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ROBUST PREDICTIONS FOR BILATERAL CONTRACTING WITH EXTERNALITIES

BY ILYA SEGAL AND MICHAEL D. WHINSTON[1]

The paper studies bilateral contracting between one principal and N agents when each agent's utility depends on the principal's unobservable contracts with other agents. We show that allowing deviations to menu contracts from which the principal chooses bounds equilibrium outcomes in a wide class of bilateral contracting games without imposing ad hoc restrictions on the agents' beliefs. This bound yields, for example, competitive convergence as $N \to \infty$ in environments in which an appropriately-defined notion of competitive equilibrium exists. We also examine the additional restrictions arising in two common bilateral contracting games: the "offer game" in which the principal makes simultaneous offers to the agents, and the "bidding game" in which the agents make simultaneous offers to the principal.

KEYWORDS: Bilateral contracting, contracting externalities, menu contracts, robust predictions, common agency, competitive convergence.

1. INTRODUCTION

THIS PAPER STUDIES BILATERAL CONTRACTING between one principal and N agents in the presence of externalities among agents. Examples include (i) vertical contracting, where sales by an upstream firm to a downstream firm reduce the downstream price received by other downstream firms (Hart and Tirole (1990), McAfee and Schwartz (1994), Rey and Tirole (1996)), (ii) nonexclusive insurance, where the contract between an insured and an insurer affects the care taken by the insured and hence the profits of other insurers (Pauly (1974), Bernheim and Whinston (1986a), Kahn and Mookherjee (1998), Bisin, Gottardi, and Guaitoli (1999), Bisin and Guaitoli (2000)), (iii) lending with default, where the debt owed by the debtor to one lender affects the expected repayment to other lenders (Bizer and deMarzo (1992), Dubey, Geanakoplos, and Shubik (1999), Rajan and Parlour (2001)), and many others (see Segal (1999)).

The literature on such contracting situations is divided into two branches. One branch studies specific noncooperative contracting games. In the two most-studied games, one of the two sides makes take-it-or-leave-it offers to the other.

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The game in which the principal makes simultaneous offers to the agents, which we refer to as the *offer game*, has been studied, for example, by Hart and Tirole (1990), McAfee and Schwartz (1994), Rey and Tirole (1996), and Segal (1999). The game in which the agents make simultaneous offers to the principal, which we refer to as the *bidding game*, appears in models of common agency, including Bernheim and Whinston (1986a,b, 1998), O'Brien and Shaffer (1997), and Martimort and Stole (2002, 2000). Some multi-stage contracting games have also been considered (deMarzo and Bizer (1993), Kahn and Mookherjee (1998)).

While this branch of the literature has generated important insights about the nature of the inefficiency caused by externalities, the games studied in this branch sustain a wide range of equilibrium outcomes, both within a given game and across different games. This diversity of equilibrium outcomes can be traced to differences in agents' beliefs about the principal's contracts with other agents. For example, in the offer game an agent who observes an out-of-equilibrium offer by the principal can hold arbitrary beliefs about the principal's offers to other agents, and these beliefs can sustain a large set of equilibrium outcomes. In contrast, in the bidding game, an agent knows the equilibrium offers made to the principal by other agents when contemplating his own offer. (Throughout we use feminine pronouns for the principal, and masculine pronouns for the agents.)

The other branch of the literature studies the case with a large number of agents and postulates competitive equilibrium (Dubey, Geanakoplos, and Shubik (1999), Bisin and Guaitoli (2000)). For example, in the lending and nonexclusive insurance applications, the competitive concept demands that each lender/insurer take the principal's total borrowing/insurance, and hence her moral hazard action, as given. However, this approach has not been justified as a limit of outcomes of noncooperative contracting games as the number of agents goes to infinity, unlike in economies without externalities, in which price taking has been justified by Cournot-style competitive limit results.

This paper considers a family of noncooperative games of contracting with externalities, which we call *bilateral contracting games*, in which the principal's contracts with agents are privately observed. This family includes the offer and bidding games as special cases. We address the concern we have raised about the first branch of the literature by identifying properties of equilibrium outcomes that are *robust* in the sense that they must be satisfied by *all* equilibria of *all* bilateral contracting games. Among the properties we establish is the necessity of convergence, in certain environments, to the competitive outcomes postulated by the second branch.

Our predictions are more precise than those of the existing literature because we allow the parties to use bilateral contracts that are more general (in fact, fully general given the observability assumptions). Specifically, we show that the set of equilibrium outcomes is affected dramatically by allowing the parties to offer each other a menu contract from which the principal can then choose, rather than a simple "point" contract specifying the principal and agent's exact trade. To understand the role of such menus, consider the situation faced by the principal when making an offer to an agent (for example, in the offer game).

The principal's problem is that the agent may reject a profitable offer due to a negative inference about the principal's trades with other agents. A menu from which the principal chooses can screen the different "types" of the principal, corresponding to her trades with other agents. By inducing different types to choose different trades with the agent, such a menu can be acceptable to the agent regardless of his beliefs.

The use of such menus by the principal in the offer game is similar to the problem of mechanism design by an informed principal examined by Myerson (1983) and Maskin and Tirole (1992). The difference is that here the principal's type results from her own choices rather than being determined exogenously. In parallel to the analysis of Myerson (1983) and Maskin and Tirole (1992), we derive a menu that maximizes the payoff of all of the principal's types among all menus that the agent would accept regardless of his beliefs. This menu plays a central role in our analysis and, following Maskin and Tirole (1992), we call it the Rothschild-Stiglitz-Wilson (RSW) menu.

The requirement that deviations using an RSW menu be unprofitable imposes a significant bound on the set of equilibrium outcomes in bilateral contracting games. Specifically, if the equilibrium bilateral surplus of the principal and an agent were too low, there would exist a bilateral contract consisting of the RSW menu and a fixed payment that would guarantee each of them a higher payoff. In this case, either party would have an incentive to deviate by offering such a contract when he or she has the opportunity to make an offer. This lower bound on bilateral surplus yields a bound on equilibrium outcomes that can be quite sharp, particularly when the number of agents is large.

The idea that a party's trades with others can be viewed as her "type" on which she can be screened via a menu has been suggested in the context of the bidding game by Martimort and Stole (2002). In our analysis we apply this idea to games in which the principal makes offers as well. In such games, in contrast to the bidding game, the introduction of menu contracts can restrict rather than expand the set of equilibrium outcomes, and allows us to obtain a uniform bound on equilibrium outcomes across the family of bilateral contracting games.

The paper is organized as follows: We begin in Section 2 by considering a simple example in the setting of vertical contracting between a manufacturer and N retailers, in which we illustrate many of the paper's themes.

In Section 3, we develop a general approach to contracting with externalities, defining a general family of bilateral contracting games and bounding their equilibrium outcomes using the notion of an RSW menu. We also use this notion to fully characterize the equilibrium outcomes of the offer game.

In the rest of the paper we apply our general results to three settings with specialized payoffs. In Section 4, we focus on a setting in which a well-defined notion of competitive equilibrium exists. We show that in this environment, all bilateral contracting outcomes must converge to the competitive outcome as $N \to \infty$. This result is implied by the possibility of deviations to the "competitive menu," which is the menu that allows the principal to trade an arbitrary quantity at the competitive equilibrium price. We also show that under some additional assumptions this competitive menu is an RSW menu, and we use this fact to characterize the equilibrium outcomes of the offer game.

In Sections 5 and 6 we consider two cases in which a competitive equilibrium does not exist: one in which the principal has a "decreasing marginal cost" (Section 5), and one in which a "competitive quasi-equilibrium" exists, e.g., with a U-shaped marginal cost function (Section 6). We show that the competitive menu is again an RSW menu and use it both to bound the equilibrium outcomes of all bilateral contracting games and to characterize outcomes in the offer game. Our results for the case of a competitive quasi-equilibrium reject the concept of "convexified competitive equilibrium" used by Dubey, Geanakoplos, and Shubik (1999), instead predicting the "noncompetitive outcomes" identified by Kahn and Mookherjee (1998), Bisin and Guaitoli (2000), and Rajan and Parlour (2001).

In Section 7, we study the equilibrium outcomes of the bidding game, examining the additional restrictions that they must satisfy relative to our previously derived bound and comparing them to those of the offer game.

Finally, Section 8 offers concluding thoughts, including a comment on the realism of these menus.

2. A SIMPLE EXAMPLE

In this section we consider a simple example of contracting with externalities. To be specific, we focus on the setting of vertical contracting (Hart and Tirole (1990), McAfee and Schwartz (1994)), where one manufacturer (the "principal") sells her output to $N \geq 2$ retailers ("agents"). The retailers then resell their purchases in the downstream market.[2]

For simplicity, we assume that retailers convert each unit of the manufacturer's product into one unit of the final good at zero marginal cost. The manufacturer's cost function is $c(X) = \alpha X + \frac{1}{2}\beta X^2$, while the downstream inverse demand function is $P(X) = \max\{a - bX, 0\}$. We assume that $a > \alpha > 0$, $b > 0$, and $b + \beta > 0$.

The efficient outcome for the vertical structure is to sell the monopoly quantity:

$$X^* = \arg\max_{X \geq 0} P(X)X - c(X) = \frac{a-\alpha}{2b+\beta}.$$

On the other hand, were the vertical structure to take the retail price as given,[3] it would sell the "competitive" quantity X^c at which the marginal cost $c'(X^c)$ equals $P(X^c)$, which yields

$$X^c = \frac{a-\alpha}{b+\beta},$$

provided that its profit from doing so is nonnegative. This is so (and so a competitive equilibrium exists) if and only if $\beta \geq 0$. Figure 1 illustrates this case. The

[2] For simplicity we do not allow retailers to withhold a portion of the good from the market.

[3] The same outcome obtains when the manufacturer takes the wholesale price as given, while each retailer takes both the wholesale and retail prices as given—a situation that corresponds to Definition 3 in Section 4.

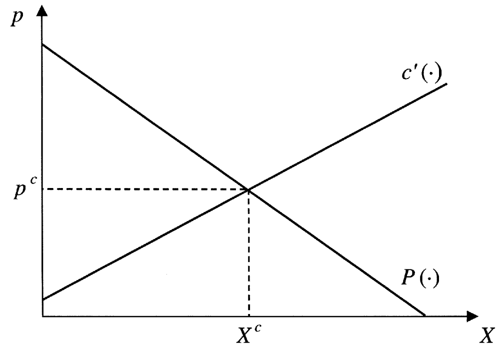

FIGURE 1.

efficient quantity X^* and the competitive quantity $X^c > X^*$ will serve as benchmarks against which to compare the outcomes of bilateral contracting.

We consider bilateral contracting games in which each retailer contracts with the manufacturer without observing other retailers' contracts with or purchases from the manufacturer.[4] A retailer's behavior in such a game depends on his beliefs about other retailers' contracts and purchases.

Here we illustrate the role of beliefs in the *offer game*, in which the manufacturer makes simultaneous offers to the downstream firms, who then accept or reject. Suppose initially that the manufacturer offers each retailer i a "point" contract (x_i, t_i), where $x_i \geq 0$ is the quantity of the product and t_i is the retailer's payment.[5] After observing the offer, retailer i forms beliefs about other retailers' contracts.

In the literature on vertical contracting, and contracting with externalities more generally, it is common to restrict attention to *passive beliefs* in which, after observing a deviation, each retailer continues to believe that other retailers receive their equilibrium offers (see, e.g., Hart and Tirole (1990), Segal (1999)). Let $(\hat{x}_1, \ldots, \hat{x}_N, \hat{t}_1, \ldots, \hat{t}_N)$ denote the equilibrium outcome. With passive beliefs, if retailer i is offered $(x_i, t_i) \neq (\hat{x}_i, \hat{t}_i)$, he still believes that other retailers make their equilibrium purchases $\hat{x}_{-i}$, and he will accept the offer if and only if $P(\widehat{X}_{-i} + x_i)x_i \geq t_i$, where $\widehat{X}_{-i} = \sum_{j \neq i} \hat{x}_i$. Given this, the manufacturer's equilibrium sale $\hat{x}_i$ to retailer i must be *pairwise stable* in the sense that

$$(1) \qquad \hat{x}_i \in \arg\max_{x_i \geq 0} \left[P(\widehat{X}_{-i} + x_i)x_i - c(\widehat{X}_{-i} + x_i) \right].$$

Condition (1) says that it is impossible to increase the *bilateral surplus* between the manufacturer and any retailer i (retailer i's revenue less the manufacturer's

[4] Likewise, we do not allow contracts that are contingent on the retail price, which would indirectly condition on the manufacturer's sales to rivals; see Hart and Tirole (1990).

[5] Contracting outcomes would not change if the manufacturer could offer a menu from which the retailer would choose without observing other contracts (see Hart and Tirole (1990)).

costs) given the purchases of all other retailers. In the present parameterized example, the unique profile of pairwise-stable trades is the symmetric profile $x^p = (\widehat{X}_N^p/N, \ldots, \widehat{X}_N^p/N)$, with the aggregate quantity[6]

$$\widehat{X}_N^p = \frac{a-\alpha}{(1+1/N)b+\beta}.$$

Note that $\widehat{X}_1^p = X^*$ (the efficient quantity), $\widehat{X}_N^p < X^c$ (the competitive quantity) for all N, and $\widehat{X}_N^p \to X^c$ as $N \to \infty$ (so there is competitive convergence).

