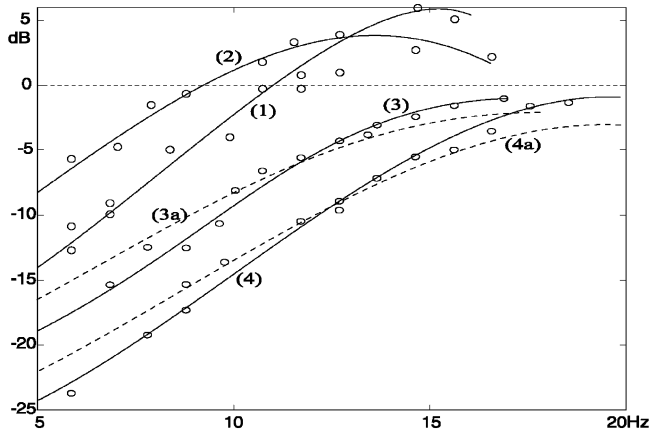

Fig. 5. Experimental and simulation sensitivity functions of the closed-loop system. (1) Linear state-feedback controller (3), (2) first-order nonlinear state-feedback controller (14a), (3) second order nonlinear state-feedback controller (16) and (4) nonlinear output-feedback controller (16) and (19). (3a) and (4a) represent the simulation responses corresponding to (3) and (4). The value of W_u puts a penalty on the control signal; small values of W_u lead to faster transient response and, hence, wider bandwidth. In all responses contained in this note γ and $W_u = 0.12$.

the now-classical linear state feedback controllers. The concept of linear H_∞ [13] has been used for Maglev control [14], [15] earlier, however, direct application of nonlinear H_∞ to deal with track disturbance in an EMS system is considered to be novel.

Although several issues require careful assessment for the real-time implementation of these nonlinear H_∞ controllers derived here, extensive range of experimental work carried out by the authors indicate that, providing a reasonable care is taken in specifying the physical parameters of the suspension magnet, the analytically derived control laws, for a given set of α, β, γ and W_u, may directly be used in assessing the performance of laboratory-scale demonstration systems. A key difference between the nonlinear state and the output feedback controllers is the execution time of the control algorithms ((16) and (19)): 50 μs for the former and 400 μs for the latter (within a sampling interval of 1 ms). In multimagnet vehicles this may impose some operational constraints. To overcome this, the embedded DSP hardware described in Section V provides communication protocols between local control loops for individual magnets and supervisory control functions to coordinate the distribution of suspension force. The dynamics of these mechanically coupled magnets on suspension stability and tracking properties are currently under investigation.

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Robust Stability and Stabilization of Discrete Singular Systems: An Equivalent Characterization

Shengyuan Xu and James Lam

***Abstract*—This note deals with the problems of robust stability and stabilization for uncertain discrete-time singular systems. The parameter uncertainties are assumed to be time-invariant and norm-bounded appearing in both the state and input matrices. A new necessary and sufficient condition for a discrete-time singular system to be regular, causal and stable is proposed in terms of a strict linear matrix inequality (LMI). Based on this, the concepts of generalized quadratic stability and generalized quadratic stabilization for uncertain discrete-time singular systems are introduced. Necessary and sufficient conditions for generalized quadratic stability and generalized quadratic stabilization are obtained in terms of a strict LMI and a set of matrix inequalities, respectively. With these conditions, the problems of robust stability and robust stabilization are solved. An explicit expression of a desired state feedback controller is also given, which involves no matrix decomposition. Finally, an illustrative example is provided to demonstrate the applicability of the proposed approach.**

***Index Terms*—Discrete-time systems, linear matrix inequality (LMI), parameter uncertainty, robust stability, robust stabilization, singular systems.**

I. INTRODUCTION

The problems of robust stability analysis and robust stabilization of linear state-space systems with parameter uncertainties have received much attention in the past decades [3], [23]. A great number of results on these topics have appeared in the literature. Among the different approaches dealing with these problems, the methods based on the concepts of *quadratic stability* and *quadratic stabilizability* have

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become popular. An uncertain system is quadratically stable if there exists a fixed Lyapunov function to check the stability of the uncertain system, while an uncertain system is quadratically stabilizable if there exists a feedback controller such that the resulting closed-loop system is quadratically stable. Many results on quadratic stability and quadratic stabilizability have been reported in both the continuous and discrete contexts; see, e.g., [1], [7], [17], [24] and the references therein.

