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Robust shape optimal design with consideration of variation

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Abstract

Design Optimization of structures is considered under variation of loading, geometry and material properties. A hybrid, genetic/nonlinear programming algorithm is coupled to a Monte Carlo simulation technique which seeks to locate an optimal design which can function adequately under the specified range of variation. Crossectional, geometric and topological changes are considered in the formulation. A traditional finite element solution is performed for each simulation where a specific value is selected for each design parameter from a statistical distribution which defines the range of variation. A design evaluation is the result of a number of simulations which produces an output distribution for each constraint imposed upon the design. The solution is executed in a parallel computing environment due to the large number of finite element analysis runs required. A specific example involving a truss structure is presented.

1 Introduction

Traditional analysis or optimization of a structure requires exact specification of loading, restraints and material properties. Since this is not truly representative of the actual structure, a safety factor is assigned which provides latitude in the true environment the structure will see versus the specific instance assumed in the computational analysis. The question one must ask is whether or not this is the most meaningful way to conduct structural design, analysis or optimization. Disasters of varying scale are well documented through history, where bridges or other structures suffered catastrophic consequences due to unforeseen loading or material failure. Translated into computational terms, this means that these structures were designed with incorrectly specified restraints or material properties, load cases which were inaccurate or incomplete or even constitutive assumptions which were invalid. Of course, the computer has been a recent invention in the scope of these events, so it would be inappropriate to focus the blame on computational analysis. The issue remains, however, as to whether the computer allows us to perform the structural design task in a fundamentally better way.

There is no doubt that structural analysis and optimization provides a far better understanding of the structure being designed as well as the ability to investigate a number of alternative designs. On the other hand, there is virtually a guarantee that the constraint set imposed will be at least partially active at the solution. This means that maximum allowed stresses and displacements will be reached in some region(s) for the final design. A safety factor is still employed, so the computational process is not that dissimilar to traditional design methodology. The final resulting design may be lighter and less costly, but is it a better or more robust design? By moving away from a deterministic analysis and considering variation, weight and cost savings may still be achieved while the fundamental character of a robust design is simultaneously achieved. Robust in this sense refers to a design which is insensitive to reasonable variation in design properties and parameters.

The concept of robust design is certainly nothing new. The basic premise of the popular Taguchi methods in experimental design [1] is to reduce the sensitivity of a design with respect to uncontrollable factors. This infers that not only must the nominal value or mean of the design parameter distribution be considered, but also the form of the distribution itself. Each design parameter, whether it is a material property, load or geometric dimension has variation. If this variation can be represented by a statistical distribution, either real or assumed, then the objective function and design constraint satisfaction for a particular design become distributions as well. This means that the stress in elements or members as well as nodal displacements and other design constraints are not single valued for each design considered. The resulting range of values is of interest, for in conjunction with optimization, they can lead to a design which is less sensitive to variation than would be possible using a conventional formulation.

The advantage of the consideration of variation is documented in Figure 1, where three different design cases are plotted. The maximum computed design stress is plotted versus frequency of occurrence based on a sampling of values from the input load and material distributions. For a conventional analysis, only the nominal stress will be known. This means that the designs producing the results documented by Case 1 and Case 2 are considered to be equal in performance as the nominal or mean stress is identical. Case 2, however, has an obvious advantage in that it has less variation than does Case 1. This has significant importance in the areas of safety and damage tolerant design. Since the design represented by Case 2 is less sensitive to loading and material

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property variation, it is less likely to fail in service even when subjected to loading beyond that assumed in the design. It might even be possible to allow for a smaller safety factor as demonstrated by Case 3 in Figure 1. Case 3 has a higher nominal stress, but the maximum stress contained in the distribution is within the design limit imposed for all three cases (40000). Of course the input parameter distribution is rarely known, but the inclusion of some variation can lead to designs which have near or even sub-optimal weight and cost values, while simultaneously being less sensitive to variation.

