Hindawi
Shock and Vibration
Volume 2019, Article ID 9273714, 9 pages
https://doi.org/10.1155/2019/9273714

Research Article

Robust Optimum Design of Multiple Tuned Mass Dampers for Vibration Control in Buildings Subjected to Seismic Excitation

Luciara Silva Vellar,[1] **Sergio Pastor Ontiveros-Pérez**,[1] **Letícia Fleck Fadel Miguel**,[1] **and Leandro Fleck Fadel Miguel**[2]

[1]*Department of Mechanical Engineering, Federal University of Rio Grande do Sul, Porto Alegre, Brazil*
[2]*Department of Civil Engineering, Federal University of Santa Catarina, Florianópolis, Brazil*

Correspondence should be addressed to Sergio Pastor Ontiveros-Pérez; sergio.ontiveros@ufrgs.br

Received 31 August 2018; Revised 5 December 2018; Accepted 30 December 2018; Published 29 January 2019

Academic Editor: Gabriele Cazzulani

Passive energy devices are well known due to their performance for vibration control in buildings subjected to dynamic excitations. Tuned mass damper (TMD) is one of the oldest passive devices, and it has been very much used for vibration control in buildings around the world. However, the best parameters in terms of stiffness and damping and the best position of the TMD to be installed in the structure are an area that has been studied in recent years, seeking optimal designs of such device for attenuation of structural dynamic response. Thus, in this work, a new methodology for simultaneous optimization of parameters and positions of multiple tuned mass dampers (MTMDs) in buildings subjected to earthquakes is proposed. It is important to highlight that the proposed optimization methodology considers uncertainties present in the structural parameters, in the dynamic load, and also in the MTMD design with the aim of obtaining a robust design; that is, a MTMD design that is not sensitive to the variations of the parameters involved in the dynamic behavior of the structure. For illustration purposes, the proposed methodology is applied in a 10-story building, confirming its effectiveness. Thus, it is believed that the proposed methodology can be used as a promising tool for MTMD design.

1. Introduction

The development of damping devices dates back to the beginning of the twentieth century when Hermann Frahm invented a device for damping vibrations in bodies, which was patented, as presented by Frahm [1].

Recently, a rapid increase in the development and application of passive energy dissipation devices, such as base isolation systems [2], viscoelastic dampers [3, 4], friction dampers [5–15], and tuned mass dampers [16–47], has occurred. Passive control systems are designed to minimize the structural response under dynamic action without using an external power source. Therefore, there are several advantages over active and semiactive systems, such as low installation and maintenance costs and large capacity to reduce vibration amplitudes, among others. The TMDs are divided into four categories: conventional TMD, pendular TMD, bidirectional and homogeneous TMD (BH-TMD) [48], and tuned liquid column dampers (TLCDs) [49].

The TMD considered in this paper is a conventional one, which is a passive control device consisting of a mass, a spring, and a viscous damper attached to a vibrating system to reduce undesirable vibrations. Due to its performance to reduce the response of structures to harmonic or random excitations, a large number of TMDs has been installed in high-rise buildings to reduce wind-induced vibrations, such as the 244 m high John Hancock Tower in Boston with a TMD consisting of two 270,000 kg lead and steel blocks, the 280 m high Citicorp Center Office Building in New York, with a TMD using a 360,000 kg concrete block, and the Terrace on the Park Building in New York City, in which a TMD was installed to reduce the vibration induced by dancing [50].

A single TMD performs well in reducing the dynamic response of a structure under external excitation when the device is tuned to the first vibration mode of the structure [22]. However, this is a disadvantage because the device has low performance controlling the response of the upper vibration modes of the structure. A simple solution to overcome these shortcomings is the installation of multiple tuned mass dampers (MTMDs) in the structure.

The performance of MTMD depends on its parameters such as mass, stiffness, and damping. However, determining the number of devices to be installed and the best position in the structure, as well as optimum parameters in terms of spring stiffness and damping constant for each TMD, is a problem of great interest to the engineer designer.

In order to solve the problem mentioned above, optimization algorithms are used to minimize an objective function and to find an optimal solution of the problem. On the other hand, it is well known that in a dynamic engineering problem, there are a high number of uncertainties involved. This leads to represent these uncertainties through probability distribution functions and involve them in the optimization process of passive dampers. Thus, the optimization process becomes more complex, and it is necessary to implement an optimization methodology capable of dealing with dynamic problems that involve uncertainties in the structural properties, in the MTMD properties, and in the seismic load.

Thus, this work presents a methodology of optimization under uncertainty to determine the optimal parameters of MTMD and its best positions in a single stage, i.e., simultaneously, in buildings subjected to earthquakes, with the aim of improving dynamic structural response in terms of minimizing maximum interstory drift. It is interesting to highlight that the optimization problem proposed in the present work is complex because (i) it is a problem of optimization of a dynamic system that involves uncertainties, (ii) it is a mixed-variable optimization problem, i.e., that involves discrete (position of each TMD) and continuous (parameters of each TMD) variables at same time, and (iii) its objective function is not convex.

Consequently, the problem of optimization under uncertainty of MTMD proposed in this work must be solved with the help of optimization methods able to deal with the complexity of this problem. In this case, the most appropriate is the implementation of a metaheuristic optimization technique, and some of its most important advantages are follows: (i) they do not require gradient information, (ii) they are not trapped in local minimums if they are adjusted correctly, (iii) they can be applied to nonconvex or discontinuous objective functions, (iv) they provide a set of optimal solutions, and (v) they can be implemented to solve optimization problems of mixed variables [51, 52].

Among the heuristic algorithms, the Search Group Algorithm (SGA), recently proposed by the last author of this paper [53], has shown to be very efficient and consequently was selected to solve the optimization problem proposed in the present work.

2. Proposed Methodology

This section presents the methodology proposed for the simultaneous optimization of MTMD taking into account the uncertainties. It presented the equations and procedures adopted to the problem formulation.

