Robust Position-based Routing for Wireless Ad Hoc Networks

Kousha Moaveninejad, Wen-Zhan Song, Xiang-Yang Li

Department of Computer Science
Illinois Institute of Technology

Abstract

We consider a wireless ad hoc network composed of a set of wireless nodes distributed in a two dimensional plane. Several routing protocols based on the positions of the mobile hosts have been proposed in the literature. A typical assumption in these protocols is that all wireless nodes have uniform transmission regions modelled by unit disk centered at each wireless node. However, all these protocols are likely to fail if the transmission ranges of the mobile hosts vary due to natural or man-made obstacles or weather conditions. These protocols may fail because either some connections that are used by routing protocols do not exist, which effectively results in disconnecting the network, or the use of some connections causes livelocks. In this paper, we describe a robust routing protocol that tolerates up to roughly 40% of variation in the transmission ranges of the mobile hosts. More precisely, our protocol guarantees message delivery in a connected ad hoc network whenever the ratio of the maximum transmission range to the minimum transmission range is at most $\sqrt{2}$.

Key words: Localized routing, planar structure, wireless ad hoc networks.

1 Introduction

Recent years saw a great amount of research in wireless networks, especially ad hoc wireless networks due to its potential applications in various situations such as battlefield, emergency relief, and so on. One of the key challenges in the design of *ad hoc* networks is the development of dynamic routing protocols

Email address: moavkoo@iit.edu, songwen@iit.edu, xli@cs.iit.edu (Kousha Moaveninejad, Wen-Zhan Song, Xiang-Yang Li).

that can efficiently find routes between two communication nodes. In recent years, a variety of routing protocols [6,16–19,22], targeted specifically for *ad hoc* environment, have been developed. The routing protocols proposed may be categorized as table-driven protocols or demand-driven protocols. Table-driven routing protocols maintain up-to-date routing information between every pair of nodes. The changes to the topology are maintained by propagating updates of the topology throughout the network. *Source-Initiated On-Demand Routing* creates routes only when desired by the source node. At this time a route discovery process is initiated within the network. For the review of the state of the art of routing protocols, see surveys by Royer and Toh [21] and by Ramanathan and Steenstrup [20]. Route discovery can be very expensive in communications costs, reducing the response time of the network. On the other hand explicit route maintenance can be even more costly in the explicit communication of substantial routing information.

Recently, many authors have proposed the use of location information to reduce the amount of control traffic. Ko and Vaidya [10] essentially use the DSR protocol, but suggest that a node forwards a packet to a node only if it is in a request zone which is likely to contain a path to the desired destination. The DREAM protocol [2] uses a limited flooding of data packets.

On the other hand, another set of greedy-based routing protocols, which completely stay away from the flooding paradigm, were proposed recently. In these protocols every node selects the next node to forward the packets based on the information in the packet header and the position of its local neighbors. This neighbor is selected by locally optimizing some criteria such as the length of the path to the destination, or the direction difference between the target and the neighbor.

Let s be the source node, t be the destination node, and u be the current node that will decide which node to forward the packets. In *Compass Routing* [11], node u finds the next relay node v such that the angle $\angle vut$ is the smallest among all neighbors of u in a given topology. A variation called *Random Compass Routing* is also proposed in [11]. In *Greedy Routing* [4], node u finds the next relay node v such that the distance $\|vt\|$ is the smallest among all neighbors of u in a given topology. In *Most Forwarding Routing* [23], node u finds the next relay node v such that $\|v't\|$ is the smallest among all neighbors of u in a given topology, where v' is the projection of v on segment ut. In *Farthest Neighbor Routing* node u finds the farthest node v as forwarding node among all neighbors of u in a given topology such that $\angle vut \leq \alpha$. Here angle α is a parameter, which is less than $\pi/2$ typically. Another variation is *Nearest Neighbor Routing*: node u finds the nearest node v as forwarding node among all neighbors of u in a given topology such that $\angle vut \leq \alpha$. Notice that it is shown in [4,11] that the compass routing, random compass routing and the greedy routing guarantee to deliver the packets from the source to

the destination if Delaunay triangulation is used as network topology. Here a network topology G over a set V of wireless nodes is Delaunay triangulation if the circumcircle of every triangle uvw (with uv, uw, wv in G) does not contain any node from V inside. However, it is expensive to construct Delaunay triangulation in a distributed manner. Obviously, there are several scenarios in which greedy routing fails if Delaunay triangulation is not used. Li *et al.* [15] proposed a structure called *local Delaunay triangulation* to approximate the Delaunay triangulation. Although these greedy protocols guarantee delivery on Delaunay triangulation, they may fail on local Delaunay triangulation.

To overcome the failure of all these greedy-based routing strategies, several researchers proposed another set of localized routing protocols which basically use the right hand rule to guarantee the delivery of the packets. A *planar* network topology that can be constructed efficiently in a distributed manner is required to guarantee the success of all localized routing protocols. Lin *et al.* [23] proposed the first localized algorithm that guarantees delivery by memorizing past traffic at nodes. Bose *et al.* [4] proposed to use the Gabriel graph as the underlying structure for the FACE routing method. Subsequently, Karp *et al.* [7] discussed in detail the medium access layer and conducted experiments with moving nodes for Face routing method. Barriére *et al.* [1] extended the scheme on graphs which are fuzzy unit graphs, that is, two nodes are connected if their distance is at most r, not connected if the distance is at least R, and may be connected otherwise. They showed that their algorithm works correctly if $R \leq \sqrt{2}r$. Routing according to the right hand rule, which guarantees delivery in planar graphs [3], is also used when simple greedy-based routing heuristics fail.

