11] gives an efficient synthesis technique for the conformal phased array antennas.

In this paper, we develop a new beamforming control method to widen nulls of adaptive antenna array. The beamforming control problem is formulated as a multi-parametric quadratic programming (mp-QP) problem. The optimal weight vector can be obtained by real-valued computation. At the same time, the presented method can guarantee that the nulling level are strictly below the prescribed threshold. This paper is organized as follows. Section 2 briefly introduces the signal model and presents the MVDR solution. The proposed method based on mp-QP is addressed in Section 3. In Section 4, simulation results are presented to verify the performance of the proposed approach. Section 5 concludes the paper.

2. BACKGROUND

Consider a uniform linear array (ULA), which consists of M elements. The output of a narrowband beamformer composed by the ULA is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k) \tag{1}$$

where k is the time index, $\mathbf{x}(k) = [x_1(k), \ldots, x_M(k)]^T$ is the complex vector of array observations, $\mathbf{w} = [w_1, \ldots, w_M]^T$ is the complex vector of beamformer weights, and the superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate transpose, respectively. Regarding the notation of this paper, lower and upper boldface letters are used for vectors and matrices, respectively. The observation (snapshot) vector at time instant k is given by

$$\mathbf{x}(k) = \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) = \mathbf{s}(k)\mathbf{a}(\theta_0) + \sum_{j=1}^{q} i_j(k)\mathbf{a}(\theta_j) + \mathbf{n}(k) \tag{2}$$

where q is the number of interference signals. Here, $s(k)$ and $i_j(k)$ are the signal and interference symbol samples. The signal and interference DOAs are θ_0 and $\theta_j, j = 1, \ldots, q$, respectively, with corresponding steering vectors $\mathbf{a}(\theta_0)$ and $\mathbf{a}(\theta_j)$.

Let $\mathbf{R}$ denote the $M \times M$ theoretical covariance matrix of the array snapshot vector. Assume that $\mathbf{R}$ is a positive definite matrix with the following form:

$$\mathbf{R} = \sigma_0^2 \mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0) + \sum_{j=1}^{q} \sigma_j^2 \mathbf{a}(\theta_j)\mathbf{a}^H(\theta_j) + \sigma_n^2 \mathbf{I}_M \tag{3}$$

where σ_0^2, σ_j^2 $(j = 1, \ldots, q)$, and σ_n^2 are the powers of the uncorrelated impinging signals $s(k)$, $i_j(k)$, and noise, respectively. $\mathbf{I}_M$ is the $M \times M$ identity matrix.

The common formulation of the beamforming problem that leads to the MVDR beamformer is as follows.

Determine the $M \times 1$ vector $\mathbf{w}_\text{o}$ that is the solution to the following linearly constrained quadratic problem:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \qquad \text{subject to } \mathbf{w}^H \mathbf{a}(\theta_0) = 1 \tag{4}$$

The solution of (4) for this particular case is

$$\mathbf{w}_{\text{MVDR}} = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\mathbf{R}^{-1}\mathbf{a}(\theta_0)} \tag{5}$$

In practice, the exact covariance matrix is not available and is replaced by the sample covariance matrix $\hat{\mathbf{R}}$

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{k=1}^{N} \mathbf{x}(k)\mathbf{x}^H(k). \tag{6}$$

3. ADAPTIVE BEAMFORMING WITH NULLING LEVEL CONTROL

Assume that the interference signal arrivals the receive array from the angle of incident θ_j $(j = 1, \ldots, J)$. When the interference moves quickly, it is possible that the mismatch occurs for adaptive weight and data due to the perturbation of the interference location.

Let $\triangle\theta$ denote the angle spread for the interference signal which come from θ. Let $\theta_k \in [\theta - \triangle\theta, \theta + \triangle\theta]$ $(k = 1, \ldots, K)$ be chosen grid that approximates the angle spread area. To control the null level for the the angle spread area $[\theta - \triangle\theta, \theta + \triangle\theta]$, we use the following multiple quadratic inequality constraints inside the the angle spread area:

$$|\mathbf{w}^H \mathbf{a}(\theta_k)|^2 \leq \xi^2, \qquad k = 1, \ldots, K \tag{7}$$

where ξ^2 is the prescribed nulling level.

