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Robust Optimization for Island Partition of Distribution System Considering Load Forecasting Error

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ABSTRACT The increase in extreme weather affects the distribution system, leading to system collapse and interruption power supply to loads. Island partition is a resilient solution when major faults happen in the distribution system. This paper proposes an island partition model considering load forecasting error. Specifically, a two-stage robust optimization island partition program is formulated to restore as many loads as possible, while satisfying operation and topology constraints. Ellipsoidal uncertainty set is built to describe the uncertainty of load forecasting error and take the temporal correlation of it into account. The two-stage robust optimization problem is transformed into a mixed integer second order cone problem to make it computationally tractable. The formed island can be more reliable and avoid secondary collapse when facing loads fluctuation, and uncertainty budget is set to control the degree of conservatism of the robust optimization result. The numerical results based on IEEE 33-node distribution system demonstrate the effectiveness of the developed model.

INDEX TERMS Distribution system, island partition, two-stage robust program, load forecasting error, resilience.

NOMENCLATURE

A. SETS

υ	Set of nodes.
$\{DG\}$	Set of distributed generations.
ε	Set of branches.
$\pi(j)$	Set of all parents of node j.
$\delta(j)$	Set of all children of node j.
Ω	Ellipsoidal uncertainty set of load forecasting error.
η	Set of nodes in islands

B. PARAMETERS

$\overline{P}_{L,j,t}$	Expected active load demand at node j in time period t.
$S_{DG,j,t}$	Capacity of DG at node j in time period t.
$e_{j,t}$	Proportion of DG active power output to DG capacity at node j in time period t.
$Q_{L,j,t}$	Reactive load demand at node j in time period t.
$S_{ij}^{\max}$	The maximum capacity of branch ij.
r_{ij}	Resistance of branch ij.
x_{ij}	Reactance of branch ij.
M	A large number, which is assigned to 33 in this paper.
U_0	Reference voltage.
C_{conf}	Uncertainty budget.
R_j^{-1}	Covariance matrix of load forecasting error at node j.
α_{RO}	Confidence coefficient of uncertainty set.

C. VARIABLES

$P_{L,j,t}$	Forecasted value of active load demand at node j in time period t.
$\Delta P_{L,j,t}$	Load forecasting error at node j in time period t.
b_j	Binary variables that is equal to 1 if node j is included in an island, being 0 otherwise.
$P_{DG,j,t}$	Active power output of DG at node$j$ in time period t.

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$Q_{DG,j,t}$	Reactive power output of DG at node$j$ in time period t.
$H_{js,t}$	Active power from node j to node s in time period t.
$Q_{SVC,j,t}$	Reactive power output of SVC at node$j$ in time period t.
$G_{js,t}$	Reactive power from node j to node s in time period t.
c_{ij}	Binary variables that is equal to 1 if branch ij is closed, being 0 otherwise.
$U_{j,t}$	Voltage of node j in time period t.
F_{ij}	Fictitious power of branch ij.
H_j	Power supplied by fictitious source at node j in an island.
N	The number of islands.

I. INTRODUCTION

Today's power systems are facing a series of natural disasters, such as hurricanes, floods, thunderstorms and blizzards. Extreme weather may result in financial loss and system collapse. To defense against the catastrophe of system-wide blackout, the resiliency of power system should be thought highly of. As defined in [1], resiliency is "the ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions." Distribution system islanding is scenario-based backup plans designed to reduce the impact of disaster, if any, and recover from disruptions as quickly as possible. IEEE encourages power supplier and users to achieve the islanding operation instead of island ban, so it has gradually become a research highlight [1]–[13].

A lot of researches of islanding problem mainly focused on search methods. A heuristic approach is introduced to solve the post-disturbance microgrid formation problem in [3]. In [4], Li *et al.* applied spanning tree search algorithms to find the candidate restoration strategies by modeling microgrids as virtual feeders and representing the distribution system as a spanning tree. As for the mathematic model of the islanding optimization, the objective functions mainly contain the maximum recovery of the important loads [5]–[7], an optimum amount of load to be shed in a fully distributed manner under large disturbances [13], the minimum of switching operations [4], [8], the number of islands as few as possible [9], and the comprehensive confidence of strategies as much as possible [8], [9].