There has been a growing interest in the control of singular systems since such systems have extensive applications in large-scale systems, economic systems, power systems, and other areas [4], [8]. Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semistate systems [4], [8]. Some fundamental results based on the theory of state-space systems have been successfully extended to singular systems. Recently, much attention has been focused on the problems of robust stability and robust stabilization of singular systems. Now, it is known that the robust stability problem for singular systems is much more complicated than that for state-space systems because it requires to consider not only stability robustness, but also regularity and absence of impulses (for continuous singular systems) or causality (for discrete singular systems) simultaneously [5], [6], [9], [16], [18], while the latter two do not arise in the state-space context. In the continuous setting, the robust stabilization problem was solved by the notion of generalized quadratic stabilization in [21], and an algebraic approach was proposed, which was extended to singular systems with time delays in [19] via a linear matrix inequality (LMI) approach. When parameter uncertainty appears in the derivative matrix, a necessary and sufficient condition for the generalized quadratic stabilization were obtained in terms of Riccati inequalities in [10]. For uncertain discrete-time singular systems, the robust stabilization problem was investigated in [22], in which a sufficient condition for generalized quadratic stabilization was obtained. It should be pointed out that only under some assumptions on the system matrices, this sufficient condition can be shown to be necessary [22]. It is noted that when there are no assumptions on the system matrices, necessary and sufficient conditions for generalized quadratic stabilization of uncertain discrete-time singular systems still have not been reported in the literature. This issue is much more complicated than that for the state-space case; one reason is that the Lyapunov matrix in discrete-time singular systems is indefinite, while in the state-space case the corresponding Lyapunov matrix is positive definite [20], [22]; this fact also leads to the requirement of system decomposition when designing stabilizing state feedback controllers for discrete-time singular systems [20], [22], which may cause some numerical problems and lacks mathematical elegance. Furthermore, it is worth pointing out that parameter uncertainties in input matrices were not considered in [22], which limits the scope of applications of the robust stabilization results in [22].

In this note, we consider the problems of robust stability analysis and robust stabilization for uncertain discrete-time singular systems with parameter uncertainties in both the state and input matrices. The parameter uncertainties are assumed to be time-invariant and unknown but norm-bounded. In terms of a strict LMI, we first present a new necessary and sufficient condition for a discrete-time singular system to be regular, causal and stable. Based on this, we then introduce the concepts of generalized quadratic stability and generalized quadratic stabilizability for uncertain discrete-time singular systems. It is shown that generalized quadratic stability and generalized quadratic stabilizability imply robust stability and robust stabilizability, respectively. A necessary and sufficient condition for generalized quadratic stability is proposed in terms of a strict LMI, which can be checked easily by recently developed standard algorithms [2]. Compared with a nonstrict LMI condition, which is always encountered in dealing with control problems for singular systems, testing a strict LMI condition can avoid some numerical problems since equality constraints are fragile and usually not met perfectly [13]. A necessary and sufficient condition for the generalized quadratic stabilizability is also proposed in terms of a set of strict matrix inequalities, which is of theoretical elegance in comparison with the sufficient condition in [22]. When these matrix inequalities are feasible, an explicit expression of a desired state feedback gain is given.

A. Notation

Throughout this note, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension, the superscript "T" represents the transpose, $\|x\|$ is the Euclidean norm of the vector x. The notation $D_{\rm int}(0,1)$ is the interior of the unit disk with center at the origin. $\lambda(E,A) = \{z \,|\det(zE - A) = 0\}$. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

II. Definition and Problem Formulation

Consider an uncertain linear discrete-time singular system described by

$$Ex(k+1) = (A + \Delta A)x(k) + (B + \Delta B)u(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular, we shall assume that $\mathrm{rank}E = r \leq n$. A and B are known real constant matrices with appropriate dimensions. ΔA and ΔB are time-invariant matrices representing norm-bounded parameter uncertainties, and are assumed to be of the following form:

$$[\,\Delta A \quad \Delta B\,] = MF(\sigma)\,[\,N_1 \quad N_2\,] \tag{2}$$

where M, N_1 and N_2 are known real constant matrices with appropriate dimensions. The uncertain matrix $F(\sigma)$ satisfies

$$F(\sigma)F(\sigma)^T \leq I \tag{3}$$

and $\sigma \in \Xi$, where Ξ is a compact set in $\mathbb{R}$. Furthermore, it is assumed that given any matrix $F : FF^T \leq I$, there exists a $\sigma \in \Xi$ such that $F = F(\sigma)$. ΔA and ΔB are said to be admissible if both (2) and (3) hold.

