

# Robust Optimization for Emergency Logistics Planning: Risk Mitigation in Humanitarian Relief Supply Chains

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This paper proposes a methodology to generate a robust logistics plan that can mitigate demand uncertainty in humanitarian relief supply chains. More specifically, we apply robust optimization (RO) for dynamically assigning emergency response and evacuation traffic flow problems with time dependent demand uncertainty. This paper studies a Cell Transmission Model (CTM) based system optimum dynamic traffic assignment model. We adopt a min-max criterion and apply an extension of the RO method adjusted to dynamic optimization problems, an Affinely Adjustable Robust Counterpart (AARC) approach. Simulation experiments show that the AARC solution provides excellent results when compared to deterministic solution and sampling based stochastic programming solution. General insights of RO and transportation that may have wider applicability in humanitarian relief supply chains are provided.

*Key words:* Robust optimization; Dynamic traffic assignment; Demand uncertainty; Emergency logistics

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## 1. Introduction

Over the past three decades, the number of reported disasters have risen threefold. Roughly, 5 billion people have been affected by disasters with an estimated damages of about 1.28 trillion dollars (Guha-Sapir et al. 2004). Although most of these disasters could not have been avoided, significant improvements in death counts and reported property losses could have been made by efficient distribution of *supplies*. The supplies here could mean personnel, medicine and food which are critical in emergency situations. The supply chains involved in providing emergency services in

the wake of a disaster are referred to as *Humanitarian Relief Supply Chains*. Humanitarian Relief supply chains are formed within short time period after a disaster with the government and the NGO's being the major drivers of the supply chain. Clearly, emergency logistics is an important component of humanitarian relief supply chains.

Most literature in emergency logistics focuses on generating transportation plans for rapid dissemination of medical supplies inbound to the disaster hit region (Sheu 2007, Ozdamar et al. 2004, Lodree Jr and Taskin 2008). There is, however, another aspect of emergency logistics which is often ignored - outbound logistics. The outbound logistics considers a situation where people and emergency supplies (e.g. medical facilities and services for special need evacuees) need to be sent from a particular location affected by disaster within a given time horizon.

In the outbound emergency logistics, the demand of traffic flows is usually highly uncertain and depends on a number of factors including the nature of disaster (natural/ man-made) and time of impact. This uncertainty in the demand causes disruptions in emergency logistics and hence disruptions in humanitarian relief supply chains leading to severe sub-optimality or even infeasibility which may ultimately lead to loss of life and property. In order to mitigate the risk of uncertain demand, we study the problem of generating evacuation transportation plans which are robust to uncertainty in outgoing demand. More specifically, we solve a dynamic (multi-period) emergency response and evacuation traffic assignment problem with uncertain demand at source nodes.

Researchers and practitioners in the field of transportation are concerned with multi-period management problems with an inherent time dependent information uncertainty. Traditional dynamic optimization approaches for dealing with uncertainty (e.g. stochastic and dynamic programming ) usually require the probability distribution for the underlying uncertain data to obtain expected objectives. However, in many cases, it may be very difficult to accurately identify the distribution required to solve a problem. Especially, this is more likely true when we are considering an evacuation transportation problem due to the inherent complexity and uncertainty. In addition, the

robust solution guaranteeing the feasible evacuation plan is important since infeasible solutions may cause the potential loss of life and property in extreme events.

We explore the potential of robust optimization (RO) as a general computational approach to manage uncertainty, feasibility, and tractability for complex transportation problems. RO approach has been originally developed to deal with static problems formulated as linear programming (LP) or conic-quadratic problems (CQP), using crude uncertainty with hard constraints. It means that uncertainty is assumed to reside in an appropriate set and RO guarantees the feasibility of the solution within the prescribed uncertainty set by adopting a min-max approach. The RO technique has been successfully applied in some complex and large scale engineering design and optimization problems similar as robust control in control theory. ([Ben-Tal and Nemirovski 1999, 2002](#)).

The original RO approach considers static problems. The underlying assumption of RO is “here and now” decisions, and all decision variables need to be determined before any uncertain data are realized. This is not typical in many transportation management problems that have the multi-period nature. In multi-period transportation problems such as dynamic traffic assignment, “wait and see” decisions are made, which means some decision variables are “adjustable” and affected by part of the realized data. Recognizing the need to account for such dynamics, [Ben-Tal et al. \(2004\)](#) have extended the RO approach and developed an Affinely Adjustable Robust Counterpart (AARC) approach to consider “wait and see” decisions.

To demonstrate the use of AARC to emergency transportation management settings, in this paper we consider a system optimum dynamic traffic assignment (SO-DTA) problem. The main contributions of this paper are summarized as follows.

- This paper develops a robust optimization framework for system optimum dynamic traffic assignment problems. The framework incorporates a linear programming (LP) formulation based on the Cell Transmission Model (CTM) ([Daganzo 1993, 1995, Ziliaskopoulos 2000](#)) and the AARC approach by considering dynamical adjustments to realizations of uncertainty with appropriate uncertainty sets. The framework is converted to LP and hence computationally tractable.

- This paper applies the proposed robust optimization framework to an emergency response and logistics planning problem. Numerical examples are provided to illustrate the value of the robust optimization in the context of emergency logistics and demonstrate the computational viability of the developed framework. Simulation experiments show that the AARC solution provides excellent results when compared to with the solutions of deterministic LP and Monte Carlo sampling based stochastic programming.
- This paper obtains some general insights that may have wider applicability for transportation managers: 1) A robust solution may improve both feasibility and performance when infeasibility costs are significant. Intuitively, the usual nominal optimal solution may be not far from the robust solution, but the usual optimal solution can perform much worse in the worst case. 2) An integration of RO and transportation modeling will improve the generation, communication, and potential use of uncertainty data in logistics transportation management. The intuition for this insight is twofold. First, in many applications in transportation, the set-based uncertainty (used by RO) is the most appropriate notion of data uncertainty. Second, computational tractability (resulting from this set-based uncertainty and dynamic traffic flow modeling in LP formulations) lead to efficient solutions for logistics transportation management under uncertainty.

The structure of the paper is as follows. In section 2, we provide a literature review. Section 3 presents a deterministic LP model for the CTM based SO-DTA problem. In section 4, AARC is formulated by considering appropriate demand uncertainty sets. We study applications in evacuation transportation and provide experiment results for two emergency logistics planning examples in section 5. Section 6 concludes and discusses future work.

## 2. Literature Review

The DTA problem describes a traffic system with time-varying flow and has been studied substantially since the seminar work of [Merchant and Nemhauser \(1978a,b\)](#). The main research can be classified into four categories: mathematical programming, optimal control, variational inequality, and simulation-based approach (see [Peeta and Ziliaskopoulos \(2001\)](#), [Friesz and Bernstein \(2000\)](#) for a review).

Daganzo (1993, 1995) proposed the CTM model, consisting of a set of linear difference equations, to develop a theoretical framework to simulate network traffic. It was assumed that the best route from origin to destination are already known to the travellers. Ziliaskopoulos (2000) relaxed this assumption by formulating a single destination SO DTA problem as a linear program with the decision variables being the route choices. Recently, the deterministic CTM based DTA model has been applied to evacuation management (e.g., Tuydes (2005), Chiu et al. (2007), Xie et al. (2010)). For example, Chiu et al. (2007) proposed a network transformation and demand modeling technique for solving an evacuation traffic assignment planning problem using the CTM based single destination SO DTA model.

Recognizing that deterministic demand or network characteristics are unrealistic in some settings, another wave of research on DTA is modeling of stochastic properties and developing robust solutions. Waller et al. (2001), Waller and Ziliaskopoulos (2006) addressed the impact of demand uncertainty and the importance of robust solution. Peeta and Zhou (1999) used Monte Carlo simulation to compute a robust initial solution for a real-time online traffic management system. Chance constraint programming for the SO DTA problem is analyzed by Waller and Ziliaskopoulos (2006). Yazici and Ozbay (2007) introduced probabilistic capacity constraint and solved the CTM based SO DTA problem for a hurricane evacuation problem. Karoonsoontawong and Waller (2007) proposed a DTA based network design problem formulated as a two stage stochastic programming and a scenario-based robust optimization (Mulvey et al. 1995). Ukkusuri and Waller (2008) proposed a two stage stochastic programming with recourse model to account for demand uncertainty.