Many studies take other factors into account because of the development of distribution system and the increased penetration of distribution generations (DGs). Reference [1] proposed a model to deal with multiple-fault scenarios, which consider the stability of microgrid and dynamic performance of DGs during the restoration process. The model developed in [10] is aimed at dealing with radial and meshed topologies of future distribution grids, in which the flows are expected to be undirected. And the model also allowed for possible mobile and fixed DGs. Hou *et al.* [2] pointed that the ac microgrid can operate in grid-connected (GC) mode and island (IS) mode as well as mode transitions, a distributed hierarchical control was applied to these modes. The frequency/voltage recovery and accurate power sharing in IS mode were achieved. Most distribution networks have unbalanced configurations that are not represented in sufficient detail by single-phase models, the work in [11] proposed a microgrid formation plan that adopts a three-phase network model to represent unbalanced distribution networks. Besides, the uncertainty of outage duration [8], the hybrid energy storage system, including the battery and the supercapacitor during islanded operation [12], and a novel distributed coordination load shedding (DCLS) approach using sub-gradient algorithm of multi-agent system [13] are also considered.

Island partition is a strategy based on the load forecasting, the works above all neglect the impact of load forecasting error, so the islands they formatted are exposed to secondary collapse. In this paper, we focus on the forecasting error of load to prevent the secondary collapse. In [3], [6], and [7], the uncertainty of load demands was modelled in microgrid formation, they were all solved as a stochastic mixed-integer linear program. Stochastic programming is the well-known and popular modeling method that deal with data uncertainty. But this approach has some practical limitations in the application to large scale power systems. Firstly, stochastic programming assumes accurate probability distributions of uncertainty, which is difficult to get. Secondly, stochastic programming finds the optimal solution relying on sampling a series of scenarios of the uncertainty realizations. It will take a long time in such a huge power system. Besides, the optimal solution the stochastic programming finds can't guarantee the best performance in any of the worst cases [14].

In order to deal with these limitations, Ben-Tal and Nemirovski [15]–[17] proposed the robust optimization. Robust optimization is an appropriate framework to model optimization problems where the optimal solution must remain feasible for some parameter variations in a given user-defined set (also called "uncertainty set"). According to this definition, robust optimization can also deal with data uncertainty. Different from stochastic programming, it only requires information about uncertainty set of random parameters, such as the mean and the extreme, instead of assuming probability distribution of uncertainty data. And robust optimization searches the best result in the worst case, so the developed model can provide feasible solution for all the scenarios. Because of these advantages, much of research along this vein, including [14], [18], [19], has focused on robust optimization for unit commitment. In [20], Yi *et al.* proposed a multi-objective robust scheduling model considering renewable energy and demand response (DR) uncertainty based on the method, which utilizes multitype DR resources to smooth fluctuations in renewable energy on different timescales. The work in [21] applied robust optimization to AC/DC hybrid microgrids.

The uncertainties associated with the real-time market price signal (buying and selling), renewable power sources and forecasted load values were considered. Robust optimization based optimal DG placement in microgrid under the uncertainties of power output of wind turbines and photovoltaics as well as load consumptions was analyzed in [22].

In this paper, the island partition of distribution system based on the two-stage robust optimization (RO) model is presented, which takes the uncertainty of load forecasting into account. However, two-stage RO model is NP-hard, it is very difficult to compute it. The comparison between a column-and-constraint generation algorithm with existing Benders-style cutting plane methods was proposed in [23]. The number of iterations in the C&CG algorithm was reduced if the second-stage decision problem is an LP, such a reduction is very significant in application. Reference [24] proposed an approach to address data uncertainty for discrete optimization and network flow problems that allows controlling the degree of conservatism of the solution. Bertsimas and Sim [24] discussed two different cases, one was that both the cost coefficients and the data in the constraints of an integer programming problem were subject to uncertainty, the other was that only the cost coefficients were subject to uncertainty, cases were proved to be computationally tractable both practically and theoretically.

This paper firstly builds a mixed integer linear model of island partition, aiming at a maximum coverage of loads. In order to avoid the secondary collapse, a two-stage robust optimization programming taking the uncertainty of load forecasting error into account is proposed. The ellipsoidal uncertainty set is built to describe the error of load forecasting, and the two-stage robust optimization problem is transformed into a mixed integer second order cone problem for its computational tractability. The contributions of the proposed method are:

1) The uncertainty of load forecasting error is considered in the island partition model. In order to describe the temporal correlation of the uncertainty parameters, the covariance matrix is found to build the ellipsoidal uncertainty set.

2) Two-stage robust optimization is applied to the island partition model. The island strategy formulated based on the model can restore the maximum loads in the worst case, so the island can be surely reliable under the fluctuation of loads. Besides, adjustable uncertainty budget is set to prevent the over-conservative of robust optimization result.

3) The two-stage robust optimization problem is transformed into a mixed integer second order cone problem. It can be computationally tractable to solve it.

The remainder of this paper is organized as follows. In Section II the island partition problem to restore loads after major faults is presented. Section III describes the MILP for the island partition optimization. Section IV transforms the island partition problem into a two-stage robust formulation and proposes the solution methodology. In Section V, the computational results are provided. Finally, some relevant conclusions are drawn in Section VI.