All previous methods (except [1]) assumed that all wireless nodes have the same transmission range, and a node u can receive the message from another node v as long as the distance between them is less than the uniform global transmission range. Consequently, all wireless nodes V together define a unit disk graph, denoted by $UDG(V)$, which has an edge uv if and only if the Euclidean distance between u and v, denoted by $\|uv\|$, is less than one unit. Several planar structures (such as RNG, GG) have been proposed to be used as the underlying network topology of UDG for localized routing protocols that guarantee the packets delivery.

However, graphs representing communication links are rarely specified as the unit disk graphs. We thus consider the general structure of arbitrary graphs defined by points in the plane, the so-called *geometric graphs*. For example, for wireless communications, different nodes may have different transmission radius. Consequently, two nodes can communicate directly if they are within the transmission range of each other (i.e., there is a communication link between these two nodes). The graph formed by all such communication links is different from the traditional disk graph, in which two nodes are connected

by a straight edge if the two corresponding disks centered at these two nodes intersect. And for wireless communications, two nodes sometimes cannot communicate directly even though they are within the transmission range of each other, due to the blocking of the signal by some barriers. Hereafter, we call the graph, formed by all wireless nodes and the edges uv, where uv represents that two nodes u and v can communicate directly, the *communication graph*. Let r_u be the transmission range of a node u. A communication graph is then called *mutual communication graph* (MG), when there is an edge uv iff $\|uv\| \leq \min\{r_u, r_v\}$.

If nodes have different transmission ranges or if some links are missing due to the blocking of signal, then it is easy to construct a configuration of a set of nodes (their positions and transmission ranges) such that there is no planar subgraph of the original communication graph. We thus have to rely on some virtual links to build a planar structure. A virtual link uv is a path consisting of a set of physical communication links connecting node u and node v where there is no direct communication link between node u and node v. Virtual edges are not desirable and we try to use them as little as possible.

In this paper, we present a new method of constructing planar topologies for communication graphs that are not UDG. Our simulations show a significant improvement in terms of the number messages used by our method compared with the previous method without losing the routing performance.

The rest of this paper is organized as follows. In Section 2, we discuss in detail the network model used in this paper and review some previous results on robust position-based routing for this model. We present our method in Section 3. In Section 4, we study the performance of our method compared with the priori art. We conclude our paper in Section 5.

2 Network Model and Preliminaries

2.1 Wireless Networks

We consider a wireless ad hoc network (or sensor network) with all nodes distributed in a two-dimensional plane. Assume that all wireless nodes have distinctive identities and each static wireless node knows its position information [1] either through a low-power Global Position System (GPS) receiver or

[1] More specifically, it is enough for our protocol when each node knows the relative position of its one-hop neighbors. The relative position of neighbors can be estimated by the *direction of arrival* and the *strength of signal*.

through some other way. By one-hop broadcasting, each node u can send its location information to all nodes within the transmission region of u. Throughout this paper, a *broadcast* by a node u means u sends the message to all nodes within its transmission region.

In wireless ad hoc networks, the transmission region of a node u is defined as the locations where the radio signal sent out by the node u can be received by some receiver node. For simplicity, it is traditionally assumed that the transmission region of each wireless node is a disk with unit radius. Here a disk centered at a node u with radius r_u, denoted by $D(u, r_u)$ is the set of points whose distance to u is at most r_u. Thus all nodes together define a unit disk graph as communication graph. However, graphs representing communication links are rarely specified as the unit disk graph. Different nodes may have different transmission radii, and more importantly, the transmission region of a node is never a perfect disk. Considering this imperfect transmission region, previous routing algorithms, which guarantee the packet delivery using some planar subgraph as network topology, are likely to fail since there might be no planar subgraph of the communication graph or some links might be missing. In the worst case, the communication graph could be very complicated. To have some meaningful study, assume that each node u has a maximum transmission radius R_u and a minimum transmission radius r_u. These two thresholds depend on both the environment and the mobile hosts' technology. Thus, the transmission region of a node u is contained inside disk $D(u, R_u)$ and contains the disk $D(u, r_u)$. See Figure 1 for an illustration. If the Euclidean distance between two mobile hosts u and v exceeds the value $\min(R_u, R_v)$ they cannot communicate directly (that is, exchange messages), Conversely, two mobile hosts are always mutually reachable if their Euclidean distance is below the value $\min(r_u, r_v)$. Otherwise, they may or may not be mutually reachable.

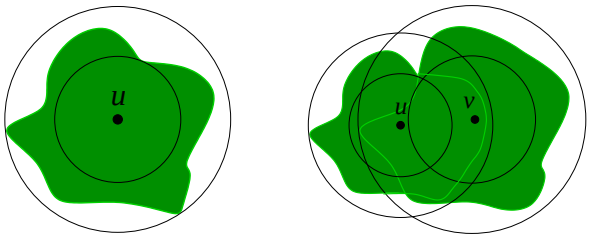


Fig. 1. The transmission region of a node is modelled by a quasi-disk.

The network is then represented by a geometric undirected graph, $G = (V, E)$, with vertices representing mobile hosts, and edges representing communication links. The set of vertices V is thus a set of points in the Euclidean plane. Let $\|uv\|$ be the Euclidean distance between the points u and v in the plane. The set of edges E satisfies $\{uv \mid u, v \in V, \|uv\| \leq \min\{r_u, r_v\}\} \subseteq E$ and $E \subseteq \{uv \mid u, v \in V, \|uv\| \leq \min\{R_u, R_v\}\}$. The nodes corresponding to the mobile hosts, and the edges between a mobile host and its neighbors form the

graph G. Two mobile hosts u and v with $\min\{r_u, r_v\} \leq \|uv\| \leq \min\{R_u, R_v\}$ may or may not be able to communicate directly.