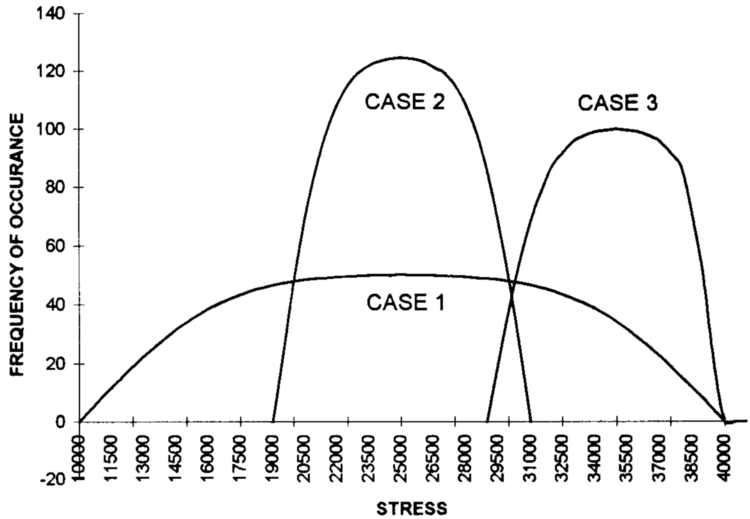

Figure 1. Output Distribution for Maximum Stress for Three Design Cases

2. Problem Formulation

Each input parameter which is allowed to vary is represented by a statistical distribution. For a normally distributed variation, this means a nominal value and standard deviation must be defined. It is not necessary to know the exact form of this distribution, but the wider the range of variation allowed, the more robust the final design. Forms of allowable input parameter variation, include load magnitude, load direction, material properties and geometric dimensions. A Monte Carlo simulation is performed to evaluate each design where all variation contributors are included. For each individual simulation of a design, a set of values are randomly selected from the specified distributions and the outputs are computed. The result from the total number of simulations is an estimate of the output distribution of the objective function and each constraint. Once again, for a normal distribution, this estimate consists of the mean and

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standard deviation. The actual number of simulations should be large enough to allow for a reasonable estimate of the output distribution, but small enough to keep the computational burden within practical limits.

The problem formulation for a minimum weight structure, subject to maximum stress and displacement constraints is as follows:

$$\text{MINIMIZE WEIGHT} = \sum \rho\ (\text{Volume})\ ;\ \text{over all elements} \qquad (1)$$

Subject to

$$\text{CON}(1) = \{\text{SLIMIT} - (\text{SMAX(MEAN)} + \text{NS*SMAX(SDEV)})\} \qquad (2)$$

and

$$\text{CON}(2) = \{\text{DLIMIT} - (\text{DMAX(MEAN)} + \text{NS*DMAX(SDEV)})\} \qquad (3)$$

Here, SLIMIT and DLIMIT are the maximum allowable stress and displacement. SMAX(MEAN) and DMAX(MEAN) are the computed mean values of the output distributions. SMAX(SDEV) and DMAX(SDEV) are the computed standard deviations of the maximum stress and displacement for the given design. The NS value simply represents the number of standard deviations desired between the mean level and the maximum allowed level. For instance, if NS is selected as three, 99.97% of the simulations for a design would have to fall below the design limit in order to satisfy the constraint. A formulation similar to that represented by Equations 1-3 was applied successfully to manufacturing tolerance optimization with the specific inclusion of variation [2].

The formulation is somewhat similar to a conventional design optimization with a deterministic evaluation employed. The difference, however, is that the formulation above captures variation in a more general fashion than does the safety factor approach. A simple case of this is when a structure is designed considering only loading in one direction. The resulting optimal structure may support this load and even an order of magnitude more before failure which is consistent with the safety factor. A load with a significant component in another direction, though, may cause failure, due to buckling or a re-distribution of loads (tension to compression) which was not considered. The inclusion of design parameter variation allows the optimization a straightforward means of compensating for these effects.

