Robust Optimal Operation of Active Distribution Network Based on Minimum Confidence Interval of Distributed Energy Beta Distribution

Yanhong Luo, Qiubo Nie, Dongsheng Yang, and Bowen Zhou

***Abstract*—With the gradual increase of distributed energy penetration, the traditional optimization model of distribution network can no longer guarantee the stable and efficient operation of the distribution network. In order to deal with the inevitable uncertainty of distributed energy, a new robust optimal operation method is proposed for active distribution network (ADN) based on the minimum confidence interval of distributed energy Beta distribution in this paper. First, an ADN model is established with second-order cone to include the energy storage device, capacitor bank, static var compensator, on-load tap changer, wind turbine and photovoltaic. Then, the historical data of related distributed energy are analyzed and described by the probability density function, and the minimum confidence interval is obtained by interval searching. Furthermore, via taking this minimum confidence interval as the uncertain interval, a less conservative two-stage robust optimization model is established and solved for ADN. The simulation results for the IEEE 33-bus distribution network have verified that the proposed method can realize a more stable and efficient operation of the distribution network compared with the traditional robust optimization method.**

***Index Terms*—Active distribution network, robust optimization, Beta distribution, second-order cone.**

I. Introduction

WITH the development of active distribution network (ADN) technology, power distribution system has changed from a traditional single system to a flexible interactive system that includes multiple distributed energy sources and flexible loads. The penetration of uncertain distributed energy will bring great challenges to the stability, economy, and flexibility of the distribution network, and the optimization model of the distribution network will change accordingly [1]-[3].

Manuscript received: March 30, 2020; accepted: July 24, 2020. Date of CrossCheck: July 24, 2020. Date of online publication: November 20, 2020.

This work was supported in part by the National Natural Science Foundation of China (No. 61703081), the Liaoning Joint Fund of National Natural Science Foundation of China (No. U1908217), the Natural Science Foundation of Liaoning Province (No. 20170520113), and the Fundamental Research Funds for the Central Universities (No. N2004016).

Y. Luo (corresponding author), Q. Nie, D. Yang, and B. Zhou are with the Department of Electric Engineering, Northeastern University, Shenyang 110004, China (e-mail: luoyanhong@ise.neu.edu.cn; nieqiubo1996@163.com; yangdongsheng@mail.neu.edu.cn; zhoubowen@ise.neu.edu.cn).

DOI: 10.35833/MPCE.2020.000198

The optimal operation of distribution network consists of two parts. The optimal operation of active power is regulated by controlling energy storage, flexible load, substation transmission, and distributed energy generation [4]-[7]. The optimal operation of reactive power is regulated by controlling capacitor banks (CBs), static var compensators (SVCs), and on-load tap changers (OLTCs). By adjusting the above devices, the network loss, cost or voltage deviation can be minimized under the condition of satisfying the power flow safety constraints [8]-[11].

Since the power flow constraints of the distribution network structure are non-linear, it brings many difficulties to the optimization process. At present, most results treat the secondary variables such as the voltage and current as linear variables since it has been proven that such process of linearization does not affect the optimization accuracy [12]-[14]. At the same time, the second-order cone (SOC) relaxation method can solve the power flow model of radial systems well, and its convergence error can be usually guaranteed within the order of 10^{-4} [15]-[18]. The power flow model after the SOC relaxation can be directly solved by the general commercial solver.

As a new optimization method to solve the uncertainty of internal structure and external environment, the robust optimization can effectively guarantee the optimal operation of distribution network in the random fluctuation environment of distributed energy [19]-[22]. Therefore, it has been widely used in the power system optimization. Reference [23] establishes a two-stage robust reactive power optimization model that considers the wind power uncertainty. This model expresses the wind power in the form of intervals. The capacitance compensator, SVC, and OLTC in the control system minimize the network loss. This method is the mainstream method to solve the robust optimization of the distribution network at this stage. Reference [24] uses photovoltaic (PV) inverters as control modules for distribution network optimization. Meanwhile, a robust optimization model considering PV uncertainty is established. Reference [25] uses the correlation of wind power and PV power as the basis for selecting the robust optimization interval, which reduces the conservative optimization of robust optimization in the scenarios involving multiple distributed energy sources.

For the above optimization methods for ADN, the interval partition of uncertainty for distributed energy is conservative or limited, which can not reflect the facts well. Meanwhile, the existing optimal operation of distributed network only considers the network loss, while the role of the energy storage device in multi-time scales is not considered, which may lead to inaccurate results. Therefore, in this paper, a new robust optimal operation model of ADN is established based on the confidence interval of distributed energy Beta distribution. The main contributions of this work are as follows.

1) An ADN optimization model including the energy storage device, capacitor compensator, SVC, OLTC, wind turbine (WT), and PV is established. The model is transformed as a linear model with the SOC constraints, and the corresponding relaxation error of the SOC is quantitatively analyzed to ensure the accuracy of the model.

2) The probability density function of distributed energy output is obtained by fitting the historical output data of distributed energy through Beta distribution, and the minimum confidence interval of distributed energy output is obtained by interval search, which can better describe the uncertainty of distributed energy output and make the robust optimization of distribution network less conservative.

3) Taking the minimum confidence interval of distributed energy output as the uncertain interval of robust optimization of ADN, a two-stage robust optimization model of ADN is established. The optimal operation results show that the proposed model is more in line with the actual situation than the traditional robust optimization (TRO) model.

II. ADN Model

According to the branch power flow model of distribution network, in this paper, the energy storage device is seen as the adjustable device of active power, and the SVC, capacitance compensator, and OLTC are regarded as the adjustable devices of reactive power. In the following, the power flow model will be relaxed and linearized through the SOC to make it easier to solve.