Recently, robust optimization has witnessed a significant growth (Ben-Tal and Nemirovski 1998, 1999, 2000, El Ghaoui et al. 1997, 2003, Bertsimas and Sim 2003, 2004). For a summary of the state of art in RO, please refer to Ben-Tal et al. (2009), Bertsimas et al. (2007) and references therein. RO has been proposed to apply in network and transportation systems (Bertsimas and Perakis (2005), Ordóñez and Zhao (2007), Atamturk and Zhang (2007), Mudchanatongsuk et al. (2008), Erera et al. (2009), Yin et al. (2008, 2009) to name a few). Related to our work, Atamturk and Zhang (2007) propose a robust optimization approach for two stage network flow and design. Erera et al.

(2009) develop a two-stage robust optimization approach for repositioning empty transportation resources. Both of the studies are in the spirit of Ben-Tal et al. (2004) where the second stage variables are determined as recourse or recovery actions while maintaining feasibility after the uncertain data is realized.

### 3. CTM for the DTA Problem

In this section, we summarize and reformulate the prior work on the deterministic linear program (DLP) based on the traditional CTM model (Ziliaskopoulos 2000). The CTM, named by Daganzo (1993, 1995), models freeway traffic flow using simple difference equations. It approximates the kinematic wave model under the assumption of a piecewise linear relationship between flow and density on the link. More formally, the following equation shows the relationship between traffic flow,  $q$ , and density on a link,  $k$ , in a traffic network.

$$q = \min(vk, q_{\max}, w(k_{\max} - k)),$$

where  $v$  is free flow velocity,  $k_{\max}$  is maximum possible density,  $w$  is backward wave speed and  $q_{\max}$  is maximum allowable flow on the link.

The LP based CTM model of Ziliaskopoulos (2000) is a simplification of the original CTM model. In the CTM model, a segment of a freeway is decomposed into cells based on the free flow velocity and length of discrete time step. By this division, vehicles can move only to adjacent cells in unit time. The connectors between cells are dummy arcs indicating the direction of flow between cells. The demand of the CTM model represents the vehicular trips for each OD pair. In other words, each demand has its own origin and destination node in the network. The demand for each OD pair is assumed to be known at the beginning and used as input data of the CTM model. However, in our model, demand at the source node is uncertain.

We provide the reformulation of the deterministic LP based CTM model. The model includes the characteristics of time-space dependent cost and an adjacent matrix. In the traditional CTM research, it is assumed that the coefficient of cost is a constant value within the time-space network.

**Table 1**    **Notations**

Symbol	Description
$\mathfrak{S}$ :	set of time intervals, $\{1, \dots, T\}$
$C$ :	set of cells, $\{1, \dots, I\}$
$C_S$ :	set of sink cells
$C_R$ :	set of source cells
$A$ :	adjacency matrix representing transportation network connectivity.
$c_i^t$ :	time-space dependent cost in cell $i$ at time $t$
$x_i^t$ :	number of vehicle contained in cell $i$ at time $t$
$y_{ij}^t$ :	number of vehicle flowing from cell $i$ to cell $j$ at time $t$
$d_i^t$ :	demand generated in cell $i$ at time $t$
$N_i^t$ :	capacity of cell $i$ at time $t$
$Q_i^t$ :	inflow/outflow capacity of cell $i$ at time $t$
$\delta_i^t$ :	traffic flow parameter for cell $i$ at time $t$
$\hat{x}_i$ :	initial occupancy of cell $i$

However, in this paper, the coefficient is assumed to be dependent on time horizon and demand nodes. It is a more common situation and is necessary to study emergency logistics management. An adjacency matrix  $A = [a_{ij}]$  is defined for representing the connectivity of the cells. The value of  $a_{ij}$  is equal to 1 if cell  $i$  is connected to cell  $j$ , otherwise  $a_{ij}=0$ .

Based on the notations in Table 1, we present the deterministic linear programming (DLP) model:

$$\min_{x,y} \sum_{t \in \mathfrak{S}} \sum_{i \in C \setminus C_S} c_i^t x_i^t \quad (\text{M-DLP}) \quad (1)$$

subject to

$$x_i^t - x_i^{t-1} - \sum_{k \in C} a_{ki} y_{ki}^{t-1} + \sum_{j \in C} a_{ij} y_{ij}^{t-1} \geq d_i^{t-1}, \quad \forall i \in C, t \in \mathfrak{S} \quad (2)$$

$$\sum_{k \in C} a_{ki} y_{ki}^t \leq Q_i^t, \quad \forall i \in C, t \in \mathfrak{S} \quad (3)$$

$$\sum_{k \in C} a_{ki} y_{ki}^t + \delta_i^t x_i^t \leq \delta_i^t N_i^t, \quad \forall i \in C, t \in \mathfrak{S} \quad (4)$$

$$\sum_{j \in C} a_{ij} y_{ij}^t \leq Q_i^t, \quad \forall i \in C, t \in \mathfrak{S} \quad (5)$$

$$\sum_{j \in C} a_{ij} y_{ij}^t - x_i^t \leq 0, \quad \forall i \in C, t \in \mathfrak{S} \quad (6)$$

$$x_i^0 = \hat{x}_i, \quad \forall i \in C \quad (7)$$

$$y_{ij}^0 = 0, \quad \forall (i, j) \in C \times C \quad (8)$$

$$x_i^t \geq 0, \forall i \in C, \quad \forall t \in \mathfrak{T} \quad (9)$$

$$y_{ij}^t \geq 0, \forall (i, j) \in C \times C, \quad \forall t \in \mathfrak{T} \quad (10)$$

The cost parameter  $c_i^t$  depends on time in order to give a penalty when any people cannot arrive at the destination at the end of time horizon  $T$ . i.e.

$$c_i^t = \begin{cases} 1 & i \in C \setminus C_s, t \neq T \\ M & i \in C \setminus C_s, t = T, \end{cases}$$

where  $M$  is assumed to be a positive large number to represent the unsatisfied demand cost. By using the time dependent cost parameter, the objective function measures the total cost incurred, which consists of travel cost and penalty cost. The objective function of the LP based CTM model (M-DLP) provides an optimistic estimate or lower bound of total cost as it simplifies the original CTM model by Daganzo (1993, 1995) and allows vehicle holding.

The dynamics of the system is that the change of traffic level is determined by traffic flow and demand at each node and in each time period. By letting demand be 0 everywhere except source cells, the formulation can be generalized by Eq. (2). The total inflow into a cell is bounded by not only the inflow capacity (Eq. (3)) but the remaining capacity of the cell (Eq. (4)). Similarly, total output flow from a cell is limited by the outflow capacity (Eq. (5)) and the current occupancy of the cell (Eq. (6)). It is assumed that the capacities of source and sink cells are infinite. The initial conditions and non-negativity conditions are considered at the remaining constraints. Note that Eq. (9) is a redundant constraint, since  $\sum_{j \in C} a_{ij} y_{ij}^t - x_i^t \leq 0, y_{ij}^t \geq 0$  and  $a_{ij} \geq 0$ . It is evident that  $0 \leq \sum_{j \in C} a_{ij} y_{ij}^t \leq x_i^t$  and the Eq.(9) can be eliminated.

#### 4. Robust Optimization Formulation of CTM

The CTM based SO DTA problem is a generic multi-period linear programming problem. In this section, we apply AARC methodology to deal with the uncertainty in demand and find a robust solution for the multi-period emergency logistics problem.



In RO approach, it is assumed that demand  $d_i^t$  is unknown and it belongs to a prescribed uncertainty set. In particular, a box uncertainty set is generally used.

$$d_i^t \in U_d^b \equiv [\underline{d}_i^t, \bar{d}_i^t] = [\tilde{d}_i^t(1 - \theta), \tilde{d}_i^t(1 + \theta)],$$

where  $\theta$  is uncertainty level and  $\tilde{d}_i^t$  is nominal demand in cell  $i$  during time interval  $t$ .