While the restriction to passive beliefs provides a simple story for the inefficiency of bilateral contracting and for competitive convergence, it has at least two problems. First, a passive-beliefs equilibrium need not exist. For instance, in the present example any passive-beliefs equilibrium involves an aggregate trade that converges to X^c as $N \to \infty$. But we have already seen that if $\beta < 0$ this would involve negative profits for the manufacturer, and so cannot be an equilibrium outcome.[7]

Second, and more fundamentally, in many circumstances the ad hoc restriction to passive beliefs may not be compelling. To take an extreme example, suppose the manufacturer has only $\overline{X}$ units for sale (i.e., $c(X) = \infty$ for $X > \overline{X}$); then a retailer who is offered $\overline{X}$ units can be sure that other retailers get none of the good, regardless of what the equilibrium allocation was supposed to be. More generally, for our payoff specifications with $\beta \neq 0$, retailers should be aware that the manufacturer's optimal contract offer to one retailer depends on her contracts with other retailers.

Once retailers are allowed to hold arbitrary beliefs after observing out-of-equilibrium offers, a large set of outcomes can be sustained in a weak perfect Bayesian equilibrium.[8] For example, as noted by McAfee and Schwartz (1994), the efficient outcome X^* can be sustained for any N by endowing retailers with *symmetry beliefs*, under which each retailer believes that the manufacturer offers the same contract to all retailers. Therefore, with symmetry beliefs, competitive convergence does *not* obtain. More generally, if each retailer i believes that $X_{-i} > a/b$ following any observed deviation, then when $\beta > 0$ we can support any outcome in which each retailer has a nonnegative payoff, and pays the manufacturer at least his incremental cost, $c(\widehat{X}) - c(\widehat{X}_{-i})$. For example, any aggregate quantity $\widehat{X} \leq X^c$ is sustainable for any N.

A key idea of this paper is that the manufacturer may be able to avoid the problem of negative inferences by retailers by using contracts that specify a *menu* of possible trades from which she herself later chooses. Here we illustrate this

[6] This coincides with the Cournot outcome for N retail firms, each facing marginal cost $c'(\widehat{X}_N^p)$.

[7] This illustrates the general point that a pairwise-stable trade profile may not be supportable in a passive-beliefs equilibrium due to the principal's deviation to several agents at once (here, the deviation is to null contracts). This point is also made by Segal (1999) and Rey and Verge (2002).

[8] This is also true for stronger solution concepts such as sequential equilibrium.

idea in the case where $\beta > 0$. Suppose that the manufacturer deviates from an equilibrium by offering retailer i a contract that gives the manufacturer the right to choose from the *competitive menu*

$$C = \{(x,t) : x \in [0, X^c], t = p^c x\}.$$

Observe that retailer i is *guaranteed a zero payoff if he accepts the offer, regardless of his belief about the aggregate quantity X_{-i} sold to other retailers*. Indeed, if $X_{-i} < X^c$, then the manufacturer will optimally choose $x_i = X^c - X_{-i}$ from the menu, and the retailer's payoff is $P(X^c)x_i - p^c x_i = 0$. On the other hand, if $X_{-i} \geq X^c$, then the manufacturer will choose $x_i = 0$ from the menu, and the retailer's payoff is again zero. By paying in addition an arbitrarily small amount $\varepsilon > 0$ to the retailer, the manufacturer can convince him to accept the menu regardless of his beliefs.

Allowing the manufacturer to deviate by offering menus dramatically constrains weak perfect Bayesian equilibrium outcomes.[9] To see this in our example, let $(\hat{x}_1, \ldots, \hat{x}_N, \hat{t}_1, \ldots, \hat{t}_N)$ be an equilibrium outcome of the offer game in which the manufacturer can offer menus, with $\widehat{X} = \sum_i \hat{x}_i \leq X^c$ (equilibria with an aggregate trade above X^c can be ruled out with similar arguments). Consider the manufacturer's deviation to retailer i with the competitive menu (plus an arbitrarily small fixed payment), while keeping her offers to other agents unchanged. As argued above, retailer i will accept this deviation, and the manufacturer will sell him $x_i = X^c - \widehat{X}_{-i}$. Note that the manufacturer's equilibrium revenue from retailer i was *at most* $P(\widehat{X})\hat{x}_i$ (otherwise the retailer would have rejected the equilibrium contract), and therefore the revenue loss on existing sales due to the deviation is at most $[P(\widehat{X}) - p^c]\hat{x}_i$. On the other hand, the manufacturer's revenue on the new sales is $p^c[X^c - \widehat{X}]$. Finally, she incurs the extra production cost $c(X^c) - c(\widehat{X})$. Adding up, we see that for the deviation not to be profitable we must have

$$p^c[X^c - \widehat{X}] - [P(\widehat{X}) - p^c]\hat{x}_i \leq c(X^c) - c(\widehat{X})$$

or equivalently

$$[p^c X^c - c(X^c)] - [p^c \widehat{X} - c(\widehat{X})] \leq [P(\widehat{X}) - p^c]\hat{x}_i. \tag{2}$$

This inequality implies competitive convergence as $N \to \infty$. Intuitively, the manufacturer can find a retailer i whose purchase $\hat{x}_i$ does not exceed $\widehat{X}/N$, and deviate to this retailer with the competitive menu. If N were large but $\widehat{X}$ were substantially below X^c, this deviation would be profitable, since the revenue loss on the existing sales to the retailer (the right-hand side of (2)) would be small,

[9] Observe that allowing menus cannot introduce *new* equilibrium outcomes in the offer game. Indeed, since in equilibrium the retailers have correct beliefs, any equilibrium outcome can be sustained with the manufacturer offering a point contract (x_i, t_i) to each retailer i on the equilibrium path.

while the gain from selling him an extra $X^c - \widehat{X}$ to achieve the manufacturer's optimal output X^c at price p^c (the left-hand side of (2)) would be substantial.

For a given aggregate quantity $\widehat{X}$, condition (2) is most easily satisfied by allocating the quantity $\widehat{X}$ equally among the retailers: $\hat{x}_i = \widehat{X}/N$ for each i. In our parameterized example this yields a quadratic inequality in $\widehat{X}$, whose solutions are given by

$$\widehat{X} \in [\underline{X}_N, X^c], \quad \text{where} \quad \underline{X}_N = \frac{a-\alpha}{(1+2b/(\beta N))(b+\beta)}.$$

In particular, the efficient (monopoly) aggregate quantity X^*, obtained with symmetry beliefs when the manufacturer could not offer menus, cannot be sustained now when $N > 2(1+b/\beta)$. On the other hand, the symmetric pairwise-stable aggregate trade $\widehat{X}_N^p$ satisfies the above condition for any N. Finally, observe that competitive convergence obtains: $\underline{X}_N \to X^c$ as $N \to \infty$.

Note that there exist many other menus that assure a retailer a nonnegative payoff. For example, consider *linear-price* menus, which give the manufacturer the right to sell to the retailer any quantity at a price p. The manufacturer's optimal choice given trades X_{-i} with other retailers can be stated with reference to her supply function $S(p) = \max\{(p-\alpha)/\beta, 0\}$: If $X_{-i} \geq S(p)$, the manufacturer will sell nothing to retailer i, and the retailer will receive zero profit. If instead $X_{-i} < S(p)$, the manufacturer will sell $x_i = S(p) - X_{-i} > 0$, which brings the aggregate sale to $S(p)$, and the retail price to $P(S(p))$. The retailer's profit will then be $[P(S(p)) - p]x_i$. This profit is nonnegative if and only if $P(S(p)) \geq p$, which in turn holds if and only if $p \leq p^c$; see Figure 2. Thus, any linear-price menu with price $p \leq p^c$ guarantees retailer i a nonnegative profit for any belief he could hold about X_{-i}. Conversely, any linear-price menu with price above p^c would entail a loss to the retailer for some belief about X_{-i}. Therefore, the *best* linear-price menu for the manufacturer among those that are acceptable to the retailer regardless of his beliefs is the competitive menu considered above. In fact, in Section 4 we show that in this setting the competitive menu is optimal for the

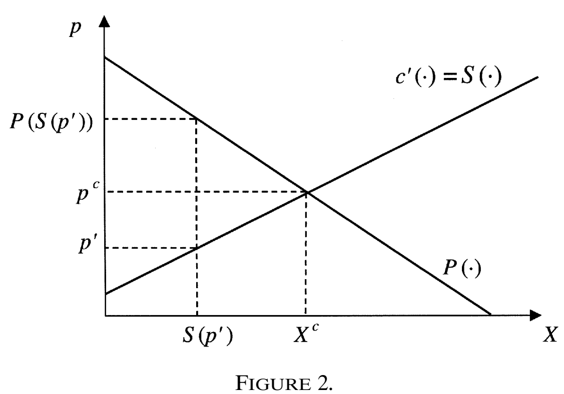

FIGURE 2.

manufacturer, whatever her trades with other retailers, among *all* (nonlinear) menus that are accepted by the retailer regardless of his belief about such trades. It is thus an RSW menu in the terminology of Maskin and Tirole (1992). In the next section, we consider a more general setting and show how deviations to menus, and in particular the RSW menu, bound the equilibrium outcomes in a large family of bilateral contracting games.

3. CHARACTERIZATION OF CONTRACTING OUTCOMES

We consider bilateral contracting games between one principal and N agents (N will also denote the set of agents).[10] The set of possible trades between the principal and agent i is $\mathscr{X}_i \subset \mathbb{R}_+$, and a typical trade will be represented by $x_i \in \mathscr{X}_i$. In addition, the agent can make a monetary transfer t_i to the principal. Let $0 \in \mathscr{X}_i$ denote the default trade between the principal and agent i. The default outcome between them is then $(x_i, t_i) = (0, 0)$.

The parties' utilities are quasilinear in money: the principal's payoff is $\sum_i t_i - c(x)$, and each agent i's payoff is $u_i(x) - t_i$, where $x = (x_1, \ldots, x_N) \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N$ is the agents' trade profile. We assume that the principal's cost $c(\cdot)$ is lower semi-continuous, which allows, e.g., for a fixed cost.[11] Externalities among agents arise because each agent's utility can depend on all agents' trades. We assume, however, that agents' *reservation utilities* do not depend on others' trades: $u_i(0, x_{-i}) = 0$ for all $x_{-i} \in \mathscr{X}_{-i}$. (In the terminology of Segal (1999), there are no externalities on nontraders.)

A bilateral contract between the principal and each agent i takes the form of a menu, i.e., a subset $M_i \subset \mathscr{X}_i \times \mathbb{R}$. After the contract is signed, the principal chooses a bundle $(x_i, t_i) \in M_i$. Her optimal choice in general depends on her contracts with other agents, which are not observed by agent i.[12] We restrict menus to be compact sets, to ensure that for any collection of menus $(M_1, \ldots, M_N)$, the principal has an optimal choice: $\arg\max_{(x,t)\in M_1\times\cdots\times M_N} [\sum_i t_i - c(x)] \neq \varnothing$.[13]

We consider the following class of *bilateral contracting games*: A game lasts for K periods. In each period $k = 1, \ldots, K$, a subset $A_k \subset N$ of agents simultaneously offer (compact) menus to the principal, and simultaneously the principal offers (compact) menus to a subset $P_k \subset N$ of agents (with $P_k \cap A_k = \varnothing$). Then the principal and agents simultaneously decide whether to accept contracts offered to them. The game then proceeds to the next period. We assume that

[10] Note that this reverses the standard terminology in the literature on bidding (common agency) games, where the single party receiving offers is called the "agent" and the parties making offers are "principals." Our terminology achieves consistency across different bargaining processes among these parties.

[11] A function $c(\cdot)$ is lower semi-continuous if $x^n \to x$ implies that $\liminf_{n\to\infty} c(x^n) \geq c(x)$.

[12] We could allow more complicated contracts in which agent i sends messages as well as the principal, affecting (x_i, t_i). However, since agents possess no private information, this would not affect the set of equilibrium outcomes.

[13] If a profile of menus from which the principal has no optimal choice could be offered in a bilateral contracting game, then there would be no continuation equilibrium following acceptance of these menus; hence the game would not have a weak perfect Bayesian equilibrium.

$\bigcup_{k=1}^{K}(P_k \cup A_k) = N$, i.e., each agent has at least one chance to contract with the principal. The principal observes the entire history, while each agent observes only menus offered to him and the principal's accept/reject decision for menus that he offers the principal. At the end of the game, for each agent i the principal chooses (x_i, t_i) from the last contract accepted with this agent (which supercedes all previous contracts), and $(x_i, t_i) = (0, 0)$ if no contract has been agreed to.[14] The parties then receive payoffs on the basis of this outcome (there is no discounting). Two examples of bilateral contracting games are given by the offer and bidding games discussed in Section 2. The solution concept we adopt is that of pure-strategy weak Perfect Bayesian equilibrium (henceforth "WPBE").[15]

We will examine some properties that all WPBE outcomes $(\hat{x}, \hat{t})$ of any bilateral contracting game must satisfy. It is clear that any such outcome must satisfy the agents' individual rationality constraints:

$$\text{(AIR)} \quad u_i(\hat{x}) - \hat{t}_i \geq 0 \qquad \text{for all } i \in N.$$

Indeed, if (AIR) were violated for some agent i, he could profitably deviate by rejecting all of the principal's offers and always offering her the menu containing only the null contract $(0, 0)$.

We now identify another necessary condition on WPBE outcomes of bilateral contracting games. This condition obtains by considering deviations using menus that guarantee an agent a nonnegative payoff regardless of his belief about the principal's trades with other agents, given that he expects the principal to choose optimally from this menu:

Definition 1: A menu $M_i \subset \mathscr{X}_i \times \mathbb{R}$ is *acceptable to agent i* if for any $\bar{x}_{-i} \in \mathscr{X}_{-i}$ and any $(\bar{x}_i, \bar{t}_i) \in \arg\max_{(x_i, t_i) \in M_i}[t_i - c(x_i, \bar{x}_{-i})]$ we have $u_i(\bar{x}_i, \bar{x}_{-i}) - \bar{t}_i \geq 0$. The set of compact menus that are acceptable to agent i is denoted by $\mathscr{A}_i \subset 2^{\mathscr{X}_i \times \mathbb{R}}$.