More precisely, let $R(u)$ be the transmission region of a node u, i.e., from where other nodes can receive the signal sent by u. Then our assumption is that $R(u)$ is contained inside the disk $D(u, R_u)$ and contains the disk $D(u, r_u)$. Two nodes u and v can communicate directly iff they are inside the transmission region of each other. Let $I(u)$ be all nodes that can send signal to u, i.e., $I(u) = \{v \mid u \in R(v)\}$. Let $T(u)$ be all nodes that can receive signal from u, i.e., $T(u) = \{v \mid v \in R(u)\}$. Let $N(u)$ be the neighbors of u in G, i.e., $N(u) = I(u) \cap T(u)$.

We note that the transmission conditions do not vary rapidly with time, compared to the speed of electronic communications. This implies that the network may change, for example, due to a change in the weather conditions, but it is at a time scale that allows an easy resetting of the network. Thus, we assume from now on that the connections between mobile hosts are fixed. However, the structure of these connections is not known, and it is the role of our protocol to ensure message delivery in this unknown network.

2.2 Our Network Model

In this paper, we model the transmission region of each node by a quasi-disk which is not our innovation: Barriére *et al.* [1] has also applied this model. They assumed, in addition, that the radius R_u is the same (say all nodes have value R) for all nodes u, and r_u has the same value r. They gave a three-phase protocol that guarantees the delivery of the packet if $R/r \leq \sqrt{2}$. Hereafter we call this graph the *FUDG* graph. Note that we only consider mutual communication links and FUDG is an *undirected* graph.

The main part of their protocol is the construction of a planar structure in a distributed manner. Since the original communication graph may not contain a planar subgraph at all, their algorithm uses some *virtual links*. To distinguish the created virtual links from communication links, we refer to the communication links in the original graph as *actual links*. Virtual links are defined using a recursive approach: given any link uv with length $\|uv\| > R$ (could be virtual or actual), if there is a node w inside the disk $disk(u, v)$, then add virtual links uw and vw to if they are not actual links. Since $R/r \leq \sqrt{2}$, obviously, one of the links uw and vw must have length at most $\sqrt{2}R/2 \leq r$, i.e., it is an actual link in the communication graph. It is easy to show that all virtual links have length at most R by induction. After collecting all virtual links, the algorithm then applies the Gabriel structure to the new graph (with all actual links and virtual links). A simple proof can show that the final structure is a

connected planar graph.

However, although the virtual links are necessary for constructing a planar structure, their protocol creates many unnecessary virtual links. They also gave an example, which shows that their protocol creates many such unnecessary virtual links even when the original communication graph is already a planar structure. [2] Figure 2 illustrates such example. There are $2n$ nodes:

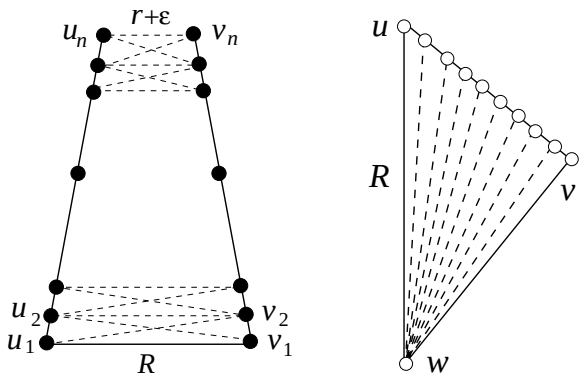


Fig. 2. Excessive virtual links are created (then removed in left case).

n nodes u_1, u_2, $\cdots$, u_n are equally distributed on the left segment; n nodes v_1, v_2, $\cdots$, v_n are equally distributed on the right segment. They are placed such that $\angle u_i u_{i+1} v_i = \pi/2$, $\angle u_i v_{i+1} v_i = \pi/2$, in addition to $u_1 v_1 = R$ and $u_n v_n = r + \epsilon$ for an arbitrarily small positive real number ϵ. A simple execution of their protocol shows that the final planar structure has all links $u_i u_{i+1}$, $v_i v_{i+1}$, $(1 \leq i \leq n-1)$ and $u_n v_n$ (but not the link $u_1 v_1$). Notice that the original communication graph has link $u_1 v_1$ instead of the virtual link $u_n v_n$. The shortest path connecting u_1 and v_1 in this final planar structure has length $O(\sqrt{n})R$. There are $O(n)$ unnecessary virtual links $u_i v_{i+1}$ and $u_{i+1} v_i$ created. Notice the original communication graph is already a planar structure.

If we let the nodes u_i and u_{i+1} (so do v_i and v_{i+1}) arbitrarily close, then their protocol will add all edges $u_i v_j$ as virtual links. Thus, in the worst case, it could add $O(n^2)$ unnecessary virtual links and then remove all these virtual links (except $u_n v_n$) even the original communication graph is planar. The right figure of Figure 2 shows that their method adds $O(n)$ virtual links to the final structure even when it is unnecessary (i.e., the original graph is already planar). In this paper, we present a new method to construct a planar structure that is more efficient in terms of communication and power spanning ratio without creating excessive virtual links.

[2] The original intention of their example is to show that the spanning ratio of the created planar structure could be arbitrarily large. We found that this example also shows that it creates many unnecessary virtual links.