II. PROBLEM FORMULATION

Generally speaking, distribution systems are configured as a radial topology. So, in this paper, a radial distribution system is considered with N nodes and L lines, the set of nodes is $\upsilon = \{1, 2, \ldots, N\}$, and each line represented by the set of edges $\varepsilon = \{(i, j)\} \subseteq \upsilon \times \upsilon$. Loads of the distribution system demand for active and reactive power. There are several DGs (photovoltaics and wind turbine generators in this paper) in distribution system, providing both active and reactive power. Static var compensators (SVCs) are also installed to provide reactive power. When a section of the distribution system is under attack and has a breakdown, it should be isolated from the main grid. The downstream of the fault area will no longer be energized. In order to reduce the economic losses, DGs are supposed to restore as many loads as possible and formulate one or several reliable islands before the restoration of the main grid. The assumptions of this paper are proposed as follows:

1) Energy storage systems are installed in distribution system to provide instant power support when a contingency event happens, such as wide fluctuation of DG output or a DG trip, preventing a cascade faults on the grid to make frequency stable. But in normal operation, the island is powered by DGs, so the models proposed in following text do not contain the energy storage system for making it computationally tractable.
2) The penetration of DGs increases rapidly, so the DGs installed in distribution system can provide energy to multiple loads. The islands in the proposed strategies can be one or several, which depends on the constraints they satisfy. Each island contains one or more DGs.
3) Each load can be energized by one island at most, and some of loads can even be discarded and not be restored at the proposed strategies. If one load is restored by two different islands, they will be coalesced as a larger island.
4) Under the situation that the remotely controlled automatic switch devices are widely applied in distribution system, the switches are installed in all lines and nodes. That means, lines can be opened or closed, and loads can also be connected or disconnected to form islands. In particular, a load can be disconnected even the lines transformed energy to it are closed.

III. MODEL FOR ISLAND PARTITION

The objective of island partition is to maximize the sum of loads picked up by DGs before fault removed, while satisfying a series of constraints. In this section, the operation constraints and topology constraints are proposed based on [25], and island partition model is formulated without forecasting error. The problem taking load forecasting error into account and the application of robust optimization will be presented in next section.

A. OBJECTIVE FUNCTION

The objective function is proposed as follows:

$$\max_{b_j} \sum_{j\in\upsilon,t} b_j \overline{P}_{L,j,t} \tag{1}$$

The islands formulated should operate stably during restoration process, so the objective function contains the total loads restored before faults removed.

B. CONSTRAINTS OF DISTRIBUTED GENERATIONS

Loads can be supplied by distributed generations (DGs) after component outages. In this paper, photovoltaics (PVs) and wind turbine generators (WTGs) are regarded as generators to supply power. The combination of PVs and WTGs will supply power more reliable in the long-term operation for wind-photovoltaic complementarity. (2)-(3) refer to the DGs output limitations.

$$P_{DG,j,t} \le e_{j,t} S_{DG,j,t}, \quad j \in \{DG\} \tag{2}$$

$$P^2_{DG,j,t} + Q^2_{DG,j,t} \le S^2_{DG,j,t}, \quad j \in \{DG\} \tag{3}$$

The maximum active power output of DGs is revealed in (2). Reactive power output of DGs is constrained in (3).

C. OPERATION CONSTRAINTS

It gives operation constraints in (4)-(9), which based on the linearized DistFlow model.

$$P_{DG,j,t} - \overline{P}_{L,j,t} * b_j = \sum_{S\in\delta(j)} H_{js,t} - \sum_{i\in\pi(j)} H_{ij,t}, \forall j \in \upsilon \tag{4}$$

$$Q_{DG,j,t} + Q_{SVC,j,t} - Q_{L,j,t} * b_j = \sum_{s\in\delta(j)} G_{js,t} - \sum_{i\in\pi(j)} G_{ij,t}, \quad \forall j \in \upsilon \tag{5}$$

$$-S^{\max}_{ij} c_{ij} \le \sum_{S\in\delta(j)} H_{js,t} - \sum_{i\in\pi(j)} H_{ij,t} \le S^{\max}_{ij} c_{ij}, \quad \forall (i,j) \in \varepsilon \tag{6}$$

$$-S^{\max}_{ij} c_{ij} \le \sum_{s\in\delta(j)} G_{js,t} - \sum_{i\in\pi(j)} G_{ij,t} \le S^{\max}_{ij} c_{ij}, \quad \forall (i,j) \in \varepsilon \tag{7}$$

$$\begin{aligned} &-M(1-c_{ij}) + 2(r_{ij}H_{ij,t} + x_{ij}G_{ij,t})/U_0 \le U_{j,t} - U_{i,t} \\ &\le M(1-c_{ij}) + 2(r_{ij}H_{ij,t} + x_{ij}G_{ij,t})/U_0, \quad \forall (i,j) \in \varepsilon \end{aligned} \tag{8}$$

$$H^2_{ij,t} + G^2_{ij,t} \le S^{\max^2}_{ij} c_{ij}, \forall (i,j) \in \varepsilon \tag{9}$$