2.1. Structural Model. The motion equation of a multi-degree-of-freedom building with MTMD possibly located in all floors of the structure (Figure 1) and subjected to earthquakes can be written as follows:

$$[M]\,\overrightarrow{\ddot{z}}\,(t) + [C]\,\overrightarrow{\dot{z}}\,(t) + [K]\,\overrightarrow{z}\,(t) = -[M]\,\overrightarrow{\ddot{x}}_{\mathrm{g}}(t), \tag{1}$$

where $[M]$, $[C]$, and $[K]$ represent the mass, damping, and stiffness matrices, respectively; $\overrightarrow{z}\,(t)$ is the relative displacement vector with respect to the base and a dot over this symbol indicates differentiation with respect to time, that is, $\overrightarrow{\dot{z}}\,(t)$ and $\overrightarrow{\ddot{z}}\,(t)$ are the velocity and acceleration vectors, respectively. $\overrightarrow{\ddot{x}}_{\mathrm{g}}\,(t)$ is the vector that represents the base acceleration.

The TMDs contribution to $[K]$ is illustrated in equation (2). The procedure is analogous for the damping matrix. On the other hand, the mass matrix is diagonal $[M] = \mathrm{diag}[\,M \;\; M_{\mathrm{TMD}}\,]$, and each damper mass ($M_{\mathrm{TMD}}$) occupies a position in the principal diagonal:

$$[K] = \begin{bmatrix} k_1 + k_2 + k_{\mathrm{TMD1}} & -k_2 & \cdots & 0 & -k_{\mathrm{TMD1}} & 0 & \cdots & 0 \\ -k_2 & k_2 + k_3 + k_{\mathrm{TMD2}} & \cdots & 0 & 0 & -k_{\mathrm{TMD2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & k_n + k_{\mathrm{TMD}n} & 0 & 0 & \cdots & -k_{\mathrm{TMD}n} \\ -k_{\mathrm{TMD1}} & 0 & \cdots & 0 & k_{\mathrm{TMD1}} & 0 & \cdots & 0 \\ 0 & -k_{\mathrm{TMD2}} & \cdots & 0 & 0 & k_{\mathrm{TMD2}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -k_{\mathrm{TMD}n} & 0 & 0 & \cdots & k_{\mathrm{TMD}n} \end{bmatrix}. \tag{2}$$

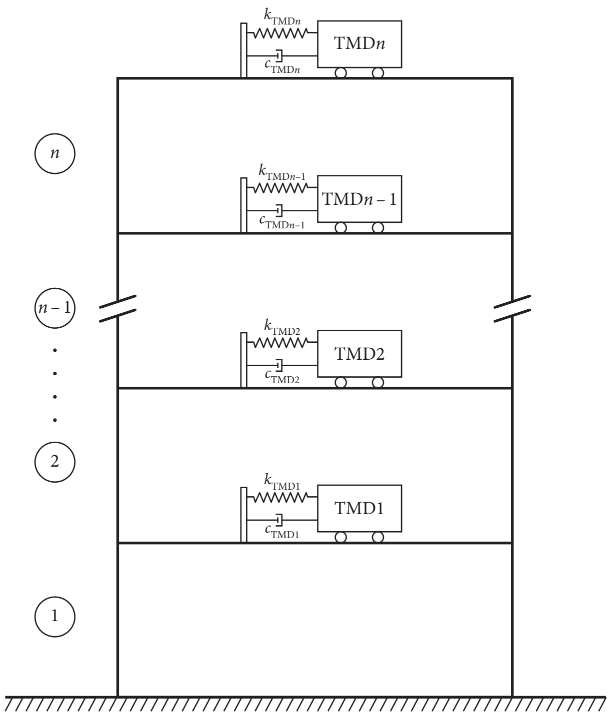

Figure 1: n-degree-of-freedom building with N tuned mass dampers vertically distributed along the building (adapted from Fadel Miguel et al. [17]).

To solve equation (1), the authors developed a computational routine based on the Newmark implicit method, which is a direct method of integration of the motion equations in the time domain [54].

2.2. Random Parameters of the Building. In this work, it is adopted the parametric probabilistic approach to model uncertainties. This methodology is similar to the used by the authors in [11], for friction dampers. Mass, stiffness, and damping of the shear building are assumed to be random variables. As these random variables cannot assume negative values, due to physical aspects, these three stochastic variables are modeled as uncorrelated random variables with Lognormal distribution, with known mean and coefficient of variation. In consequence, in each run of the subroutine, the structure presents different parameters. As the response of the building depends on these random variables, it also becomes random.

In addition, to consider uncertainties in the installed MTMD, their parameters of spring stiffness and damping constant are also assumed to be independent Lognormal random variables with known coefficients of variation and mean values given by the design variables.

2.3. Simulation of Random Seismic Excitations. It is necessary to define the seismic loading to solve equation (1). Hence, in this work, the seismic load is defined as a one-dimensional earthquake loading that is simulated by passing a Gaussian white noise process through the Kanai–Tajimi filter [55, 56] with power spectral density (PSD) function given by the following equation:

$$S(\omega) = S_0 \left[\frac{\omega_g^4 + 4\omega_g^2 \xi_g^2 \omega^2}{\left(\omega^2 - \omega_g^2\right)^2 + 4\omega_g^2 \xi_g^2 \omega^2} \right],$$
$$S_0 = \frac{0.03\xi_g}{\pi \omega_g \left(4\xi_g^2 + 1\right)}, \tag{3}$$

where S_0 is a constant spectral density, related to the peak ground acceleration (PGA), and ω_g and ξ_g are the ground frequency and damping, respectively.

Nevertheless, the optimal solution possibly will be different if the ground parameters of the Kanai–Tajimi spectrum are altered. Therefore, uncertainties in the ground excitation should be considered. Thus, to take into account the random nature of the dynamic excitation, the ground frequency ω_g, the ground damping ξ_g, and the PGA are assumed to be independent Lognormal variables with known mean and coefficients of variation. Consequently, in each run of the subroutine, a different earthquake time history is generated.