3 Multi-Phase Routing Schemes

Before we discuss our method, we first review some definitions that will be used later. For any pair of nodes u and v, let $lune(u, v)$ be the intersection of two disks centered at u and v with radius $\|uv\|$ and $disk(u, v)$ be the disk with diameter uv. Let $disk(u, v, w)$ be the circumcircle of a triangle $\triangle uvw$. The *relative neighborhood graph*[24], denoted by RNG(V), consists of all edges uv such that $lune(u, v)$ is empty of other nodes inside. The *Gabriel graph* [5] GG(V) contains all edges uv such that $disk(u, v)$ is empty of other nodes. Assume no four vertices of V are co-circular. The *Delaunay triangulation*, denoted by $Del(V)$, is the union of all triangles $\triangle uvw$ such that $disk(u, v, w)$ is empty of other nodes. Obviously, $RNG(V) \subseteq GG(V) \subseteq Del(V)$. Recently, Li *et al.* [15] also defined a sequence of structures called localized Delaunay. A triangle $\triangle uvw$ in UDG is called a *k-localized Delaunay triangle* if $disk(u, v, w)$ does not contain any vertex that is within k hops of u, v, or w. The *k-localized Delaunay graph* over V, denoted by $LDel^{(k)}(V)$, has exactly all Gabriel edges in UDG and edges of all k-localized Delaunay triangles.

Our routing scheme consists of five phases: the Link Collecting phase, the Gabriel phase, the Virtual-link Adding phase, the Extraction phase, and the Routing phase. In the Link Collecting phase, each node u will collect all actual links uv, i.e., u and v can communicate mutually and directly. The aim of the Gabriel phase is to remove some edges so the number of intersections processed in the Virtual-link Adding phase decreases. The aim of the Virtual-link Adding phase is to add some edges (called virtual edges) to the physical communication graph to guarantee the connectivity of the graph after the Extraction phase is executed. In other words the Extraction phase might disconnect the graph if we don't add virtual edges. The aim of the extraction phase is to remove the intersections from the physical communication graph and to produce a planar graph. Once the extraction phase is done, the routing phase performs message delivery between mobile hosts. All computations in all phases are local and do not require any central controller.

The Gabriel phase and the Virtual-link Adding phase are our new contributions, and will be described in detail. The routing phase is basically the same as the routing phase in [4,7], and alternates greedy routing with perimeter routing, i.e., routing around the faces of a planar graph using the right-hand-rule. Thus we include only a brief discussion of the routing phase.

The following data structures and messages are used by our method:

(1) Data Structures:
   (a) N(u)=$\{(bDeleted, bActual, bProcessed, (v, v_x, v_y), (w, w_x, w_y)\}$: the history/final (actual or virtual) neighbor list of u where v is ever or

now a neighbor of u and w is the *relay node* if link uv is virtual. In our method, each virtual edge uv is a two-hop path connecting u and v through some third node, the third node is called the relay node. Note that for virtual link uv with relay node w, both link uw and wv may be virtual or actual. *bDeleted* is a flag to show whether it was ever deleted or not; *bActual* is a flag to show whether it is an actual edge or a virtual edge; *bProcessed* is a flag to show whether it was ever processed or not. Initially, all flags are FALSE. If the neighbor is an actual neighbor then the last argument, which is the ID and coordinate of the relay node, would have no meaning.

(b) $\mathsf{L(u)} = \{bDeleted, bProcessed, (v, v_x, v_y,), (w, w_x, w_y)\}$: the set of all known edges vw by u, where neither v nor w is u. Here *bDeleted* is a flag to show whether it was ever deleted or not; *bProcessed* is a flag to show whether it was ever processed or not. Initially, all flags are FALSE.

(2) Message Format:

(a) $\mathsf{NewNode}(v, v_x, v_y)$: node v uses this message to inform other nodes its information. Here (v_x, v_y) is the coordinate of node v.

(b) $\mathsf{Confirm}(u, v)$: node u confirms node v. In other words node u confirms that it can receive signal from node v.

(c) $\mathsf{NewLink}(u, v, (u_x, u_y), (v_x, v_y))$: a node uses this message to inform other nodes of the existence of a link uv. Here uv could be an actual link or a virtual link.

3.1 Link Collecting Phase

Before we start the Gabriel Phase we have to construct the FUDG graph, so that each node will be aware of its one-hop neighbors, as follows. Initially, every mobile host broadcasts its own geometry position to the mobile hosts within its transmission region. Unlike the unit disk graph model, knowing the geometry position of another node v, if $\|uv\| > r$, a node u cannot determine whether it can communicate directly with the mobile host v. It only knows that it can receive the message from v, but is not sure whether v can receive its signal. Thus, node u has to send a message Confirm to confirm that it can receive the message from v.

Constructing FUDG Graph phase:

(1) Initially, each node u sets an empty neighbor list $\mathsf{N(u)}$ and an empty link list $\mathsf{L(u)}$. Each node u broadcasts its location information, by sending the message $\mathsf{NewNode}(u, u_x, u_y)$ to all nodes inside its transmission region using a local broadcast model.

(2) When a node v receives the message $\mathsf{NewNode}(u, u_x, u_y)$ from a node u,

(a) if $\|uv\| > r$ then node v confirms node u by sending the message Confirm (v, u).
(b) if $\|uv\| \leq r$ then node v adds the record ($NotDeleted$, $Actual$, $NotProcessed$, (u, u_x, u_y), $(null, null, null)$) to N(v).

(3) If a node u receives a confirmation message Confirm(v, u) from a node v, node u adds the record ($NotDeleted$, $Actual$, $NotProcessed$, (v, v_x, v_y), $(null, null, null)$) to N(u).

Now each node has the information of its one-hop neighbors and can continue to the next phase. Notice that, node u does not have to wait to collect all its one hop neighbors to start the Gabriel phase. It can perform Gabriel phase whenever it gets the information about a new neighbor. To avoid unnecessary recalculation of the Gabriel Phase, we set a timeout value TMax, which is the time it will start the Gabriel phase after getting the confirmation message from its first neighbor.