Adding the constraints (7) to the MVDR beamforming problem (4), we obtain the following modified MVDR problem:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{subject to} \qquad \mathbf{w}^H \mathbf{a}(\theta_0) = 1, \quad |\mathbf{w}^H \mathbf{a}(\theta_k)|^2 \leq \xi^2 \quad (k = 1, \ldots, K) \tag{8}$$

This problem has quadratic objective function. There is a single linear equality constraint and multiple quadratic inequality constraints. In the above Eq. (8), a set of K constrains is used to guarantee the beamformer can make the null fall into the whole interference angle spreading areas. That is, multiple constraints can widen nulls of the beamformer so that the interference can still be suppressed well even in the situation where the interference direction and the data mismatch. On the contrary, if only one constrain corresponding to the null position is used, the resulting width of the null is very narrow. Furthermore, the beamforming performance will deteriorate greatly in the real electromagnetic environments, such as rapidly moving jammer environments, etc. Additionally, notice that $|\mathbf{w}^H \mathbf{a}(\theta_k)|^2$ can directly determine the output power of antenna array at the interference direction θ_k (refer to Eq. (21)), thus, it can be viewed as the "directional gain" of the antenna array. To sum up, multiple quadratic constrains taken is justified.

In Section 4, we will convert this problem (8) to a multi-parametric quadratic programming problem such that the optimal weight vector is estimated by the real-value computation.

4. ALGORITHM FORMULATION

4.1. mp-QP MVDR

In this section, we present the multi-parametric programming problem for MVDR beamformor, named as mp-QP MVDR. As seen in (8), the data are in general complex valued. However, for convenience, we will work with real-valued data. To do so, a preprocessing path is taken prior to the beamforming operation.

Let

$$\mathbf{R}_1 = \mathrm{Real}\{\mathbf{R}\}, \qquad \mathbf{R}_2 = \mathrm{Imag}\{\mathbf{R}\}$$
$$\mathbf{w}_1 = \mathrm{Real}\{\mathbf{w}\}, \qquad \mathbf{w}_2 = \mathrm{Imag}\{\mathbf{w}\}$$
$$\mathbf{a}_{01}(\theta_0) = \mathrm{Real}\{\mathbf{a}(\theta_0)\}, \quad \mathbf{a}_{02}(\theta_0) = \mathrm{Imag}\{\mathbf{a}(\theta_0)\}$$
$$\mathbf{a}_{k1}(\theta_k) = \mathrm{Real}\{\mathbf{a}(\theta_k)\}, \quad \mathbf{a}_{k2}(\theta_k) = \mathrm{Imag}\{\mathbf{a}(\theta_k)\}\forall k \in (1, \ldots, K)$$

where, $\mathrm{Real}\{\cdot\}$ and $\mathrm{Imag}\{\cdot\}$ stand for the real and imaginary part of a complex matrix or vector.

By simple algebra, the cost function $\mathbf{w}^H\mathbf{R}\mathbf{w}$ can be rewritten as

$$\begin{aligned}\mathbf{w}^H\mathbf{R}\mathbf{w} &= \mathrm{Real}\{\mathbf{w}^H\mathbf{R}\mathbf{w}\} + j\mathrm{Imag}\{\mathbf{w}^H\mathbf{R}\mathbf{w}\} \\ &= \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{pmatrix}^T \begin{pmatrix} \mathbf{R}_1 & -\mathbf{R}_2 \\ \mathbf{R}_2 & \mathbf{R}_1 \end{pmatrix} \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{pmatrix} \\ &\quad + j \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{pmatrix}^T \begin{pmatrix} \mathbf{R}_2 & \mathbf{R}_1 \\ -\mathbf{R}_1 & \mathbf{R}_2 \end{pmatrix} \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{pmatrix}\end{aligned}$$