2.4. Robust Optimization Problem. In this paper, the objective function used to evaluate the effectiveness of MTMD installed in buildings under seismic excitation is the expected value of the maximum interstory drift $E[D_{\max}]$, which is obtained by solving equation (1) in the time domain through the vector $\vec{z}(t)$.

The design variables are the MTMD parameters, i.e., spring and damping constants, considered as continuous design variables, and the positions in the structure of the MTMD, considered as discrete design variables.

Therefore, given the possible positions (npTMD) in the vector $\vec{P}$ for the maximum number of devices (nTMD) to be installed in the structure, it is of interest to determine the optimum position and optimal parameters (spring and damping constants) of each TMD to minimize the expected value of the maximum interstory drift. The design variables are grouped into the vector $\vec{y} = [\vec{P}, E[k_{\text{TMD}}], E[c_{\text{TMD}}]]$. Thus, the optimization problem can be placed as follows:

$$\begin{aligned}
&\text{Find} && \vec{y}, \\
&\text{Minimizes} && J(\vec{y}) = E\left[D_{\max}(\vec{y})\right], \\
&\text{Subject to} && \begin{cases} k_{\text{TMD}}^{\min} \le E[k_{\text{TMD}}] \le k_{\text{TMD}}^{\max}, \\ c_{\text{TMD}}^{\min} \le E[c_{\text{TMD}}] \le c_{\text{TMD}}^{\max}, \\ \text{number of available positions} = np\text{TMD}, \\ \text{maximum number of dampers} = n\text{TMD}. \end{cases}
\end{aligned} \tag{4}$$

This optimization problem may be solved through the Search Group Algorithm summarized in the next section.

3. Search Group Algorithm (SGA)

As explained previously, the optimization problem presented in this work is complex, involving uncertainties and mixed variables and not convex objective function. Therefore, this sort of optimization problem must be solved by methods capable of handling these characteristics. Within optimization methods, heuristics techniques are best suited to solve such optimization problems.

The Search Group Algorithm (SGA), developed by the last author of this paper in 2015 [53], has shown to be accurate and efficient among several heuristic algorithms. Due to its characteristics, the SGA was chosen for solving the MTMD optimization problem proposed in this work. A brief explanation of the SGA is presented below.

The SGA has a good balance between the exploration (the search of promising regions on the domain at the first iterations of the optimization process) and exploitation (the algorithm refines the best design in each of these promising regions at each iteration).

The first step in the optimization process is the random generation of the initial population PP on the search domain; the second step is the objective function evaluation for each individual of the PP population, and after that, the search group R is constructed by selecting n_g individuals from PP applying a standard tournament selection; the mutation of the search group is the third step and it consists in replacing n_{mut} individuals from R by new individuals away from the current position, generated based on the statistics of the current search group, and the probability of a member to be replaced depends on its rank in the current search group, i.e., the worse the design is, the more likely it is to be replaced; in the fourth step, each member of the search group generates a family, this is, the set comprised by each member of the search group and the individuals that it generated, in which the number of individuals that each member of the search group generates depends on the quality of its objective function; finally, when the iteration number is higher than $\mathrm{it}_{\mathrm{global}}^{\max}$, the selection scheme is modified: the new search group is formed by the best n_g individuals among all the families. This phase is called local because the algorithm will tend to exploit the region of the current best design.

For more details about the SGA, refer [53]. It is interesting to highlight that the authors have made available for implementation the MATLAB codes of SGA for free download on the MathWorks site.

4. Results and Discussions

4.1. Simulated Structure. To illustrate the effectiveness of the proposed method in optimum design of MTMD, as well as to evaluate the capacity of MTMD in improving the performance of structures under seismic excitation, a 10-story building, modeled as shear building (Figure 1) is studied.

4.2. Random Parameters of the Building and Excitation. As explained previously in Sections 2.2 and 2.3, the structure parameters, such as mass, stiffness, and damping, the MTMD parameters, as spring and damping constants and also the seismic load parameters, such as PGA, ground frequency, and ground damping, are all modeled as uncorrelated random variables with Lognormal distribution, with known coefficients of variation and mean values. These mean values and coefficients of variation of each parameter are presented in Table 1.

Table 1: Mean value and coefficient of variation of each input random variable.

Random variable	Mean value	Coefficient of variation (%)
Mass per story	360 t	5
Stiffness per story	650 MN/m	5
Damping per story	6.2 MNs/m	5
Spring constant for each TMD	Design variable	5
Damping constant for each TMD	Design variable	5
PGA	0.475 g	10
ω_g	18 rad/s	10
ξ_g	0.6	10

4.3. Latin Hypercube Sampling (LHS). This work proposes a methodology for robust optimization of MTMD installed in structures subjected to artificial seismic excitation taking into account the uncertainties present in both structure and excitation, in order to minimize the expected value of the maximum interstory drift. Thus, as explained previously, many parameters are modeled as random variables.

In this context, in order to reduce computational cost, the Latin hypercube sampling (LHS) is used, which provides an efficient way of generating variables from their multivariate distributions, taking samples from equally probable intervals [57, 58]. The scheme developed by McKay et al. [59] selects different values of a random variable as follows: the domain of the random variable is divided into n nonoverlapping intervals of equal probability. A value of each interval is chosen randomly with respect to the probability density in the interval. The choice must be made in a random manner with respect to the density in each interval; i.e., the selection should reflect the height of the density across the range. For more information about the LHS, refer [59, 60].

4.4. Robust Optimization of MTMD. The robust design optimization of MTMD in order to minimize the expected value of the maximum interstory drift $E[D_{\max}]$ is developed in this section. As previously explained, the objective function requires the determination of vector $\overrightarrow{z}(t)$ which is obtained by solving equation (1). For this, a computational routine was developed based on the Newmark method.