In order to find a less conservative solution, we consider a joint constraint where the demands are upper bounded. Let's consider  $\sum_{t \in T} d_i^t \leq D_i, \forall i \in C_R$ , which refers to a joint budget for demand uncertainty. This represents the situation that the total demand ( $\sum_{t \in T} d_i^t$ ) from a source node is limited by an upper bound ( $D_i$ ). The box uncertainty set in conjunction with the budget uncertainty set becomes a polyhedral uncertainty set, which can be a more realistic assumption in emergency logistics management. Now, we have the following uncertain data set.

$$d_i^t \in U_d^p \equiv \left\{ d_i^t : \underline{d}_i^t \leq d_i^t \leq \bar{d}_i^t, \sum_{t \in T} d_i^t \leq D_i \right\}$$

Next, a specific form of linear decision rules is assumed to convert M-DLP to AARC formulation. The linear rules are used to derive a computationally tractable problem by approximating the robust solution. We note that the solution from AARC is optimal in worst case from the predetermined uncertainty set. However, there is no guarantee that the robust solution is close to optimal in the other cases since the relationship between uncertain parameters and decision variables may not be linear. Specifically, the adjustable control variables,  $y_{ij}^t$ , can be represented as an affine function of previously observed demand values, i.e.,  $y_{ij}^t = \pi_{ijt}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \pi_{ijt}^{s\tau} d_s^\tau$ , where  $\pi_{ijt}^{-1}$  and  $\pi_{ijt}^{s\tau}$  are non-adjustable variables and  $I_t = \{0, \dots, t-1\}$ .

Along with the affine rule of control variables, the state variables also becomes a affine function of the previously realized data,  $x_i^t = \eta_{it}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \eta_{it}^{s\tau} d_s^\tau$ , by the linear structure of CTM model.

By substituting the state and control variables, we have the following AARC formulation.

$$\min_{\sigma, \pi^{-1}, \pi, \eta^{-1}, \eta} z \quad (M - AARC) \tag{11}$$

s.t.

$$\begin{aligned}
& \sum_{t \in \mathfrak{S}} \sum_{i \in C} c_i^t (\eta_{it}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \eta_{it}^{s\tau} d_s^\tau) \leq z, \quad \forall d_i^t \in U_d \\
& (\eta_{it}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \eta_{it}^{s\tau} d_s^\tau) - (\eta_{it-1}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_{t-1}} \eta_{it-1}^{s\tau} d_s^\tau) - \sum_{k \in C} a_{ki} (\pi_{kit-1}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_{t-1}} \pi_{kit-1}^{s\tau} d_s^\tau) \\
& + \sum_{j \in C} a_{ij} (\pi_{ijt-1}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_{t-1}} \pi_{ijt-1}^{s\tau} d_s^\tau) \geq d_i^{t-1}, \quad \forall i \in C, t \in \mathfrak{S} \\
& \sum_{k \in C} a_{ki} (\pi_{kit}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \pi_{kit}^{s\tau} d_s^\tau) \leq Q_i^t, \quad \forall d_i^t \in U_d, i \in C, t \in \mathfrak{S} \\
& \sum_{k \in C} a_{ki} (\pi_{kit}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \pi_{kit}^{s\tau} d_s^\tau) + \delta_i^t (\eta_{it}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \eta_{it}^{s\tau} d_s^\tau) \leq \delta_i^t N_i^t, \quad \forall d_i^t \in U_d, i \in C, t \in \mathfrak{S} \\
& \sum_{j \in C} a_{ij} (\pi_{ijt}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \pi_{ijt}^{s\tau} d_s^\tau) \leq Q_i^t, \quad \forall d_i^t \in U_d, i \in C, t \in \mathfrak{S} \\
& \sum_{j \in C} a_{ij} (\pi_{ijt}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \pi_{ijt}^{s\tau} d_s^\tau) - (\eta_{it}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \eta_{it}^{s\tau} d_s^\tau) \leq 0, \quad \forall d_i^t \in U_d, i \in C, t \in \mathfrak{S} \\
& \eta_{i0}^{-1} = \hat{x}_i, \quad \forall i \in C \\
& \pi_{ij0}^{-1} = 0, \quad \forall (i, j) \in C \times C \\
& \pi_{ijt}^{-1} + \sum_{s \in C_R} \sum_{\tau \in I_t} \pi_{ijt}^{s\tau} d_s^\tau \geq 0, \quad \forall (i, j) \in C \times C, t \in \mathfrak{S}, d_i^t \in U_d
\end{aligned}$$

The formulation (M-AARC) is intractable since it is a semi-infinite program, and it can be reformulated as a tractable optimization problem as shown in the Theorem 1. The minimum objective value  $z$ , denoted as  $z_{AARC}^*$ , is a guaranteed upper bound value for all realization of uncertain data under the assumption of linear dependency. The objective value,  $z_{AARC}^*$ , also can be interpreted as the optimistic estimate of total travel cost in worst case, which can be lower than the optimistic estimate from robust counterpart(RC),  $z_{RC}^*$  as AARC has a larger robust feasible region (Ben-Tal et al. 2004). The decision variables of AARC are not adjustable control and state variables ( $y_{ij}^t$  and  $x_i^t$ , respectively), but a set of coefficient of affine function of the control variables including  $\pi_{ijt}^{-1}, \pi_{ijt}^{s\tau}, \eta_{it}^{-1}$  and  $\eta_{it}^{s\tau}$ . It means that the solution of AARC is the linear decision rule. Specific values of  $y_{ij}^t$  and  $x_i^t$  are calculated after the realization of the demand at time  $t - 1$ .

Theorem 1. Given polyhedral uncertainty set,  $U_d^p$ , the affinely adjustable robust counterpart of the CTM based SO DTA problem becomes the following linear programming problem and thus computationally tractable. Note that  $\lambda$  is a set of dual variables and the numerical indexes are used for notational simplicity.