A. Objective Function

In this paper, the optimization objective of the ADN mainly consists of three parts in the operation cycle. The first part is the cost of network loss. As is known, reducing the network loss can improve the operation efficiency of distribution network. The second part is the power purchasing cost of substation. Since the system considers the time-of-use electricity price and energy storage, the cost can be reduced by rationally mobilizing resources. The third part is the penalty cost of the bus voltage fluctuation. The fluctuation of the bus voltage usually affects the users' experience. Therefore, the penalty of voltage fluctuation should be added to the objective function. The total objective function is set as:

$$\min \sum_{t=1}^{T}\left(\sum_{(i,j)\in X} c_1 I_{ij,t}^2 r_{ij} + \sum_{m\in Y} c_2 P_{m,t} + \sum_{n\in Z} c_3 \left| V_{n,t}^2 - V_N^2 \right|\right) \tag{1}$$

where T is the period of operation; X, Y, and Z are the sets of the distribution network branches, substation buses, and distribution network buses, respectively; c_1, c_2, and c_3 are the factors of the network loss cost, power purchasing cost, and voltage fluctuation penalty, respectively; $I_{ij,t}$ is the current of branch (i, j) at time t; r_{ij} is the resistance of branch (i, j); $P_{m,t}$ is the active power flowing into bus m at time t; $V_{n,t}$ is the voltage of bus n at time t; V_N is the rated voltage; and Z is the set of branches containing all bus.

B. Power Flow Constraints of Distribution Network

In this paper, the branch power flow is used to describe the power flow constraints of distribution network. The expressions are as follows.

$$\sum_{i\in\pi(i)}[(P_{i,t} - I_{ij,t}^2 r_{ij}) + P_{i,W,t} + P_{i,PV,t} + P_{i,ESS,t}] = \sum_{j\in\partial(j)} P_{j,t} + P_{i,load} - P_{in,t} \tag{2}$$

$$\sum_{i\in\pi(i)}[(Q_{i,t} - I_{ij,t}^2 x_{ij}) + Q_{i,CB,t} + Q_{i,SVC,t}] = \sum_{j\in\partial(j)} Q_{j,t} + Q_{i,load} - Q_{in,t} \tag{3}$$

where $\pi(i)$ is the parent branch set of bus i; $\partial(j)$ is the child branch set of bus i; $P_{i,t}$ and $Q_{i,t}$ are the active and reactive power flowing into bus i at time t, respectively; $P_{j,t}$ and $Q_{j,t}$ are the active and reactive power flowing out of bus j at time t, respectively; $P_{i,W,t}$ is the output of WT at bus i at time t; $P_{i,PV,t}$ is the output of PV at bus i at time t; $P_{i,ESS,t}$ is the output of the energy storage device at bus i at time t; $P_{i,load}$ and $Q_{i,load}$ are the active and reactive load power of bus i, respectively; $P_{in,t}$ and $Q_{in,t}$ are the active and reactive power for transformer bus at time t, respectively; x_{ij} is the reactance of branch (i, j); $Q_{i,CB,t}$ is the reactive power compensated by the capacitor at bus i at time t; and $Q_{i,SVC,t}$ is the reactive power compensated by the SVC at bus i at time t.

OLTC can adjust the voltage of transformer at the secondary side by changing its tap ratio, thus the voltage relationship between the child bus and parent bus is:

$$V_{j,t}^2 = V_{i,t}^2 - 2(P_{j,t} r_{ij} + Q_{j,t} x_{ij}) + I_{ij,t}^2 (r_{ij}^2 + x_{ij}^2) \quad \forall (i,j)\in X/T_B \tag{4}$$

$$\frac{V_{j,t}^2}{\alpha_{ij}^2} = V_{i,t}^2 - 2(P_{j,t} r_{ij} + Q_{j,t} x_{ij}) + I_{ij,t}^2 (r_{ij}^2 + x_{ij}^2) \quad \forall (i,j)\in T_B \tag{5}$$

where $V_{j,t}$ is the child bus voltage at time t; $V_{i,t}$ is the parent bus voltage at time t; α_{ij} is the transformer ratio; and T_B is the set of branches containing OLTC.

C. Transformation of Power Flow Constraints

In order to solve the power flow model of the distribution network, the bilinear and quadratic variables in the power flow constraints need to be transformed.

The linearization process of the voltage and current variables are:

$$V_{i,t}^2 = \tilde{V}_{i,t} \tag{6}$$

$$I_{ij,t}^2 = \tilde{I}_{ij,t} \tag{7}$$

Through the above transformations, there are no quadratic variables in the power flow constraints.

For the bilinear problem brought by OLTC, the processing method of [23] is to transform the bilinear problem by the big M method. We introduce a set of auxiliary variables $\{c_k \,|\, k=1,2,...,n\}$, and $c_k\in\{0,1\}$, $\sum_{k=1}^{n} c_k = 1$. The bus voltage containing the transformer is transformed as:

$$\frac{\tilde{V}_{i,t}}{\alpha_{ij}^2}=\frac{\tilde{V}_{i,t}}{\alpha_{ij,1}^2}c_1+\frac{\tilde{V}_{i,t}}{\alpha_{ij,2}^2}c_2+\ldots+\frac{\tilde{V}_{i,t}}{\alpha_{ij,n}^2}c_n \tag{8}$$

where $\alpha_{ij,k}\ (k=1,2,\ldots,n)$ is the transformer ratio in case k where the transformer can be adjusted. Further, we introduce a set of linear variables $\left\{p_k \middle| k=1,2,\ldots,n\right\}$, $p_k\in\mathbf{R}$ to replace $\tilde{V}_{i,t}c_k$:

$$\frac{\tilde{V}_{i,t}}{\alpha_{ij}^2}=\frac{p_1}{\alpha_{ij,1}^2}+\frac{p_2}{\alpha_{ij,2}^2}+\ldots+\frac{p_n}{\alpha_{ij,n}^2} \tag{9}$$

A set of bilinear variables containing the transformer ratio and voltage are transformed into the sum of a set of linear variables.