We then show that the link information collected by all nodes are consistent: if node u has a link uv, then node v will also have a link vu. If $\|uv\| \leq r$, then u and v are mutually reachable so the NewNode message sent by node u is received by node v and node v adds node u to N(v) and vice versa. In this case no confirmation is needed. Otherwise, when node u has a link uv, is means that the message sent by u has been received by v and the confirmation message from v has been received by u. Consequently, the message from v will be received by u and the confirmation message sent by u will be received by v. Thus, node v will also create an actual link vu.

3.2 Gabriel Phase

The unit disk graph, defined on all nodes with transmission ranges r, is a subgraph of FUDG graph. We use UDG(FUDG) to denote such unit disk graph. The Gabriel Phase applies Gabriel Topology on UDG(FUDG) and builds GG(UDG(FUDG)). In other words, any edge uv with length less than r is removed if there exists a node w such that $\angle uwv \geq \pi/2$. Removing edge uv is performed by setting the flag *bDeleted* of node u to be True in the adjacency list of node v and vice versa. These edges will be removed in the Extraction Phase, if we don't remove them before the Extraction Phase starts. We remove them at this stage so we will have less number of intersections in the Virtual-link Adding Phase.

Then, each node, say u, sends out the information of its neighbors, say v, whose *bDeleted* flag is False by sending the message NewLink$((u, u_x, u_y), (v, v_x, v_y))$. When node w, which is neither u nor v, receives NewLink$((u, u_x, u_y), (v, v_x, v_y))$ message, if (a) the link uv doesn't belong to L(w) and (b) either $\|uw\| \leq r$

or $\|vw\| \leq r$, then it adds the record ($NotDeleted$, $NotProcessed$, (u, u_x, u_y), (v, v_x, v_y)) to $\mathsf{L}(\mathsf{w})$.

Notice that, after the Gabriel phase, we only have two sets of links (1) a subset links with length at most r, and these links form a planar graph (not necessarily connected). (2) all actual links that have length larger than r, but no more than R (these links may intersect themselves or with the links from the first subset). Our next phase will add virtual links so we can remove intersections later.

3.3 Virtual-link Adding Phase

Consider any two intersected links xy and uv. Either of these two links could be a virtual link or an actual link. Removing the intersection without disconnecting the network is based on the following observation illustrated by Figure 3. Assume $\angle xuy$ is the largest angle of the quadrilateral. Then we can

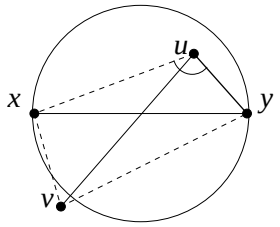


Fig. 3. Two intersected links: one will be removed.

remove link xy and add link xu (if it is not added before) to remove this intersection and preserve the connectivity. If the added link xu causes the new intersections, we continue to process the new intersections.

Notice that if we don't keep track of deleted links, we may end up generating many messages or even dead-lock due to the following phenomena: link xy could be added back when we process some other intersections, and then link xy will cause new intersections again (although those intersections have been processed). To avoid this loop of intersection processing, we will only mark edge xy as deleted but do not actually remove it from the graph now. Later, if we need to add link xy while processing some other intersections, we first check if link xy exists or not (it could be marked as deleted). If it exists, we do nothing; otherwise, we add the link information to node x and node y.

Since any pair of nodes with distance at most r can communicate directly, the added virtual links have length larger than r. We also prove that all introduced virtual links have length at most R.

Lemma 1 *All introduced virtual links have length at most R.*

PROOF. We prove this by induction on the time of the virtual link introduced to the structure. When the first virtual link is introduced, all links have length at most R. The virtual link is introduced since there are two previous known links intersecting each other. Consider any two intersecting links uv and xy. See Figure 3 for an illustration. The largest angle among the four corner angles $\angle xuy$, $\angle uyv$, $\angle yvx$ and $\angle vxu$ is at least $\pi/2$ from the pigeonhole principal. Assume that $\angle xuy$ is the largest angle (ties are broken by the largest apex node ID). Obviously, the shortest edge of xu and uy is at most $\frac{\sqrt{2}}{2}\|xy\| \leq \frac{\sqrt{2}}{2}R \leq r$. W.l.o.g., assume that $\|uy\| \leq \|ux\|$. Consequently, node u and y can receive the message from each other. Then node u will know the existence of link xy through node y and node y will know the existence of link uv through node u. Then according to our algorithm, node y will decide to remove link xy and proposes to add links xu and uy (actually uy already existed as an actual link, thus only link xu could be a virtual link introduced). Obviously, the length of link xu is less than that of xy, which is at most R.

Links xy, or uv, or both could be virtual links, by induction, the virtual links introduced from their intersection has length at most R also. This finishes the proof. □

We then discuss in detail our method of adding virtual links. First of all, consider any two intersected links, say uv and xy, in the current configuration of the network. See Figure 3 for an illustration. W.l.o.g., assume that $\angle xuy$ is the largest angle of the quadrilateral $xuyv$ (ties are broken by ID of the apex). Then it is easy to show that either uy or ux has length at most r, i.e., it is an actual link. Assume that $\|uy\| \leq r$. Consequently, both u and y can detect such intersection since they know the existence of these two links uv and xy (node u knows xy through node y and node y knows uv through node u). To avoid that both of them process this intersection, we only let node u (with the largest angle $\angle xuy$) process it. Thus, we have the following procedure of adding virtual links.