$$\min_{z, \eta, \pi, \lambda} \quad z \quad (M - AARC1) \quad (12)$$

s.t

$$\begin{aligned} & \sum_{\tau \in \mathfrak{S}} \sum_{s \in C_R} (\bar{d}_s^\tau \lambda_{s\tau}^{11} - \underline{d}_s^\tau \lambda_{s\tau}^{12}) + \sum_{s \in C_R} D_s \lambda_s^{13} \leq z - \sum_{t \in \mathfrak{S}} \sum_{i \in C \setminus C_s} c_i^t \eta_{it}^{-1} \\ & \lambda_{s\tau}^{11} - \lambda_{s\tau}^{12} + \lambda_s^{13} = \sum_{t=\{\tau+1 \dots T\}} \sum_{i \in C \setminus C_s} c_i^t \eta_{it}^{s\tau}, \quad \forall \tau = \{0, \dots, T-1\}, s \in C_R \\ & \lambda_{s\tau}^{11} - \lambda_{s\tau}^{12} + \lambda_s^{13} = 0, \quad \forall \tau = \{T\}, s \in C_R \\ & \lambda_{s\tau}^{11}, \lambda_{s\tau}^{12}, \lambda_s^{13} \geq 0, \quad \forall \tau = \{T\}, s \in C_R \\ & \sum_{\tau \in \mathfrak{S}} \sum_{s \in C_R} (\bar{d}_s^\tau \lambda_{its\tau}^{21} - \underline{d}_s^\tau \lambda_{its\tau}^{22}) + \sum_{s \in C_R} D_s \lambda_{its}^{23} \leq \eta_{it}^{-1} - \eta_{it-1}^{-1} - \sum_{k \in C} a_{ki} \pi_{kit-1}^{-1} + \sum_{j \in C} a_{ij} \pi_{ijt-1}^{-1}, \quad \forall i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{21} - \lambda_{its\tau}^{22} + \lambda_{its}^{23} = I_{\{\tau=t-1, s=i\}} - \eta_{it}^{s\tau} + (\eta_{it-1}^{s\tau} + \sum_{k \in C} a_{ki} \pi_{kit-1}^{s\tau} - \sum_{j \in C} a_{ij} \pi_{ijt-1}^{s\tau}) I_{\{\tau < t-1\}}, \\ & \quad \forall \tau = \{0 \dots t-1\}, s \in C_R, i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{21} - \lambda_{its\tau}^{22} + \lambda_{its}^{23} = 0, \quad \forall \tau = \{t \dots T\}, s \in C_R, i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{21}, \lambda_{its\tau}^{22}, \lambda_{its}^{23} \geq 0, \quad \forall \tau = \{t \dots T\}, s \in C_R, i \in C, t \in \mathfrak{S} \\ & \sum_{\tau \in \mathfrak{S}} \sum_{s \in C_R} (\bar{d}_s^\tau \lambda_{its\tau}^{31} - \underline{d}_s^\tau \lambda_{its\tau}^{32}) + \sum_{s \in C_R} D_s \lambda_{its}^{33} \leq Q_i^t - \sum_{k \in C} a_{ki} \pi_{kit}^{-1}, \quad \forall i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{31} - \lambda_{its\tau}^{32} + \lambda_{its}^{33} = \sum_{k \in C} a_{ki} \pi_{kit}^{s\tau}, \quad \forall \tau = \{0 \dots t-1\}, s \in C_R, i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{31} - \lambda_{its\tau}^{32} + \lambda_{its}^{33} = 0, \quad \forall \tau = \{t \dots T\}, s \in C_R, i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{31}, \lambda_{its\tau}^{32}, \lambda_{its}^{33} \geq 0, \quad \forall \tau = \{t \dots T\}, s \in C_R, i \in C, t \in \mathfrak{S} \\ & \sum_{\tau \in \mathfrak{S}} \sum_{s \in C_R} (\bar{d}_s^\tau \lambda_{its\tau}^{41} - \underline{d}_s^\tau \lambda_{its\tau}^{42}) + \sum_{s \in C_R} D_s \lambda_{its}^{43} \leq \delta_i^t (N_i^t - \eta_{it}^{-1}) - \sum_{k \in C} a_{ki} \pi_{kit}^{-1}, \quad \forall i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{41}, \lambda_{its\tau}^{42}, \lambda_{its}^{43} \geq 0, \quad \forall i \in C, t \in \mathfrak{S} \\ & \sum_{\tau \in \mathfrak{S}} \sum_{s \in C_R} (\bar{d}_s^\tau \lambda_{its\tau}^{51} - \underline{d}_s^\tau \lambda_{its\tau}^{52}) + \sum_{s \in C_R} D_s \lambda_{its}^{53} \leq Q_i^t - \sum_{j \in C} a_{ij} \pi_{ijt}^{-1}, \quad \forall i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{51} - \lambda_{its\tau}^{52} + \lambda_{its}^{53} = \sum_{j \in C} a_{ij} \pi_{ijt}^{s\tau}, \quad \forall \tau = \{0 \dots t-1\}, s \in C_R, i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{51} - \lambda_{its\tau}^{52} + \lambda_{its}^{53} = 0, \quad \forall \tau = \{t \dots T\}, s \in C_R, i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{51}, \lambda_{its\tau}^{52}, \lambda_{its}^{53} \geq 0, \quad \forall \tau = \{t \dots T\}, s \in C_R, i \in C, t \in \mathfrak{S} \\ & \sum_{\tau \in \mathfrak{S}} \sum_{s \in C_R} (\bar{d}_s^\tau \lambda_{its\tau}^{61} - \underline{d}_s^\tau \lambda_{its\tau}^{62}) + \sum_{s \in C_R} D_s \lambda_{its}^{63} \leq \eta_{it}^{-1} - \sum_{j \in C} a_{ij} \pi_{ijt}^{-1}, \quad \forall i \in C, t \in \mathfrak{S} \\ & \lambda_{its\tau}^{61} - \lambda_{its\tau}^{62} + \lambda_{its}^{63} = \sum_{j \in C} a_{ij} \pi_{ijt}^{s\tau} - \eta_{it}^{s\tau}, \quad \forall \tau = \{0 \dots t-1\}, s \in C_R, i \in C, t \in \mathfrak{S} \end{aligned}$$

$$\lambda_{its\tau}^{61} - \lambda_{its\tau}^{62} + \lambda_{its}^{63} = 0, \quad \forall \tau = \{t \dots T\}, s \in C_R, i \in C, t \in \mathfrak{S}$$

$$\lambda_{its\tau}^{61}, \lambda_{its\tau}^{62}, \lambda_{its}^{63} \geq 0 \quad \forall \tau = \{t \dots T\}, s \in C_R, i \in C, t \in \mathfrak{S}$$

$$\sum_{\tau \in \mathfrak{S}} \sum_{s \in C_R} (\bar{d}_s^\tau \lambda_{ijts\tau}^{71} - \underline{d}_s^\tau \lambda_{ijts\tau}^{72}) + \sum_{s \in C_R} D_s \lambda_{ijts}^{73} \leq \pi_{ijt}^{-1}, \quad \forall (i, j) \in C \times C, t \in \mathfrak{S}$$

$$\lambda_{ijts\tau}^{71} - \lambda_{ijts\tau}^{72} + \lambda_{ijts}^{73} = -\pi_{ijt}^{s\tau}, \quad \forall \tau = \{0 \dots t-1\}, s \in C_R, (i, j) \in C \times C, t \in \mathfrak{S}$$

$$\lambda_{ijts\tau}^{71} - \lambda_{ijts\tau}^{72} + \lambda_{ijts}^{73} = 0, \quad \forall \tau = \{t \dots T\}, s \in C_R, (i, j) \in C \times C, t \in \mathfrak{S}$$

$$\lambda_{ijts\tau}^{71}, \lambda_{ijts\tau}^{72}, \lambda_{ijts}^{73} \geq 0, \quad \forall \tau = \{t \dots T\}, s \in C_R, (i, j) \in C \times C, t \in \mathfrak{S}$$

$$\eta_{i0}^{-1} = \hat{x}_i, \quad \forall i \in C$$

$$\pi_{ki0}^{-1} = 0, \quad \forall (i, j) \in C \times C$$

Proof) By using the following relationship, we can reformulate each constraint affected by uncertain data as an equivalent LP problem.

$$\begin{aligned} \sum_{\tau \in I_t} \alpha_i^\tau d_i^\tau &\leq v \quad \forall d_i^t \in U_d = \{\underline{d}_i^t < d_i^t < \bar{d}_i^t, \sum_{t \in T} d_i^t \leq D_i\} \\ &\Leftrightarrow \max_{d_i^\tau} \left( \sum_{\tau \in I_t} \alpha_i^\tau d_i^\tau \right) \leq v \end{aligned}$$

Without loss of generality, the polyhedral uncertainty set is represented as  $Ad \leq b$  and  $\max_{d_i^\tau} (\sum_{\tau \in I_t} \alpha_i^\tau d_i^\tau)$  is written as (P). By applying strong duality property, we can derive an equivalent constraint with dual problem (D) on the left hand side (Bertsimas and Sim 2004).

$$\begin{array}{ll} \max & \alpha d \leq v \quad (P) \\ \text{s.t.} & \\ & Ad \leq b \\ & d \geq 0 \end{array} \quad \Longleftrightarrow \quad \begin{array}{ll} \min & b\lambda \leq v \quad (D) \\ \text{s.t.} & \\ & A^T \lambda = \alpha \\ & \lambda \geq 0 \end{array}$$

Therefore, there exists  $\lambda$  satisfying  $b\lambda \leq v, A^T \lambda = \alpha$  and  $\lambda \geq 0$  and the equivalent AARC of the CTM based SO DTA becomes tractable.