We introduce a large real number M and give the following constraints:

$$-M(1-c_k)+\tilde{V}_{i,t}\le p_k\le M(1-c_k)+\tilde{V}_{i,t} \tag{10}$$

$$-Mc_k\le p_k\le Mc_k \tag{11}$$

In this way, the bilinear variables are directly changed to linear variables. Similarly, the compensation power of the capacitive reactive power compensator can be rewritten as:

$$Q_{i,CB,t}=\frac{1}{2}C_i\tilde{V}_{i,t} \tag{12}$$

where C_i is the capacitance value of the capacitive reactive power compensator, which is a discrete variable. It can be seen that $C_i\tilde{V}_{i,t}$ is also a bilinear variable about the voltage and capacitance. In the same way, we introduce a set of auxiliary variables $\left\{d_k \middle| k=1,2,\ldots,m\right\}$, and $d_k\in\{0,1\}$, $\sum_{k=1}^{m}d_k=1$. Therefore, the following equation can be obtained:

$$C_i\tilde{V}_{i,t}=C_{i,1}\tilde{V}_{i,t}d_1+C_{i,2}\tilde{V}_{i,t}d_2+\ldots+C_{i,m}\tilde{V}_{i,t}d_m \tag{13}$$

where $C_{i,k}\ (k=1,2,\ldots,m)$ is the all discrete reactive power of capacitor i.

Then, we further introduce a set of linear variables $\left\{q_k \middle| k=1,2,\ldots,m\right\}$, $q_k\in\mathbf{R}$ to replace $\tilde{V}_{i,t}d_k$:

$$C_i\tilde{V}_{i,t}=C_{i,1}q_1+C_{i,2}q_2+\ldots+C_{i,m}q_m \tag{14}$$

The constraints of this set of variables are:

$$-M(1-d_k)+\tilde{V}_{i,t}\le q_k\le M(1-d_k)+\tilde{V}_{i,t} \tag{15}$$

$$-Md_k\le q_k\le Md_k \tag{16}$$

In this way, the bilinear variables are transformed into the sum of a set of linear variables, which can be directly solved by a general commercial solver.

D. Energy Storage Device Constraint

The energy storage device is used to suppress the fluctuation of distributed energy sources, reduce the pressure of the power distribution network, and ensure the economical and stable operation of the system. The corresponding model can be expressed as:

$$-\Delta P_{\text{ess}}\le P_{\text{ess},t+1}-P_{\text{ess},t}\le\Delta P_{\text{ess}} \tag{17}$$

$$\underline{P}_{\text{ess}}\le P_{\text{ess},t}\le\bar{P}_{\text{ess}} \tag{18}$$

$$P_{\text{ess},1}=P_{\text{ess},T} \tag{19}$$

where ΔP_{ess} is the maximum change rate of the charging and discharging power of the energy storage device; P_{ess} is the capacity of the energy storage device; and $\underline{P}_{\text{ess}}$ and $\bar{P}_{\text{ess}}$ are the lower and upper bounds of the capacity of the energy storage device, respectively.

E. SOC and Stability Constraints of Distribution Network

For the ADN, the voltage, branch current, active power, and reactive power of the distribution network are expressed by the SOC constraint as:

$$\left\|\begin{bmatrix}2P_{ij,t}\\2Q_{ij,t}\\\tilde{V}_{i,t}-\tilde{I}_{ij,t}\end{bmatrix}\right\|_2\le\tilde{V}_{i,t}+\tilde{I}_{ij,t} \tag{20}$$

For the bus voltage of the distribution network, the security constraints can be formulated as:

$$\underline{V}^2\le\tilde{V}_{i,t}\le\bar{V}^2\quad\forall i\in Z \tag{21}$$

where $\underline{V}$ and $\bar{V}$ are the lower and upper bounds of the bus voltage fluctuation, respectively.

For the branch power flow of the distribution network, the security constraint is as follows.

$$\underline{P}\le P_{ij,t}\le\bar{P}\quad\forall(i,j)\in X \tag{22}$$

where $\underline{P}$ and $\bar{P}$ are the lower and upper bounds of the branch power flow of the distribution network, respectively.

III. Establishment and Solution of Robust Optimization Model

The above model (1) is a traditional optimization model, which is only suitable for the case that the distributed energy output is determined. Considering the uncertainty of distributed energy output, this paper further proposes the interval distribution probability of distributed energy, and establishes a robust optimization model for ADN based on the minimum confidence interval of distributed energy Beta distribution.

A. Uncertainty Description of Distributed Energy Based on Beta Distribution

The uncertain distribution of wind power is represented by the set S_t:

$$S_t=\left\{P_{W,t}\middle|P'_{W,t}-\varepsilon_t\le P_{W,t}\le P'_{W,t}+\varepsilon_t\right\} \tag{23}$$

where $P_{W,t}$ is the actual wind power value; $P'_{W,t}$ is the average value of the wind power interval; and ε_t is obtained from the historical distribution of power prediction error at time t.