It is easy to know that $\mathrm{Imag}\{\mathbf{w}^H\mathbf{R}\mathbf{w}\} = 0$ since $\left(\mathbf{w}^H\mathbf{R}\mathbf{w}\right)^H = \mathbf{w}^H\mathbf{R}\mathbf{w} \in \mathbb{R}$ for $\forall \mathbf{w} \in \mathbb{C}^M$. Thus, the modified MVDR problem (8) can by algebraic manipulation be reformulated as the following mp-QP problem

$$\min_z \frac{1}{2}\mathbf{z}^T\mathbf{H}\mathbf{z} \qquad \text{subject to } \mathbf{G}\mathbf{z} \leq \mathbf{b} \tag{9}$$

where the matrices and vectors in (9) have the following forms:

$$\begin{aligned}\mathbf{z} &= [\mathbf{w}_1^T, \mathbf{w}_2^T]^T \in \mathbb{R}^{2M} \\ \mathbf{H} &= \begin{pmatrix} \mathbf{R}_1 & -\mathbf{R}_2 \\ \mathbf{R}_2 & \mathbf{R}_1 \end{pmatrix} \in \mathbb{R}^{2M\times 2M} \\ \mathbf{G} &= [\mathbf{G}_0^T, \mathbf{G}_1^T, \ldots, \mathbf{G}_K^T]^T \\ \mathbf{G}_k &= [\mathbf{B}_k^T, -\mathbf{B}_k^T]^T \ (k = 0, 1, \ldots, K) \\ \mathbf{B}_k &= \begin{bmatrix} \mathbf{a}_{k1}^T(\theta_k) & \mathbf{a}_{k2}^T(\theta_k) \\ \mathbf{a}_{k2}^T(\theta_k) & -\mathbf{a}_{k1}^T(\theta_k) \end{bmatrix} \ (k = 0, 1, \ldots, K) \\ \mathbf{b} &= [\mathbf{b}_0^T, \mathbf{b}_1^T, \ldots, \mathbf{b}_K^T]^T\end{aligned}$$

$$\begin{aligned}\mathbf{b}_0 &= [1, 0, -1, 0]^T \\ \mathbf{b}_k &= [\sqrt{\lambda_\xi}\xi, \sqrt{1-\lambda_\xi}\xi, -\sqrt{\lambda_\xi}\xi, -\sqrt{1-\lambda_\xi}\xi]^T \ \lambda_\xi \in [0, 1]\end{aligned}$$

Then, we have the following Theorem 1.

Theorem 1. Suppose that there are q uncorrelated signals $s_k(t)$ $(k = 1, \ldots, q)$ with distinct DOAs arriving an ULA with M isotropic sensors. Assume that the additive noise of kth sensor $n_k(t)$ $(k = 1, \ldots, M)$ is a complex Gaussian random process with zero-mean and equal variance σ_n^2 and the noise $n_k(t)$ is uncorrelated with $s_k(t)$ $(k = 1, \ldots, q)$. Then the matrix $\mathbf{H}$ (which is defined in (9)) is a positive definite matrix.

Proof. Let $(\lambda, \mathbf{e})$ be an eigenpair for the matrix $\mathbf{R}$, where $\mathbf{e} = \mathbf{e}_1 + j\mathbf{e}_2$ with $\mathbf{e}_k \in \mathbb{R}^M$ $(k = 1, 2)$, $\mathbf{R} = \mathbf{R}_1 + j\mathbf{R}_2$ with $\mathbf{R}_k \in \mathbb{R}^{M\times M}$ $(k = 1, 2)$. It is clear that the eigenvalue λ is a real number since $\mathbf{R}$ is a hermitian matrix. Thus, we have the following expression:

$$\mathbf{R}(\mathbf{e}_1 + j\mathbf{e}_2) = (\mathbf{R}_1 + j\mathbf{R}_2)(\mathbf{e}_1 + j\mathbf{e}_2) = \lambda(\mathbf{e}_1 + j\mathbf{e}_2) \tag{10}$$

From Eq. (10), we have the following equation

$$\mathbf{H} \times \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1 & -\mathbf{R}_2 \\ \mathbf{R}_2 & \mathbf{R}_1 \end{bmatrix} \times \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} = \lambda \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} \tag{11}$$

Starting from the eigenvalue decomposition of the covariance matrix $\mathbf{R}$, $\mathbf{R}$ can be expressed as

$$\mathbf{R} = \mathbf{E}\Gamma\mathbf{E}^H \tag{12}$$

where $\mathbf{E} = [\mathbf{e}_1, \ldots, \mathbf{e}_M]$ with $\mathbf{e}_k$ is the orthonormal eigenvector of the matrix $\mathbf{R}$. $\Gamma = \mathrm{diag}\{\lambda_1, \ldots, \lambda_M\}$ with $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_q > \lambda_{q+1} = \ldots = \lambda_M = \sigma_n^2 > 0$, namely, every eigenvalue $\lambda_k > 0$ for $\forall k \in [1, \ldots, M]$.