3. Optimization

The desire to locate design solutions which are robust or insensitive to variation in design parameters requires an optimization approach which is global in nature. Designs which are far apart in the design space, may have similar

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objective function and constraint values, but may have significantly different sensitivities to load or material property variation. This global optimization should also allow for topological change, as this is the most likely means of influencing the robust character of a design. This capability is an attribute of genetic algorithms and thus the application of genetic or evolutionary optimization is made to control all topological issues. Alternatively, the geometric and sizing issues require the capability of continuous variable optimization which is not conveniently addressed by genetics. The continuous variable design optimization problem is better resolved by a conventional gradient based nonlinear programming algorithm. The total process employed herein is a hybrid approach where the top level design topology is determined by the genetic algorithm and the geometric refinement is handled by a gradient based penalty function method. The combination has been proven effective on a wide range of problems spanning a number of Engineering disciplines.

3.1 Genetic Algorithm

Genetic Optimization methods emulate the process of natural selection in nature. Each design topology is represented by a string or chromosome which can be combined with other chromosomes to produce a series of design populations where both the average performance as well as the best design continuously improves. The chromosome contains strings of decision values which for a beam or truss structure define which members are active and for a plate, shell or solid structure define which elements are active. The technique operates on a population of designs, rather than a single design which is where the global nature of the process is introduced. Since the encoding of the design is operated on instead of modifications to an existing design, topological issues are easily included. The rules governing the transition of population to population are probabilistic rather than deterministic which includes randomness due to mutation.

To see how the genetic encoding is implemented, consider the structural topology pictured in Figure 2. The design involves a truss structure which is to support two loads at specified locations. The ground attachment points may move, but must remain on the vertical line shown. Only topological issues are addressed by the genetic algorithm which include the definition of the active members and potentially how many ground attachment and intermediate nodal points are used. In order to keep the maximum number of truss members fixed which simplifies the genetic representation, the number of nodes is fixed and only the active member selection is addressed. Figure 2 contains two floating nodes (points 1 and 3) and two ground attachment points (5 and 6). The floating nodes may or may not be utilized, depending upon the active members selected.

With the above assumptions, a total of fourteen truss members are possible. The most basic encoding then becomes a string of fourteen binary digits which for a particular topology might be:

$$\text{Encoding} = \{0,0,0,0,0,1,1,1,0,0,1,0,1,1\} \quad (4)$$

A zero in a position indicates that a member is inactive while a one in a position indicates it is active. The encoding represented in Equation 4, then, identifies members 6,7,8,11,13 and 14 as active which produces a six bar truss design. The analysis begins with a population of encodings generated randomly and operates on them with the genetic algorithm to update the population from generation to generation until convergence is achieved. Details on the algorithmic procedures employed are provided by Goldberg [3] and Davis [4].