In order to obtain the wind power distribution in this interval, we assume that there are N data in the set and normalize the data as:

$$\begin{cases}\theta_{i,t}=\dfrac{P_{W,t}-(P'_{W,t}-\varepsilon_t)}{(P'_{W,t}+\varepsilon_t)-(P'_{W,t}-\varepsilon_t)}=\dfrac{P_{W,t}-P'_{W,t}+\varepsilon_t}{2\varepsilon_t}\\ \theta_{i,t}\in[0,1]\end{cases} \tag{24}$$

Then, the normalized wind power distribution is represented by the following set:

$$S'_t=\left\{\theta_{1,t},\theta_{2,t},\ldots,\theta_{i,t},\ldots,\theta_{N,t}\right\} \tag{25}$$

According to the method of [26], we can fit the historical output error of the WT at time t by Beta distribution as:

$$f(\theta; a, b)=\frac{1}{\operatorname{Beta}(a, b)}\theta^{a-1}(1-\theta)^{b-1} \tag{26}$$

$$\operatorname{Beta}(a, b)=\int_0^1 \theta^{a-1}(1-\theta)^{b-1}\mathrm{d}\theta \tag{27}$$

where θ is the statistical random variable; and a and b are the shape parameters of the Beta distribution. The uncertainty interval of the wind power is not evenly distributed and the wind power value of this interval can be well fitted by Beta distribution.

Further, the mean value and variance of the uncertain set of wind power (μ and σ^2) are obtained:

$$\mu=\frac{1}{N}\sum_{i=1}^{N}\theta_{i,t} \tag{28}$$

$$\sigma^2=\frac{1}{N-1}\sum_{i=1}^{N}(\theta_{i,t}-\mu)^2 \tag{29}$$

According to the nature of the Beta distribution, the relationship among a, b, μ, and σ^2 can be formulated as:

$$\mu=\frac{a}{a+b} \tag{30}$$

$$\sigma^2=\frac{ab}{(a+b+1)(a+b)^2} \tag{31}$$

Combining (28)-(31), the shape parameters a and b can be expressed as:

$$a=\frac{(1-\mu)\mu^2}{\sigma^2}-\mu \tag{32}$$

$$b=\frac{1-\mu}{\mu}a \tag{33}$$

In this way, the probability density function of Beta distribution of the wind power is obtained. The above model represents the uncertainty interval of distributed energy in the form of probability distribution. The distribution function can be formulated as:

$$F(\theta; a, b)=\int_0^{\theta} f(\theta; a, b)\mathrm{d}\theta \tag{34}$$

By further expanding (34), we can obtain:

$$F(\theta; a, b)=\frac{\int_0^{\theta}\theta^{a-1}(1-\theta)^{b-1}\mathrm{d}\theta}{\int_0^{1}\theta^{a-1}(1-\theta)^{b-1}\mathrm{d}\theta} \tag{35}$$

The probability of the normalized distributed energy output falling in the interval $[\Gamma_1, \Gamma_2]$ is obtained as:

$$\Pr(\Gamma_1\le\theta\le\Gamma_2)=F(\Gamma_2; a, b)-F(\Gamma_1; a, b) \tag{36}$$

The interval length is obtained as:

$$l=\Gamma_2-\Gamma_1 \tag{37}$$

To reduce the conservativeness of the robust optimization interval, the distribution probability of the minimum confidence interval is assumed to be p. Therefore, the following optimization problem is formulated:

$$\min(\Gamma_2-\Gamma_1) \tag{38}$$

The constraint is set as:

$$\Pr(\Gamma_1\le\theta\le\Gamma_2)\ge p \tag{39}$$

The optimal confidence interval $[\Gamma_1, \Gamma_2]$ can be obtained by calling the common solver. The detailed solution process is not discussed in this paper. According to the optimal confidence interval, the uncertainty interval of wind power is $[P'_{W,t}-\varepsilon_t+2\varepsilon_t\Gamma_1, P'_{W,t}-\varepsilon_t+2\varepsilon_t\Gamma_2]$.

The above results are the robust uncertainty intervals of a WT at one hour. For each distributed energy, the same processing is done for the whole running period. The uncertainty of PV is also considered as Beta distribution, thus we have the same treatment for PV. The process is shown in Fig. 1.

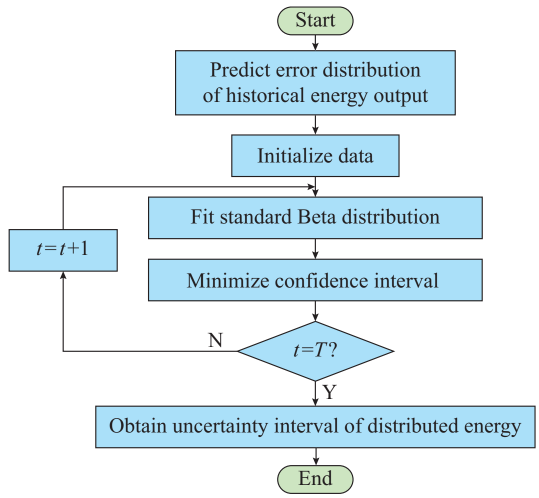

Fig. 1. Uncertainty interval of distributed energy output.

B. Description and Solution of Robust Optimization Model

The objective of the robust optimization is to find the optimal strategy under the condition where random variables have the greatest interference with the objective function. Since the OLTC and compensation capacitor cannot respond to the random fluctuations of distributed energy sources in real time, they are used as the first-stage decision variables of the robust optimization. And the SVC and energy storage device are used as the second-stage decision variables, which can quickly respond to the output deviation of distributed energy to achieve the optimal goal. The above optimization objectives correspond to the solution of the min-max-min mathematical model. Therefore, the robust optimization model of ADN is described as:

$$\begin{cases}\min\limits_{P_{i,ESS}, Q_{i,SVC}}\max\limits_{P_{i,PV}, P_{i,W}}\left\{\min\limits_{\alpha_i, Q_{i,CB}}\sum_{t=1}^{T}\left(\sum_{(i,j)\in X}c_1\tilde{I}_{ij,t}r_{ij}+\right.\right.\\ \left.\left.\sum_{m\in Y}c_2P_{m,t}+\sum_{n\in Z}c_3\left|\tilde{V}_{n,t}-V_N^2\right|\right)\right\}\\ \text{s.t. (2)-(22)}\end{cases} \tag{40}$$