Specifically, (4)-(5) imply active and reactive power constraints of each node, as shown in Fig. 1 (take active power as an example), b_j denotes whether load j is connected to islands. (6)-(7) are the input power limits of each node, c_{ij} is the status of line ij, if line ij is closed, the input of node j is limited by the maximum capacity of line ij, otherwise, it equals to zero. (8) gives the DistFlow equation. If line ij is closed, the voltage difference of this line is limited by power flow; otherwise, it is arbitrary. (9) refers to the branch capacity limits.

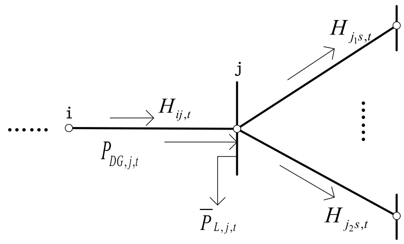

FIGURE 1. DistFlow model for radial distribution network.

D. TOPOLOGY CONSTRAINTS

Different from the transmission systems, the topology of distribution system should be radial. According to [26], the graph is radial when the following two conditions are satisfied: (i) each sub-graph is a connected graph; (ii) the number of branches equals to the number of nodes minus the given number of sub-graphs. And topology constraints are presented as follows. Specifically, (10) and (11) are the virtual flow equation; (12) limits the power injected by fictitious sources. (13) denotes the bound of the fictitious power on each line; (14) limits the number of fictitious network branches.

$$\sum_{s\in\delta(j)} F_{js} - \sum_{i\in\pi(j)} F_{ij} = -b_j, \quad j \in \upsilon \backslash \{DG\} \tag{10}$$

$$\sum_{s\in\delta(j)} F_{js} - \sum_{i\in\pi(j)} F_{ij} = H_j, \quad j \in \{DG\} \tag{11}$$

$$b_j \le H_j \le M * b_j, j \in \{DG\} \tag{12}$$

$$-M * c_{ij} \le F_{ij} \le M * c_{ij}, \quad \forall ij \in \varepsilon \tag{13}$$

$$\sum_{ij\in\varepsilon} c_{ij} = \sum_j b_j - N \tag{14}$$

In order to achieve the connectivity and radiality, a fictitious network with the same topology structure as distribution system is designed. Each island is regarded as a sub-graph, where DGs are chosen as the fictitious sources and all other nodes have unit load demand.

Constraints (10)-(13) guarantee the first condition. The input fictitious power of each node (except the source nodes) equals to 1 if the node is connected to an island, otherwise, it equals to 0. Each source nodes provides fictitious power if it is connected to an island. It means that there is at least one path between source nodes and the other nodes in the fictitious network.

Constraints (14) guarantees the second condition. Assumption in section II mentioned that there will be more than one DGs in an island, so the number of sub-graphs equals to the number of islands. Thus, the number of connected branches equals to the number of connected nodes minus the number of islands.

In general, the model of island partition is formulated as follows, which is a MILP problem

$$\text{Objective Function: (1)} \quad (15\text{-a})$$
$$\text{s.t. DG Constraints: (2)-(3)} \quad (15\text{-b})$$
$$\text{Operation Constraints: (4)-(9)} \quad (15\text{-c})$$
$$\text{Topology Constraints: (10)-(14)} \quad (15\text{-d})$$

IV. TWO-STAGE ROBUST OPTIMIZATION

A. UNCERTAINTY SET OF LOAD FORECASTING ERROR

Actually, it is difficult to forecast the load accurately in existing studies. The load forecasting is described as follows:

$$P_{L,j,t} = \overline{P}_{L,j,t} + \Delta P_{L,j,t} \tag{16}$$

The prediction of load is divided into two parts, load forecasting expectation and load forecasting error, the latter is uncertain. It can be difficult to get the probability distribution of load forecasting error, so the uncertainty set is used to describe it. In robust optimization, uncertainty set often takes the form of a finite discrete set, a polytope, or an ellipsoid. In order to prevent over-conservative of the result and consider temporal correlation of load forecasting error in each node, the ellipsoidal uncertainty set is proposed as follows:

$$\Omega = \{\Delta P_{L,j,t} : \Delta P_{L,j,t}^T R_j^{-1} \Delta P_{L,j,t} \leq C_{conf}\} \tag{17}$$

The expected value of the uncertainty parameter determines the center of the ellipsoid, and it is set to zero in this paper. $\Delta P_{L,j,t}$ represents any vector in the ellipsoid. R_j is the covariance matrix of uncertainty parameters. It is a positive definite matrix, which determines the expansion extent of the ellipsoid from the center in all directions. Ellipsoidal uncertainty set can describe the temporal correlation of load forecasting error at node j using R_j, which can acquire from historical data [27]. C_{conf} is related to the confidence coefficient, which can adjust the size of the uncertainty set.