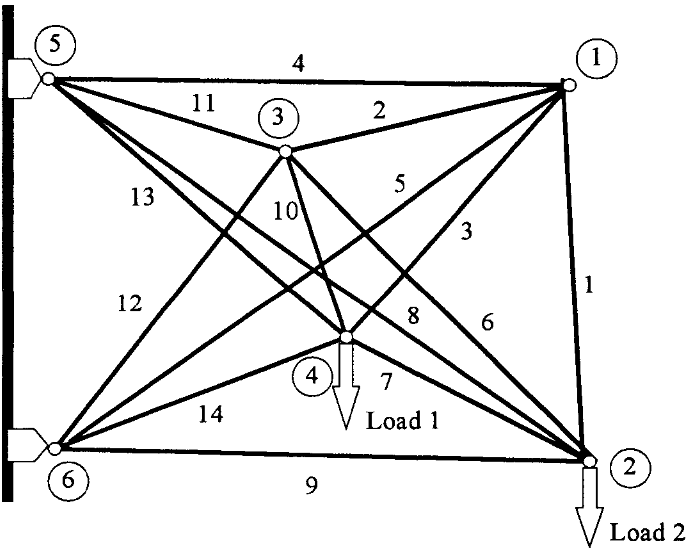

Figure 2. Possible Design Topologies for a Two Load, Six Node Truss Structure

3.2 Nonlinear Programming Algorithm

Once the design topology is known, the optimization problem becomes one of determining geometric dimensions involving size and location. These issues are concerned with continuous variables and as such are easily handled by conventional nonlinear programming techniques. Whether the optimization is performed on a conventional formulation or one involving variation, design constraints must be included. This gives rise to a penalty type method. The general form of the penalty function is given as:

$$P(x) = F(x) + \Omega\,(R,CON) \qquad (5)$$

Here F(x) is the objective function (weight), Ω is a particular penalty function form which penalizes an infeasible design, R is a penalty parameter which is used to force convergence to a feasible design and CON is the vector of constraint values. The exact penalty form varies widely in practice. The particular form utilized here, is a biased penalty function [5] which seeks to reduce the distortion of the original objective function contours.

Once again referring to Figure 2, the design variables for the nonlinear programming formulation include the crossectional area of each active member as well as the location of the ground and free nodes. For the genetic encoding represented by Equation 4 there are a total of twelve variables, six crossectional areas, the y location of each ground point and the x and y location for each floating node.

3.3 Integration

The general flow through the optimization is quite straightforward. The genetic optimizer operates at the highest level and determines the topology of the design. Control is then passed to the penalty function algorithm to optimize the geometry and crossectional areas for the topology selected. Each evaluation of an objective function or constraint requires a number of evaluations, based on the number of simulations required to estimate the output distributions. This results in a large number of finite element analysis runs. The dual parallel nature, at the genetic population level and at the simulation level, can be exploited to reduce solution time. In addition, design sensitivity information can be utilized to further reduce the computational effort.

A number of other issues deserve some mention. The topology of a particular design represented by the genetic encoding may not be a structure at all. This situation is handled by assigning a small crossectional area to the non-active members. This ensures a non-singular stiffness matrix for the finite element solution. It also has the added benefit of maintaining a consistent size and form in the stiffness matrix. The other issue is related to the specific values selected for the input parameter variation distributions. In order to maintain an equivalence between design comparisons, the same set of randomly generated input value sets are use for all design evaluations. This allows a reasonably small number of simulations for each design evaluation.

4.0 Example

In order to illustrate the capability of the robust design methodology, consider the example pictured in Figure 3. Two loads must be supported at a distance of 360 and 720 units from the vertical supporting plane. Several solutions to the problem considering minimum weight have been presented for this example [6]. To maintain consistency with the formulation developed for input parameter variation, two ground points and two free nodes will be included. Input parameter variation is considered in load magnitude, load direction and material modulus.

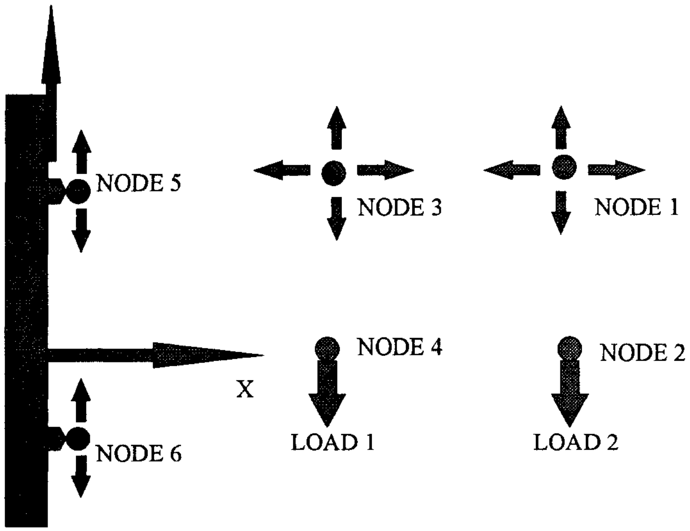

Figure 3. Two Load Design Example

The variation introduced is summarized in Table 1. The constraints are required to be satisfied to at least one sigma value of the output distributions, with a maximum stress limit of 25,000 and displacement limit of 2.5 units.