where $P_{i,ESS}$, $P_{i,PV}$, and $P_{i,W}$ are the outputs of the energy storage device, PV and WT at bus i, respectively; and $Q_{i,SVC}$ and $Q_{i,CB}$ are the reactive power compensated by the SVC and capacitor at bus i, respectively. This model is a linear robust optimization model including an SOC, which does not include bilinear and non-linear variables. The first "min" optimization model is deterministic. According to the output of distributed energy, the "min" optimization model is solved directly by the linear YALMIP-CPLEX solver to obtain the OLTC and discrete capacitor, which will be also used as the initial value of the second-stage "min-max" optimization model. The second "min-max" optimization model is the common form of robust optimization. The optimal control variables are the output of SVC and energy storage device, and the robust uncertainty variable is the output of distributed energy. The optimization goal of this model is to give the best decision when the distributed energy output has the greatest influence on the objective function. There are some difficulties in solving the "min-max" optimization model directly. The mathematical processing method is to convert the model to its dual max problem through duality theory, and solve the max model equivalent to the original model [27], [28]. Reference [25] has proven that for the distribution network optimization model after the SOC relaxation, the distributed energy output based on robust optimization is always on the boundary of the uncertain interval. Therefore, the boundary of uncertainty interval is used in this paper to describe the uncertainty of distributed energy output. Then, the uncertain variable number of the robust optimization model changes from infinite to finite. Thus, the CPLEX solver can be directly used to solve the optimization problem after the transformation. The solution process of the robust optimization model is shown in Fig. 2.

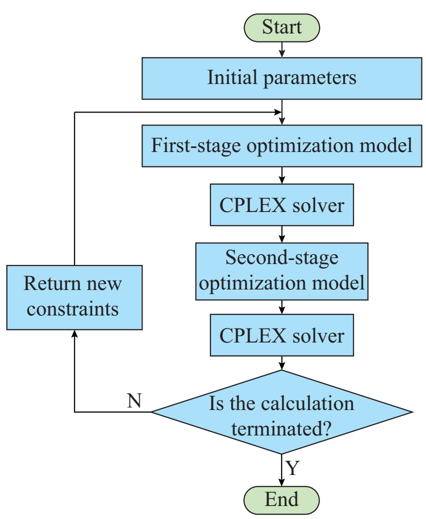

Fig. 2. Solution process of robust optimization model.

IV. Simulation Study

A. System Configuration

In order to verify the effectiveness of the robust optimization model proposed in this paper, the YAMLIP in MATLAB R2015b and the commercial CPLEX solver are used to solve the robust optimization problem of IEEE 33-bus system.

The IEEE 33-bus system is shown in Fig. 3, where ESS stands for energy storage system. The total load is (3.715+j1.86)MVA, and the optimized period is set as $T = 24$ hours. The OLTC is installed between buses 9 and 10, and its ratio range is [0.98, 1.02], which is divided into 5 switching positions and the variation of each switching position is 0.01.

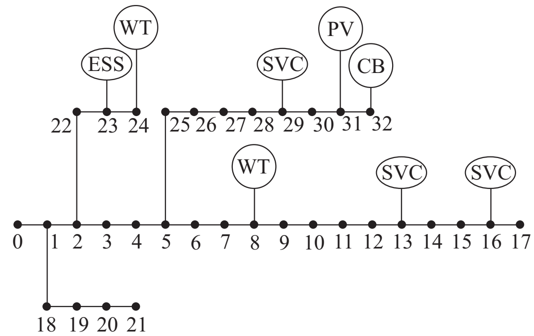

Fig. 3. Topology of IEEE 33-bus system.

The ESS is installed on bus 23. Its capacity is 0.2 MW and the power change rate is 0.04 MW/h. Three SVC compensators are installed on buses 13, 16, and 29, and the adjustable range is [–0.1 Mvar, 0.3 Mvar]. One capacitor compensator is installed on bus 32, with a capacity range of [0 Mvar, 0.2 Mvar]. There are two WTs connected to buses 8 and 24, with apparent power of 0.6 MVA. There is also one PV connected to bus 31 and its apparent power is 0.2 MVA. The output data of distributed energy come from an area in Australia, and the two-year history data are selected as the interval selection basis for the random distribution of distributed energy output. The minimum confidence interval is obtained by the prediction errors of distributed energy power of back propagation (BP) neural network. To ensure the accuracy of the model, the converted electricity price of the network loss is set as 0.1 $/kWh, and the penalty coefficient of voltage fluctuations is set as 0.5. The penalty cost of voltage fluctuations is calculated from voltage per unit. The time-of-use electricity price of the transformer bus is shown in Fig. 4.

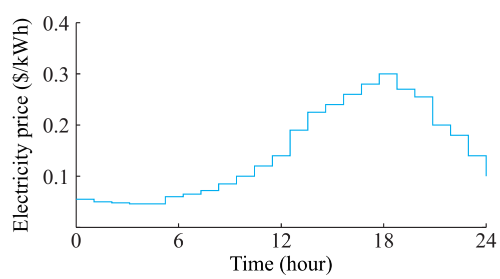

Fig. 4. Time-of-use electricity price of transformer bus.

B. Generation of Minimum Confidence Interval for Distributed Energy

According to the proposed method, the output data of distributed energy are normalized. At T = 1 hour, the frequency distribution histogram of the output data of a WT is shown in Fig. 5.

The distribution interval of the power normalization value is [0, 1]. According to (28), the mean value of this distribution is 0.6238 and the variance is 0.0222. The corresponding Beta distribution parameters $a = 5.97$ and $b = 3.60$ are calculated using (29). For this distribution, the confidence probability is 96%, and the distribution function value and the minimum confidence interval are shown in Fig. 6.