The degree of conservatism of robust optimization result can be easily controlled by changing the uncertainty budget C_{conf}. The larger C_{conf} is, the more conservative the result will be. C_{conf} can be decided by the confidence coefficient of uncertainty set, and confidence coefficient represents the probability of an uncertain quantity falling into the uncertainty set. In general, $\Delta P_{L,j,t}$ conforms to multivariate normal distribution, the relationship between C_{conf} and confidence coefficient is proposed as follows [28]:

$$C_{conf} = \chi^2_{1-\alpha_{RO}}(24) \tag{18}$$

TABLE 1. Corresponding Values of α_{RO} and C_{conf}.

α_{RO}	C_{conf}
90%	33.20
80%	29.55
60%	25.11
30%	19.94
20%	18.06
10%	15.66

where $\chi^2_{1-\alpha_{RO}}(24)$ represents $1-\alpha_{RO}$ quantile of chi-square distribution with 24 degrees of freedom. Table 1 reveals the value of C_{conf} corresponding to each α_{RO}.

B. TWO-STAGE ROBUST OPTIMIZATION MODEL AND SOLUTION METHODOLOGY

In order to consider load forecasting error, the optimal problem is formulated as a two-stage robust optimization program. The first stage is to optimize the base island partition strategy based on the expected load forecasting, and the second stage is to adjust the result according to the uncertainty set of load forecasting error. The island proposed will stay reliable even loads demand fluctuates. The model proposed in Section III is transformed in this part.

Constraint (4) changes as follows when load forecasting is described as equation (16)

$$P_{DG,j,t} - (\overline{P}_{L,j,t} + \Delta P_{L,j,t}) * b_j = \sum_{S\in\delta(j)} H_{js,t} - \sum_{i\in\pi(j)} H_{ij,t} \tag{19}$$

Constraint (19) is robust because of the uncertainty parameters $\Delta P_{L,j,t}$, it can be written as follows:

$$\overline{P}_{L,j,t} * b_j + \max_{\Delta P_{L,j,t}} \{\Delta P_{L,j,t} * b_j\} = P_{DG,j,t} - (\sum_{S\in\delta(j)} H_{js,t} - \sum_{i\in\pi(j)} H_{ij,t}) \tag{20}$$

Constraint (4) is replaced with constraint (20), so the island partition model in Section III is changed to be a two-stage robust optimization model when considering load forecasting error. The decision variables in each stage are different, so, it is difficult to solve the robust optimization model. For computational tractability, the two-stage model is transformed into the certainty robust counterpart as followings.

Firstly, $\Delta P_{L,j,t}$ in constraint (20) can be replaced with the ellipsoidal uncertainty set. Define that $L = R_j^{1/2}$, ξ is the variable substitution of $\Delta P_{L,j,t}$, Ω can be written equivalently as

$$\Omega = \{\Delta P_{L,j,t} : \Delta P_{L,j,t} = \sqrt{C_{conf}} L_j \xi, \|\xi\|_2 \leq 1\} \tag{21}$$

where $\| \|_2$ is the 2-norm. Thus, constraint (20) can be changed into

$$\overline{P}_{L,j,t} b_j + \max_{\|\xi\|_2\leq 1} \{\sqrt{C_{conf}} L_j b_j \xi\} = P_{DG,j,t} - (\sum_{S\in\delta(j)} H_{js,t} - \sum_{i\in\pi(j)} H_{ij,t}) \tag{22}$$

In constraint (22), the uncertainty budget C_{conf} can be set in advance, covariance matrix R_j can acquire from historical data, and decision variables b_j are decided in the first stage of robust optimization program. Therefore, ξ is the only uncertainty decision variables in the second stage. Constraint (22)

TABLE 2. Load demand of 33 nodes and maximum active power output of all DGs at each time period.