For example, consider the offer game, and imagine the principal deviating by offering menu $M_i \in \mathscr{A}_i$ to agent i (plus an arbitrarily small payment to ensure acceptance), while following her equilibrium contracting with other agents. The principal's maximum profit from this deviation is[16]

$$\Pi_i^{M_i}(\hat{x}_{-i}) \equiv \max_{(x_i, t_i) \in M_i} [t_i - c(x_i, \hat{x}_{-i})].$$

For the deviation not to be profitable, we must have

$$\hat{t}_i - c(\hat{x}) \geq \Pi_i^{M_i}(\hat{x}_{-i}).$$

[14] Our results would not be affected if the principal could choose from menus immediately upon their acceptance rather than at the end of the game.

[15] See Mas-Colell, Whinston, and Green (1995). Our results would be unaffected by adopting instead stronger notions of perfect Bayesian equilibrium.

[16] We define this profit not including the (fixed) transfer from agents $j \neq i$, $\sum_{j \neq i} \hat{t}_j$.

Summing this inequality with agent i's (AIR), we see that any WPBE trade profile of the offer game must satisfy

$$u_i(\hat{x}) - c(\hat{x}) \geq \Pi_i^{M_i}(\hat{x}_{-i}) \qquad \text{for all } i \in N.$$

This inequality bounds from below the bilateral surplus $u_i(\hat{x}) - c(\hat{x})$ of the principal and each agent i.

Observe, moreover, that the above inequality must be satisfied by a WPBE trade profile in *any* bilateral contracting game. Indeed, if it were violated, menu M_i would increase the bilateral surplus of the principal and agent i by giving the agent a payoff of at least 0 and the principal a payoff of at least $\Pi_i^{M_i}(\hat{x}_{-i})$ (since she has the option of leaving her trades with other agents unchanged). Then either the principal or agent i could deviate by offering M_i to the other, combined with a lump-sum transfer chosen to make each of them better off. A formalization of this argument yields the following result.

PROPOSITION 1: *Any pure-strategy WPBE trade profile $\hat{x}$ of a bilateral contracting game must satisfy the "menu deviation" condition*

$$\text{(MD)} \qquad u_i(\hat{x}) - c(\hat{x}) \geq \sup_{M_i \in \mathcal{A}_i} \Pi_i^{M_i}(\hat{x}_{-i}) \qquad \textit{for all } i \in N.$$

PROOF: Suppose in negation that there exists an equilibrium with outcome $(\hat{x}, \hat{t})$ in which for some $i \in N$ and some $M_i \in \mathcal{A}_i$,

$$(3) \qquad u_i(\hat{x}) - c(\hat{x}) < \Pi_i^{M_i}(\hat{x}_{-i}).$$

Let $\bar{k}$ be the largest k for which $i \in P_k \cup A_k$ (i.e., the last period in which the principal and agent i contract).

Suppose first that $i \in P_{\bar{k}}$. Consider a deviation by the principal in contracting with agent i in which she rejects all of agent i's offers, offers the null contract $\{(0, 0)\}$ to the agent in periods $k < \bar{k}$, and offers him M_i plus a small payment $\varepsilon > 0$ in period $k = \bar{k}$. Agent i will accept this contract for any beliefs about the principal's contracting with other agents. Suppose that the principal uses her equilibrium strategy in contracting with other agents. Then her payoff from the deviation is

$$\sum_{j \neq i} \hat{t}_j + \Pi_i^{M_i}(\hat{x}_{-i}) - \varepsilon > \sum_{j \neq i} \hat{t}_j + u_i(\hat{x}) - c(\hat{x}) \geq \sum_j \hat{t}_j - c(\hat{x}),$$

where the first inequality is by (3), and the second by agent i's (AIR). Thus, the deviation makes the principal better off.

Suppose instead that $i \in A_{\bar{k}}$. Consider a deviation by the agent in which he uses the equilibrium strategy in periods $k < \bar{k}$ and offers M_i in period $\bar{k}$ minus a payment

$$\Delta = \Pi_i^{M_i}(\hat{x}_{-i}) - [\hat{t}_i - c(\hat{x})] - \varepsilon,$$

where $\varepsilon > 0$ is small. By construction, this deviation makes the principal better off than in equilibrium even if she continues to use her equilibrium strategy with other agents. (She may do even better by changing her strategies with other agents.) Since rejecting the agent's deviation cannot make the principal better off than in equilibrium (this option was available to her in equilibrium), she will strictly prefer to accept it. Since M_i is acceptable, the agent's payoff from this deviation will be at least Δ for any contracts ultimately signed by the principal with other agents. Condition (3) implies that $\Delta > u_i(\hat{x}) - \hat{t}_i$ when ε is sufficiently small; hence the agent's deviation is profitable. *Q.E.D.*

In principle, computing the right-hand side of (MD) involves identifying a distinct profit-maximizing compact acceptable menu for each type x_{-i} (assuming that such an optimal menu exists—i.e., that the supremum is actually attained). It turns out, however, that there often exists a *single* compact acceptable menu that maximizes the principal's profit for *every* type x_{-i} that she may have. We call it an *RSW menu*, in accord with the terminology introduced by Maskin and Tirole (1992):

DEFINITION 2: A menu $R_i \in \mathscr{A}_i$ is an *RSW menu* for agent i if

$$\Pi_i^{R_i}(x_{-i}) = \sup_{M_i \in \mathscr{A}_i} \Pi_i^{M_i}(x_{-i}) \qquad \text{for all } x_{-i} \in \mathscr{X}_{-i}.$$

That the existence of an RSW menu need not be an unlikely coincidence is suggested by the following observation:

LEMMA 1: *The union of acceptable menus is an acceptable menu.*

PROOF: Suppose $M = \bigcup_{s \in S} M_s$, where M_s is an acceptable menu to agent i for each $s \in S$. Take any $\bar{x}_{-i} \in \mathscr{X}_{-i}$, and any $(\bar{x}_i, \bar{t}_i) \in \arg\max_{(x_i, t_i) \in M}[t_i - c(x_i, \bar{x}_{-i})]$. By construction, $(\bar{x}_i, \bar{t}_i) \in M_s$ for some $s \in S$, and $(\bar{x}_i, \bar{t}_i) \in \arg\max_{(x_i, t_i) \in M_s}[t_i - c(x_i, \bar{x}_{-i})]$ because $M_s \subset M$. Since M_s is acceptable, we must have $u_i(\bar{x}_i, \bar{x}_{-i}) - \bar{t}_i \geq 0$. *Q.E.D.*

Since the principal's payoff can only increase when she chooses from a larger menu, Lemma 1 suggests that a natural candidate for an RSW menu is the union $\Omega_i \equiv \bigcup_{M_i \in \mathscr{A}_i} M_i$ of all compact acceptable menus to agent i. Indeed, if Ω_i is compact, then

$$\Pi_i^{\Omega_i}(x_{-i}) = \max_{(x_i, t_i) \in \Omega_i}[t_i - c(x_i, x_{-i})] = \sup_{M_i \in \mathscr{A}_i} \max_{(x_i, t_i) \in M_i}[t_i - c(x_i, x_{-i})]$$
$$= \sup_{M_i \in \mathscr{A}_i} \Pi_i^{M_i}(x_{-i}).$$

A technical complication may arise when the menu Ω_i is not compact, in which case the principal may not have an optimal choice from it. Indeed, there is usually an infinite number of compact acceptable menus, so the union of these menus need not be compact. In our cases of interest, however, there will always exist a compact RSW menu $R_i \subset \Omega_i$.[17] We will find it convenient to use the smallest RSW menu, obtained by eliminating from Ω_i all the bundles that are never optimal for the principal.

We note the relation of condition (MD) to pairwise stability. A trade profile $\hat{x}$ is *pairwise stable* if it maximizes the bilateral surplus of each principal-agent pair; that is,

$$\hat{x}_i \in \arg\max_{x_i \in \mathscr{X}_i} [u_i(x_i, \hat{x}_{-i}) - c(x_i, \hat{x}_{-i})] \qquad \text{for each } i \in N.$$

PROPOSITION 2: *Any pairwise-stable trade profile $\hat{x}$ satisfies* (*MD*).

PROOF: Suppose in negation that there exist $i \in N$ and $M_i \in \mathscr{A}_i$ such that

$$\Pi_i^{M_i}(\hat{x}_{-i}) > u_i(\hat{x}) - c(\hat{x}).$$

Taking $(\tilde{x}_i, \tilde{t}_i) \in \arg\max_{(x_i,t_i)\in M_i}[t_i - c(x_i, \hat{x}_{-i})]$, the inequality means

$$\tilde{t}_i - c(\tilde{x}_i, \hat{x}_{-i}) > u_i(\hat{x}) - c(\hat{x}).$$

Since M_i is acceptable, $u_i(\tilde{x}_i, \hat{x}_{-i}) - \tilde{t}_i \geq 0$. Adding to the above inequality, we obtain

$$u_i(\tilde{x}_i, \hat{x}_{-i}) - c(\tilde{x}_i, \hat{x}_{-i}) > u_i(\hat{x}) - c(\hat{x}),$$

which contradicts pairwise stability. *Q.E.D.*

Intuitively, pairwise stability is stronger than condition (MD), because it requires that the bilateral surplus of any principal-agent pair cannot be increased by *any* deviation, while condition (MD) restricts deviations to be acceptable menus. Thus, despite our criticisms of passive beliefs, the resulting pairwise stability condition is compatible with our condition (MD). However, some trade profiles that are not pairwise stable may also satisfy condition (MD), because some deviations that increase bilateral surplus may be rejected due to an agent's adverse beliefs about the principal's trades with other agents.

We now ask whether conditions (AIR) and (MD) are *sufficient* for a trade profile $\hat{x}$ to emerge in a given bilateral contracting game. The answer in general is no, but the reasons for additional restrictions on equilibrium trade profiles

[17] Were this not the case, we would proceed by approximating the RSW profit arbitrarily closely using compact acceptable subsets of Ω_i. Maskin and Tirole (1992) perform such approximation in a model with finite domains $\mathscr{X}_i$. The RSW profit in their model is achieved by a menu on the boundary of Ω_i, which is not acceptable because not all of the principal's optimal choices satisfy the agent's participation constraint. They then construct an arbitrarily close compact acceptable menu in which the principal's optimal choices are unique.

differ from game to game.[18] Here we illustrate this point by characterizing the WPBE outcomes of the offer game, which can be done using RSW menus. We discuss bidding games in Section 7.

In the offer game, even when the principal's deviation offering an acceptable menu to one agent is not profitable, a deviation in which she offers acceptable menus to several agents at once may be. The principal's supremum profit from such a deviation to a set $D \subset N$ of agents is given by

$$\overline{\Pi}_D(\hat{x}_{-D}) \equiv \sup_{M_i \in \mathcal{A}_i \forall i \in D} \max_{(x_D, t_D) \in \prod_{i \in D} M_i} \left[\sum_{i \in D} t_i - c(x_D, \hat{x}_{-D})\right].$$

Note that when an RSW menu R_i exists for each agent i, this profit can be computed as

$$\overline{\Pi}_D(\hat{x}_{-D}) = \max_{(x_D, t_D) \in \prod_{i \in D} R_i} \left[\sum_{i \in D} t_i - c(x_D, \hat{x}_{-D})\right]. \tag{4}$$

Immunity to such multilateral deviations by the principal, along with the agents' participation constraints, fully characterizes the WPBE outcomes of the offer game:

PROPOSITION 3: *Suppose that $u_i(\cdot)$ is bounded below for all i. Then $(\hat{x}, \hat{t})$ is a WPBE outcome of the offer game if and only if the agents' individual rationality constraints (AIR) hold, and the following "multilateral menu deviation" condition holds:*

(MMD) $\sum_{i \notin D} \hat{t}_i - c(\hat{x}) \geq \overline{\Pi}_D(\hat{x}_{-D})$ *for all* $D \subset N$.

PROOF: *Necessity:* If (AIR) did not hold, an agent would deviate by rejecting the principal's offer. If (MMD) did not hold for some $D \subset N$, the principal could profitably deviate by offering each agent $i \in D$ a menu $M_i \in \mathcal{A}_i$ plus a small payment, while following her equilibrium strategies with all other agents. All agents from D would accept, and the principal's payoff could be arbitrarily close to $\sum_{i \notin D} \hat{t}_i + \overline{\Pi}_D(\hat{x}_{-D}) > \sum_i \hat{t}_i - c(\hat{x})$.

Sufficiency: A WPBE sustaining $(\hat{x}, \hat{t})$ is described by the following strategies and beliefs. The principal's strategy is to offer the point menu $\widehat{M}_i = \{(\hat{x}_i, \hat{t}_i)\}$ to each agent i. Following any accepted menu profile $(M_1, \ldots, M_N)$ in which only one agent, say j, accepts a nondegenerate menu, the principal chooses from among those elements of $(x, t) \in \arg\max_{(x,t) \in \prod_i M_i} [\sum_i t_i - c(x)]$ that have the lowest values of $u_j(x) - t_j$.[19] Each agent i's strategy is to accept $\widehat{M}_i$ and all menus

[18] One can in fact derive some additional restrictions that all WPBE's of all bilateral contracting games must satisfy. For example, the parties' individual rationality constraints rule out any trade profile with negative surplus. This rules out the pairwise stable trade profile in the example of Section 2 when $\beta < 0$ and N is large, even though it satisfies (MD) by Proposition 2.