## 5. Emergency Logistics Management

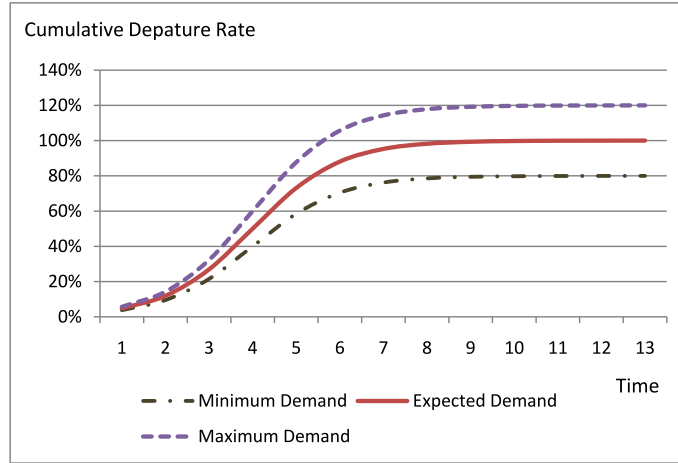
Emergency management is one of the best application areas for applying robust optimization due to the uncertainty of human beings and disaster. Robust solution, especially AARC solution, can play an important role for emergency logistics planning for several reasons. First of all, the role of hard constraint is emphasized since the penalty cost for an infeasible solution is loss of life or

property. Next, it is very difficult to estimate or forecast the demand model in the to-be-affected areas due to unexpected human behavior and nature of disaster. Finally, we can take advantage of updated or realized data on demand by employing AARC solution. When we solve M-AARC1, the optimal coefficients of the Linear Decision Rule (LDR) are computed offline. Going online, the actual decision variables (flows) are determined for period  $t$  by inserting the revealed uncertainties from previous periods in the LDR. A fully online version of the method can be also implemented. In such version, at period  $t$  only the  $t$ -period design variables are activated. The horizon is then rolled forward and the problem is resolved after adjusting the state variables revealed in previous periods.

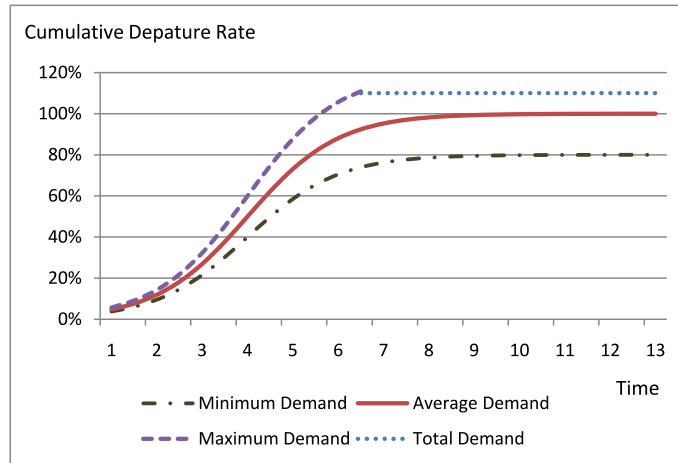
In this section, an emergency logistics planning problem is considered and the meaning of demand uncertainty sets is explained. Then, we present a summary of experiments to test the performance of the AARC approach. The AARC solution is benchmarked against an ideal solution with complete future information, deterministic LP, and sampling based stochastic programming. Two test networks are chosen from [Chiu et al. \(2007\)](#) and [Yazici and Ozbay \(2007\)](#) for the numerical analysis.

### 5.1. Demand modeling

In an emergency logistics problem, a general approach to model time-varying evacuee demand is captured by the following steps: The first step of demand modeling is calculation of total demand. Next, demand arrival or vehicle departure rate is determined for describing a dynamic environment. For example, S-shape curve can be used for representing cumulative percentage of demand arrival. In most studies, it is assumed that the parameters (e.g. slope) of S-curve are unknown but deterministic value. Since the parameters can be estimated with empirical data or simulation results, different research showed different values ([Radwan et al. 1985](#), [Lindell 2008](#)). However, in real world, both total number of demand and departure rate are uncertain. By considering box uncertainty or polyhedral uncertainty set, we can overcome the limitation of deterministic S-curve and cover infinite number of S-curve including fast, medium and slow response. Figure 1 shows *the S-curve with upper and lower bound* defined by box uncertainty. In Figure 2, polyhedral uncertainty



**Figure 1 S-curve: box uncertainty**



**Figure 2 S-curve: polyhedral uncertainty**

set (box uncertainty & budget uncertainty) is shown and the upper bound of S-curve is limited by total demand.

## 5.2. Small Network Example

In the first numerical experiment, a small network configuration is drawn in Figure 3 to verify the performance of AARC from the illustrative example of Chiu et al. (2007). The network consists of 14 nodes including 3 source nodes (1,5, and 9) and 1 super sink cell (14).

The data of the transportation network is adopted from Chiu et al. (2007) except demand data since deterministic demand was used in the original model. Also, we assume that the penalty

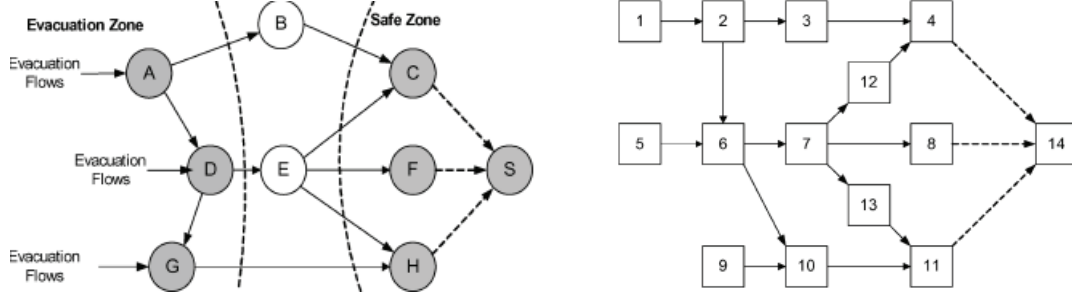


Figure 3 CTM Example (Chiu et al. 2007)

Table 2 Time Invariant Cell Properties

Cell	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ni	8	20	20	20	8	20	20	20	8	20	20	20	20	8
Qi	12	12	*	12	12	12	12	12	12	12	12	12	12	12
$\hat{x}_i$	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3 Time Dependent Data

Time	0	1	2	3	4	5	6	7	8	9	10
$Q_3^t$	12	6	6	0	0	0	12	12	12	12	12

cost(M) for unmet demand is 100. Table 2 and 3 show time invariant data and time dependent data, respectively. In the example, the flow capacity of node 3 is time dependent and changes from time 1 to 6.

As mentioned before, we consider uncertain multi-period demand. In particular, the following mathematical formulation of S-curve (Radwan et al. 1985) is adopted for demand loading.

$$P(t) = 1 / (1 + \exp(-\alpha(t - \beta))), \quad (13)$$

where  $P(t)$  is the cumulative distribution with  $\alpha = 1$ , the slope of curve, and  $\beta = 3$ , the median departure time. In both box and polyhedral uncertainty set, nominal demand at time  $t$  is calculated by multiplying  $(P(t) - P(t - 1))$  with expected total demand. Also, the joint budget of demand uncertainty is assumed one and half times of the sum of expected total demand.

**5.2.1. AARC vs. DLP** Based on the nominal data, uncertain demand in a polyhedral set is generated and tested. The uncertainty level  $\theta$  is increased from 2.5% to 30%. First, objective values are calculated and emergency logistics plans are generated using M-DLP and M-AARC.

Next, given the uncertainty level and evacuation plan, simulated (or realized) objective value from Eq.(1) is computed by generating random demand in the specified uncertainty set. Average values, standard deviation and worst case solution of 1000 simulated objective values are used to compare the traffic assignment solutions.

Our first object of experiments is comparison of AARC and DLP under a polyhedral uncertainty set. Objective values of robust optimization approaches, which measure the worst case solution of the vehicle control plan, are computed and compared at Table 4 by changing uncertainty level. DLP solution shows the cost when only deterministic nominal demand is dealt with. It is natural that the objective value of AARC with larger uncertainty level is bigger. Also, the objective value of DLP is smaller since it is equivalent to AARC with zero uncertainty level.