Remark 1: It should be pointed out that the structure of the uncertainty with the form (2) and (3) has been widely used when dealing with the problem of robust stabilization for state-space and singular uncertain systems in both continuous and discrete time contexts; see e.g., [14] and [22].

The nominal unforced discrete-time singular system of (1) can be written as:

$$Ex(k+1) = Ax(k). \tag{4}$$

Definition 1 [4], [12], [20]:

I) System (4) is said to be regular if $\det(zE - A)$ is not identically zero.

II) System (4) is said to be causal if $\deg(\det(zE - A)) = \mathrm{rank}E$.

III) System (4) is said to be stable if $\lambda(E,A) \subset D_{\rm int}(0,1)$.

IV) System (4) is said to be admissible if it is regular, causal, and stable.

The purpose of this note is to develop conditions for robust stability and robust stabilization for the uncertain discrete-time singular system (1). More specifically, the objective of the robust stability problem is to

develop conditions such that the unforced discrete-time singular system of (1) is admissible, while the aim of the robust stabilization problem is to determine a linear state feedback control law for the uncertain singular system (1) such that the resulting closed-loop system is admissible for all parameter uncertainties satisfying (2) and (3).

III. Main Results

In this section, we give solutions to the robust stability and stabilization problems formulated previously in terms of some strict matrix inequalities. First, we present the following result for the singular discrete-time system (4) to be admissible, which will play an important role in the derivation of our main results.

Theorem 1: Discrete-time singular system (4) is admissible if and only if there exist matrices $P > 0$ and Q such that

$$A^T P A - E^T P E + Q S^T A + A^T S Q^T < 0 \tag{5}$$

where $S \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column and satisfies $E^T S = 0$.

Proof:

Necessity: Suppose the discrete-time singular system (4) is admissible, then it follows from [4] that there exist two nonsingular matrices $\bar{M}$ and $\bar{N}$ such that

$$E = \bar{M}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\bar{N} \quad A = \bar{M}\begin{bmatrix} \bar{A} & 0 \\ 0 & I \end{bmatrix}\bar{N} \tag{6}$$

where $\bar{A} \in \mathbb{R}^{r\times r}$ and

$$\lambda(I, \bar{A}) \subset D_{\text{int}}(0,1). \tag{7}$$

Then, S can be expressed as

$$S = \bar{M}^{-T}\begin{bmatrix} 0 \\ I \end{bmatrix} H \tag{8}$$

where H is any nonsingular matrix. Considering (7), we have that there exists a matrix $\bar{P} > 0$ such that $\bar{A}^T \bar{P} \bar{A} - \bar{P} < 0$. Now, define

$$P = \bar{M}^{-T}\begin{bmatrix} \bar{P} & 0 \\ 0 & I \end{bmatrix}\bar{M}^{-1} \quad Q = \bar{N}^T\begin{bmatrix} 0 \\ -I \end{bmatrix} H^{-T}. \tag{9}$$

Then, it is easy to verify that the matrices P and Q in (9) satisfy (5).

Sufficiency: Under the condition of the theorem, we first show that the discrete-time singular system (4) is regular and causal. To this end, we follow a similar line as in the proof of [20, Th. 1], and choose two nonsingular matrices $\hat{M}$ and $\hat{N}$ such that [4]

$$E = \hat{M}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\hat{N} \quad A = \hat{M}\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}\hat{N}. \tag{10}$$

Then, S can be given as

$$S = \hat{M}^{-T}\begin{bmatrix} 0 \\ I \end{bmatrix}\hat{H} \tag{11}$$

where $\hat{H}$ is any nonsingular matrix. Write

$$\hat{M}^T P \hat{M} = \begin{bmatrix} \hat{P}_1 & \hat{P}_2 \\ \hat{P}_2^T & \hat{P}_3 \end{bmatrix} \quad \hat{N}^{-T} Q \hat{H}^T = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \tag{12}$$

where the partition is compatible with that of A in (10). Now, substituting (10)–(12) to (5) gives

$$\hat{N}^T\begin{bmatrix} * & * \\ * & W \end{bmatrix}\hat{N} < 0 \tag{13}$$

where $*$ represents matrices that are not relevant in the following discussion, and

$$W = A_2^T \hat{P}_1 A_2 + A_4^T \hat{P}_2^T A_2 + A_2^T \hat{P}_2 A_4 + A_4^T \hat{P}_3 A_4 + Q_2 A_4 + A_4^T Q_2^T.$$