	Load Magnitude	Load 1/2 Direction	Material Modulus E
Mean	100,000	-90 Deg	10E+06
Std. Deviation	20,000	15 Deg	2E+05

Table 1. Variation in Input Parameters Imposed for Example

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The solution located for the robust design formulation is compared to previously generated solutions in Table 2. The resulting design was a four bar truss with active members 7,8,13 and 14. The final topology is pictured in Figure 4.

	Robust Design	Design 1 [6]	Design 3 [6]
Mean Stress	12,226	13,240	14,181
St. Dev. Stress	2438	2038	3051
Mean Disp.	1.86	2.72	2.59
St. Dev. Disp.	0.645	0.808	0.692
Elements	4	5	4
Weight	2574	3210	1916

Table 2. Comparison of Solution Results

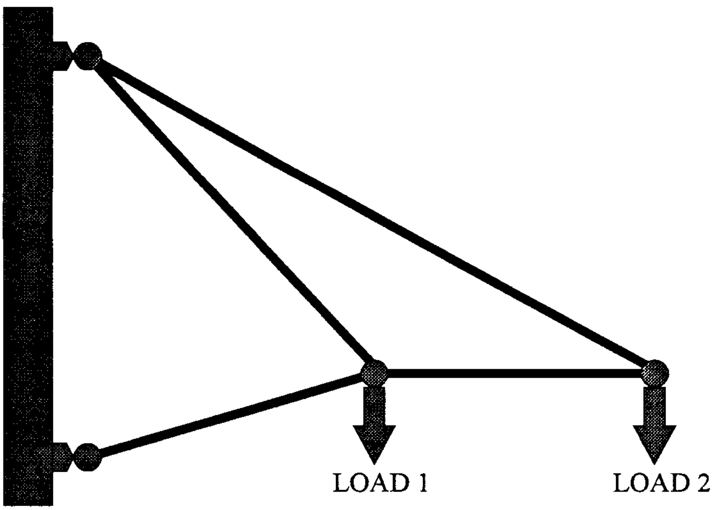

Figure 4. Optimal Solution Topology

A direct comparison to the previously reported results is difficult as when variation is considered, both of the previous results have displacements above the allowable, even at their mean level. The results do show, however that the mean values for the constraints are lower and the standard deviation for the active displacement constraint is less for the robust design. The standard deviation with regard to stress is a little higher for the robust design compared to Design 1, but it should be noted that the nominal stress is over ten standard deviations below the maximum. When the robust design is allowed to approach a 2 unit displacement, the weight is reduced to 1492, which is actually a much lighter structure than was previously found.

It is interesting to point out that Designs 1 and 2 reported in Table 2 were actually optimized under a multiple-objective formulation where sensitivity was a design consideration. Minimum weight designs generated by a standard formulation in the same weight range (+50% of robust design optimal) typically have standard deviations in the range of 5000-10,000 for stress and 0.80 to 1.25 for displacement. This certainly places the robust design optimal solution in a very good comparative position. It is less sensitive to the critical constraint and has a weight equal to or better than the best previously reported design.

5.0 Summary and Conclusions

A new structural design methodology has been presented which considers the robust character of the design and has been demonstrated on a simple example. The key attribute of the procedure is the inclusion of variability in the input parameters and their effect on the desired output performance. The algorithm employed is a hybrid combination of genetic and conventional nonlinear programming algorithms. A Monte Carlo simulation approach is applied to address the variation in the input parameters. The parallel nature of the algorithm compensates for the additional computational burden inherent in the approach. The results on the simple example, clearly demonstrate the advantage of the approach.

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