From Eq. (10) $\sim$ (12), it's not difficult to prove the matrix $\mathbf{H}$ has the same eigenvalues $(\lambda_1, \lambda_1, \ldots, \lambda_M, \lambda_M$, namely, each eigenvalue repeats twice) as $\mathbf{R}$. So, all eigenvalues of $\mathbf{H}$ are positive. Finally, observe that $\mathbf{H}$ is a real-symmetric matrix since $\mathbf{R}$ is a hermitian matrix which implies that $\mathbf{R}_1^T = \mathbf{R}_1$ and $\mathbf{R}_2^T = -\mathbf{R}_2$. Therefore, the matrix $\mathbf{H}$ is a positive definite matrix.

This concludes the proof. ■

4.2. The Optimal Solution

As shown in [12, 13], the multi-parametric quadratic programming (mp-QP) problem (9) can be solved by applying the Karush-Kuhn-

Tucker (KKT) conditions

$$\mathbf{H}\mathbf{z} + \mathbf{G}^T \boldsymbol{\lambda} = \mathbf{0}, \qquad \boldsymbol{\lambda} \in \mathbb{R}^{4(K+1)} \tag{13}$$

$$\lambda_i \mathbf{G}^i \mathbf{z} - \mathbf{b}^i = 0, \qquad i = 1, \ldots, 4(K+1) \tag{14}$$

$$\boldsymbol{\lambda} \geq \mathbf{0} \tag{15}$$

$$\mathbf{G}\mathbf{z} - \mathbf{b} \leq \mathbf{0}. \tag{16}$$

In the sequel, let the superscript index denote a subset of the rows of a matrix or vector. Theorem 1 shows that $\mathbf{H}$ has full rank, (13) gives

$$\mathbf{z} = -\mathbf{H}^{-1}\mathbf{G}^T \boldsymbol{\lambda} \tag{17}$$

Definition 1 Let $\mathbf{z}^*$ be the optimal solution to (9). We define active constraints the constraints with $\mathbf{G}^i\mathbf{z}^* - \mathbf{b}^i = 0$, and inactive constraints the constraints with $\mathbf{G}^i\mathbf{z}^* - \mathbf{b}^i < 0$. The optimal active set $\mathcal{A}^* = \{i|\mathbf{G}^i\mathbf{z}^* = \mathbf{b}^i\}$.

Definition 2 For an active set, we say that the linear independence constraint qualification (LICQ) holds if the set of active constraint gradients are linearly independent, i.e., $\mathbf{G}^{\mathcal{A}}$ has full row rank.

Assuming that LICQ holds, (14) and (17) lead to

$$\boldsymbol{\lambda}^{\mathcal{A}} = -\left(\mathbf{G}^{\mathcal{A}}\mathbf{H}^{-1}\left(\mathbf{G}^{\mathcal{A}}\right)^T\right)^{-1}\mathbf{b}^{\mathcal{A}} \tag{18}$$

Equation (18) can now be substituted into (17) to obtain

$$\mathbf{z} = \mathbf{H}^{-1}\left(\mathbf{G}^{\mathcal{A}}\right)^T\left(\mathbf{G}^{\mathcal{A}}\mathbf{H}^{-1}\left(\mathbf{G}^{\mathcal{A}}\right)^T\right)^{-1}\mathbf{b}^{\mathcal{A}} \tag{19}$$

Partition, now, the vector $\mathbf{z}$ into $\mathbf{z}_1$, $\mathbf{z}_2 \in \mathbb{R}^M$, by $\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}$, and define $\mathbf{w}_o$ as follows.

$$\mathbf{w}_o = \mathbf{z}_1 + j\mathbf{z}_2 \in \mathbb{C}^M \tag{20}$$

It is clear that the optimal solution of (8) for this particular case is $\mathbf{w}_o$.