[19] We can specify any optimal choices by the principal for all other profiles of accepted contracts.

from $\mathscr{A}_i$, and to reject all other menus. As for agent i's beliefs following an offer $M_i \neq \widehat{M}_i$, for any menu $M_i \notin \mathscr{A}_i$ we can find $\bar{x}_{-i}(M_i) \in \mathscr{X}_{-i}$ such that there exists $(x_i', t_i') \in \arg\max_{(x_i, t_i) \in M_i}[t_i - c(x_i, \bar{x}_{-i}(M_i))]$ with $u_i(x_i', \bar{x}_{-i}(M_i)) - t_i' < 0$. Following any menu $M_i \neq \widehat{M}_i$ such that $M_i \notin \mathscr{A}_i$, let agent i believe that the principal's offer to each other agent $j \neq i$ is $M_j = \{(\bar{x}_j(M_i), \bar{t}_j)\}$, where $\bar{t}_j < \inf_{x \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N} u_j(x)$ for all j. (Observe that $M_j \in \mathscr{A}_j$, so that such an offer will be accepted by agent j.)

Conditions (AIR) ensure that agents have no profitable deviations. The principal's deviation to any menu $M_i \notin \mathscr{A}_i$ will be rejected by agent i given his beliefs and the principal's strategy. The principal's deviation to any compact acceptable menus (including the null contract) with agents from $D \subset N$ cannot give her a higher payoff than $\sum_{i \notin D} \hat{t}_i + \overline{\Pi}_D(\hat{x}_{-D})$, which by (MMD) does not exceed her equilibrium payoff. *Q.E.D.*

This section's results can be illustrated in the case without externalities, i.e., where $u_i(x_i, x_{-i}) \equiv u_i(x_i)$. In this case, agent i will accept a menu M_i if and only if the principal's optimal choices (x_i, t_i) from this menu for any x_{-i} satisfy $u_i(x_i) - t_i \geq 0$. Eliminating the principal's suboptimal choices, we see that $\{(x_i, u_i(x_i)) : x_i \in \mathscr{X}_i\}$ is an RSW menu. This menu, in which agent i "sells out" to the principal, is similar to "truthful" menus considered by Bernheim and Whinston (1986b).[20] Condition (MD) is then equivalent to pairwise stability, saying that the bilateral surplus of each principal-agent pair is maximized given the principal's trades with other agents. In general, even without externalities, this does not imply that the total surplus $\sum_i u_i(\hat{x}) - c(\hat{x})$ is maximized, since there may be "coordination failures" among different principal-agent pairs. Bernheim and Whinston (1986b) note that such coordination failures are common in the bidding game. As for the offer game, adding condition (MMD) for $D = N$ and all agents' (AIR) implies that any WPBE trade profile $\hat{x}$ must maximize total surplus (Segal (1999, Proposition 3) offers a similar result). Thus, coordination failures are eliminated when the principal can resort to multilateral deviations.

4. A (STRICT) COMPETITIVE EQUILIBRIUM EXISTS

In the remainder of the paper we apply the results of Section 3 to a particular class of payoff functions with externalities. In particular, we suppose that there are $N \geq 2$ agents, $\mathscr{X}_i = \mathbb{R}_+$ for each agent i, and his payoff is $u_i(x_i, x_{-i}) - t_i = \alpha(X)x_i - t_i$, where $X = \sum_{i \in N} x_i$. The principal's payoff is $\sum_{i \in N} t_i - c(X)$.[21] We assume that $\alpha(\cdot)$ is bounded below and that $c(\cdot)$ is lower semicontinuous. We also normalize $c(0) = 0$.

[20] To relate our model formally to Bernheim and Whinston's (1986b), let each x_i represent the principal's promise of the whole public decision, and let $c(x)$ be prohibitively high unless $x_1 = \cdots = x_N$. Note the absence of externalities in this model.

[21] These payoffs are described by Segal's (1999) Condition L and the absence of externalities on nontraders.

Note that this setting includes as a special case the vertical contracting example discussed in Section 2, studied by Hart and Tirole (1990) and McAfee and Schwartz (1994). Another model that belongs to this class is the insurance with moral hazard model studied by Pauly (1974), Kahn and Mookherjee (1998), Bisin, Gottardi, and Guaitoli (1999), and Bisin and Guaitoli (2000) (see Segal (1999) for other examples). In the insurance model, a single risk-averse individual with constant absolute risk aversion $r \geq 0$ and initial wealth W (the principal in our terminology) may contract with any of N risk-neutral firms. The individual chooses the unobservable probability $\pi \in \Pi$ of a loss $L > 0$ at a private cost $\gamma(\pi)$. Letting $x_i \geq 0$ denote the payment promised by agent i in the event of a loss, the certainty equivalent of the principal's payoff can be written as

$$W + \sum_i t_i - \gamma(\pi) - \frac{1}{r} \ln[1 + \pi \cdot (e^{-r(X-L)} - 1)].$$

The principal chooses the probability of loss $\pi(X)$ to maximize this expression. By the Monotone Selection Theorem of Milgrom and Shannon (1994), $\pi(X)$ is nondecreasing in X. The payoff of each agent i is then $-\pi(X)x_i - t_i$.

We use the following notion of competitive equilibrium:

DEFINITION 3: A price-quantity pair $(p^c, X^c) \in \mathbb{R} \times \mathbb{R}_+$ is a *strict competitive equilibrium* if (i) $\arg\max_{X \in \mathbb{R}_+} [p^c X - c(X)] = \{X^c\}$, and (ii) $p^c = \alpha(X^c)$.

Condition (i) says that X^c is the principal's *unique* optimal trade given price p^c (this uniqueness is indicated by the modifier "strict"). Condition (ii) ensures that agents are willing to demand an aggregate quantity of X^c if each agent takes price p^c and aggregate trade X^c as given. In this section, we assume that a strict competitive equilibrium exists. Figure 1 in Section 2 depicts a strict competitive equilibrium in the vertical contracting example, while Figure 3 depicts such an equilibrium in the insurance with moral hazard model when $\Pi = \{\underline{\pi}, \bar{\pi}\}$ (note that *area(B)* must be at least as large as *area*(A)).

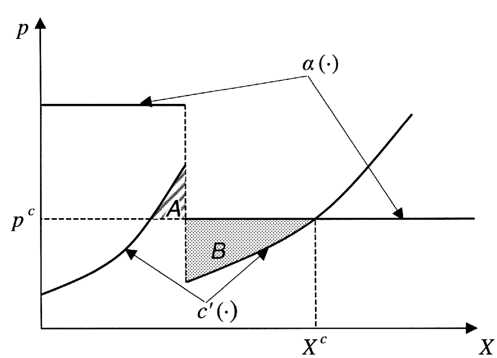

FIGURE 3.

We begin by establishing a "competitive limit" result as the number of agents grows infinitely large. We do this using Proposition 1, by requiring immunity to deviations involving one particular acceptable menu—the *competitive menu* $C = \{(x, p^c x) : x \in [0, X^c]\}$. Faced with this menu, the principal whose aggregate trade with other agents is $X_{-i} \leq X^c$ will choose $x_i = X^c - X_{-i}$. We also ensure that the principal with $X_{-i} > X^c$ chooses $x_i = 0$, by assuming that $p^c X - c(X)$ is strictly decreasing for $X \geq X^c$. Then agent i receives a zero payoff in both cases, so C is indeed an acceptable menu. Given this fact, Proposition 1 bounds from below the bilateral surplus of the principal and agent i in any WPBE outcome $(\hat{x}, \hat{t})$ of a bilateral contracting game:

$$\alpha(\widehat{X})\hat{x}_i - c(\widehat{X}) \geq \Pi^C(\widehat{X}_{-i}) = \begin{cases} p^c(X^c - \widehat{X}_{-i}) - c(X^c) & \text{when } \widehat{X}_{-i} \leq X^c, \\ -c(\widehat{X}_{-i}) & \text{otherwise.} \end{cases} \tag{5}$$

In the example in Section 2 we noted that this bound imposes a significant restriction on the set of equilibrium outcomes. Here we show more generally that when a strict competitive equilibrium exists, under mild additional assumptions this condition *completely determines* the equilibrium aggregate trade as the number of agents N grows large:

PROPOSITION 4: *Suppose that*:

(i) (p^c, X^c) *is a strict competitive equilibrium*,
(ii) $p^c X - c(X)$ *is strictly decreasing for* $X \geq X^c$, *and*
(iii) $\alpha(X) \leq p^c$ *for all* $X \geq X^c$.

If $\{\widehat{X}^N\}_{N=1}^{\infty}$ *is a sequence of WPBE aggregate trades in a sequence of bilateral contracting games with* N *agents*, *then*:

(a) $\widehat{X}^N \leq X^c$ *for all* N,
(b) *if the aggregate surplus* $W(X) = \alpha(X)X - c(X)$ *is bounded above on* $X \in \mathbb{R}_+$, *then* $\widehat{X}^N \to X^c$ *as* $N \to \infty$.

PROOF: Suppose in negation that (a) does not hold. Consider an agent i with $\hat{x}_i^N > 0$. By (iii),

$$\alpha(\widehat{X}^N)\hat{x}_i^N - c(\widehat{X}^N) \leq p^c \hat{x}_i^N - c(\widehat{X}^N).$$

If $\widehat{X}_{-i}^N > X^c$, then the right-hand side is less than $-c(\widehat{X}_{-i}^N)$ by (ii), which contradicts (5). If instead $\widehat{X}_{-i}^N \leq X^c$, then the right-hand side is less than $p^c(X^c - \widehat{X}_{-i}^N) - c(X^c)$ by (i), which again contradicts (5). Therefore, (a) must hold.

Part (a) implies that for each i, $\widehat{X}_{-i}^N \leq \widehat{X}^N \leq X^c$ and consequently $\Pi^C(\widehat{X}_{-i}^N) = p^c(X^c - \widehat{X}_{-i}^N) - c(X^c)$. Adding up (5) over $i \in N$ yields:

$$\alpha(\widehat{X}^N)\widehat{X}^N - Nc(\widehat{X}^N) \geq p^c(NX^c - (N-1)\widehat{X}^N) - Nc(X^c). \tag{6}$$

Dividing by $N-1$, the inequality can be rewritten as

$$\begin{aligned} p^c\widehat{X}^N - c(\widehat{X}^N) &\geq \frac{N}{N-1}[p^c X^c - c(X^c)] - \frac{1}{N-1}W(\widehat{X}^N) \\ &\geq \frac{N}{N-1}[p^c X^c - c(X^c)] - \frac{1}{N-1}\sup_{X\in\mathbb{R}_+} W(X) \\ &\to p^c X^c - c(X^c) \quad \text{as} \quad N\to\infty. \end{aligned}$$

Part (b) follows since $c(\cdot)$ is lower semi-continuous and $\arg\max_{X\in\mathbb{R}_+}[p^c X - c(X)] = X^c$. *Q.E.D.*

In fact, under a mild additional assumption, the competitive menu is an RSW menu:

LEMMA 2: *Suppose that*:

(i) (p^c, X^c) *is a strict competitive equilibrium*,
(ii) $c'(\cdot)$ *exists and is bounded*, $c'(X) > p^c$ *for* $X > X^c$, *and there exists* $\overline{X} < X^c$ *such that* $c'(X) < p^c$ *for* $X \in [\overline{X}, X^c)$, *and*
(iii) $\alpha(X) \leq p^c$ *for* $X \geq X^c$.

Then the competitive menu C *is an RSW menu*.

PROOF: See Appendix A.

The Lemma is proven using the first-order approach to incentive-compatibility. To illustrate the idea behind the proof, let us take for granted that there exists a piecewise differentiable RSW menu R_i to agent i and that in this menu the principal of each type X_{-i} captures all of the bilateral surplus, setting the agent's utility to zero. Letting $(x_i(X_{-i}), t_i(X_{-i}))$ denote the principal's optimal choice from this menu, we can then write her profit as

$$\Pi_i^{R_i}(X_{-i}) = x_i(X_{-i})\alpha(x_i(X_{-i}) + X_{-i}) - c(x_i(X_{-i}) + X_{-i}).$$

On the other hand, $\Pi_i^{R_i}(X_{-i})$ is the principal's value function when choosing from menu R_i, whose derivative by the Envelope Theorem can be calculated holding her optimal choice fixed: $d\Pi_i^{R_i}(X_{-i})/dX_{-i} = -c'(x_i(X_{-i}) + X_{-i})$. Comparing with the derivative of the above expression, we obtain

$$\begin{aligned} -c'(x_i(X_{-i})+X_{-i}) = {}& x_i'(X_{-i})\alpha(x_i(X_{-i})+X_{-i}) - \{x_i'(X_{-i})+1\} \\ & \times [x_i(X_{-i})\alpha'(x_i(X_{-i})+X_{-i}) - c'(x_i(X_{-i})+X_{-i})]. \end{aligned}$$

This is a differential equation for the function $x_i(X_{-i})$. An initial condition for this differential equation is provided by observing that we must have $x_i(X_{-i}) = 0$ for $X_{-i} \geq X^c$, since otherwise the profit of the principal of type X_{-i} would be negative. This pins down the unique solution $x_i(X_{-i}) = X^c - X_{-i}$ for $X_{-i} < X^c$, which is implemented by the competitive menu.