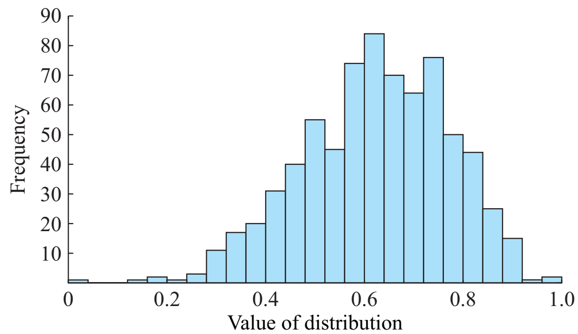

Fig. 5. Frequency distribution histogram of output data of a WT.

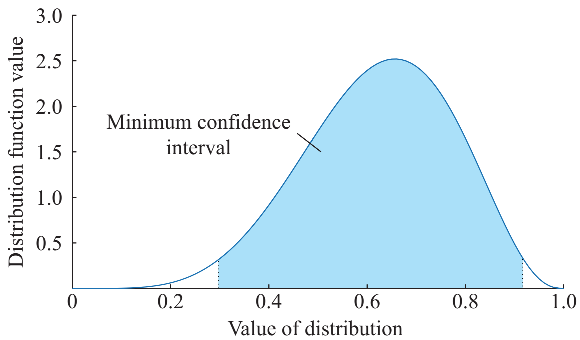

Fig. 6. Beta distribution and minimum confidence interval.

The range of its minimum confidence interval is [0.281, 0.932]. Figure 6 shows that the robust confidence interval based on the Beta distribution is smaller than the uncertainty interval of TRO, which has less conservativeness and can cover most of scenarios. The above result is the robust confidence interval of the WT at one hour, and the similar process is carried out for 24 operation hours to obtain the uncertainty interval.

C. Model Economy Analysis

In order to verify the economy of the proposed model, the distribution network structure is optimized by the deterministic optimization (DO), TRO, and robust optimization with minimum confidence interval (ROMCI) methods, respectively. The network loss cost, power purchasing cost, penalty cost for voltage fluctuations, and total cost are represented by C_1, C_2, C_3, and C_4, respectively. The results are shown in Table I.

TABLE I
COST COMPARISON OF VARIOUS METHODS

Method	C_1 (\$)	C_2 (\$)	C_3 (\$)	C_4 (\$)
DO	310.39	8091.05	19.30	8420.74
TRO	377.46	9346.21	23.69	9747.36
ROMCI	344.40	8770.20	21.46	9136.06

The results show that the DO method does not consider the randomness of distributed energy sources, thus there is no cost increase caused by distributed energy fluctuations. The total cost of the DO method is lower than that of the other two optimization methods. The TRO method has the highest total cost, which indicates that its excessive conservatism will often lead to poor economy. Although the TRO method can cover all scenarios, the occurrence of extreme scenarios is usually a low-probability event, that is, the practical application of the method is poor. The total cost of the ROMCI method is between the DO method and the TRO method. Meanwhile, the minimum confidence interval can cover the vast majority of scenarios, and the optimization result is less conservative, which indicates that the proposed model is more practical.

D. Role of Energy Storage Device in Optimization Process

In the proposed model, the capacity and charging and discharging power of the energy storage device are shown in Fig. 7. In order to ensure the long-term stable operation of the system, the energy of the energy storage device returns to the initial value after one operation period. From the 1st hour to the 6th hour, the electricity price is lower and the energy storage device stores energy. From the 16th hour to the 22th hour, the electricity price is higher and the energy storage device is discharged. That is, the energy storage device can reduce the operation cost of the distribution network and realize the economic operation of the distribution network.

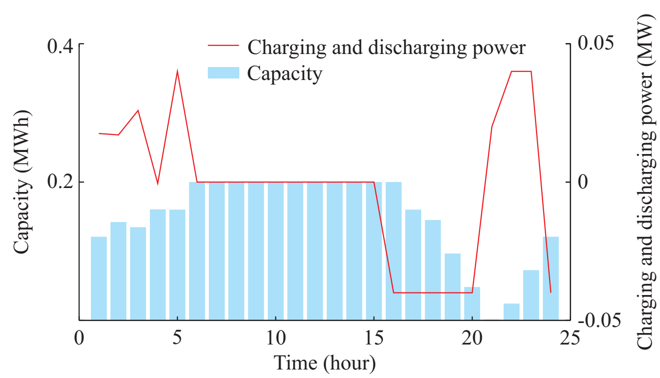

Fig. 7. Capacity and charging and discharging power of energy storage device in proposed model.

E. Model Stability Analysis

On the other hand, the excessive conservatism suggests a need for more tunable devices for the system. Therefore, for the TRO and the proposed methods, the voltage fluctuation of the distribution network is analyzed in the same reactive power compensation equipment. In the operation period of $T=24$ hours, the voltage waveforms of the remaining nodes of the distribution network that do not contain the substation are shown in Figs. 8 and 9.

The voltage data of the TRO method are compared with those of proposed method. The bus voltage fluctuation interval of the TRO method is [0.9362 p.u., 0.9985 p.u.], and that of the proposed method is [0.9491 p.u., 0.9987 p.u.]. This indicates that the voltage fluctuation range of the TRO method is larger with the same reactive compensation capacity. That is, more reactive compensation devices need to be configured if the same voltage fluctuation range is to be achieved.

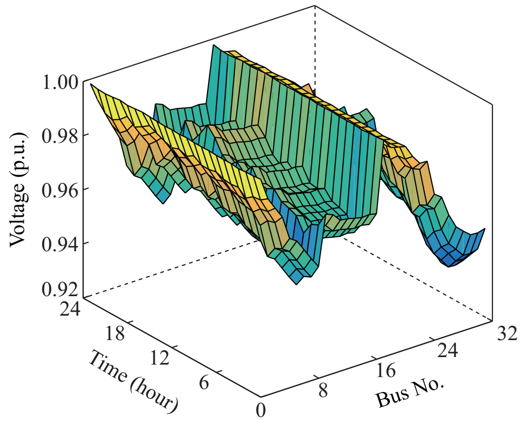

Fig. 8. Voltage distribution of TRO method.