In simulation, the emergency logistics plan from inequality flow constraints has to be adjusted in some way since we relaxed the constraint in Eq. (2). We assumed that if there are fewer vehicles in a node than the vehicle flow plan, proportionate flow is allocated to each path. Also, any vehicles exceeding the plan will remain at the node and pay penalty for not planning them. Table 4 shows the simulated objective value of ideal DLP, DLP, and AARC. Ideal DLP is the case where perfect future demand information is known at the beginning of planning horizon. It is the lower bound of simulated objective value. The average improvement of AARC over DLP is significant at higher uncertainty level.

The AARC problem with 14 nodes and 15 planning horizon has 36,600 constraints and 190,428 variables. It is solved in about 44 seconds on a PC with Intel processor 1.87 Ghz and 2GB of memory.

**5.2.2. AARC vs. Sampling Based Stochastic Programming** Sampling based stochastic programming (SSP), or Monte Carlo sampling method, is an important approximation approach. Stochastic problems are solved by generating random samples and solving a deterministic problem to optimize sample average objective value (Shapiro 2003). For comparison with stochastic programming, beta distribution is assumed, and if the sum of the sampling demand is bigger than



**Table 4 Objective Value – Polyhedral Uncertainty**

$\theta$	obj		avg			sd			worst		
	DLP	AARC	Ideal	DLP	AARC	Ideal	DLP	AARC	Ideal	DLP	AARC
0.025	350.10	358.18	354.35	417.13	355.34	1.08	15.14	0.91	356.54	452.33	357.17
0.05	350.10	366.27	358.59	484.25	359.70	2.16	30.19	1.76	362.97	554.56	363.27
0.075	350.10	375.24	362.85	551.49	364.88	3.26	45.10	2.97	369.83	656.80	370.73
0.1	350.10	384.35	367.33	618.81	370.72	4.57	59.95	3.78	377.14	759.03	378.44
0.15	350.10	402.57	376.78	753.49	381.79	7.20	89.63	5.35	391.76	963.49	393.29
0.2	350.10	420.80	386.35	888.23	397.50	9.70	119.29	6.14	406.38	1167.95	410.64
0.25	350.10	439.03	395.95	1023.00	410.17	12.17	148.94	7.90	421.00	1372.42	427.02
0.3	350.10	457.25	405.55	1157.77	420.57	14.63	178.59	10.12	435.61	1576.88	441.57

the upper bound of total demand, it is ignored and re-sampled. The following equation represents SSP.

$$\begin{aligned}
& \text{Min}_{x,y} \frac{1}{L} \sum_{l \in \Lambda} \sum_{t \in \mathfrak{S}} \sum_{i \in C \setminus C_s} c_i^t x_{il}^t \\
& s.t. \\
& x_{il}^t - x_{il}^{t-1} - \sum_{k \in C} a_{ki} y_{ki}^{t-1} + \sum_{j \in C} a_{ij} y_{ij}^{t-1} \geq d_{il}^{t-1}, \quad \forall l \in \Lambda, i \in C, t \in \mathfrak{S} \\
& \sum_{k \in C} a_{ki} y_{ki}^t \leq Q_i^t, \quad \forall i \in C, t \in \mathfrak{S} \\
& \sum_{k \in C} a_{ki} y_{ki}^t + \delta_i^t x_{il}^t \leq \delta_i^t N_i^t, \quad \forall l \in \Lambda, i \in C, t \in \mathfrak{S} \\
& \sum_{j \in C} a_{ij} y_{ij}^t \leq Q_i^t, \quad \forall i \in C, t \in \mathfrak{S} \\
& \sum_{j \in C} a_{ij} y_{ij}^t - x_{il}^t \leq 0, \quad \forall l \in \Lambda, i \in C, t \in \mathfrak{S} \\
& x_{il}^0 = \hat{x}_i, \quad \forall i \in C, l \in \Lambda \\
& y_{ij}^0 = 0, \quad \forall (i, j) \in C \times C \\
& x_{il}^t \geq 0, \quad \forall i \in C, t \in \mathfrak{S}, l \in \Lambda \\
& y_{ij}^t \geq 0, \quad \forall (i, j) \in C \times C, t \in \mathfrak{S}
\end{aligned} \tag{14}$$

where independent sampling scenario  $l \in \Lambda = \{1, 2, \dots, L\}$ ,  $x_{il}^t$  is the number of vehicles contained in cell  $i$  at time  $t$  for sampling scenario  $l$ ,  $d_{il}^t$  is demand generated in cell  $i$  at time  $t$  for sampling scenario  $l$ .

**Table 5 AARC vs. SSP when  $\theta$  changes. (Beta(5,2), L=50, M=100)**

	obj		avg		gap		sd		worst	
$\theta$	AARC	SSP	AARC	SSP	AARC	SSP	AARC	SSP	AARC	SSP
0.025	358.18	350.50	355.34	368.84	0.28%	4%	0.91	9.02	357.17	392.49
0.05	366.27	350.91	359.70	387.57	0.31%	8%	1.76	18.04	363.27	434.88
0.075	375.24	351.31	364.88	406.31	0.56%	12%	2.97	27.06	370.73	477.27
0.1	384.35	351.71	370.72	425.04	0.92%	16%	3.78	36.08	378.44	519.66
0.15	402.80	352.58	381.79	463.30	1.33%	23%	5.35	54.30	393.29	605.5
0.2	420.80	353.82	397.50	501.69	2.89%	30%	6.14	72.44	410.64	691.37
0.25	439.03	355.59	410.17	540.09	3.95%	36%	7.90	90.59	427.02	777.27
0.3	457.25	357.74	420.57	578.50	3.70%	43%	10.12	108.72	441.57	863.15

**Table 6 AARC vs. SSP when  $\theta$  changes. (Beta(1,1), L=50, M=100)**

	obj		avg		gap		sd		worst	
$\theta$	AARC	SSP	AARC	SSP	AARC	SSP	AARC	SSP	AARC	SSP
0.025	358.18	350.20	350.86	352.51	0.41%	0.88%	2.16	6.13	356.04	357.97
0.05	366.27	350.30	350.68	354.93	0.55%	1.77%	4.15	12.26	361.62	401.84
0.075	375.24	350.40	350.29	357.34	0.63%	2.65%	6.39	18.40	367.70	427.71
0.1	384.35	350.50	360.15	359.75	3.66%	3.54%	7.12	24.53	377.49	453.59
0.15	402.80	350.74	362.09	364.78	4.61%	5.38%	10.39	36.99	389.53	505.90
0.2	420.80	351.34	364.82	370.16	5.75%	7.30%	14.43	48.78	403.76	560.14
0.25	439.03	352.80	376.34	375.96	9.37%	9.26%	15.80	60.93	419.08	618.25
0.3	457.25	354.63	380.24	381.92	10.71%	11.20%	20.33	73.06	433.76	677.22

For the comparison, at first, 50 samples are generated using beta distribution function, Beta(1,2). Next, Beta(5,2) and Uniform distribution (i.e. Beta(1,1)) are used for generating uncertain demand for simulation. This may be reasonable when we do not have exact information on the distribution. The objective value of SSP is lower than AARC since it finds the average of minimum cost with given sample data. It has different meaning from the RO approach generating best worst case solution. However, the simulated objective values can be compared since they show the performance of emergency logistics plan. When Beta(5,2) is used for simulation, we can see that AARC is better than SSP in terms of the average of the simulated objective value in Table 5. The gap between AARC and the ideal solution is very small even with higher uncertainty, e.g. it is less than 4% when the uncertain level is 30 %! In contrast for SSP, the gap is increased drastically. As shown in Table 6, the average values of the simulated objective value from AARC and SSP are comparable with the random demand from Beta (1,1).