Lemma 2 implies that condition (5) represents the tightest bound obtainable using Proposition 1. The lemma also allows us to fully characterize the WPBE outcomes of the offer game using Proposition 3. While in general the characterization in Proposition 3 involves the principal's multilateral deviations, it turns out that in the present setting it suffices to consider unilateral deviations to the RSW (competitive) menu:

PROPOSITION 5: *Under the assumptions of Lemma* 2, $\hat{x} \in \mathbb{R}^N_+$ *is sustainable as a WPBE trade profile in the offer game if and only if it satisfies condition* (5).

PROOF: The necessity of (5) obtains by Proposition 1. For sufficiency, note first that (5) implies $\widehat{X} \leq X^c$ by Proposition 4(a) (whose assumptions follow from those of Lemma 2). Next, since C is an RSW menu, by (4),

$$\overline{\Pi}_D(\hat{x}_{-D}) = \pi^c - p^c(\widehat{X} - \widehat{X}_D), \tag{7}$$

where $\pi^c = p^c X^c - c(X^c)$. Summing (5) over agents $i \in D$ yields

$$\begin{aligned}\sum_{i \in D} \alpha(\widehat{X})\hat{x}_i - |D| c(\widehat{X}) &\geq |D|\big[\pi^c - p^c \widehat{X}\big] + p^c \widehat{X}_D \\ &= \big[\pi^c - p^c(\widehat{X} - \widehat{X}_D)\big] + (|D| - 1)\big(\pi^c - p^c \widehat{X}\big).\end{aligned}$$

Since $(\pi^c - p^c \widehat{X}) \geq -c(\widehat{X})$, this implies

$$\sum_{i \in D} \alpha(\widehat{X})\hat{x}_i - c(\widehat{X}) \geq \pi^c - p^c(\widehat{X} - \widehat{X}_D) = \overline{\Pi}_D(\hat{x}_{-D}).$$

Therefore, letting $\hat{t}_i = \alpha(\widehat{X})\hat{x}_i$ for each i, the outcome $(\hat{x}, \hat{t})$ satisfies conditions (AIR) and (MMD), and Proposition 3 implies that it is a WPBE outcome of the offer game. *Q.E.D.*

Intuitively, the result holds because once the principal has access to a choice from menu C with one agent, having choices from menu C with other agents has no incremental value for her. Since condition (5) is satisfied by both competitive and pairwise-stable trade profiles (the former trivially, the latter by Proposition 2), Proposition 5 implies the following corollary.

COROLLARY 1: *Under the assumptions of Lemma* 2, *any pairwise-stable trade profile* $\hat{x} \in \mathbb{R}^N_+$, *and any trade profile* $\hat{x} \in \mathbb{R}^N_+$ *such that* $\sum_i \hat{x}_i = X^c$, *is sustainable in a WPBE of the offer game*.

In the example in Section 2, the set of aggregate trades satisfying condition (5) is the interval $[\underline{X}_N, X^c]$, which contains both the pairwise-stable aggregate trade $\widehat{X}^p_N$ and the competitive aggregate trade X^c.

5. NONINCREASING MARGINAL COST

We now consider a situation in which the marginal cost $c'(\cdot)$ is nonincreasing, and so a strict competitive equilibrium does not exist. (This covers the case in which marginal cost is constant and a competitive equilibrium exists, but is not strict.) We show that in this case, under a mild additional assumption, allowing menus does not restrict the equilibrium set, because the null contract is an RSW menu:

LEMMA 3: *Suppose that*:

(i) $c'(\cdot)$ *exists and is nonincreasing*,
(ii) *there exists* $\overline{X} \geq 0$ *such that for* $X \geq \overline{X}$, $\alpha(X)$ *is nonincreasing and* $\alpha(X) \leq c'(X)$.

Then the null contract $Z = \{(0,0)\}$ *is an RSW menu*.

PROOF: See Appendix A.

Given this result, condition (MD) reduces to the inequality $\alpha(\widehat{X})\hat{x}_i \geq c(\widehat{X}) - c(\widehat{X}_{-i})$: each agent's gross payoff must at least equal the incremental cost of providing $\hat{x}_i$ units to him. In the offer game, the principal can deviate by offering the null contract to any subset of agents, hence any such subset must be "subsidy-free," in the sense of generating a nonnegative surplus (Faulhaber (1975)). It turns out, however, that with nonincreasing marginal cost any trade profile in which the total surplus is nonnegative can be made subsidy-free and so can be sustained in the offer game:

PROPOSITION 6: *Suppose the assumptions of Lemma* 3 *hold*. *Then* $\hat{x} \in \mathbb{R}^N_+$ *is sustainable as a WPBE trade profile in the offer game if and only if*

$$\alpha(\widehat{X})\widehat{X} \geq c(\widehat{X}). \tag{8}$$

PROOF: Necessity is obvious. For sufficiency, since nonincreasing marginal cost implies nonincreasing average cost, (8) implies that for any $D \subset N$,

$$\frac{c(\widehat{X}) - c(\widehat{X}_{-D})}{\widehat{X}_D} \leq \frac{c(\widehat{X})}{\widehat{X}} \leq \alpha(\widehat{X}).$$

Thus, letting $\hat{t}_i = \alpha(\widehat{X})\hat{x}_i$ for all i, we satisfy not only (AIR), but also condition (MMD) for the case where the RSW menu is a null menu. Proposition 3 then yields the result. *Q.E.D.*

6. A COMPETITIVE QUASI-EQUILIBRIUM EXISTS

In this section, we consider a class of situations in which a competitive equilibrium does not exist due to nonconvexities, yet deviations to menus do restrict the set of equilibrium outcomes. Specifically, we assume that the following modification of competitive equilibrium does exist:

DEFINITION 4: A price-quantity pair $(p^c, X^c) \in \mathbb{R} \times \mathbb{R}_+$ is a *strict competitive quasi-equilibrium* relative to $\widetilde{X}$ if (i) $p^c X - c(X) < p^c \widetilde{X} - c(\widetilde{X}) = p^c X^c - c(X^c)$ for all $X \in (\widetilde{X}, X^c)$, and (ii) $p^c = \alpha(X^c)$.

Part (i) of the definition means that when the principal faces price p^c, she will choose aggregate quantity X^c if forced to choose $X > \widetilde{X}$, and she is indifferent between $\widetilde{X}$ and X^c. Figures 4(a) and (b) depict cases in which (p^c, X^c) is a strict competitive quasi-equilibrium in the vertical contracting example and the insurance example where the principal has a binary moral hazard choice $\pi \in \{\underline{\pi}, \bar{\pi}\}$. By the definition of $\widetilde{X}$, in both figures $area(A) = area(B)$. In the insurance example in Figure 4(b), considered by Kahn and Mookherjee (1998) and Bisin and Guaitoli (2000), the strict competitive quasi-equilibrium price is actuarially fair given the low level of care when the individual has insurance level X^c, but at this price the individual would prefer not to buy any insurance. Similar situations occur in the lending models of Rajan and Parlour (2001), deMarzo and Bizer (1993), and Dubey, Geanakoplos, and Shubik (1999).

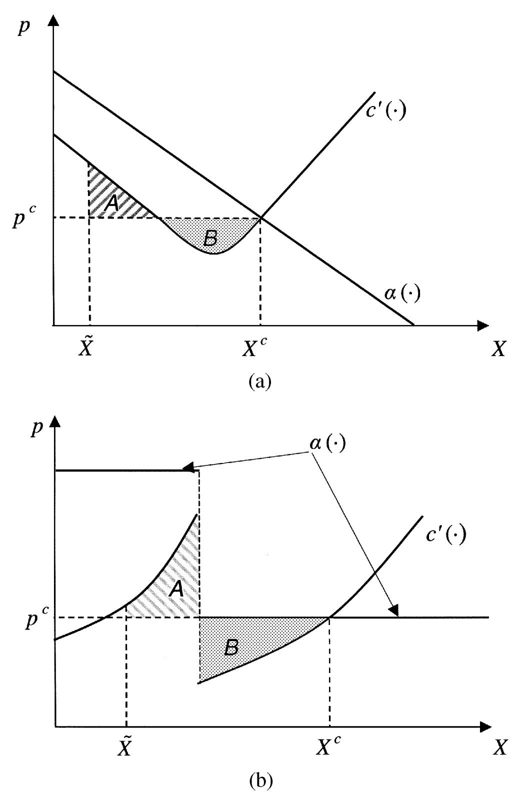

FIGURE 4.

Different papers in the literature have made different predictions for the outcomes of bilateral contracting in such situations. Dubey, Geanakoplos, and Shubik (1999) consider a continuum of anonymous principals (borrowers) along with a continuum of lenders (agents), which convexifies the principals' aggregate demand for credit and thus ensures the existence of a "convexified competitive equilibrium." Other papers such as Rajan and Parlour (2001), Kahn and Mookherjee (1998), and deMarzo and Bizer (1993), however, predict very "noncompetitive" outcomes even when the number of parties on each side is large. We examine the predictions of our contracting model for such situations. Note that our model is consistent with the existence of many principals who deal with the agents personally rather than through an anonymous market, so that the contracting problem separates across the principals.

Observe first that the competitive menu C is still an acceptable menu provided that $p^c X - c(X)$ is strictly decreasing on $[0, \widetilde{X}] \cup [X^c, +\infty)$. This assumption ensures that when $X_{-i} < \widetilde{X}$ or $X_{-i} > X^c$, the principal faced with menu C will choose $x_i = 0$. On the other hand, when $X_{-i} \in (\widetilde{X}, X^c]$, the principal will choose $x_i = X^c - X_{-i}$. (When $X_{-i} = \widetilde{X}$, the principal is indifferent between $x_i = 0$ and $x_i = X^c - \widetilde{X}$.) In all these cases, the agent receives a zero payoff, and so C is an acceptable menu. Proposition 1 then implies that any WPBE of a bilateral contracting game must satisfy

$$\alpha(\widehat{X})\hat{x}_i - c(\widehat{X}) \geq \Pi^C(\widehat{X}_{-i}) = \begin{cases} p^c(X^c - \widehat{X}_{-i}) - c(X^c) & \text{when } \widehat{X}_{-i} \in (\widetilde{X}, X^c), \\ -c(\widehat{X}_{-i}) & \text{otherwise.} \end{cases} \tag{9}$$

We can now bound the limiting behavior of aggregate trades as N goes to infinity:

PROPOSITION 7: *Suppose that*:

(i) (p^c, X^c) *is a strict competitive quasi-equilibrium relative to* $\widetilde{X} > 0$,
(ii) $p^c X - c(X)$ *is strictly decreasing on* $[0, \widetilde{X}] \cup [X^c, +\infty)$,
(iii) $\alpha(X) \leq p^c$ *for* $X \geq X^c$, *and* $\alpha(X)$ *is continuous at* $X = X^c$, *and*
(iv) *the aggregate surplus* $W(X) = \alpha(X)X - c(X)$ *is bounded above*.

Then for any sequence $\{\widehat{X}^N\}_{N=1}^\infty$ *of WPBE aggregate trades*, $\limsup_{N\to\infty} \widehat{X}^N \leq \widetilde{X}$.[22]

PROOF: Suppose in negation that there exists a subsequence $\{\widehat{X}^K\}_{K=1}^\infty \subset \{\widehat{X}^N\}_{N=1}^\infty$ such that $\widehat{X}^K \to X^0 > \widetilde{X}$ as $K \to \infty$. By the same arguments as those in the proof of Proposition 4, (9) implies that

$$p^c \widehat{X}^K - c(\widehat{X}^K) \to p^c X^c - c(X^c) \quad \text{as} \quad K \to \infty.$$

[22] This means that for any $X' > \widetilde{X}$ there exists N' such that $\widehat{X}^N < X'$ for all $N \geq N'$.

Since $c(\cdot)$ is lower semicontinuous, (p^c, X^c) is a strict competitive quasi-equilibrium relative to $\widetilde{X}$, and $X^0 = \lim_{K\to\infty} \widehat{X}^K > \widetilde{X}$, we must then have $X^0 = X^c$. Then, by continuity of $\alpha(\cdot)$ at X^c, we must also have $\alpha(\widehat{X}^K) \to \alpha(X^c) = p^c$ as $K \to \infty$, and therefore

$$\lim_{K\to\infty}\big[\alpha\big(\widehat{X}^K\big)\widehat{X}^K - c\big(\widehat{X}^K\big)\big] = \lim_{K\to\infty}\big[p^c\widehat{X}^K - c\big(\widehat{X}^K\big)\big] = p^c X^c - c(X^c).$$

However, our assumptions imply that $p^c X^c - c(X^c) = p^c\widetilde{X} - c(\widetilde{X}) < 0$. But then the equilibrium total surplus $\alpha(\widehat{X}^K)\widehat{X}^K - c(\widehat{X}^K)$ must be negative for K large enough, which contradicts the fact that all parties must have nonnegative payoffs in an equilibrium. *Q.E.D.*

Under somewhat stronger assumptions, the competitive menu C in fact constitutes an RSW menu, which allows us to characterize the equilibrium outcomes of the offer game as follows:

PROPOSITION 8: *Suppose that*:

(i) (p^c, X^c) *is a strict competitive quasi-equilibrium relative to* $\widetilde{X}$,
(ii) $c'(\cdot)$ *exists and is bounded*, $c'(X) > p^c$ *for* $X > X^c$, *there exists* $\overline{X} < X^c$ *such that* $c'(X) < p^c$ *for* $X \in [\overline{X}, X^c)$, *and* $c'(X) > p^c$ *and is nonincreasing for* $X \le \widetilde{X}$, *and*
(iii) $\alpha(X) \le p^c$ *for* $X \ge X^c$.