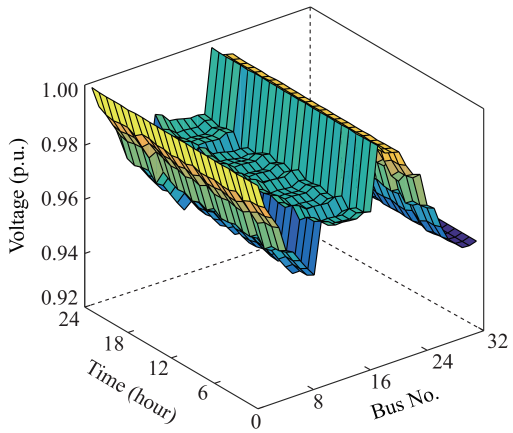

Fig. 9. Voltage distribution of proposed method.

F. Model Accuracy Analysis

In order to verify the accuracy of the SOC relaxation of (20). The relaxation error of the SOC is defined as:

$$\Delta_{ij,t} = \left| \tilde{I}_{ij,t}\tilde{V}_{ij,t} - P_{ij,t}^2 - Q_{ij,t}^2 \right| \tag{41}$$

The relaxation error of each branch is shown in Fig. 10.

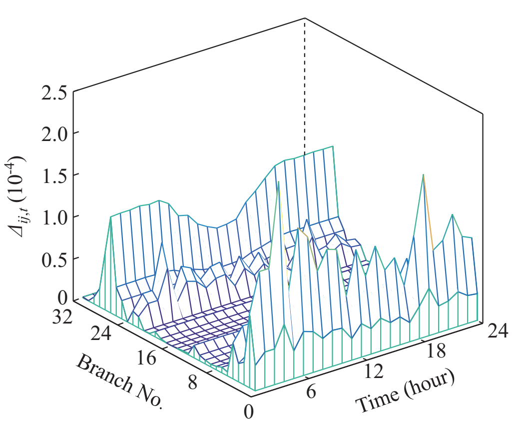

Fig. 10. Relaxation error distribution.

The maximum value of the relaxation error in Fig. 10 is 1.82×10^{-4}, and the relaxation error is distributed on the order of 10^{-4} or less. The effect of this relaxation error on power balance is small. That is, the model satisfies the equivalence of power balance within the allowed error range.

V. Conclusion

This paper proposes a robust optimization method for distribution networks based on the minimum confidence interval of Beta distribution. We fit the distributed energy output by Beta distribution, find the minimum confidence interval under the distribution, and use it as the uncertainty interval of distributed energy output for the ADN. Compared with the TRO method, the method proposed in this paper is less conservative and has been well verified in the IEEE 33-bus system.

References

[1] X. Zhu, H. Han, S. Gao *et al.*, "A multi-stage optimization approach for active distribution network scheduling considering coordinated electrical vehicle charging strategy," *IEEE Access*, vol. 6, pp. 50117-50130, Sept. 2018.

[2] Y. Tan, Y. Cao, Y. Li *et al.*, "Optimal day-ahead operation considering power quality for active distribution networks," *IEEE Transactions on Automation Science and Engineering*, vol. 14, no. 2, pp. 425-436, Apr. 2017.

[3] Y. Li, D. W. Gao, W. Gao *et al.*, "Double-mode energy management for multi-energy system via distributed dynamic event-triggered Newton-Raphson algorithm," *IEEE Transactions on Smart Grid*, vol. 11, no. 6, pp. 5339-5356, Nov. 2020.

[4] L. H. Macedo, J. F. Franco, M. J. Rider *et al.*, "Optimal operation of distribution networks considering energy storage devices," *IEEE Transactions on Smart Grid*, vol. 6, no. 6, pp. 2825-2836, Nov. 2015.

[5] Q. Li, R. Ayyanar, and V. Vittal, "Convex optimization for DES planning and operation in radial distribution systems with high penetration of photovoltaic resources," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 3, pp. 985-995, Jul. 2016.

[6] Z. Lin, Z. Hu, H. Zhang *et al.*, "Optimal ESS allocation in distribution network using accelerated generalised Benders decomposition," *IET Generation, Transmission & Distribution*, vol. 13, no. 13, pp. 2738-2746, Jul. 2019.

[7] Y. Luo, X. Zhang, D. Yang *et al.*, "Emission trading based optimal scheduling strategy of energy hub with energy storage and integrated electric vehicles," *Journal of Modern Power Systems and Clean Energy*, vol. 8, no. 2, pp. 267-275, Mar. 2020.

[8] J. Ma, L. Yuan, Z. Zhao *et al.*, "Transmission loss optimization-based optimal power flow strategy by hierarchical control for DC microgrids," *IEEE Transactions on Power Electronics*, vol. 32, no. 3, pp. 1952-1963, Mar. 2017.

[9] J. Liu, X. Huang, and Z. Li, "Multi-time scale optimal power flow strategy for medium-voltage DC power grid considering different operation modes," *Journal of Modern Power Systems and Clean Energy*, vol. 8, no. 1, pp. 46-54, Jan. 2020.

[10] Y. Li, H. Zhang, X. Liang *et al.*, "Event-triggered based distributed cooperative energy management for multienergy systems," *IEEE Transactions on Industrial Informatics*, vol. 15, no. 14, pp. 2008-2022, Dec. 2019.