Considering the 10-story building, modeled as shear building, i.e., 10 degree of freedom, there are ten possible locations to install a maximum of ten TMD (one for each story). Thus, constraints are the number of possible locations for the dampers ($np\mathrm{TMD} = 10$), the maximum number of devices to be installed in the structure ($n\mathrm{TMD} = 10$), the allowed limit for the expected value of the spring constant of each device ($5\ \mathrm{kN/m} \le E[k_{\mathrm{TMD}}] \le 5000\ \mathrm{kN/m}$), and the allowed limit for the expected value of the damping constant of each device ($1\ \mathrm{kNs/m} \le E[c_{\mathrm{TMD}}] \le 1000\ \mathrm{kNs/m}$). Positions and parameters (stiffness and damping constants) of the MTMD are discrete and continuous design variables, respectively. The total mass of the MTMD is assumed to be 3% of the total mass of the building. The random earthquakes are

Table 2: Robust design of MTMD.

Run	Positions $\overrightarrow{P}$	$E[k_{TMD}]$ (kN/m)	$E[c_{TMD}]$ (kNs/m)	$E[D_{max}]$ (m)
		Uncontrolled structure		0.03941
1	[0000000111]	1313.857; 915.187; 1468.914	43.358; 200.407; 11.058	0.01588
2	[0000000111]	1439.044; 914.560; 1426.387	43.400; 205.440; 11.314	0.01595

simulated through the Kanai–Tajimi spectrum, for a duration time of 20 s. The integration step is 0.02 s.

Regarding the parameters of the SGA, it is considered that the population $n_{pop} = 100$ individuals, the number of iterations $it^{max} = 100$, the percentage of it^{max} dedicated to the global phase is 30%, and the percentage of n_{pop} that makes up the search group is 30% of n_{pop}. Thus, two independent runs were performed, and the results are presented in Table 2.

Table 2 shows that the expected value of D_{max} in the two independent simulations is practically the same (around 1.59 cm), indicating that the proposed methodology is robust. It is also interesting to note that the optimal positions obtained in the two independent simulations are the same, as indicated in $\overrightarrow{P}$; these positions are the eighth, ninth, and tenth stories, and the expected values of the spring and damping constants are also similar in the two simulations. For purposes of illustration, Table 3 presents the statistical moments of D_{max} for the case of an uncontrolled structure and for the case of the structure equipped with the robust design of MTMD shown in Table 2.

The percentage reduction of the expected value and the variance of D_{max} obtained after the installation of the three optimized TMDs are also shown in Table 3, resulting in values greater than 59% for the expected value and for the variance.

In addition, Figure 2 shows the frequency diagrams (unit area histograms) constructed with the D_{max} observations for the case of the uncontrolled structure (red histogram) and for the case of the controlled structure (blue histogram) and with the help of the statistical moments shown in Table 2 was possible to adjust a Lognormal probability density function to the two histograms representing the random variable D_{max} for the case of the uncontrolled structure (red curve) and for the case of the structure equipped with the robust design of MTMD (blue curve).

Looking at Figure 2, it is interesting to note how the blue curve is slenderer compared to the red curve due to the reduced D_{max} variance after installing the robust design of MTMD. This demonstrates the performance of the methodology, since even though it is a robust optimization methodology of a single objective function, the capacity in terms of reduction of a second parameter (in this case the variance) was satisfactory.

To demonstrate the effectiveness of the proposed method in different ways, the optimum solution obtained in simulation 1 of Table 2 is compared to the response of the structure analyzed with five alternative methods (Table 4). Alternative 1 is to locate the optimized MTMD (robust solution of simulation 1) at others positions than optimal locations, however keeping the same parameters, that is, the same spring constant k_{TMD} and damping constant c_{TMD}. Alternative 2 is to add one TMD in each story, totalizing 10 TMDs, however keeping the same total spring and damping constants. Alternative 3 is to add just one TMD at the top, keeping the same total spring and damping constants. Alternative 4 is to perform a robust optimization of the mechanical parameters ($E[k_{TMD}]$ and $E[c_{TMD}]$) using the methodology proposed in this paper, however considering a single tuned mass damper located at the top of the structure, using the SGA with a population $n_{pop} = 100$ individuals and the number of iterations $it^{max} = 100$, i.e., the same SGA parameters of the robust design of MTMD. Finally, alternative 5 is to perform the robust optimization of MTMD using Genetic Algorithm with the same parameters in terms of population and iterations utilized in the robust design and in alternative 4. It is important to note that the total TMD mass is the same in all cases, equal to 3% of the total building mass.

As can be seen in Table 4, the objective function ($E[D_{max}]$) obtained with the alternative method 1 is 15.11%

Table 3: Statistical moments of maximum interstory drift.

Uncontrolled	Robust design	Reduction (%)
$E[D_{max}]$ (m)		
0.03941	0.01588	59.71
$\text{var}[D_{max}]$ (m^2)		
$2.3971E-5$	$4.7513-6$	80.17

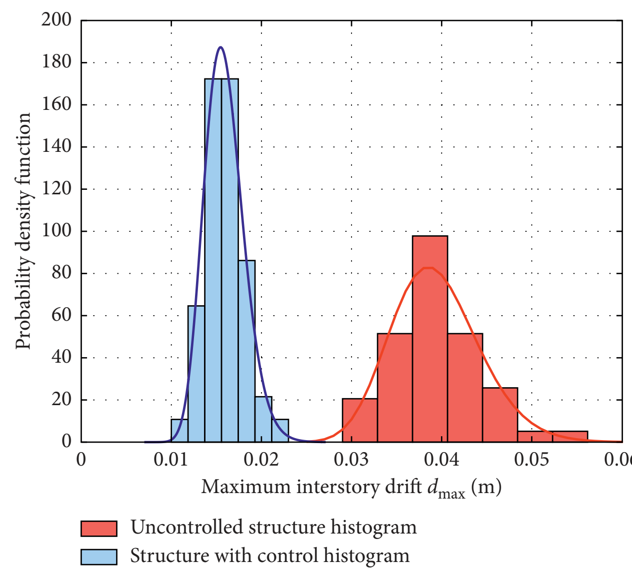

Figure 2: Probability density function of maximum interstory drift d_{max} for uncontrolled structure (red curve) and with control (blue curve).

Table 4: Comparison between robust design and alternative methods.