Then $\hat{x} \in \mathbb{R}^N_+$ *is sustainable as a WPBE trade profile in the offer game if and only if condition* (9) *holds for all* $i \in N$ *such that* $\widehat{X}_{-i} \ge \widetilde{X}$, *and*

(10) $$\alpha(\widehat{X})\widehat{X}_D \ge c(\widehat{X}) - c(\widehat{X}_{-D}) \quad \text{for all sets } D \subset N \text{ such that } \widehat{X}_{-D} < \widetilde{X}.$$

PROOF: See Appendix A.

Proposition 8 can be viewed as combining the results for the cases in which a strict competitive equilibrium exists and in which marginal costs are nonincreasing.[23]

Our results are consistent with those of Kahn and Mookherjee (1998), who call the threshold $\widetilde{X}$ the "third-best" outcome (Bisin and Guaitoli (2000), Rajan and Parlour (2001), and deMarzo and Bizer (1993) make similar predictions). In the equilibrium of the particular bilateral contracting game they consider, the principal trades precisely the aggregate quantity $\widetilde{X}$, making him exactly indifferent about trading an additional quantity $X^c - \widetilde{X}$ at the competitive price p^c. Proposition 7 establishes that under the appropriate assumptions, the equilibrium aggregate trade in *any* bilateral contracting game cannot exceed $\widetilde{X}$ when

[23] For symmetric equilibria, Proposition 8 implies that $\widehat{X}$ can be sustained as the aggregate trade in a WPBE of an offer game if and only if either $((N-1)/N)\widehat{X} < \widetilde{X}$ and $\alpha(\widehat{X})\widehat{X} - c(\widehat{X}) \ge 0$, or $((N-1)/N)\widehat{X} \ge \widetilde{X}$ and $(1/N)\alpha(\widehat{X})\widehat{X} + ((N-1)/N)p^c\widehat{X} - c(\widehat{X}) \ge p^c X^c - c(X^c)$.

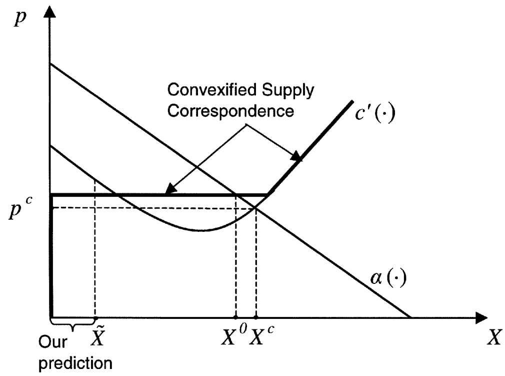

FIGURE 5.

the number of agents is sufficiently large. Intuitively, if $\widehat{X} \in (\widetilde{X}, X^c)$, then there would exist a profitable bilateral deviation between the principal and an agent i whose trade with the principal is sufficiently small. In this deviation the parties would sign the competitive contract C, from which the principal would then choose quantity $X^c - \widehat{X}_{-i}$, so that the aggregate trade becomes X^c. On the other hand, the aggregate trade X^c itself cannot be sustained in equilibrium, since it entails a negative total surplus. Therefore, asymptotically the aggregate quantity $\widehat{X}$ cannot exceed $\widetilde{X}$.

This prediction is quite different from the "convexified competitive equilibrium quantity" X^0, obtained by assuming a continuum of anonymous principals, and intersecting their convexified supply correspondence with the agents' demand curve $\alpha(\cdot)$ (see Figure 5). The anonymity assumption is innocuous in large economies without externalities, where a seller is indifferent between, say, selling 5 apples to buyers A and B each, and selling all 10 apples to buyer A. However, in economies with externalities, such as the lending model of Dubey, Geanakoplos, and Shubik (1999), a lender's decision whether to split \$10 equally between borrowers A and B or lend it all to borrower A will affect the default rate on his loans. (For example, a borrower who borrows more than \$7 may be more likely to default.) For this reason, lenders will not treat borrowers as anonymous, as the convexification approach assumes, but rather will contract with each borrower individually, resulting in the noncompetitive outcomes described by our model.

7. BIDDING GAMES

In this section, we examine the equilibrium outcomes of the "bidding game" (studied by Bernheim and Whinston (1986a,b, 1998) and Martimort and Stole (2002), among others), in which agents simultaneously offer menus to the principal, who then chooses which menus to accept and makes choices from the accepted menus. Specifically, we identify the restrictions that equilibrium trade profiles of the bidding game have to satisfy in addition to (AIR) and (MD), and compare these trade profiles to those sustainable in the offer game studied above. Note that in the bidding game, the only information sets that may not be reached in equilibrium are the singletons at which the principal makes choices, and therefore the concept of WPBE coincides with that of subgame-perfect Nash equilibrium (henceforth "SPNE").

Note that in contrast to the offer game, in a bidding game we need to consider only unilateral deviations, just as in Proposition 1. The additional constraints on equilibrium outcomes relative to condition (MD) come from a different source: since an agent in the bidding game knows the equilibrium menus offered by other agents, he can predict the principal's choices from them following his deviation. Thus, his beliefs about the principal's trades with other agents are not arbitrary, which creates the possibility for additional profitable deviations and hence reduces the set of sustainable outcomes.

In this section we focus on the case in which $c'(\cdot)$ is increasing and $\alpha(\cdot)$ is nonincreasing. To see the usefulness to an agent of his knowledge of other agents' equilibrium menus in planning a deviation, consider an agent who offers a point contract with a trade level x_i' that is above his equilibrium trade level $\hat{x}_i$. This agent can be assured that the principal's aggregate trade X_{-i} with the other agents will not increase as a result of this deviation (if she accepts it) because her payoff has increasing differences in $(x_i, -X_{-i})$ (see, e.g., Milgrom and Shannon's (1994) Monotone Selection Theorem). If this deviation increased bilateral surplus, then the agent could be assured of a higher payoff by deviating to a point contract (x_i', t_i) where t_i gives the principal a payoff equal to her supposed equilibrium payoff if she does not alter her trades with other agents (if the principal prefers to reduce her aggregate trade with other agents, this makes both parties better off still). Thus, it must be impossible for any principal-agent pair to raise their bilateral surplus by increasing their trade—i.e., the trade profile must be pairwise stable to upward deviations. Formally, any SPNE trade profile $\hat{x} \in \mathbb{R}_+^N$ of the bidding game must satisfy

$$(11) \qquad \hat{x}_i \in \arg\max_{x_i \geq \hat{x}_i} \alpha(x_i + \widehat{X}_{-i})x_i - c(x_i + \widehat{X}_{-i}) \qquad \text{for each } i \in N.$$

In fact, equilibrium trade profiles of the bidding game can be characterized as follows:

PROPOSITION 9: *If $\alpha(\cdot)$ is nonincreasing and $c'(\cdot)$ is strictly increasing and (p^c, X^c) is a strict competitive equilibrium, then $\hat{x} \in \mathbb{R}_+^N$ is sustainable as a SPNE trade profile in the bidding game if and only $\sum_i \hat{x}_i \leq X^c$ and condition* (11) *holds.*

PROOF: See Appendix B.

Observe that condition (11) implies condition (5) when $\widehat{X} \leq X^c$ (by taking $x_i = X^c - \widehat{X}_{-i} \geq \hat{x}_i$). Therefore, comparing Proposition 9 to Proposition 5 (whose assumptions are weaker than those in Proposition 9), we have the following corollary.[24]

COROLLARY 2: *Under the assumptions of Proposition* 9, *the set of equilibrium trade profiles of the bidding game is a subset of those in the offer game.*

Nevertheless, Proposition 9 implies the following result, parallel to Corollary 1 for offer games.

COROLLARY 3: *Under the assumptions of Proposition* 9, *any pairwise-stable trade profile* $\hat{x} \in \mathbb{R}_+^N$, *and any trade profile* $\hat{x} \in \mathbb{R}_+^N$ *such that* $\sum_i \hat{x}_i = X^c$, *is sustainable as a SPNE trade profile in the bidding game.*

In the example of Section 2, Proposition 9 in fact implies that the set of aggregate trade levels $\widehat{X}$ arising in symmetric equilibria of the bidding game is precisely the set $[\widehat{X}_N^p, X^c]$. Martimort and Stole (2000) independently obtain a similar characterization in the context of "intrinsic" common agency (where the principal in our terminology cannot accept offers from only a subset of agents).

8. CONCLUSION

In this paper, we have studied bilateral contracting between one principal and N agents when each agent's utility depends on the principal's unobservable trades with other agents. In such settings, letting the principal choose from a menu can make her trade with an agent depend on her trades with other agents, just as an informed principal can make the allocation depend on her exogenous type in Maskin and Tirole's (1992) analysis of mechanism design by an informed principal. Moreover, such menus can be designed to guarantee the agent a utility level regardless of his beliefs. We have seen that requiring immunity to deviations to such menu contracts often yields a significant bound on equilibrium outcomes in a wide class of bilateral contracting games without imposing ad hoc restrictions on agents' beliefs. Indeed, in settings in which a competitive equilibrium exists, this bound yields, under certain mild assumptions, competitive convergence as $N \to \infty$. To highlight how specific contracting processes lead to further restrictions on equilibrium outcomes, we have also examined the additional restrictions that arise in two commonly studied bilateral contracting processes: the "offer game," in which the principal makes simultaneous offers to the agents (e.g., Hart and Tirole (1990), Segal (1999)), and the "bidding game," in which the agents make simultaneous offers to the principal (e.g., Bernheim and Whinston (1986a, b)).

A natural question is whether we see such menus used in practice. We do see firms that are in the role of agents, such as upstream suppliers and credit

[24] We can obtain a similar conclusion for the case in which $c'(\cdot)$ is strictly decreasing by observing that a "downward deviation" version of condition (11) holds in this case (recall from Proposition 6 that any trade profile with nonnegative total surplus is sustainable in the offer game in this case).

card companies, offering purchasers menus of options from which the purchasers then choose.[25] Perhaps less common are menus offered by a party acting as a principal from which the party itself chooses. (It should be noted that in our theory principals need not offer menus on the equilibrium path—e.g., in the offer game such menus need only be used when making deviations.) Nevertheless, such menus can be observed. One example is the case of academic publishing contracts where the publisher retains an option of whether to continue publishing the book; if the publisher fails to reprint the book for a given amount of time, the copyright is returned to the author. This option is a natural response to the author's concern that the publisher will decide to devote its resources to promoting other books instead of the author's: if the publisher were to do so, and demand for the author's book would subsequently decline, the publisher will opt not to incur the expenses of reprinting, and the book's copyright will be returned to the author.

A second natural question about our theory is the role that common knowledge of the principal's cost function plays. We used this common knowledge, for example, to construct the competitive menu using the deterministic competitive price p^c. One can see that as long as the support of the principal's privately observed cost function shock is small, our bound will continue to hold approximately. For example, in a case for which a strict competitive equilibrium exists, the principal would simply need to add a small additional fixed payment to the competitive menu for it to be accepted by an agent regardless of his beliefs. When the support of the shock is large, however, our results may not continue to hold.[26]

Department of Economics, Stanford University, Stanford, CA 94305-6072, U.S.A.; ilya.segal@stanford.edu; http://www.stanford.edu/~isegal

and

Department of Economics, Northwestern University, Evanston, IL 60208-2600, U.S.A.; mwhinston@northwestern.edu; http://www.faculty.econ.northwestern.edu/faculty/whinston

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APPENDIX A

The Bilateral Surplus Function: For the proofs in this appendix it will prove useful to define the function

$$B(X, X_{-i}) = \alpha(X)(X - X_{-i}) - c(X),$$

[25] Note with regard to the credit card example that, in some cases (e.g., in the bidding game), unchosen offers made by multiple firms can be viewed as forming a menu for the purposes of the theory.