Remarks. Similar to the array synthesis technique based on the support vector regression [14] (for convenience, this method is named as SVR-AS), the proposed mp-QP MVDR method need also find an optimal weight vector for an antenna array. However, the differences of the two methodologies are as follows:

(1) The proposed mp-QP MVDR gives the optimal weight vector of an antenna array for the optimal problem of the receiving beam. While, the SVR-AS gives the weight vector of an antenna array for the radiation pattern problem.

(2) The SVR-AS needs the pairs voltages/radiation pattern as the train data. The mp-QP MVDR doesn't need the train data.

The benefits of the proposed approach are as follows:

(1) It can widen the nulling extent.

(2) It can guarantee that the nulling level in the specified areas are strictly below the prescribed threshold.

(3) It can provide not only an improved robustness against the interference angle shaking but also a better array gain.

4.3. Array Gain

To investigate the performance of the proposed method, this section gives the definition of array gain for an adaptive antenna array system. From Eq. (1), the mean square power output of the beamformer can be expressed as

$$\begin{aligned} P &= \mathrm{E}\{|y(k)|^2\} = \mathrm{E}\{|\mathbf{w}^H\mathbf{x}(k)|^2\} = \sigma_0^2|\mathbf{w}^H\mathbf{a}(\theta_0)|^2 + \mathbf{w}^H\mathbf{R}_{j+n}\mathbf{w} \\ &= \sigma_0^2|\mathbf{w}^H\mathbf{a}(\theta_0)|^2 + \sum_{j=1}^{q}\sigma_j^2|\mathbf{w}^H\mathbf{a}(\theta_j)|^2 + \sigma_n^2\mathbf{w}^H\boldsymbol{\rho}_n\mathbf{w} \end{aligned} \tag{21}$$

where $\mathrm{E}\{\cdot\}$ denotes the statistical expectation. The $M \times M$ interference-plus-noise covariance matrix $\mathbf{R}_{j+n} = \mathrm{E}\{(\sum_{j=1}^{q} i_j(k)\mathbf{a}(\theta_j)+\mathbf{n}(k))(\sum_{j=1}^{q} i_j(k)\mathbf{a}(\theta_j) + \mathbf{n}(k))^H\} = \sum_{j=1}^{q}\sigma_j^2\mathbf{a}(\theta_j)\mathbf{a}^H(\theta_j) + \sigma_n^2\boldsymbol{\rho}_n$, σ_0^2 is the desired signal power, σ_j^2 $(j = 1, \ldots, q)$ and σ_n^2 are the interfering signal power and noise power, respectively. $\boldsymbol{\rho}_n$ is the Hermitian cross-spectral density matrix of the noise normalized to have its trace equals to M.

The array gain is defined as follows.

Definition 3 For an adaptive antenna array system, the array gain is defined as the output signal-to-interference-and-noise ratio (SINR) divided by the input SINR and is given by

$$G = \frac{\sigma_0^2|\mathbf{w}^H\mathbf{a}(\theta_0)|^2/(\mathbf{w}^H\mathbf{R}_{j+n}\mathbf{w})}{\sigma_0^2/(\sum_{j=1}^{q}\sigma_j^2 + \sigma_n^2)}. \tag{22}$$

To calculate the output due to the desired signal, a distortionless constraint is imposed on $\mathbf{w}$ that $\mathbf{w}^H\mathbf{a}(\theta_0) = 1$. Consider the special case of spatial white noise and identical noise spectra at each sensor, the noise cross-spectral density matrix $\boldsymbol{\rho}_n$ reduces to an identity matrix.