Method	Positions $\overrightarrow{P}$	$E[k_{\mathrm{TMD}}]$ (kN/m)	$E[c_{\mathrm{TMD}}]$ (kNs/m)	$E[D_{\max}]$ (m)
Robust design	[0000000111]	1313.857; 915.187; 1468.914	43.358; 200.407; 11.058	0.01588
Alternative 1	[0010010010]	1313.857; 915.187; 1468.914	43.358; 200.407; 11.058	0.01828
Alternative 2	[1111111111]	369.796 for each one of the 10 TMDs	25.482 for each one of the 10 TMDs	0.02712
Alternative 3	[0000000001]	3697.958	254.823	0.02121
Alternative 4	[0000000001]	4296.981	115.874	0.01617
Alternative 5	[0000000111]	1419.331; 1448.020; 1447.742	39.478; 312.519; 9.683	0.01603

greater than that obtained with the proposed method (robust design). Additionally, $E[D_{\max}]$ obtained with the alternative methods 2, 3, 4, and 5 is 70.78%, 33.56%, 1.83%, and 0.94%, respectively, greater than the value obtained in the proposed robust design. Therefore, the proposed methodology achieves better results than all the tested alternative methods. The second and third best results were obtained with alternative methods 5 and 4, respectively; it is important to note that these two alternative methods (4 and 5) perform a robust optimization following the proposed methodology, only changing the SGA by GA (in the case of alternative method 5) and fixing only 1 TMD at the top and optimizing its parameters with the proposed methodology for the case of the alternative method 4. It is interesting to note that alternative method 5, despite reaching values close to those obtained with SGA, required a higher computational time (for the same population size and iteration number). In addition, alternative method 4, which considers only 1 TMD at the top, despite achieving results close to those obtained with MTMD, has the disadvantage of controlling only the first mode, and it concentrates all the additional mass of the TMD at the top of the structure, whereas MTMD are able to control more vibration modes, and they distribute the total mass of the TMD according to the number of TMDs.

For purposes of illustration, considering only the expected value of the structural properties, that is, coefficient of variation equal to zero for all parameters, and a seismic excitation generated using the expected value of the parameters and assuming that the coefficient of variation is zero, Figure 3 shows the maximum interstory drift before and after the installation of the MTMD robust design.

Next, in Table 5, the maximum interstory drift per floor before and after the installation of the robust design is presented, evidencing the effectiveness of the MTMD, reaching reductions between 45% and 64%.

Thus, as can be seen in Table 5, the greatest maximum interstory drift is at the first story. Therefore, in order to observe the behavior of the structure in terms of the relative displacement between the first floor and the ground, Figure 4 shows the structural response over the duration of the earthquake and Figure 5 shows the displacement at the top of the building, for the uncontrolled and controlled structure.

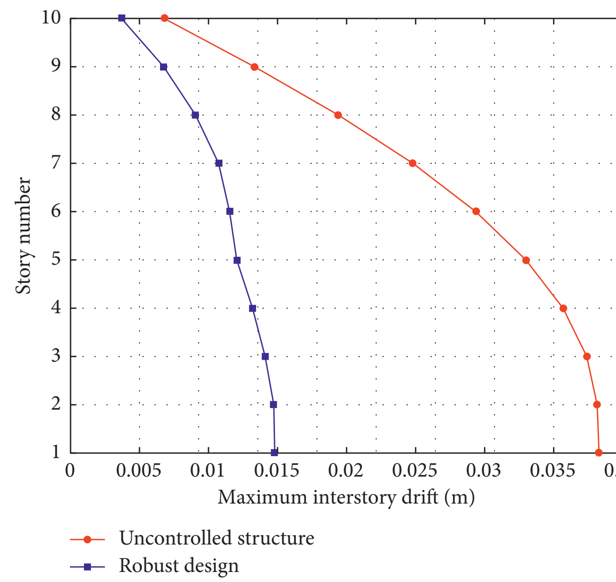

Figure 3: Maximum interstory drift per story for uncontrolled structure (red curve) and controlled structure (blue curve), for coefficient of variation equal to zero for all parameters.

Table 5: Comparison between maximum interstory drift.

Story	Uncontrolled structure (m)	With control (m)	Reduction (%)
1	0.0383	0.0148	61.35
2	0.0381	0.0147	61.39
3	0.0374	0.0141	62.30
4	0.0357	0.0132	63.05
5	0.0330	0.0120	63.50
6	0.0294	0.0115	60.80
7	0.0248	0.0107	56.74
8	0.0194	0.0091	53.29
9	0.0133	0.0068	49.32
10	0.0068	0.0037	45.65

5. Conclusions

It is well known that passive dampers increase the energy dissipation capacity in buildings. Thus, in recent years, engineers have been concerned with the optimal implementation of passive energy dissipation devices and among the most used passive devices is the TMD.

However, until nowadays, several research works have not considered the uncertainties present in the structure and in the parameters of the device. For this reason, the main contribution of this research is a methodology that provides an optimal and robust design of multiple tuned mass dampers (MTMDs). The methodology developed considers the uncertainties in the mechanical properties of the structure, in the mechanical properties of the MTMD, and also in the properties used for the generation of artificial earthquakes.

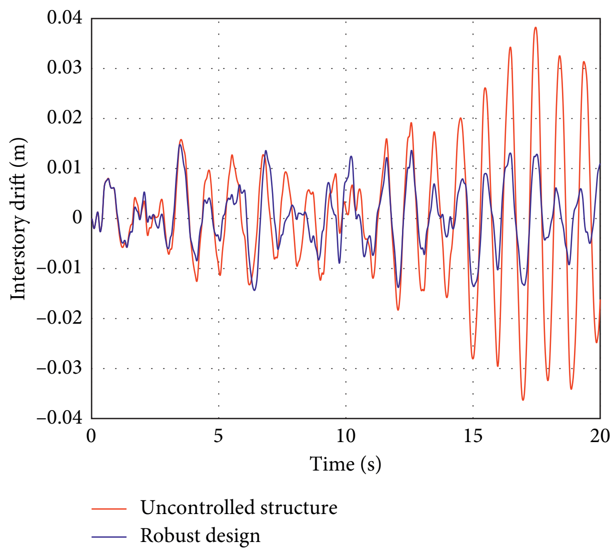

Figure 4: Interstory drift at first story for uncontrolled structure (red curve) and controlled structure (blue curve).