[26] In general, the screening problem involved in designing a menu becomes two-dimensional since the menu must screen both the principal's trades and the principal's cost function shock. Moreover, the problem has both private and common value elements, in the language of Maskin and Tirole (1990, 1992).

giving the bilateral surplus when the aggregate trade with agents $j \neq i$ is X_{-i} and the aggregate trade is X (and so the trade with agent i is $x_i = X - X_{-i}$). Note that if the principal of type X_{-i} offers agent i an acceptable menu M and chooses $x_i = X - X_{-i}$ from it, (AIR) can be written as

$$\Pi^M(X_{-i}) \leq B(X, X_{-i}). \tag{12}$$

The proofs of several results utilize the following properties of bilateral surplus:

LEMMA 4: *Suppose that (p^c, X^c) is a strict competitive equilibrium and $\alpha(X) \leq p^c$.*
(a) *If $X \geq X^c \geq X_{-i}$, then $B(X, X_{-i}) \leq B(X^c, X_{-i})$.*
(b) *If $X \geq X_{-i} \geq X_c$ and $p^c X - c(X)$ is nonincreasing for $X \geq X^c$, then $B(X, X_{-i}) \leq B(X_{-i}, X_{-i})$.*

PROOF: For part (a), observe that

$$\begin{aligned} B(X, X_{-i}) &= \alpha(X)(X - X_{-i}) - c(X) \\ &= [p^c X - c(X)] + [\alpha(X) - p^c](X - X_{-i}) - p^c X_{-i} \\ &\leq [p^c X^c - c(X^c)] - p^c X_{-i} = B(X^c, X_{-i}), \end{aligned}$$

where the inequality uses the fact that (p^c, X^c) is a strict competitive equilibrium. For part (b), we can write

$$\begin{aligned} B(X, X_{-i}) &= [p^c X - c(X)] + [\alpha(X) - p^c](X - X_{-i}) - p^c X_{-i} \\ &\leq [p^c X_{-i} - c(X_{-i})] - p^c X_{-i} = B(X_{-i}, X_{-i}), \end{aligned}$$

where the inequality follows from the fact that $p^c X - c(X)$ is nonincreasing for $X \geq X^c$. *Q.E.D.*

LEMMA 5: *Suppose that $\alpha(\cdot)$ is nonicreasing, $c(\cdot)$ is differentiable, and $\alpha(X) \leq c'(X)$ for all $X > X_{-i}$. Then $B(X, X_{-i}) \leq B(X_{-i}, X_{-i})$ for all $X > X_{-i}$.*

PROOF:

$$\begin{aligned} B(X, X_{-i}) - B(X_{-i}, X_{-i}) &= \alpha(X)(X - X_{-i}) - c(X) + c(X_{-i}) \\ &= \int_{X_{-i}}^{X} [\alpha(X) - c'(Y)]\, dY. \end{aligned}$$

The result follows since $c'(Y) \geq \alpha(Y) \geq \alpha(X)$ for all $Y \in (X_{-i}, X)$. *Q.E.D.*

PROOF OF LEMMA 2: The assumptions of the lemma imply the assumptions of Proposition 4, under which C is an acceptable menu, with the associated profit function $\Pi^C(\cdot)$ described in (5). We will show that for an arbitrary compact acceptable menu M, $\Pi^M(X_{-i}) \leq \Pi^C(X_{-i})$ for all $X_{-i} \in \mathbb{R}_+$. For $X_{-i} \geq X^c$, this follows from the agent's individual rationality constraint (12) and Lemma 4(b):

$$\Pi^M(X_{-i}) \leq B(X, X_{-i}) \leq B(X_{-i}, X_{-i}) = \Pi^C(X_{-i}).$$

Now suppose in negation that there exists $X'_{-i} < X^c$ and an acceptable menu M such that $\Pi^M(X'_{-i}) > \Pi^C(X'_{-i})$. By Lemma 1, the union menu $U = M \cup C$ is also acceptable, and, moreover, it is compact. Its associated profit function is $\Pi^U(X_{-i}) = \max\{\Pi^M(X_{-i}), \Pi^C(X_{-i})\}$. Let

$$X^0_{-i} = \min\{X_{-i} \in [X'_{-i}, X^c] : \Pi^U(X_{-i}) = \Pi^C(X_{-i})\}. \tag{13}$$

(The minimum is achieved because $\Pi^U(\cdot)$ and $\Pi^C(\cdot)$ are absolutely continuous under our assumptions—see, e.g., Milgrom and Segal (2002, Theorem 2).) By construction, $X^0_{-i} > X'_{-i}$.

Take $\varepsilon = \min\{X^0_{-i} - X'_{-i}, X^c - \overline{X}\}$. Let $(x^M(X_{-i}), t^M(X_{-i}))$ be an optimal choice from a menu M for the principal of type X_{-i}, and let $X^M(X_{-i}) = x^M(X_{-i}) + X_{-i}$. Standard envelope theorem arguments of mechanism design (formalized by Milgrom and Segal (2002, Corollary 1)) imply that for any acceptable menu M,

$$\Pi^M(X^0_{-i}) = \Pi^M(X^0_{-i} - \varepsilon) - \int_{X^0_{-i}-\varepsilon}^{X^0_{-i}} c'(X^M(X_{-i}))\, dX_{-i}.$$

Therefore,

$$\Pi^U(X^0_{-i} - \varepsilon) - \Pi^C(X^0_{-i} - \varepsilon) = \int_{X^0_{-i}-\varepsilon}^{X^0_{-i}} \left[c'(X^U(X_{-i})) - c'(X^C(X_{-i}))\right] dX_{-i}. \tag{14}$$

Since $\Pi^U(X^0_{-i} - \varepsilon) > \Pi^C(X^0_{-i} - \varepsilon)$, we must have $c'(X^U(X''_{-i})) > c'(X^C(X''_{-i})) = c'(X^c)$ for some $X''_{-i} \in (X^0_{-i} - \varepsilon, X^0_{-i})$. Therefore, we must have either (a) $X^U(X''_{-i}) > X^c$ or (b) $X^U(X''_{-i}) < \overline{X}$. At the same time, by (13), we must have $\Pi^U(X''_{-i}) > \Pi^C(X''_{-i})$.

Case (a) can be ruled out because then the agent's participation constraint (12) and Lemma 4(a) would imply

$$\Pi^U(X''_{-i}) \le B(X^U(X''_{-i}), X''_{-i}) \le B(X^c, X''_{-i}) = \Pi^C(X''_{-i}).$$

Thus, we can focus on case (b), where $X''_{-i} \le X^U(X''_{-i}) < \overline{X}$. Let $(x'', t'') \equiv (x^U(X''_{-i}), t^U(X''_{-i}))$. By (13), we have

$$t'' - c(X''_{-i} + x'') = \Pi^U(X''_{-i}) > \Pi^C(X''_{-i}) \ge p^c x'' - c(X''_{-i} + x'').$$

Therefore, $t'' > p^c x''$. But then, since $(x'', t'') \in U$, we have

$$\Pi^U(X^c - x'') \ge t'' - c(X^c - x'' + x'') > p^c x'' - c(X^c) = \Pi^C(X^c - x'').$$

Since

$$X^c - x'' = X^c - X^U(X''_{-i}) + X''_{-i} > X^c - \overline{X} + X''_{-i} > X^c - \overline{X} + X^0_{-i} - \varepsilon \ge X^0_{-i},$$

this contradicts (13). *Q.E.D.*

PROOF OF LEMMA 3: Z is trivially an acceptable menu, with the associated profit function $\Pi^Z(X_{-i}) = -c(X_{-i})$. To see that Z is an RSW menu, take an arbitrary acceptable menu M. By Lemma 1, the union menu $U = M \cup Z$ is also acceptable, with $\Pi^U(X_{-i}) = \max\{\Pi^M(X_{-i}), \Pi^Z(X_{-i})\}$. We will show that $\Pi^U(X_{-i}) = \Pi^Z(X_{-i})$, and therefore $\Pi^M(X_{-i}) \le \Pi^Z(X_{-i})$, for all X_{-i}.

First, note that if (x, t) is the principal's optimal choice from U when $X_{-i} \ge \overline{X}$, the agent's participation constraint (12) and Lemma 5 imply that

$$\Pi^U(X_{-i}) \le B(X_{-i} + x, X_{-i}) \le B(X_{-i}, X_{-i}) = \Pi^Z(X_{-i}).$$

Therefore, $(0, 0)$ is an optimal choice from U for the principal when $X_{-i} \ge \overline{X}$.

Note that with $c'(\cdot)$ nonincreasing, the principal's payoff has increasing differences in (x_i, X_{-i}). Therefore, by Topkis's Monotonicity Theorem (see., e.g., Milgrom and Shannon (1994)), $(0, 0)$ must be an optimal choice from U for any $X_{-i} \le \overline{X}$, which implies that $\Pi^U(X_{-i}) = \Pi^Z(X_{-i})$ for all such X_{-i}. *Q.E.D.*

PROOF OF PROPOSITION 8: The proof of C being an RSW menu follows the same logic as that of Lemma 2, with one modification: $X^C(X''_{-i}) = X''_{-i}$ for $X''_{-i} < \widetilde{X}$. Our assumptions imply that it is again not possible to have $X^U(X''_{-i}) \in [X''_{-i}, X^c]$ such that $c'(X^U(X''_{-i})) > c'(X^C(X''_{-i}))$.

The necessity part of the Proposition follows in a manner similar to previous arguments. For the sufficiency part, (9) holding for all $i \in N$ such that $\widehat{X}_{-i} \geq \widetilde{X}$ implies that $\widehat{X} \leq X^c$ (as in the proof of Proposition 4). Observe next that for all sets $D \subset N$ such that $\widehat{X}_{-D} \in [\widetilde{X}, X^c]$, $\overline{\Pi}_D(\hat{x}_{-D})$ is given by (7), while for sets $D \subset N$ such that $\widehat{X}_{-D} < \widetilde{X}$, we have $\overline{\Pi}_D(\hat{x}_{-D}) = -c(\widehat{X}_{-D})$. Let $\hat{t}_i = \alpha(\widehat{X})\hat{x}_i$ for all i. Then the proof that condition (MMD) in Proposition 3 is satisfied for sets $D \subset N$ such that $\widehat{X}_{-D} \geq \widetilde{X}$ is the same as in Proposition 5. For sets $D \subset N$ such that $\widehat{X}_{-D} < \widetilde{X}$, (10) ensures that if $\hat{t}_i = \alpha(\widehat{X})\hat{x}_i$ for all i, then condition (MMD) is satisfied for these sets as well. *Q.E.D.*

APPENDIX B

PROOF OF PROPOSITION 9: Necessity follows from the argument in the text.[27] To establish sufficiency, we observe that each agent i offering a menu

$$\widehat{M}_i = \left\{ (x_i, t_i) : x_i \in [0, X^c] \quad \text{and} \quad t_i = \begin{cases} p^c x_i & \text{if } x_i \leq \hat{x}_i \\ p^c x_i + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})\} & \text{if } x_i \geq \hat{x}_i \end{cases} \right\}, \tag{15}$$

combined with the choice by the principal of $(\hat{x}, \hat{t})$ from these menus and any optimal choices when facing any other profile of offered menus, constitutes a SPNE of the bidding game under the assumptions of the proposition. To verify this claim, we use the general characterization of equilibrium outcomes in bidding games given in Proposition 10 below:

PROPOSITION 10: *Take a strategy profile $M = (M_1, \ldots, M_N)$. For $S \subset N$ and $x_{-S} \in \prod_{j \in N \setminus S} \mathscr{X}_j$, define*[28]

$$\pi_S(x_{-S}) = \max_{(x_S, t_S) \in \prod_{j \in S}[M_j \cup (0,0)]} \left[\sum_{j \in S} t_j - c(x_S, x_{-S}) \right].$$

If the outcome $(\hat{x}, \hat{t})$ arises in a SPNE in which agents use strategies M, then:

(i) $\sum_j \hat{t}_j - c(\hat{x}) = \pi_N$;

(ii) *agents' participation constraints (AIR) hold*;

(iii) *for all $i \in N$ and $x_i \in \mathscr{X}_i$, there exists*

$$(x_{-i}, t_{-i}) \in \underset{(\tilde{x}_{-i}, \tilde{t}_{-i}) \in \prod_{j \in N \setminus i}[M_j \cup (0,0)]}{\arg\max} \left[\sum_{j \neq i} \tilde{t}_j - c(x_i, \tilde{x}_{-i}) \right]$$

such that

$$u_i(x_i, x_{-i}) + \pi_{-i}(x_i) \leq u_i(\hat{x}) + \sum_{j \neq i} \hat{t}_j - c(\hat{x}). \tag{16}$$

If, in addition, the principal's choice is invariant to additive transformations in the menus from which she chooses,[29] *then*:

(iv) *for all $i \in N$,*

$$\sum_j \hat{t}_j - c(\hat{x}) = \pi_{-i}(0).$$

Moreover, if conditions (i)–(iv) *hold, then $(\hat{x}, \hat{t})$ can be supported as a SPNE outcome in which the agents use strategies M.*

[27] It may also be established formally using condition (iii) of Proposition 10 below.

[28] For the case $S = N$, we write simply π_N (it has no arguments).

[29] An additive transformation of a menu is one that adds a lump-sum payment (of either sign) to the menu.

PROOF OF PROPOSITION 10: *Necessity:* (i) says that the principal's choice from agents' equilibrium menus is optimal, which is necessary for subgame perfection. If condition (ii) did not hold for agent i, then he could profitably deviate to $M_i' = \{(0,0)\}$. If (iii) did not hold for some i and x_i, agent i could deviate to $M_i' = \{(x_i, t_i)\}$, where $t_i = \pi_{-i}(0) - \pi_{-i}(x_i) + \varepsilon$ for a small $\varepsilon > 0$. The principal would accept this deviation, and letting (x_{-i}, t_{-i}) denote her ensuing choice from other agents' menus, agent i's payoff would be

$$u_i(x_i, x_{-i}) - t_i = u_i(x_i, x_{-i}) - \pi_{-i}(0) + \pi_{-i}(x_i) - \varepsilon$$
$$> u_i(\hat{x}) + \sum_{j \neq i} \hat{t}_j - c(\hat{x}) - \pi_{-i}(0) - \varepsilon \geq u_i(\hat{x}) - \hat{t}_i - \varepsilon,$$

where the strict inequality is by the violation of (iii) and the weak inequality is by (i) and the fact that $\pi_N \geq \pi_{-i}(0)$. Therefore, for small enough ε, the deviation would make agent i better off. Finally, if the principal's choice is invariant to additive transformations of menus, then if (iv) did not hold, agent i would deviate to offer menu M_i minus a small $\varepsilon > 0$. The principal would accept the deviation and choose $(x_i, t_i) = (\hat{x}_i, \hat{t}_i - \varepsilon)$ and $(x_{-i}, t_{-i}) = (\hat{x}_{-i}, \hat{t}_{-i})$, making agent i better off.