Adding Virtual Link Phase:

(1) Assume that node u creates a new link uv. Node u checks whether link uv causes intersections with some links stored in list $\mathsf{L(u)}$. If it does cause intersection, say with a link xy, it goes to Step 3.
(2) Assume node u receives $\mathsf{NewLink}(u, v, (x_x, x_y), (y_x, y_y))$ from some neighbor. Node u checks whether link xy causes intersections with some links stored in list $\mathsf{N(u)}$. If it does cause intersection, say with a link uv, goes to Step 3.
(3) Node u checks whether the following conditions are satisfied: (1) $\angle xuy$ is the largest among the quadrilateral $xuyv$, and (2) link ux does not exist. If the conditions are satisfied, node u will add virtual link ux, i.e., adds

record ($notDeleted$, $notProcessed$, $(x, x_x, x_y,)$, (y, y_x, y_y)) to $\mathsf{L(u)}$.

Notice that node y can also detect this intersection. Node y then sends a message to node x (through the relay of some other nodes if link yx is virtual) asking it to form a virtual link ux. Node y also marks link xy as deleted, i.e., set $bDeleted$ of link xy to TRUE.

When node x receives the message of forming virtual link xu from node y, node x adds node u to its list $\mathsf{N(u)}$ and mark link xy deleted also.

Notice that here link xy could be virtual link also. The communication through link xy will then through the relay of a sequence of actual links. In addition, since we check whether the link ux exists or not before we decide to add this link, all added virtual links have length at least r.

3.4 Extraction Phase

This phase is very similar to the Gabriel phase but with some differences. Edges whose length are greater than r might be removed but edges whose length are smaller than r will not be removed. Remember that after the Gabriel phase, we only have two set of edges: (1) edges in GG(UDG(FUDG)) forming a planar graph and with length at most r, and (2) edges in the original communication graph with length in the range $(r, R]$. In the virtual-link adding phase, we only add virtual links with length larger than r.

Extract Edges

(1) Each node u checks every incident link uv whose flag $bDeleted$ is FALSE and $\|uv\| > r$. If there is another neighbor w such that $\angle uwv \geq \pi/2$ and there is a link between v and w, then the flag $bDeleted$ of link uv is set to TRUE (i.e., edge uv is marked to be deleted).
(2) All edges whose $bDeleted$ flag is FALSE form the final structure.

3.5 Correctness

In the remainder of the section, we show that our algorithm does terminate and generate a planar structure.

We first show that the final structure is indeed a connected planar graph. First of all, if the original communication graph is a connected planar graph, our algorithm will not add *any* virtual links. The final structure is just the original communication graph. If the algorithm terminates with two intersected links, say xy and uv, then according to the proof of Lemma 1, we know that this intersection will be detected by at least two of these four nodes x, y, u, and v

(in our proof, we show that u and y will detect the intersection by assuming the $\angle xuy$ is the largest angle among the quadrilateral $xuyv$). We also showed that one of the nodes will decide to add virtual links (nodes u and x will add virtual link ux in our assumption). Thus link xy will be removed in the Extraction phase according to our algorithm. Thus, if the algorithm terminates, there are no intersected links. It is obvious that the final structure is connected if the original communication graph is connected since, every time when we decide to remove a link that causes intersection, we already added some virtual links (if necessary) to form a path connecting the two end-points of the removed link.

It is obvious that the final structure is connected if the original communication graph is connected since we remove an edge uv iff links uw and wv exist (although either of them could be a virtual link).

We then show that our algorithm does terminate. Notice that although our algorithm is divided to many phases, we do not have to strictly follow these phases, i.e., different nodes may be in different phases, and some nodes may come back to some earlier phases when necessary. For example, the Virtual-link Adding phase and the Extraction Phase can be interleaved.

We show that the total edge length of the structure is decreased whenever we process an intersected pair of links. Again, assume that intersected links xy and uv are processed. Similar to the proof of Lemma 1, we know that the removed link (edge xy in our proof) has length larger than the possibly added virtual link (edge xu in our proof). Notice that, when we remove a link, it is possible that we do not have to add any virtual link (when xu is already actual link). Since the there are at most $n^2/2$ links in a n-node network, the number of connected graphs is also a finite number. Thus, our algorithm is guaranteed to terminate.

3.6 Routing

After a planar structure is constructed, we then apply some previously developed routing protocols on top of this structure. For completeness, we review some localized routing protocols that rely on a planar structure.

3.6.1 Right Hand Rule and Face Routing

Right hand rule is long-known method for traversing a graph (in analogy to following the right hand wall in a maze). And it has been used in some wireless routing protocols [4,23,9,8]. The rule states that when arriving at node x from node y, the next edge traversed is the next one sequentially counterclockwise

about x from edge xy. In the example shown in Figure 4, x will forward the packet to z following right hand rule, traversing face P. It is known that the right hand rule traverses the interior of a closed polygonal region (a face) in clockwise edge order. And it traverses an exterior region in counterclockwise edge order. In general, right hand rule is applied in planar graphs (in which no two edges intersect each other). [8] gives a *no-crossing heuristic* to deal with the case where edges cross.

Applying the right hand rule in planar graphs, a routing protocol called *Face Routing* is proposed by [11] (in the paper they call the algorithm *Compass Routing II*). We consider a planar graph G. The nodes and edges of graph G partition the Euclidean plane into contiguous regions called the *faces* of G. The main idea of the face routing is to walk along the faces which are intersected by the line segment st between the source s and the destination t. In each face, it uses the right hand rule to explore the boundaries. On its way around a face, the algorithm keeps track of the points where it crosses the line st. Having completely surrounded a face, the algorithm returns to one of these intersections lying closest to t, where it proceeds by exploring the next face closer to t. Figure 4 gives an illustration. See [11,13] for detailed algorithms. They also proved that the face routing algorithm guarantees to reach the destination t after traversing at most $O(n)$ edges where n is the number of nodes.