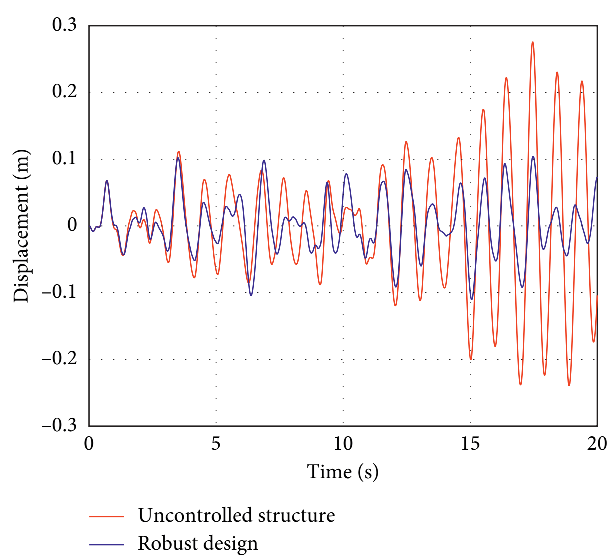

Figure 5: Displacement at top of the structure for uncontrolled structure (red curve) and controlled structure (blue curve).

The proposed methodology is constituted by the SGA optimization algorithm that is able to provide in a single stage, i.e., simultaneously, the optimum values of the mechanical parameters of MTMD and their positions in the structure. On the other hand, the performance of the proposed methodology is evaluated with a computational routine developed by the authors based on the Newmark method that allows computing the structural response of buildings subjected to seismic excitation and equipped with MTMD. To consider uncertainties in the parameters involved, the Monte Carlo simulation was used to determine the expected value of the maximum interstory drift in the structure, that is, the objective function to be minimized.

It is interesting to note that the response reduction performance was expressed in terms of reduction of the expected value of the maximum interstory drift of the building; however, the proposed methodology is flexible, allowing the user to change the objective function.

Additionally, the methodology proved to be robust, since, after two independent runs, it delivered two very similar solutions, that is, the same number of TMDs with similar mechanical parameters and located in the same positions (floors 8, 9, and 10). Both solutions allowed reducing the objective function around 60%.

Moreover, the comparison of the proposed methodology with five alternative methods showed that the proposed method resulted in the lowest maximum interstory drift in all cases. The second and third best results were obtained with alternative methods 5 and 4, respectively; it is important to note that these two alternative methods (4 and 5) perform a robust optimization following the proposed methodology, only changing the SGA by GA (in the case of alternative method 5) and fixing only 1 TMD at the top and optimizing its parameters with the proposed methodology for the case of the alternative method 4. Therefore, these two alternative methods (4 and 5) also serve to prove the effectiveness of the methodology proposed in this work.

It is also interesting to highlight that, for a usual PC (an Intel Core i7-4700MQ 2.4 GHz CPU and 12 GB RAM), the computational cost required to carry out the proposed robust optimization was satisfactory for this sort of dynamic problem, highlighting another advantage of the developed methodology.

Finally, due to its performance, the proposed methodology can be recommended as an effective tool to carry out the optimum design of MTMD. Thus, this work showed that the design of passive devices for the vibration control as MTMD can be accomplished in an economic and safe way, reducing costs and optimizing the resources.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The authors acknowledge the financial support of CAPES and CNPq, Brazil.

References

[1] H. Frahm, "Device for damping of bodies," US Patent No 989, 1911.

[2] M. Dolce, D. Cardone, and F. C. Ponzo, "Shaking-table tests on reinforced concrete frames with different isolation systems," *Earthquake Engineering & Structural Dynamics*, vol. 36, no. 5, pp. 573–596, 2007.

[3] Z.-D. Xu, H.-T. Zhao, and A.-Q. Li, "Optimal analysis and experimental study on structures with viscoelastic dampers,"

Journal of Sound and Vibration, vol. 273, no. 3, pp. 607–618, 2004.

[4] Z.-D. Xu, "Earthquake mitigation study on viscoelastic dampers for reinforced concrete structures," *Journal of Vibration and Control*, vol. 13, no. 1, pp. 29–43, 2016.

[5] W. L. Qu, Z. H. Chen, and Y. L. Xu, "Dynamic analysis of wind-excited truss tower with friction dampers," *Computers & Structures*, vol. 79, no. 32, pp. 2817–2831, 2001.

[6] K.-W. Min, J.-Y. Seong, and J. Kim, "Simple design procedure of a friction damper for reducing seismic responses of a single-story structure," *Engineering Structures*, vol. 32, no. 11, pp. 3539–3547, 2010.

[7] A. Filiatrault, "*Performance evaluation of friction damped braced steel frames under simulated earthquake loads*," M.S. thesis, University of British Columbia, Vancouver, BC, USA, 1985.

[8] L. F. F. Miguel, L. F. F. Miguel, and R. H. Lopez, "Robust design optimization of friction dampers for structural response control," *Structural Control and Health Monitoring*, vol. 21, no. 9, pp. 1240–1251, 2014.

[9] L. F. F. Miguel, L. F. Fadel Miguel, and R. H. Lopez, "A firefly algorithm for the design of force and placement of friction dampers for control of man-induced vibrations in footbridges," *Optimization and Engineering*, vol. 16, no. 3, pp. 633–661, 2015.

[10] L. F. F. Miguel, L. F. F. Miguel, and R. H. Lopez, "Simultaneous optimization of force and placement of friction dampers under seismic loading," *Engineering Optimization*, vol. 48, no. 4, pp. 582–602, 2016.