P(MW) / T	$\sum_{j\in\{DG\}} e_{j,t}S_{DG,j,t}$	$\sum_{j\in v}\bar{P}_{L,j,t}$	P(MW) / T	$\sum_{j\in\{DG\}} e_{j,t}S_{DG,j,t}$	$\sum_{j\in v}\bar{P}_{L,j,t}$
T1	7.2	4.8	T13	10.8	8.73
T2	7.2	4.03	T14	8.4	8.96
T3	6	3.93	T15	8.4	8.9
T4	7.2	3.98	T16	8.4	8.56
T5	6	3.93	T17	7.2	9.2
T6	6	3.56	T18	4.8	9.43
T7	7.2	4.01	T19	4.8	9.34
T8	7.2	4.93	T20	4.8	9.43
T9	9.6	5.62	T21	7.2	9.26
T10	8.4	6.65	T22	7.2	9.26
T11	7.2	8.94	T23	7.2	7.8
T12	9.6	8.96	T24	7.2	5.56

TABLE 3. Statuses for nodes and lines in two island partition models.

Models	Connected Nodes	Closed Lines
MILP Model	2,3,4,5,6,7,8,9,15,16, 17,18,21,26,27,32,33	2,3,4,5,6,7,8,15,16,17, 25,26,32,33,34,36
Robust Model	2,3,4,5,6,7,8,9,10,15 16,17,18,19,21	2,3,4,5,6,7,8,9,15,16, 17,18,33,34

can be transformed as follows [29]:

$$\max_{\|\xi\|_2\le 1}\{\sqrt{\left(\sqrt{C_{conf}}L_jb_j\xi\right)^2}\}$$
$$=\sqrt{\left(P_{DG,j,t}-(\sum_{S\in\delta(j)}H_{js,t}-\sum_{i\in\pi(j)}H_{ij,t})-\bar{P}_{L,j,t}b_j\right)^2} \quad (23)$$

$\|\xi\|_2 \le 1$, constraint (23) is transformed into a second order cone robust counterpart as follows:

$$\bar{P}_{L,j,t}b_j+\|\sqrt{C_{conf}}L_jb_j\|_2 = P_{DG,j,t}-(\sum_{S\in\delta(j)}H_{js,t}-\sum_{i\in\pi(j)}H_{ij,t}) \quad (24)$$

Two-stage robust optimization model, which contains the uncertainty parameters, is transformed into deterministic optimization model, and a mixed-integer second order cone robust optimization model is proposed through above transformation.

In general, robust optimization model of island partition considering load forecasting error is formulated as follows:

Objective Function: (1) (25-a)
s.t. DG Constraints: (2)-(3) (25-b)
Operation Constraints: (5)-(9), (24) (25-c)
Topology Constraints: (10)-(14) (25-d)

V. CASE STUDIES

Results from several case studies are presented in this section. The proposed approach has been applied to an IEEE 33-node test system. Two PVs and two WTGs are located at nodes

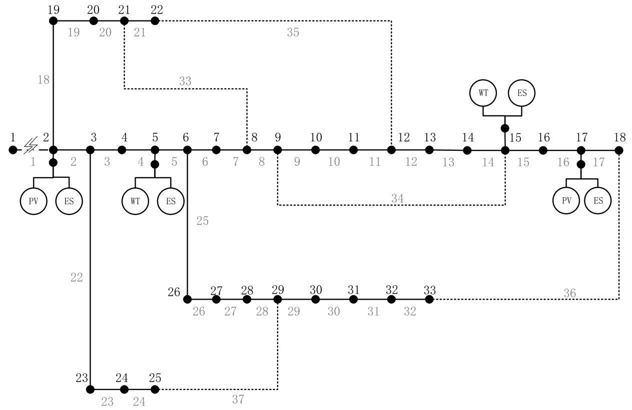

FIGURE 2. IEEE 33-node system with four DGs.

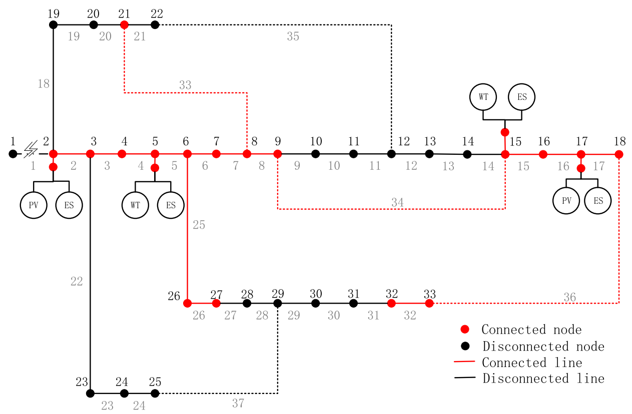

FIGURE 3. Island partition result based on MILP model.