Under both demand scenarios, AARC provides more stable and robust solution than SSP in

**Table 7 AARC vs. SSP when  $M$  changes. (Beta(5,2), L=50,  $\theta=0.1$ )**

$M$	obj		avg		gap		sd		worst	
	AARC	SSP	AARC	SSP	AARC	SSP	AARC	SSP	AARC	SSP
25	384.35	345.04	370.48	435.43	0.86%	18.54%	3.91	19.88	379.35	474.46
50	384.35	347.63	370.27	474.35	0.80%	29.17%	4.22	31.82	379.13	539.28
75	384.35	349.46	371.06	470.81	1.02%	28.18%	3.87	36.19	379.61	552.61
100	384.35	350.50	373.69	430.37	1.73%	17.16%	3.34	37.03	380.42	524.02

**Table 8 Cell Properties**

	Nodes 11-16	The others
$N_i^t$	600	450
$Q_i^t$	1440	1080
$\hat{x}_i$	0	0

the aspect of standard deviation and worst case solution. In all cases, the worst case costs of SSP exceed the worst case value of AARC. Moreover, the AARC solution guarantees the feasibility and provides a guaranteed upper bound on the optimal cost. The SSP solution does not guarantee neither of the above.

Next, we test and summarize the effect of penalty value on the performance of each approach. Table 7 shows that as the value of  $M$  changes, AARC always provides more stable and robust solution than SSP in the aspect of standard deviation and worst case solution, and provides an evacuation solution that leads to small gap from the ideal solution and can meet all the demand.

### 5.3. Cape May County Network Example

We select another network from Yazici and Ozbay (2007) to increase the size of the problem. Official evacuation routes of Cape May county, New Jersey are considered in Figure 4, which is composed of 27 nodes including 3 origin nodes (1, 2, and 3) and 1 super destination node (27). All data except the uncertain demand set are adopted from Yazici and Ozbay (2007) and listed at Table 8. For departure time distribution function, Eq. (13) is used with  $\alpha = 1$  and  $\beta = 6$ . Also, the penalty cost( $M$ ) for unmet demand is set to be 100.

Tables 9 - 10 show similar results as the pervious small example. AARC approach improves the transportation solution compared to the deterministic model. Also, we can observe that AARC solution provides better results than SSP in terms of the worse case solution as well as solution stability.

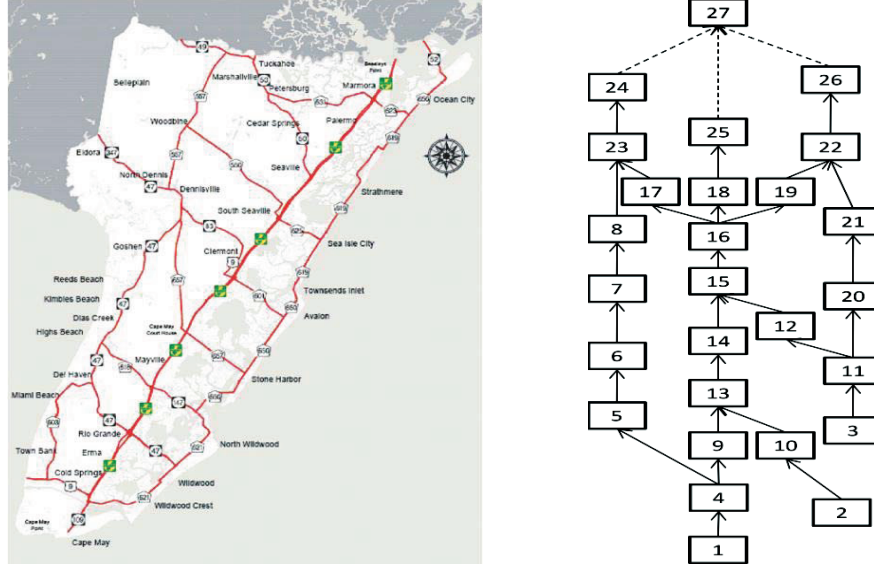


Figure 4 CTM Example (Yazici and Ozbay 2007)

Table 9 objective value – Polyhedral uncertainty

$\theta$	obj		simulation avg			simulation sd			simulation worst		
	DLP	AARC	Ideal	DLP	AARC	Ideal	DLP	AARC	Ideal	DLP	AARC
0.025	360901	370641	366938	387104	368196	1041	4367	598	369062	395998	369472
0.05	360901	379965	373144	413517	375368	2131	8769	1437	377521	431356	378343
0.075	360901	389686	379451	439826	382618	3222	13154	2143	386006	466584	387086
0.1	360901	399438	385697	466134	389101	4306	17538	3307	394492	501812	396030
0.15	360901	419351	398391	518751	403235	6533	26307	5449	411802	572267	414668
0.2	360901	439908	411315	571368	417660	8844	35077	7731	429603	642723	433757
0.25	360901	460738	424475	623985	432095	11218	43846	10066	447677	713178	452758
0.3	360901	511177	437901	676602	456290	13700	52615	17039	467532	783633	497991

The AARC problem with 27 nodes and 45 planning horizon has 4,096,941 constraints and 9,079,890 variables. It is solved in about 4 hours on a PC with Intel processor 3.0 Ghz and 32 GB of memory.

## 6. Conclusion and Future Work

This paper applied the RO methodology to the CTM based SO DTA model under demand uncertainty. In particular, AARC was formulated for dealing with a multi-period transportation problem to find an robust and uncertainty immunized solution, which is especially important in an emergency logistics problem. Two S-shaped curves with upper and lower bound was introduced by considering uncertainty sets, which are appropriate for modeling uncertain demand. With the lin-

**Table 10 AARC vs. SSP when  $\theta$  changes. (Beta(5,2), L=50, M=100)**

	obj		avg		gap		sd		worst	
$\theta$	AARC	SSP	AARC	SSP	AARC	SSP	AARC	SSP	AARC	SSP
0.025	370641	356996	368196	387661	0.34%	6%	598	4363	369472	396562
0.05	379965	353143	375368	414563	0.60%	11%	1437	8761	378343	432436
0.075	389686	349344	382618	441409	0.84%	16%	2143	13141	387086	468218
0.1	399438	345584	389101	468254	0.88%	21%	3307	17522	396030	503999
0.15	419351	338247	403235	521959	1.22%	31%	5449	26282	414668	575578
0.2	439908	331121	417660	575665	1.54%	40%	7731	35043	433757	647151
0.25	460738	324304	432095	629294	1.79%	48%	10066	43805	452758	718651
0.3	511177	317763	456290	682927	4.20%	56%	17039	52565	497991	709139

ear decision rule for an approximated solution and the appropriate reformulation technique, AARC becomes a linear programming problem and hence computationally tractable. The objective value obtained is guaranteed upper bound within a prescribed uncertainty set. Although the AARC solution does not guarantee optimality, we find that the AARC approach leads to high quality solutions compared to the deterministic problem and the sampling based stochastic problem.

However, we do not argue that AARC approach always outperforms the stochastic programming. The proposed AARC method is favorable when either reliable information on probability distribution of uncertain parameter is not available or decision makers want to find a strongly guaranteed performance without facing infeasible solution even in extreme case. In those cases, RO can outperform the traditional stochastic programming approach. Also, the purpose of RO is quite different from sensitivity analysis with variation of parameters. RO finds uncertainty immunized solution for pre-described uncertainty set, while sensitivity analysis is a post-optimization tool to test the stability or perturbation of optimal solution ([Ben-Tal and Nemirovski 2000](#)).

Our work has focused on the CTM based SO-DTA problem by using affine control rule for uncertain demand. The reason for using the linear decision rule is to derive computational tractable problem. However, theoretically, we do not know how the approximation makes the robust solution be deviated from the optimal solution. The approximation approach is used based on the belief that it is important to provide a solvable problem in emergency logistics field ([Shapiro and Nemirovski \(2005\)](#), Remark 2). The scope of future work could be extended to consider control beyond linear decision rule and to explore large scale examples. Moreover, robust optimization approach can

be applied to different uncertainty sources (e.g. capacity uncertainty or cost uncertainty) and alternative transportation problems like dynamic network design.

There are other issues raised from this paper. One of these issues is that LP based CTM model allows vehicle holding, which may be unrealistic. RO approach can be applied to alternative deterministic mathematical formulations (e.g. Nie (2010)) to overcome this issue. Extension to considering unbounded uncertainty set with globalized robust optimization (Ben-Tal et al. 2006) is another interesting research direction.