[11] L. F. F. Miguel, L. F. F. Miguel, and R. H. Lopez, "Failure probability minimization of buildings through passive friction dampers," *Structural Design of Tall and Special Buildings*, vol. 25, no. 17, pp. 869–885, 2016.

[12] S. P. Ontiveros-Pérez, L. F. F. Miguel, and L. F. F. Miguel, "A new assessment in the simultaneous optimization of friction dampers in plane and spatial civil structures," *Mathematical Problems in Engineering*, vol. 2017, Article ID 6040986, 18 pages, 2017.

[13] L. F. F. Miguel, L. F. Fadel Miguel, and R. H. Lopez, "Methodology for the simultaneous optimization of location and parameters of friction dampers in the frequency domain," *Engineering Optimization*, vol. 50, no. 12, pp. 2108–2122, 2018.

[14] L. F. F. Miguel and J. D. Riera, "Controle de vibrações de estruturas utilizando amortecedores por atrito," *Revista Internacional de Desastres Naturales, Accidentes e Infraestructura Civil*, vol. 8, no. 1, 2008.

[15] S. P. Ontiveros-Pérez, L. F. F. Miguel, and L. F. F. Miguel, "Optimization of location and forces of friction dampers," *REM-International Engineering Journal*, vol. 70, no. 3, pp. 273–279, 2017.

[16] L. F. Fadel Miguel, R. H. Lopez, and L. F. F. Miguel, "Discussion of paper: "Estimating optimum parameters of tuned mass dampers using harmony search" [Eng. Struct. 33 (9) (2011) 2716–2723]," *Engineering Structures*, vol. 54, pp. 262–264, 2013.

[17] L. F. Fadel Miguel, R. H. Lopez, L. F. F. Miguel, and A. J. Torii, "A novel approach to the optimum design of MTMDs under seismic excitations," *Structural Control and Health Monitoring*, vol. 23, no. 11, pp. 1290–1313, 2016.

[18] L. F. F. Miguel, R. H. Lopez, A. J. Torii, L. F. F. Miguel, and A. T. Beck, "Robust design optimization of TMDs in vehicle-bridge coupled vibration problems," *Engineering Structures*, vol. 126, pp. 703–711, 2016.

[19] Y. Arfiadi and M. N. S. Hadi, "Optimum placement and properties of tuned mass dampers using hybrid genetic algorithms," *International Journal of Optimization in Civil Engineering*, vol. 1, pp. 167–187, 2011.

[20] M. Mohebbi, K. Shakeri, Y. Ghanbarpour, and H. Majzoub, "Designing optimal multiple tuned mass dampers using genetic algorithms (gas) for mitigating the seismic response of structures," *Journal of Vibration and Control*, vol. 19, no. 4, pp. 605–625, 2012.

[21] M. G. Soto and H. Adeli, "Tuned mass dampers," *Archives of Computational Methods in Engineering*, vol. 20, no. 4, pp. 419–431, 2013.

[22] L. S. Vellar, L. F. F. Miguel, and L. F. F. Miguel, "Controle de vibrações de estruturas através do uso de amortecedor de massa sintonizado (AMS)," in *Proceedings of XXXVII Jornadas Sul Americanas de Engenharia Estrutural*, pp. 1168–1182, Asunción, Paraguay, 2016.

[23] A. Mohtat and E. Dehghan-Niri, "Generalized framework for robust design of tuned mass damper systems," *Journal of Sound and Vibration*, vol. 330, no. 5, pp. 902–922, 2011.

[24] K. Xu and T. Igusa, "Dynamic characteristics of multiple substructures with closely spaced frequencies," *Earthquake Engineering & Structural Dynamics*, vol. 21, no. 12, pp. 1059–1070, 1992.

[25] Q. Wu, W. Zhao, W. Zhu, R. Zheng, and X. Zhao, "A tuned mass damper with nonlinear magnetic force for vibration suppression with wide frequency range of offshore platform under earthquake loads," *Shock and Vibration*, vol. 2018, Article ID 1505061, 18 pages, 2018.

[26] Y. Luo, H. Sun, X. Wang, L. Zuo, and N. Chen, "Wind induced vibration control and energy harvesting of electromagnetic resonant shunt tuned mass-damper-inerter for building structures," *Shock and Vibration*, vol. 2017, Article ID 4180134, 13 pages, 2017.

[27] Q. Wu, X. Zhao, S. He, W. Tang, and R. Zheng, "A bufferable tuned-mass damper of an offshore platform against stroke and response delay problems under earthquake loads," *Shock and Vibration*, vol. 2016, Article ID 9702152, 12 pages, 2016.

[28] J. S. Bae, J. S. Park, J. H. Hwang, J. H. Roh, B. D. Pyeon, and J. H. Kim, "Vibration suppression of a cantilever plate using magnetically multimode tuned mass dampers," *Shock and Vibration*, vol. 2018, Article ID 3463528, 13 pages, 2018.

[29] Q. Wu, X. Zhao, R. Zheng, and K. Minagawa, "High response performance of a tuned-mass damper for vibration suppression of offshore platform under earthquake loads," *Shock and Vibration*, vol. 2016, Article ID 7383679, 11 pages, 2016.

[30] A. A. Farghaly and M. S. Ahmed, "Optimum design of TMD System for tall buildings," *ISRN Civil Engineering*, vol. 2012, Article ID 716469, 13 pages, 2012.

[31] L. Zuo and S. A. Nayfeh, "Optimization of the individual stiffness and damping parameters in multiple-tuned-mass-damper systems," *Journal of Vibration and Acoustics*, vol. 127, no. 1, pp. 77–83, 2005.

[32] S. Elias and V. Matsagar, "Wind response control of tall buildings with a tuned mass damper," *Journal of Building Engineering*, vol. 15, pp. 51–60, 2018.

[33] M. Abé and Y. Fujino, "Dynamic characterization of multiple tuned mass dampers and some design formulas," *Earthquake Engineering & Structural Dynamics*, vol. 23, no. 8, pp. 813–835, 1994.

[34] W. E. D. Sánchez, S. M. Avila, and J. L. V. Brito, "Optimal placement of damping devices in buildings," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 40, p. 337, 2018.