[11] H. Zhang, Y. Li, D. W. Gao *et al.*, "Distributed optimal energy management for Energy Internet," *IEEE Transactions on Industrial Informatics*, vol. 13, no. 6, pp. 3081-3097, Dec. 2017.

[12] M. Baradar, M. R. Hesamzadeh, and M. Ghandhari, "Second-order cone programming for optimal power flow in VSC-type AC-DC grids," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4282-4291, Nov. 2013.

[13] W. Zheng, W. Wu, B. Zhang *et al.*, "A fully distributed reactive power optimization and control method for active distribution networks," *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 1021-1033, Mar. 2016.

[14] J. A. Taylor and F. S. Hover, "Convex models of distribution system reconfiguration," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1407-1413, Aug. 2012.

[15] S. Huang, Q. Wu, J. Wang *et al.*, "A sufficient condition on convex relaxation of AC optimal power flow in distribution networks," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1359-1368, Mar. 2017.

[16] P. Du, Z. Chen, Y. Chen *et al*., "A bi-level linearized dispatching model of active distribution network with multi-stakeholder participation based on analytical target cascading," *IEEE Access*, vol. 7, pp. 154844-154858, Oct. 2019.

[17] Q. Li and V. Vittal, "Convex hull of the quadratic branch AC power flow equations and its application in radial distribution networks," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 839-850, Jan. 2018.

[18] T. Ding, S. Liu, Z. Wu *et al*., "Sensitivity-based relaxation and decomposition method to dynamic reactive power optimisation considering DGs in active distribution networks," *IET Generation, Transmission & Distribution*, vol. 11, no. 1, pp. 37-48, Jan. 2017.

[19] P. Shen, Q. Liu, G. Liu *et al*., "Robust reconfiguration of distribution network considering the access of photovoltaic electric vehicle charging tower," in *Proceesings of 2019 IEEE Innovative Smart Grid Technologies – Asia (ISGT Asia)*, Chengdu, China, Oct. 2019, pp. 1655-1658.

[20] Y. Luo, D. Yang, Z. Yin *et al*., "Optimal configuration of hybrid-energy microgrid considering the correlation and randomness of the wind power and photovoltaic power," *IET Renewable Power Generation*, vol. 14, no. 4, pp. 616-627, Mar. 2020.

[21] Y. Zhou, Y. Tian, K. Wang *et al*., "Robust optimisation for AC-DC power flow based on second-order cone programming," *The Journal of Engineering*, vol. 2017, no. 13, pp. 2164-2167, Oct. 2017.

[22] L. F. Bertuccelli, A. Wu, and J. P. How, "Robust adaptive markov decision processes: planning with model uncertainty," *IEEE Control Systems Magazine*, vol. 32, no. 5, pp. 96-109, Oct. 2012.

[23] T. Ding, S. Liu, W. Yuan *et al*., "A two-stage robust reactive power optimization considering uncertain wind power integration in active distribution networks," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 1, pp. 301-311, Jan. 2016.

[24] T. Ding, C. Li, Y. Yang *et al*., "A two-stage robust optimization for centralized-optimal dispatch of photovoltaic inverters in active distribution networks," *IEEE Transactions on Sustainable Energy*, vol. 8, no. 2, pp. 744-754, Apr. 2017.

[25] Y. Zhang, X. Ai, J. Wen *et al*., "Data-adaptive robust optimization method for the economic dispatch of active distribution networks," *IEEE Transactions on Smart Grid*, vol. 10, no. 4, pp. 3791-3800, Jul. 2019.

[26] H. Yang, J. Sha, and F. Zhang, "Model and algorithm for minimum probability interval of wind power prediction error based on Beta distribution," *Proceedings of the CSEE*, vol. 35, no. 9, pp. 2135-2142, Sept. 2015.

[27] R. Jiang, J. Wang, M. Zhang *et al*., "Two-stage minimax regret robust unit commitment," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2271-2282, Aug. 2013.

[28] C. Roldán, R. Minguez, R. García-Bertrand *et al*., "Robust transmission network expansion planning under correlated uncertainty," *IEEE Transactions on Power Systems*, vol. 34, no. 3, pp. 2071-2082, May 2019.

Yanhong Luo received the B.S. degree in automation control, the Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2003 and 2009, respectively. She is currently with Northeastern University as an Associate Professor. Her research interests include adaptive dynamic programming, renewable energy, distributed control and optimization of Energy Internet and microgrid, and data-driven self-learning control and its industrial applications.

Qiubo Nie received the B.S. degree from Shenyang University of Technology, Shenyang, China, in 2018. He is currently pursuing the M.S. degree with the College of Information Science and Engineering, Northeastern University, Shenyang, China. His current research interests include reactive power optimization and active distribution system operation.

Dongsheng Yang received the B.S., M.S., and Ph.D. degrees from Northeastern University, Shenyang, China, in 1999, 2004, and 2007, respectively. He is currently the Deputy Dean at Intelligent Electrical Science and Technology Institute, College of Information Science and Engineering, Northeastern University, Shenyang, China. His research interests include complex network theory, energy efficiency optimization control and intelligent allocate electricity technology.

Bowen Zhou received the B.Sc. and M.Sc. degrees from Wuhan University, Wuhan, China, in 2010 and 2012, respectively, and the Ph.D. degree from Queen's University Belfast, Belfast, UK, in 2016, all in electrical engineering. He joined Institute of Electric Automation, College of Information Science and Engineering, Northeastern University, Shenyang, China, in 2016, where he is currently working as a Lecturer. He is currently a Member of IEEE, IET, IAENG, CSEE, CAA, and CCF. He is also a Standing Director or Director of several IEEE PES China Committees and Subcommittees. His research interests include power system operation, stability and control, vehicle to grid, energy storage and virtual energy storage, demand response, renewable energy, and Energy Internet.