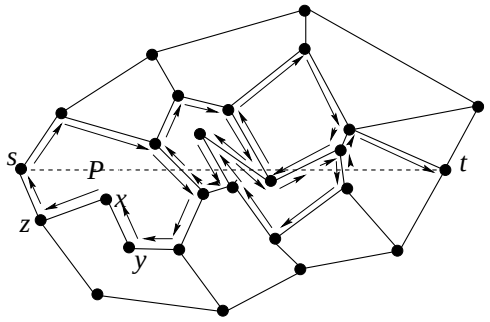


Fig. 4. An illustration of the face routing algorithm.

Though face routing terminates in linear time, it is not satisfactory, since already a very simple flooding algorithm will terminate in $O(n)$ steps. Then in [13], Kuhn *et al.* proposed a new method called *Adaptive Face Routing* (AFR), in which, restricted search areas are used to avoid exploring the complete boundary of faces. The idea is as follows. The exploration of faces is restricted to an ellipse area. The ellipse size is set to an initial estimate of the optimal path length. If face routing fails to reach the destination (when it reach the ellipse, it has to turn back), the algorithm will restart with a bounding ellipse of doubled size.

Recently, Kuhn *et al.* [14] extend the *Adaptive Face Routing* to a routing algorithm called *Other Adaptive Face Routing* (OAFR). Instead of changing

to the next face at the "best" intersection of the face boundary with st, OAFR returns to the boundary point closest to the destination.

3.6.2 Combine Face Routing with Greedy Routing

Greedy routing was used in early routing protocols for wireless networks. However, it is easy to construct a simple example to show that greedy algorithms will not succeed to reach the destination but fall into a local minimum, a node without any "better" neighbors. Then a natural approach to improve the potential of greedy routing for practical purposes is to combine greedy routing and face routing (or right hand rule) to recover the routing after simple greedy routing fails in local minimum. Many wireless protocols used this approach [4,9,14,12,23,25].

Greedy Perimeter Stateless Routing (GPRS) [8,9] is one of the famous routing protocols for wireless networks. It uses RNG or GG as the planar routing topology, then combines greedy and right hand rule to forward packets in the network. It works as follows. When a node receives a greed-mode packet, it searches its neighbor table for a neighbor who is closer to the destination t. If there is one, it will forward it to that neighbor. When no neighbor is closer, the node marks the packet into perimeter mode. GPSR forwards perimeter-mode packets using a simple planar graph traversal (right-hand rule). When a packet enters perimeter mode, GPSR records in the packet the location L_p. Then when receiving a perimeter-mode packet, GPSR will first compare it with forwarding node's location. GPRS returns a packet to greedy mode of the distance from the forwarding node to t is less than that from L_p to t. For more detail, please refer to [9,8]. GPRS can guarantee the delivery of the packets when the underlying network topology is a planar graph.

Recently, Kuhn *et al.* [14] proposed a new algorithm to combine greedy routing with their *Other Adaptive Face Routing* (OAFR). They called the new method *Greedy Other Adaptive Face Routing* (GOAFR). The idea is similar to GPSR. When greedy method falls in local minimum, GOAFR uses OAFR to recover the routing. Same as for AFR, they proved the cost of GOAFR is bounded by $O(c^2(p^*))$ which is asymptotically optimal. In addition, they show that the algorithm is also average-case efficient through extensive simulations. In [14], the authors showed simulations of a variety of face routing algorithms and their combinations with a greedy approach. Notice that unlike GPSR, when doing face routing in GOAFR, it does not return to greedy method until OAFR completely finishes the exploration of the face. This may affect the efficiency of routing. In [12], Kuhn *et al.* uses an "early fallback" technique to return to greedy routing as soon as possible. The new algorithm called GOAFR$^+$. It employs two counters p and q to keep track of how many of the nodes visited during the current face routing phase are located closer (counted by p) and

how many are not closer (counted by q) to the destination than the starting point of the current face routing phase. When a certain fallback condition holds, GOAFR$^+$ directly falls back to greedy mode. This modification makes an obvious improvement for the average case performance. Their theoretical analysis also proves that GOAFR$^+$ is asymptotically optimal in the worst case.

4 Experiments

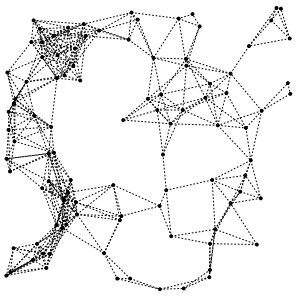
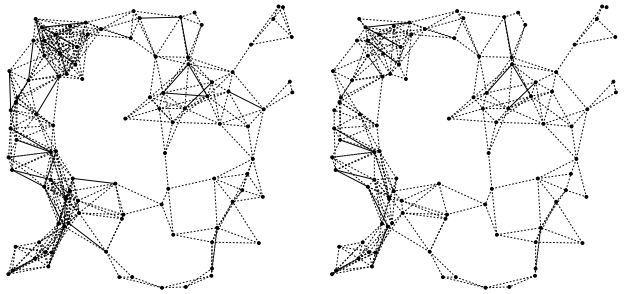
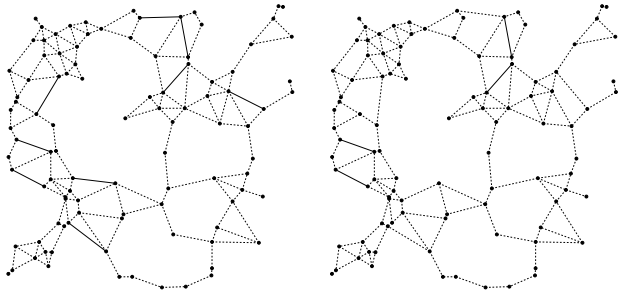


Fig. 5. A quasi Unit Disk Graph.