[35] C.-L. Lee, Y.-T. Chen, L.-L. Chung, and Y.-P. Wang, “Optimal design theories and applications of tuned mass dampers,” *Engineering Structures*, vol. 28, no. 1, pp. 43–53, 2006.

[36] A. Ghosh and B. Basu, “A closed-form optimal tuning criterion for TMD in damped structures,” *Structural Control and Health Monitoring*, vol. 14, no. 4, pp. 681–692, 2007.

[37] N. B. Desu, S. K. Deb, and A. Dutta, “Coupled tuned mass dampers for control of coupled vibrations in asymmetric buildings,” *Structural Control and Health Monitoring*, vol. 13, no. 5, pp. 897–916, 2006.

[38] J.-F. Wang, C.-C. Lin, and C.-H. Lian, “Two-stage optimum design of tuned mass dampers with consideration of stroke,” *Structural Control and Health Monitoring*, vol. 16, no. 1, pp. 55–72, 2009.

[39] P. Brzeski, P. Perlikowski, and T. Kapitaniak, “Numerical optimization of tuned mass absorbers attached to strongly nonlinear duffing oscillator,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 1, pp. 298–310, 2014.

[40] N. Hoang, Y. Fujino, and P. Warnitchai, “Optimal tuned mass damper for seismic applications and practical design formulas,” *Engineering Structures*, vol. 30, no. 3, pp. 707–715, 2008.

[41] B. Farshi and A. Assadi, “Development of a chaotic nonlinear tuned mass damper for optimal vibration response,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 11, pp. 4514–4523, 2011.

[42] G. C. Marano, R. Greco, and B. Chiaia, “A comparison between different optimization criteria for tuned mass dampers design,” *Journal of Sound and Vibration*, vol. 329, no. 23, pp. 4880–4890, 2010.

[43] C. Li and W. Qu, “Optimum properties of multiple tuned mass dampers for reduction of translational and torsional response of structures subject to ground acceleration,” *Engineering Structures*, vol. 28, no. 4, pp. 472–494, 2006.

[44] G. Chen and J. Wu, “Optimal placement of multiple tune mass dampers for seismic structures,” *Journal of Structural Engineering*, vol. 127, no. 9, pp. 1054–1062, 2001.

[45] P. Warnitchai and N. Hoang, “Optimal placement and tuning of multiple tuned mass dampers for suppressing multi-mode structural response,” *Smart Structures and Systems*, vol. 2, no. 1, pp. 1–24, 2006.

[46] O. Lavan and Y. Daniel, “Full resources utilization seismic design of irregular structures using multiple tuned mass dampers,” *Structural and Multidisciplinary Optimization*, vol. 48, no. 3, pp. 517–532, 2013.

[47] E. Dehghan-Niri, S. M. Zahrai, and A. Mohtat, “Effectiveness-robustness objectives in MTMD system design: an evolutionary optimal design methodology,” *Structural Control and Health Monitoring*, vol. 17, no. 2, pp. 218–236, 2010.

[48] J. L. Almazán, J. C. De la Llera, J. A. Inaudi, D. López-García, and L. E. Izquierdo, “A bidirectional and homogeneous tuned mass damper: a new device for passive control of vibrations,” *Engineering Structures*, vol. 29, no. 7, pp. 1548–1560, 2007.

[49] J. Mondal, H. Nimmala, S. Abdulla et al., “Tuned liquid damper,” in *Proceedings of the 3rd International Conference on Mechanical Engineering and Mechatronics*, no. 68, Prague, Czech Republic, August 2014.

[50] F. Sadek, B. Mohraz, A. W. Taylor, and R. M. Chung, “A method of estimating the parameters of tuned mass dampers for seismic applications,” *Earthquake Engineering & Structural Dynamics*, vol. 26, no. 6, pp. 617–635, 1997.

[51] L. F. F. Miguel and L. F. Fadel Miguel, “Shape and size optimization of truss structures considering dynamic constraints through modern metaheuristic algorithms,” *Expert Systems with Applications*, vol. 39, no. 10, pp. 9458–9467, 2012.

[52] L. F. F. Miguel, R. H. Lopez, and L. F. F. Miguel, “Multimodal size, shape, and topology optimisation of truss structures using the Firefly algorithm,” *Advances in Engineering Software*, vol. 56, pp. 23–37, 2013.

[53] M. S. Gonçalves, R. H. Lopez, and L. F. F. Miguel, “Search group algorithm: a new metaheuristic method for the optimization of truss structures,” *Computers & Structures*, vol. 153, pp. 165–184, 2015.

[54] A. G. Groehs, *Mecânica Vibratória*, Editora Unisinos, São Leopoldo, Brazil, 2001.

[55] K. Kanai, “An empirical formula for the spectrum of strong earthquake motions,” *Bulletin Earthquake Research Institute*, vol. 39, pp. 85–95, 1961.

[56] H. Tajimi, “A statistical method of determining the maximum response of a building structure during an earth-quake,” in *Proceedings of 2nd World Conference in Earthquake Engineering*, pp. 781–797, Tokyo, Japan, July 1960.

[57] D. E. Huntington and C. S. Lyrintzis, “Improvements to and limitations of Latin hypercube sampling,” *Probabilistic Engineering Mechanics*, vol. 13, no. 4, pp. 245–253, 1998.

[58] R. L. Iman, “Latin hypercube sampling,” in *Encyclopedia of Quantitative Risk Analysis and Assessment*, John Wiley & Sons, Hoboken, NJ, USA, 2008.

[59] M. D. McKay, R. J. Beckman, and W. J. Conover, “Comparison of three methods for selecting values of input variables in the analysis of output from a computer code,” *Technometrics*, vol. 21, no. 2, pp. 239–245, 1979.

[60] G. D. Wyss and K. H. Jorgensen, “A user’s guide to LHS: Sandia’s Latin hypercube sampling software,” Report SAND98-0210, Sandia National Laboratories, Albuquerque, NM, USA, 1998.