2,17,5,15, respectively. and capacity of these DGs is 6 MW. Energy storage systems are connected to the same nodes as DGs to provide instant power support. Load demand of the whole 33 nodes and maximum active power output of all DGs in different time period are listed in Table 2. The interruption is happened between node 1 and node 2, and the distribution system is disconnected to grid before the fault removed. IEEE 33-node text system with four DGs is shown in Fig. 2. All studies are tested on a PC with an Intel Core i5-8250U CPU@1.6GHz and 8GB RAM. GAMS is used to formulate the models and link the CPLEX solver.

In order to evaluate the effectiveness of the robust optimization model, the MILP model without considering load forecasting error in reference [25] is compared with the model proposed in this paper.

For better comparison, assumptions listed in Section II are applied to both the two models. Island partition strategy based on the MILP model proposed in reference [25] (for ease of description, we use "MILP model" in the following text) is shown in Fig. 3, and Fig. 4 shows the island boundary based on the robust optimization model proposed in Section IV (for ease of description, we use "robust model" in the following text). In Fig. 3, $C_{conf} = 33.20$. Table 3 represents closed lines and connected nodes in two models. Both of the two

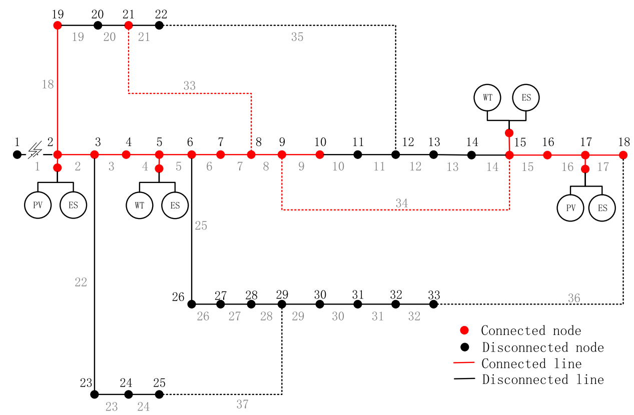

FIGURE 4. Island partition result based on robust model.

TABLE 4. Data for DGs and restored loads in two models at each time period.

$P(MW)$ / T	MILP Model		Robust Model	
Time Period	$\sum_{j\in\{DG\}} P_{DG,j,t}$	$\sum_{j\in\eta} \overline{P}_{L,j,t}$	$\sum_{j\in\{DG\}} P_{DG,j,t}$	$\sum_{j\in\eta} \overline{P}_{L,j,t}$
T1	2.44	2.44	2.251	2.17
T2	2.07	2.07	2.16	1.84
T3	2.00	2.00	2.113	1.79
T4	2.03	2.03	2.112	1.81
T5	2.00	2.00	2.135	1.79
T6	1.83	1.83	1.93	1.63
T7	2.06	2.06	1.902	1.83
T8	2.54	2.54	2.624	2.27
T9	2.86	2.86	3.071	2.56
T10	3.39	3.39	3.486	3.03
T11	4.59	4.59	4.597	4.08
T12	4.59	4.59	4.442	4.09
T13	4.44	4.44	4.032	3.96
T14	4.59	4.59	4.578	4.09
T15	4.57	4.57	4.523	4.07
T16	4.39	4.39	4.339	3.91
T17	4.71	4.71	4.693	4.21
T18	4.80	4.80	4.791	4.28
T19	4.76	4.76	4.324	4.25
T20	4.80	4.80	4.702	4.28
T21	4.74	4.74	4.663	4.24
T22	4.74	4.74	4.628	4.24
T23	3.97	3.97	3.886	3.55
T24	2.86	2.86	2.83	2.55
$\sum_t$	85.77	85.77	84.812	76.52

models can restore loads after faults happened in main grid. But the boundary of two models is different, and it is worth remarking that there are less loads in island when considering load forecasting error.

We assume that islands will operate for 24 hours, it insures the boundary of island will remain unchanged before restoration of the main grid, output of DGs and demand of restored loads at each time period in two different models are contrasted in Table 4. It can be observed that the output of DGs is equal to the demand of restored loads at each time period in MILP model. While the output of DGs is larger than the demand of restored loads in robust model. In the actual situation, load demand often fluctuates, and will be over the expected value sometimes. Therefore, island based

TABLE 5. Data for DGs and restored loads in 24 hours of different uncertainty budget.