Thus the array gain for white noise is given by

$$\begin{aligned} G &= \frac{\sum_{j=1}^{q}\sigma_j^2+\sigma_n^2}{\mathbf{w}^H\mathbf{R}_{j+n}\mathbf{w}} \\ &= \frac{\sum_{j=1}^{q}\sigma_j^2+\sigma_n^2}{\sum_{j=1}^{q}\sigma_j^2|\mathbf{w}^H\mathbf{a}(\theta_j)|^2+\sigma_n^2\mathbf{w}^H\mathbf{w}} \\ &= \frac{\sum_{j=1}^{q}\sigma_j^2/\sigma_n^2+1}{\sum_{j=1}^{q}\sigma_j^2/\sigma_n^2\|\mathbf{w}^H\mathbf{a}(\theta_j)\|^2+\|\mathbf{w}\|^2} \end{aligned} \tag{23}$$

where $\|\cdot\|$ stands for the Euclidean norm.

From Eq. (23), the array gain of the mp-QP MDVR beamformer and pure MVDR beamformer are given as follows.

$$G_{mp\text{-}Qp} = \frac{\sum_{j=1}^{q}\sigma_j^2/\sigma_n^2+1}{\sum_{j=1}^{q}\sigma_j^2/\sigma_n^2\|\mathbf{w}_{mp\text{-}Qp}^H\mathbf{a}(\theta_j)\|^2+\|\mathbf{w}_{mp\text{-}Qp}\|^2} \tag{24}$$

$$G_{MVDR} = \frac{\sum_{j=1}^{q}\sigma_j^2/\sigma_n^2+1}{\sum_{j=1}^{q}\sigma_j^2/\sigma_n^2\|\mathbf{w}_{MVDR}^H\mathbf{a}(\theta_j)\|^2+\|\mathbf{w}_{MVDR}\|^2} \tag{25}$$

where $\mathbf{w}_{mp\text{-}Qp}$ and $\mathbf{w}_{MVDR}$ are the weight vectors given by (5) and (20), respectively.

5. SIMULATION RESULTS

In this section, we conduct some simulations to validate the proposed approach. Assume that the uniform linear array (ULA) consists of ten isotropic sensors ($M = 10$) equispaced by half-wavelength. The number of snapshots at each sensor is taken to $N = 256$.

Case 1: Consider that an ideal scenario without the angle spread for the interference signal. Assume that the desired signal and two interference signals are plane waves impinging on the ULA from the directions $0°$, $-40°$, and $40°$, respectively. In this simulation, the signal-to-noise-ratio (SNR) is set to 0 dB, 2 dB and 5 dB, for the desired signal and the two interferer signals, respectively. It is assume that $\xi^2 = 10^{-7}$, i.e., we require the beampattern nulling level below -70 dB.

For Case 1, the complex vectors of beamformer weights calculated by the aforementioned two methods are presented in Table 1, while the beampatterns that they generate are also plotted in Fig. 1. From Fig. 1, we observe that, all the beampatterns have nulls at the DOAs of the interference signals and maintain a distorionless response for the SOI. However, the mp-QP MVDR places deep nulls (its nulling level is equal to -80 dB) at the DOAs of two interference signal sources. The MVDR response presents lower nulling levels compared to the mp-QP

Table 1. Weighting values calculated for Case 1.

Sensor ♯	MVDR	mp-QP MVDR
1	$0.1097 + 0.0473j$	$0.1077 + 0.0469j$
2	$0.1275 + 0.0076j$	$0.1269 + 0.0072j$
3	$0.0780 + 0.0256j$	$0.0808 + 0.0264j$
4	$0.0463 - 0.0005j$	$0.0441 - 0.0007j$
5	$0.1117 - 0.0180j$	$0.1106 - 0.0188j$
6	$0.1008 - 0.0204j$	$0.1043 - 0.0196j$
7	$0.1021 - 0.0274j$	$0.1015 - 0.0267j$
8	$0.0861 + 0.0002j$	$0.0846 - 0.0012j$
9	$0.1268 + 0.0068j$	$0.1293 + 0.0069j$
10	$0.1110 - 0.0212j$	$0.1102 - 0.0204j$