Sufficiency: (i) ensures that the principal does not want to deviate following $(M_1, \ldots, M_N)$. Let the principal's strategy in response to a deviation M_i' by agent i be to choose some $(x_i, t_i) \in \arg\max_{(\tilde{x}_i, \tilde{t}_i) \in M_i'} [\tilde{t}_i + \pi_{-i}(\tilde{x}_i)]$ and

$$(x_{-i}, t_{-i}) \in \arg\min_{(\bar{x}_{-i}, \bar{t}_{-i}) \in \arg\max_{(\tilde{x}_{-i}, \tilde{t}_{-i}) \in \prod_{j \in N \setminus i} [M_j \cup (0,0)]} \left[\sum_{j \neq i} \tilde{t}_j - c(x_i, \tilde{x}_{-i})\right]} u_i(x_i, \bar{x}_{-i}).$$

If agent i's deviation M_i' is accepted by the principal (and she chooses $(x_i, t_i) \in M_i'$), we must have $\pi_{-i}(x_i) + t_i \geq \pi_{-i}(0)$. But then, agent i's payoff from the deviation is

$$u_i(x_i, x_{-i}) - t_i \leq u_i(x_i, x_{-i}) + \pi_{-i}(x_i) - \pi_{-i}(0) \leq u_i(\hat{x}) + \sum_{j \neq i} \hat{t}_j - c(\hat{x}) - \pi_{-i}(0) = u_i(\hat{x}) - \hat{t}_i,$$

where the second inequality uses (iii) and the last equality uses (iv). Therefore, the deviation cannot be profitable. *Q.E.D.*

REMARK: Condition (iii) of Proposition 10 captures the fact that an agent i can predict the principal's reaction to his deviation based on his knowledge of the menus offered in equilibrium by other agents. Observe that an agent i who contemplates deviating with a menu from which the principal will choose (x_i, t_i) can achieve the same result by offering the point menu (x_i, t_i). Condition (iii) says that for any such deviation there must exist an optimal response by the principal that lowers the bilateral surplus of agent i and the principal (which includes transfers earned from agents $j \neq i$), given the menus offered by the other agents.[30] Condition (iv) reflects the fact that in situations in which the principal is indifferent among a number of trade profiles she may be able to prevent an agent from reducing his transfer by (effectively) threatening to choose a less desirable outcome from among these profiles.[31] This possibility is ruled out if the principal's choice is invariant to additive transformations in the menus from which she chooses.

We now use Proposition 10 to verify that each agent i offering the menu $\widehat{M}_i$ given by (15), combined with the choice by the principal of $(\hat{x}, \hat{t})$ from these menus and any optimal choices

[30] It can be verified that condition (iii), along with (ii), implies the condition (MD).

[31] For example, suppose that the principal's cost function is separable (i.e., $c(x) = \sum_i c_i(x_i)$) and negative externalities are present. Then there is an equilibrium in which each agent i offers the menu $M_i = \{(x_i, t_i) : t_i = c_i(x_i) + K\}$, where $K = u_i(x^*)$, agent i's gross payoff at the first-best trade profile x^*, and the principal chooses x^* given these offers and punishes any agent for deviating with a very large choice of x_{-i}. The agents earn zero in this equilibrium and the principal earns the entire first-best surplus.

when facing any other profile of offered menus, constitutes a SPNE of the bidding game under the assumptions of the proposition.

The verification begins with two lemmas characterizing the principal's optimal trades from menus $\widehat{M}_j$ with agents $j \neq i$ given that she trades x_i with agent i.

LEMMA 6: *Suppose we fix x_i. Any solution to*

$$\max_{(x_j,t_j)\in \widehat{M}_j\,\forall j\neq i} \sum_{j\neq i} t_j - c\Big(x_i + \sum_{j\neq i} x_j\Big) \tag{17}$$

has at most one element k with $x_k > \hat{x}_k$.

PROOF: Observe first that any solution to (17) must have $X \leq X^c$: Were this not true, we would have an $x_j > \hat{x}_j$ (since $\widehat{X} \leq X^c$). A differential reduction in x_j of $dx_j < 0$ would then increase the objective function in (17) by $[p^c - c'(X)]\,dx_j > 0$.

Suppose now that there is a solution to (17) in which $(x_j, x_k) \gg (\hat{x}_j, \hat{x}_k)$ for some j and k. Consider a change in which only j and k's trades are changed, and these are changed to $[\hat{x}_j, x_k + (x_j - \hat{x}_j)]$ (this is feasible since $x_k + (x_j - \hat{x}_j) < X \leq X^c$). Given the transfers associated with these trades, the change in profit is $\{p^c(X^c - \widehat{X}) - [c(X^c) - c(\widehat{X})]\} > 0$, which yields a contradiction. *Q.E.D.*

LEMMA 7: *Suppose we fix x_i. The solution and value of the solution* [*denoted $\pi_{-i}(x_i)$*] *to problem* (17) *take the following form*:

1. *If $x_i \leq \hat{x}_i$: Any solution has $\sum_{j\neq i} x_j = X^c - x_i$ and $\pi_{-i}(x_i) = p^c(\widehat{X} - x_i) - c(\widehat{X})$.*
2. *If $x_i = \hat{x}_i$: Any solution has either $\sum_{j\neq i} x_j = X^c - x_i$ or $\sum_{j\neq i} x_j = \widehat{X}_{-i}$ and $\pi_{-i}(x_i) = p^c(\widehat{X} - x_i) - c(\widehat{X}) = p^c\widehat{X}_{-i} - c(x_i + \widehat{X}_{-i})$.*
3. *If $x_i \in (\hat{x}_i, X^c - \widehat{X}_{-i})$: Any solution has $\sum_{j\neq i} x_j = \widehat{X}_{-i}$ and $\pi_{-i}(x_i) = p^c\widehat{X}_{-i} - c(x_i + \widehat{X}_{-i})$.*
4. *If $x_i \in [X^c - \widehat{X}_{-i}, X^c]$: Any solution has $\sum_{j\neq i} x_j = X^c - x_i$ and $\pi_{-i}(x_i) = p^c(X^c - x_i) - c(X^c)$.*
5. *If $x_i > X^c$: Any solution has $x_{-i} = 0$ and $\pi_{-i}(x_i) = -c(x_i)$.*

PROOF: Given x_i, call a profile x_{-i} with $x_i \leq \hat{x}_j$ for all $j \neq i$ a "type A configuration." Its value is $p^c \sum_{j\neq i} x_j - c(x_i + \sum_{j\neq i} x_j)$. Call a profile x_{-i} with $x_k > \hat{x}_k$ for some k and $x_j \leq \hat{x}_j$ for all $j \notin \{i, k\}$ a "type B_k configuration." Its value is

$$p^c \sum_{j\neq i} x_j - c\Big(x_i + \sum_{j\neq i} x_j\Big) + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})\}.$$

The set of profiles x_{-i} that are type B_k configurations for some $k \neq i$ are called "type B configurations."

Now consider the optimal choices in the five cases of the lemma:

1. If $x_i \leq \hat{x}_i$: Note, first, that $\hat{x}_i \leq X^c - \widehat{X}_{-i} \leq X^c - \hat{x}_k$ for all k. Since $x_i \leq X^c - \widehat{X}_{-i}$, the best type A configuration has $x_j = \hat{x}_j$ for all j and has value $p^c\widehat{X}_{-i} - c(x_i + \widehat{X}_{-i})$. On the other hand, since $x_i \leq X^c - \hat{x}_k$ for all k, the best type B_k configuration has $\sum_{j\neq i} x_j = X^c - x_i$ and yields a value of $p^c(\widehat{X} - x_i) - c(\widehat{X})$ for any $k \neq i$. Since $\widehat{X}_{-i} < \widehat{X} - x_i \leq X^c$, the best type B configuration is better and any optimal solution to problem (17) has $\sum_{j\neq i} x_j = X^c - x_i$.

2. If $x_i = \hat{x}_i$: The best type A and type B configurations are the same as in case 1, but now they yield the same value.

3. If $x_i \in (\hat{x}_i, X^c - \widehat{X}_{-i})$: The best type A and type B configurations are the same as in case 1. However, since $\widehat{X} - x_i < \widehat{X}_{-i} \leq X^c$, the best type A configuration is now better and any optimal solution to problem (17) has $\sum_{j\neq i} x_j = \widehat{X}_{-i}$.

4. If $x_i \in [X^c - \widehat{X}_{-i}, X^c]$: In this case the best type A configuration has $x_j \leq \hat{x}_j$ for all j and $\sum_{j\neq i} x_j = X^c - x_i$, and has value $p^c(X^c - x_i) - c(X^c)$. If $x_i \leq X^c - \hat{x}_k$ for all k, then the best type B_k solution is the same as in cases 1–3. If, instead, $x_i > X^c - \hat{x}_k$ for some k, then all type B_k configurations yield a value strictly less than $p^c(X^c - x_i) - c(X^c) + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})\} = p^c(\widehat{X} - x_i) - c(\widehat{X})$. Hence, a type A configuration is optimal and any solution to problem (17) has

$\sum_{j\neq i} x_j = X^c - x_i$. If $\widehat{X} = X^c$, then a type B configuration is also optimal, and also has $\sum_{j\neq i} x_j = X^c - x_i$.

5. If $x_i > X^c$: The best type A configuration has $x_{-i} = 0$ and has value $-c(x_i)$. Since $x_i > X^c - \max_{k\neq i}\{\hat{x}_k\}$, the best type B configuration has value strictly less than $\max_k p^c \hat{x}_k - c(x_i + \hat{x}_k)$. Since $x_i > X^c$, the best type A configuration is better and any optimal solution to problem (17) has $x_{-i} = 0$.

Q.E.D.

We now prove the result by verifying that conditions (i)–(iv) of Proposition 10 are satisfied:

To check condition (i) we show that the principal does not improve her payoff by deviating to some $x \neq \hat{x}$. Suppose first that she deviates to an x with $x_i < \hat{x}_i$ for some i. Then according to case 1 of Lemma 7 her payoff is bounded above by $p^c x_i + \pi_{-i}(x_i) = p^c \widehat{X} - c(\widehat{X})$, which is her equilibrium payoff. Suppose instead that she deviates to an x with $x_i > \hat{x}_i$ for some i. Then according to cases 3–5 of Lemma 7 her payoff is bounded above by

$$\begin{aligned} &p^c x_i + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})\} + \pi_{-i}(x_i) \\ &\quad \leq p^c X^c - c(X^c) + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})\} \\ &\quad = p^c \widehat{X} - c(\widehat{X}). \end{aligned}$$

Hence, condition (i) is satisfied.

Condition (ii) is satisfied since agent i earns $\alpha(\widehat{X})\hat{x}_i - p^c \hat{x}_i \geq 0$.

To see that condition (iii) holds, note that the right-hand side of condition (16) can be written as

$$\alpha(\widehat{X})\hat{x}_i + \sum_{j\neq i} t_j(\hat{x}_j) - c(\widehat{X}) = \alpha(\widehat{X})\hat{x}_i + p^c \widehat{X}_{-i} - c(\widehat{X}). \tag{18}$$

Now consider five cases:

1. If $x_i < \hat{x}_i$: For any optimal choice x_{-i} by the principal, we have

$$\begin{aligned} \alpha\Big(x_i + \sum_{j\neq i} x_j\Big)x_i + \pi_{-i}(x_i) &= \alpha\big(x_i + (X^c - x_i)\big)x_i + p^c(\widehat{X} - x_i) - c(\widehat{X}) \\ &\leq \alpha\big(x_i + (X^c - x_i)\big)x_i + p^c(X^c - x_i) - c(X^c) \\ &= p^c X^c - c(X^c) \\ &= [p^c(X^c - \widehat{X}_{-i}) - c(X^c)] + p^c \widehat{X}_{-i} \\ &\leq [\alpha(\widehat{X})\hat{x}_i - c(\widehat{X})] + p^c \widehat{X}_{-i} \end{aligned}$$

where the last inequality follows from condition (11).

2. If $x_i = \hat{x}_i$: This follows from the same arguments as in cases 1 and 3.
3. If $x_i \in (\hat{x}_i, X^c - \widehat{X}_{-i})$: For any optimal choice by the principal, we have

$$\begin{aligned} \alpha\Big(x_i + \sum_{j\neq i} x_j\Big)x_i + \pi_{-i}(x_i) &= \alpha(x_i + \widehat{X}_{-i})x_i + p^c \widehat{X}_{-i} - c(x_i + \widehat{X}_{-i}) \\ &\leq \alpha(\hat{x}_i + \widehat{X}_{-i})\hat{x}_i + p^c \widehat{X}_{-i} - c(\hat{x}_i + \widehat{X}_{-i}) \end{aligned}$$

by condition (11).

4. If $x_i \in [X^c - \widehat{X}_{-i}, X^c]$: For any optimal choice by the principal, we have

$$\begin{aligned} \alpha\Big(x_i + \sum_{j\neq i} x_j\Big)x_i + \pi_{-i}(x_i) &= \alpha(x_i + (X^c - x_i))x_i + p^c(X^c - x_i) - c(X^c) \\ &= p^c X^c - c(X^c) \\ &\leq [\alpha(\widehat{X})\hat{x}_i - c(\widehat{X})] + p^c \widehat{X}_{-i} \end{aligned}$$

where the last inequality follows from condition (11).

5. If $x_i > X^c$: Now

$$\begin{aligned}\alpha\Big(x_i + \sum_{j\neq i} x_j\Big)x_i + \pi_{-i}(x_i) &= \alpha(x_i)x_i - c(x_i)\\ &\leq p^c x_i - c(x_i)\\ &< p^c X^c - c(X^c)\\ &\leq \big[\alpha(\widehat{X})\hat{x}_i - c(\widehat{X})\big] + p^c \widehat{X}_{-i}\end{aligned}$$

where the last inequality follows from condition (11).

For condition (iv), note that Lemma 7 tells us that the principal's optimal payoff when she does not trade with one agent i (i.e., when $x_i = 0$) is $p^c\widehat{X} - c(\widehat{X})$, which exactly equals her equilibrium payoff. *Q.E.D.*

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