(a) Barriere's method (b) Our method



(c) Barriere's method (d) Our Method

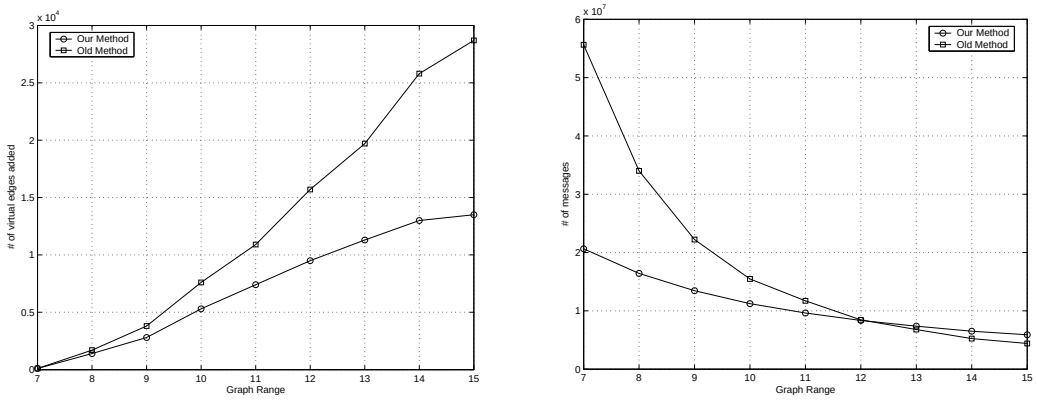
Fig. 6. (a) and (b): virtual edges added by Barriere's method and our method; (c) and (d): Final structure created by these two methods.

We conducted extensive simulations to study the performance of our method compared with the previous method proposed in [1]. We compare them in terms of both the structure construction performance and the routing performance. We randomly put 500 nodes in a square with side length varying from

7 to 15 and the transmission range of each node is fixed to one unit. First of all, we want to know how many messages used by these two methods respectively. Figure 5 illustrates a quasi-UDG graph with 100 nodes in a square region of 7. Figure 6 (a) and (b) show the virtual edges added during the process by priori art and our method. Figure 6 (c) and (d) show the final topologies generated by these two methods.

Our simulations show that our method uses significantly less messages than the method by Barriére *et al.* [1] for dense networks. The used messages are similar for sparse networks. Similar observation holds for the number of virtual links created: our method creates significantly less virtual links for dense graphs. See Figure 7.

Notice that, a structure that can be constructed with less messages does not imply that the routing performance based on it is better than other structure. We continue to study the routing performances of the structures constructed by our method and by the method of Barriére *et al.* [1]. We test the Greedy-Face routing method on structures constructed by both methods (given the same original communication graphs). Surprisingly, we found that the routing performances based on these two structures are almost the same. See Figure 8 for an illustration. We measured both the average route hops and average route lengths. Notice that, if a route uses a constructed virtual link, we have to measure the hops and lengths of the actual path stored at the end-points to connect them.

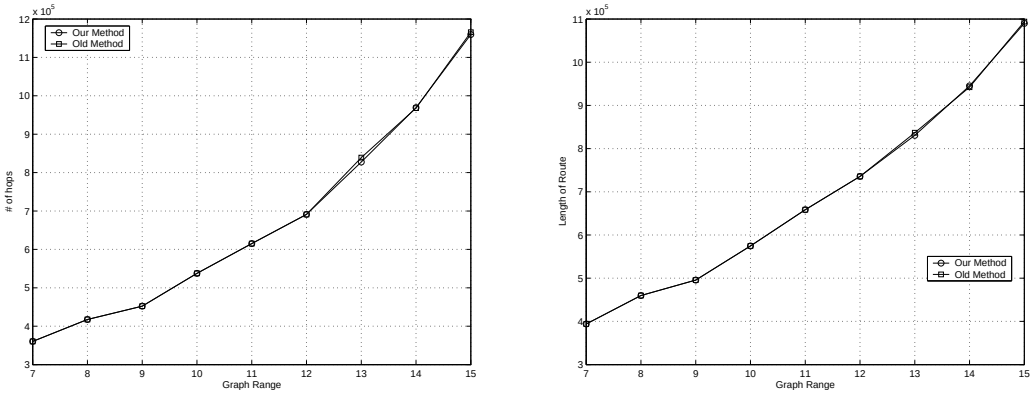


(a) Virtual edges added (b) Messages used

Fig. 7. Comparison of our method with Barriere's method.

5 Conclusion

We consider a wireless ad hoc network composed of a set of wireless nodes distributed in a two dimensional plane. Several protocols have been devel-



(a) Average route hops (b) Average route length

Fig. 8. Comparison of our method with Barriere's method.

oped to perform routing in ad hoc wireless networks based on the positions of the mobile hosts. A typical assumption in these protocols is that all wireless nodes have uniform transmission regions modelled by unit disk centered at each wireless node. However, all these protocols are likely to fail if the transmission regions of the mobile hosts are not unit disks due to natural or man-made obstacles or weather conditions. In this paper, we describe a robust routing protocol that tolerates up to roughly 40% of variation in the transmission ranges of the mobile hosts. More precisely, our protocol guarantees message delivery in a connected ad hoc network whenever the ratio of the maximum transmission range to the minimum transmission range is at most $\sqrt{2}$. Our protocol saved the communication cost without losing the performances compared with the previous method that can tolerate the same transmission variance. Our simulations showed that our protocol out-performs the previous method for dense networks significantly: it uses much less messages and creates much less virtual links, while the routing performances of our method is almost the same as the previous method.

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