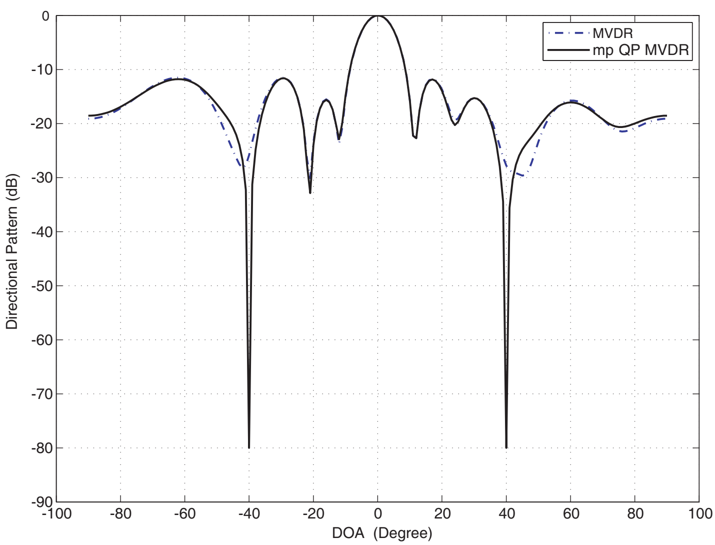

Figure 1. The beamforming performances without angle spread scenario.

MMVDR method. This can result in deep degradations in case of unexpected interferences or increase in the noise power.

Figure 2 gives the array gain curves (which are computed by (24) and (25), respectively) of the aforementioned beamformers for the ideal scenario without the angle spread, with SNRs ranging from -5 dB to 20 dB. It can be seen that the mp-QP MVDR shows better array gain.

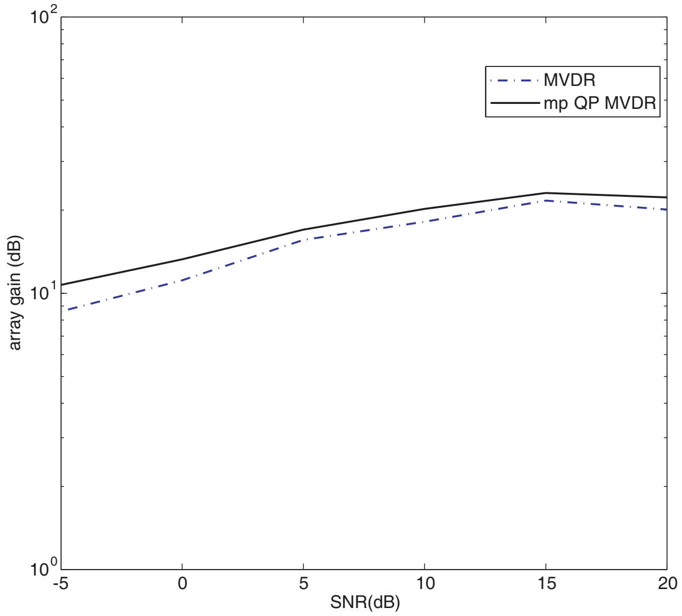

Figure 2. Array gain versus SNR for various beamformers without angle spread scenario.

Case 2: Consider that an angle spread scenario for the interference signal. Assume that the desired signal and two interference signals are plane waves impinging on the ULA from the directions $0°$, $-40°$, and $40°$ respectively. Assume that the interference angle spreads from $35°$ to $50°$. In this simulation, the SNR is set to 0 dB, 2 dB and 5 dB, for the desired signal and the two interferer signals, respectively. It is assume that $\xi^2 = 10^{-3.8}$, i.e., we require the beampattern nulling level below -38 dB.

For Case 2, the complex vectors of beamformer weights calculated by the aforementioned two methods are presented in Table 2, while the beampatterns that they generate are also plotted in Fig. 3. Fig. 3 gives the nulling performance of MVDR and mp-QP MVDR, with interference angle spreading. From Fig. 3, we can observe that the mp-QP MVDR can widen the nulling extent and guarantee the nulling level (inside the interference angle spread areas) strictly below the prescribed threshold.

Figure 4 gives the array gain curves (which are computed by (24) and (25), respectively) of the aforementioned beamformers for the angle spread scenario of the interference signal, with SNRs ranging

Table 2. Weighting values calculated for Case 2.