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## References

- Atamturk, A., M. Zhang. 2007. Two-stage robust network flow and design under demand uncertainty. *Operations Research-Baltimore* **55**(4) 662–673.
- Ben-Tal, A., S. Boyd, A. Nemirovski. 2006. Extending scope of robust optimization: Comprehensive robust counterparts of uncertain problems. *Mathematical Programming* **107**(1) 63–89.
- Ben-Tal, A., L. El Ghaoui, A. Nemirovski. 2009. *Robust optimization*. Princeton Univ Pr.
- Ben-Tal, A., A. Goryashko, E. Guslitzer, A. Nemirovski. 2004. Adjustable robust solutions of uncertain linear programs. *Mathematical Programming* **99**(2) 351–376.
- Ben-Tal, A., A. Nemirovski. 1998. Robust convex optimization. *Mathematics of Operations Research* **23**(4) 769–805.
- Ben-Tal, A., A. Nemirovski. 1999. Robust solutions of uncertain linear programs. *Operations Research Letters* **25**(1) 1–14.
- Ben-Tal, A., A. Nemirovski. 2000. Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming* **88**(3) 411–424.
- Ben-Tal, A., A. Nemirovski. 2002. Robust optimization—methodology and applications. *Mathematical Programming* **92**(3) 453–480.

- 
- Bertsimas, D., D.B. Brown, C. Caramanis. 2007. Theory and applications of Robust Optimization. *Submitted*. Available at [users.ece.utexas.edu/~cmcaram/pubs/RobustOptimizationLV.pdf](http://users.ece.utexas.edu/~cmcaram/pubs/RobustOptimizationLV.pdf).
- Bertsimas, D., G. Perakis. 2005. Robust and adaptive optimization: A tractable approach to optimization under uncertainty. *NSF/CMMI/OR 0556106*.
- Bertsimas, D., M. Sim. 2003. Robust discrete optimization and network flows. *Mathematical Programming* **98**(1) 49–71.
- Bertsimas, D., M. Sim. 2004. The price of robustness. *Operations Research* **52**(1) 35–53.
- Chiu, Y.C., H. Zheng, J. Villalobos, B. Gautam. 2007. Modeling no-notice mass evacuation using a dynamic traffic flow optimization model. *IIE Transactions* **39**(1) 83–94.
- Daganzo, C.F. 1993. The cell transmission model. part I: A simple dynamic representation of highway traffic. *Transportation Research B* **28**(2) 269–287.
- Daganzo, C.F. 1995. The cell transmission model, part II: Network traffic. *Transportation Research Part B* **29**(2) 79–93.
- El Ghaoui, L., M. Oks, F. Oustry. 2003. Worst-case value-at-risk and robust portfolio optimization: A conic programming approach. *Operations Research* **51**(4) 543–556.
- El Ghaoui, L., F. Oustry, M. AitRami. 1997. A cone complementarity linearization algorithm for static output-feedback and related problems. *IEEE Transactions on Automatic Control* **42**(8) 1171–1176.
- Erera, A.L., J.C. Morales, M. Savelsbergh. 2009. Robust optimization for empty repositioning problems. *Operations Research* **57**(2) 468–483.
- Friesz, T.L., D. Bernstein. 2000. *Handbook of transport modelling*, chap. Analytical dynamic traffic assignment models. Pergamon, 181–195.
- Guha-Sapir, D., G.S. Debarati, D. Hargitt, P. Hoyois, R. Below, D. Brechet. 2004. *Thirty years of natural disasters 1974-2003: The Numbers*. Presses univ. de Louvain.
- Karoonsoontawong, A., S.T. Waller. 2007. Robust dynamic continuous network design problem. *Transportation Research Record: Journal of the Transportation Research Board* **2029**(-1) 58–71.
- Lodree Jr, E.J., S. Taskin. 2008. An insurance risk management framework for disaster relief and supply chain disruption inventory planning. *Journal of the Operational Research Society* **59**(5) 674–684.

- Merchant, D.K., G.L. Nemhauser. 1978a. A model and an algorithm for the dynamic traffic assignment problems. *Transportation Science* **12**(3) 183.
- Merchant, D.K., G.L. Nemhauser. 1978b. Optimality conditions for a dynamic traffic assignment model. *Transportation Science* **12**(3) 200.
- Mudchanatongsuk, S., F. Ordonez, J. Liu. 2008. Robust solutions for network design under transportation cost and demand uncertainty. *Journal of the Operational Research Society* **59**(5) 652–662.
- Mulvey, J.M., R.J. Vanderbei, S.A. Zenios. 1995. Robust optimization of large-scale systems. *Operations Research* **43**(2) 264–281.
- Nie, Y. 2010. A Cell-based Merchant-Nemhauser Model for the System Optimum Dynamic Traffic Assignment Problem. *Transportation Research Board 89th Annual Meeting*.
- Ordóñez, F., J. Zhao. 2007. Robust capacity expansion of network flows. *Networks* **50**(2) 136–145.
- Ozdamar, L., E. Ekinici, B. Kucukyazici. 2004. Emergency logistics planning in natural disasters. *Annals of Operations Research* **129**(1) 217–245.
- Peeta, S., C. Zhou. 1999. Robustness of the off-line a priori stochastic dynamic traffic assignment solution for on-line operations. *Transportation Research Part C* **7**(5) 281–303.
- Peeta, S., A.K. Ziliaskopoulos. 2001. Foundations of dynamic traffic assignment: The past, the present and the future. *Networks and Spatial Economics* **1**(3) 233–265.
- Radwan, A.E., AG Hobeika, D. Sivasailam. 1985. A computer simulation model for rural network evacuation under natural disasters. *ITE journal* **55**(9) 25–30.
- Shapiro, A. 2003. Monte Carlo sampling methods. *Handbooks in Operations Research and Management Science* **10** 353–426.
- Shapiro, A., A. Nemirovski. 2005. *On complexity of stochastic programming problems*. Springer.
- Sheu, J.B. 2007. An emergency logistics distribution approach for quick response to urgent relief demand in disasters. *Transportation Research Part E* **43**(6) 687–709.
- Tuydes, H. 2005. Network traffic management under disaster conditions. Ph.D. thesis, Northwestern University, Evanston, IL.
- Ukkusuri, S.V., S.T. Waller. 2008. Linear programming models for the user and system optimal dynamic



- 
- network design problem: formulations, comparisons and extensions. *Networks and Spatial Economics* **8**(4) 383–406.
- Waller, S.T., J.L. Schofer, A.K. Ziliaskopoulos. 2001. Evaluation with traffic assignment under demand uncertainty. *Transportation Research Record: Journal of the Transportation Research Board* **1771** 69–74.
- Waller, S.T., A.K. Ziliaskopoulos. 2006. A chance-constrained based stochastic dynamic traffic assignment model: Analysis, formulation and solution algorithms. *Transportation Research Part C* **14**(6) 418–427.
- Xie, C., D.Y. Lin, S. Travis Waller. 2010. A dynamic evacuation network optimization problem with lane reversal and crossing elimination strategies. *Transportation Research Part E* **46**(3) 295–316.
- Yazici, M.A., K. Ozbay. 2007. Impact of Probabilistic Road Capacity Constraints on the Spatial Distribution of Hurricane Evacuation Shelter Capacities. *Transportation Research Record: Journal of the Transportation Research Board* **2022** 55–62.
- Yin, Y., S. Lawphongpanich, Y. Lou. 2008. Estimating investment requirement for maintaining and improving highway systems. *Transportation Research Part C* **16**(2) 199–211.
- Yin, Y., S.M. Madanat, X.Y. Lu. 2009. Robust improvement schemes for road networks under demand uncertainty. *European Journal of Operational Research* **198**(2) 470–479.
- Ziliaskopoulos, A.K. 2000. A linear programming model for the single destination system optimum dynamic traffic assignment problem. *Transportation Science* **34**(1) 37–49.