$P(MW)$ / C_{conf}	$\sum_{j\in\{DG\},t} P_{DG,j,t}$	$\sum_{j\in\eta,t} \overline{P}_{L,j,t}$	$\Delta = \sum_{j\in\{DG\},t} P_{DG,j,t} - \sum_{j\in\eta,t} \overline{P}_{L,j,t}$
$C_{conf} = 33.20$	84.812	76.52	8.292
$C_{conf} = 29.55$	84.878	77.11	7.768
$C_{conf} = 25.11$	85.008	77.75	7.258
$C_{conf} = 19.94$	85.002	78.42	6.582
$C_{conf} = 18.06$	84.889	78.71	6.179
$C_{conf} = 15.66$	85.338	79.24	6.098

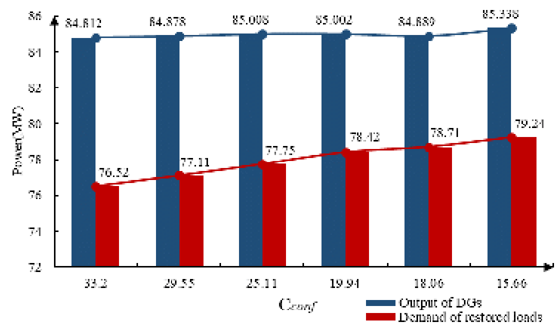

FIGURE 5. Variation trend of DGs output and restored loads demand in 24 hours of different uncertainty budget.

on MILP model will face the secondary collapse, especially in T18 and T20 when the demand of restored loads reaches the maximum active power output of DGs. It means, DGs may not satisfy the demands of restored loads in T18 and T20, and island based on MILP model may not operate normally if load demand fluctuates. Different from it, the demand of restored loads doesn't reach the maximum active power output of DGs at each time period in island based on robust model. It means, island will operate normally even if load demand fluctuates. If the secondary collapse happens in distribution system, it will suffer serious economic losses. By contrast, it is more reliable and economical for island based on robust model, it will operate stably even if some of loads fluctuate.

C_{conf} is defined as the uncertainty budget to control the degree of conservatism of robust model. Total output of DGs and demand of restored loads in 24 hours of different C_{conf} are revealed in Table 5, and Fig. 5 shows variation trend of them. When C_{conf} decreases, the difference value between output of DGs and demand of restored loads decreases. The difference value represents the output margin reserved by DGs for load fluctuation. The smaller the difference value, the less conservative the result will be. Table 6 shows statuses of each line and node of different uncertainty budget.

Islands boundary of each C_{conf} changes in order to satisfy different conservatism. It can be proved that C_{conf} can control the conservatism of two-stage robust optimization result. The over-conservative result will be economically damaging by losing some important loads. Therefore, it is necessary to choose a suitable value of C_{conf}.

TABLE 6. Statuses for nodes and lines of different uncertainty budget.

Cases	Connected Nodes	Closed Lines
$C_{conf} = 33.20$	2,3,4,5,6,7,8,9,10,15 16,17,18,19,21	2,3,4,5,6,7,8,9,15,16, 17,18,33,34
$C_{conf} = 29.55$	2,3,4,5,6,7,8,9,10,14, 15,16,17	2,3,4,5,6,7,8,9,14,15, 16,34
$C_{conf} = 25.11$	2,3,4,5,6,7,8,9,10,15, 16,17,18,21	2,3,4,5,6,7,8,9,15,16, 17,33,34
$C_{conf} = 19.94$	2,3,4,5,6,7,8,9,10,15, 16,17,23	2,3,4,5,6,7,8,9,15,16, 22,34
$C_{conf} = 18.06$	2,3,5,6,7,8,9,14,15,16, 17,18,19,20,21	2,5,6,7,8,14,15,16,17,18, 19,20,33,34
$C_{conf} = 15.66$	2,3,4,5,6,7,8,9,10,15, 16,17,18,33	2,3,4,5,6,7,8,9,15,16, 17,34,36

VI. CONCLUSION

This paper proposed a two-stage robust optimization model for island partition problem, which takes load forecasting error into account. It satisfies operation and topology constraints while aiming at restoring as many loads as possible. Then the two-stage model is transformed into a mixed-integer second order cone robust optimization model for computational tractability. Comparing the island boundary as well as output of DGs and restored load demand of the model proposed in this paper with existing model, it can be proved that the model in this paper is more reliable and can avoid the secondary collapse when loads fluctuate. The degree of conservatism of the robust optimization result can be controlled by adjusting the uncertainty budget. The robust island partition model can be applied to reliability assessment or planning of distribution network in the future work.

This paper only focuses on load forecasting error, actually, there is forecasting error in output of DGs too. And this paper assumes that the priority of loads in distribution system are all the same. Future work should further consider the forecasting error of DGs output as well as take different priority of loads into account to improve the reliability of critical loads. It is mentioned at the end of Section V that it is necessary to choose a suitable value of C_{conf}. However, this paper doesn't concentrate on it. Choosing a suitable uncertainty budget to balance the economy and reliability of the island operation will be an issue deserved to study in the future work.

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