Sensor ♯	MVDR	mp-QP MVDR
1	$0.1262 - 0.0236j$	$0.0765 - 0.0158j$
2	$0.0674 + 0.0075j$	$0.0963 + 0.0356j$
3	$0.0986 - 0.0301j$	$0.1097 - 0.0431j$
4	$0.1240 + 0.0014j$	$0.1383 - 0.0176j$
5	$0.0972 - 0.0069j$	$0.0888 - 0.0187j$
6	$0.1090 - 0.0089j$	$0.0984 + 0.0091j$
7	$0.0777 + 0.0208j$	$0.1029 + 0.0163j$
8	$0.1100 + 0.0125j$	$0.1264 - 0.0126j$
9	$0.1521 + 0.0249j$	$0.1274 + 0.0159j$
10	$0.0377 + 0.0023j$	$0.0353 + 0.0309j$

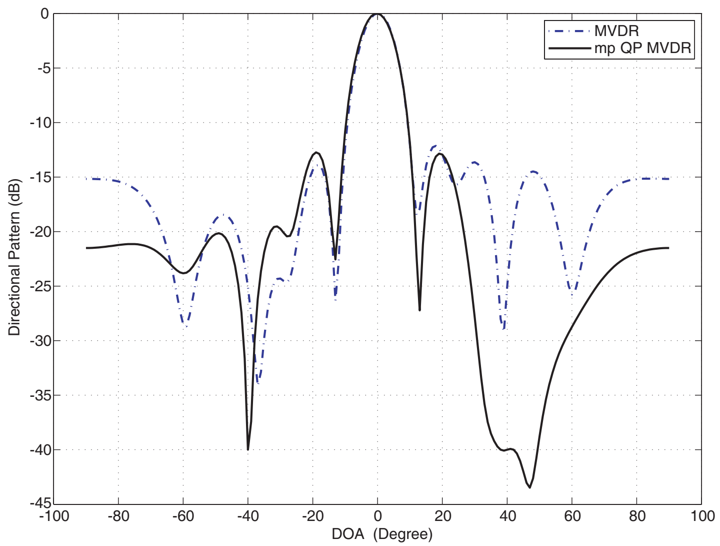

Figure 3. The nulling performance with angle spreading from 35° to 50°.

from -5 dB to 20 dB. It can be seen that the mp-QP MVDR shows better performance since it guarantees the nulling level of all interference signals strictly below prescribed threshold.

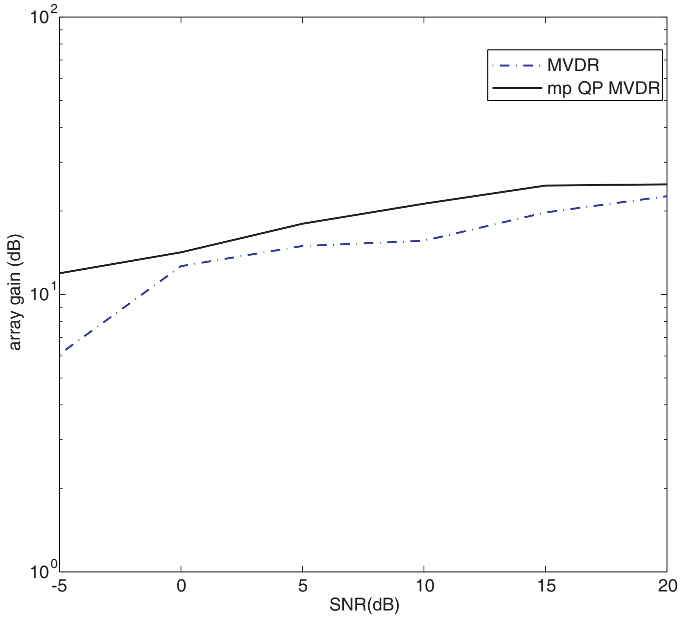

Figure 4. Array gain versus SNR for various beamformers with angle spread scenario.

6. CONCLUSIONS

This paper presents an effective robust beamforming method based on the multi-parametric quadratic programming for nulling level control. The optimal weight vector can be estimated by real-valued computation. The presented method can guarantee that the nulling level inside the interference angle spread areas are strictly below the prescribed threshold. Simulation results are presented to verify the performance of the proposed mp-QP MVDR beamformer with good reliability.

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