From (13), it is easy to see

$$W < 0. \tag{14}$$

Now, we assert that the matrix A_4 is nonsingular. Suppose, by contradiction, A_4 is singular. Then, there exists a vector $\xi \neq 0$ such that $A_4 \xi = 0$. Pre- and postmultiplying (14) by ξ^T and ξ result in

$$\xi^T A_2^T \hat{P}_1 A_2 \xi < 0. \tag{15}$$

On the other hand, since $P > 0$, we have $\hat{P}_1 > 0$. Then, it is easy to see that (15) is a contradiction since $\xi^T A_2^T \hat{P}_1 A_2 \xi \geq 0$. Therefore, A_4 is nonsingular, which implies that the discrete-time singular system (4) is regular and causal.

Next, we show that (4) is stable. Since (4) is regular and causal, there exist two nonsingular matrices $\tilde{M}$ and $\tilde{N}$ such that [4]

$$E = \tilde{M}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\tilde{N} \quad A = \tilde{M}\begin{bmatrix} \tilde{A} & 0 \\ 0 & I \end{bmatrix}\tilde{N}. \tag{16}$$

In this case, S can be written as

$$S = \tilde{M}^{-T}\begin{bmatrix} 0 \\ I \end{bmatrix}\tilde{H} \tag{17}$$

where $\tilde{H}$ is any nonsingular matrix. Write

$$\tilde{M}^T P \tilde{M} = \begin{bmatrix} \tilde{P}_1 & \tilde{P}_2 \\ \tilde{P}_2^T & \tilde{P}_3 \end{bmatrix} \quad \tilde{N}^{-T} Q \tilde{H}^T = \begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \end{bmatrix} \tag{18}$$

where the partition is compatible with that of A in (16). Now, substituting (16)–(18) into (5) yields

$$\tilde{N}^T\begin{bmatrix} \tilde{A}^T \tilde{P}_1 \tilde{A} - \tilde{P}_1 & \tilde{A}^T \tilde{P}_2 + \tilde{Q}_1 \\ \tilde{P}_2^T \tilde{A} + \tilde{Q}_1^T & \tilde{P}_3 + \tilde{Q}_2 + \tilde{Q}_2^T \end{bmatrix}\tilde{N} < 0$$

which implies $\tilde{A}\tilde{P}_1\tilde{A}^T - \tilde{P}_1 < 0$. Noting this and $\tilde{P}_1 > 0$, we have that $\lambda(I, \tilde{A}) \subset D_{\text{int}}(0,1)$. Therefore, (4) is stable. This together with the regularity and causality of (4) gives that the discrete-time singular system (4) is admissible. This completes the proof. □

Remark 2: Theorem 1 provides a necessary and sufficient condition for the discrete-time singular system (4) to be admissible. It is noted that the condition in (5) is a strict LMI, which is in contrast to that in [20] where a nonstrict LMI was reported. As is known, some numerical problems may arise when checking nonstrict LMI conditions. Therefore, the strict LMI condition in (5) is more desirable than the nonstrict one. Furthermore, the LMI condition in (5) will make it quite easy in finding stabilizing state feedback controllers, which will be demonstrated in the sequel.

It is well-known that the concepts of quadratic stability and quadratic stabilization have played important roles in dealing with the problems of robust stability and stabilization for uncertain state-space systems. Considering this and Theorem 1, we introduce the following definitions for uncertain discrete-time singular systems.

Definition 2: Uncertain discrete-time singular system (1) is said to be generalized quadratically stable if there exist matrices $P > 0$ and Q such that

$$(A + \Delta A)^T P (A + \Delta A) - E^T P E + Q S^T (A + \Delta A) + (A + \Delta A)^T S Q^T < 0 \tag{19}$$

where $S \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column and satisfies $E^TS = 0$.

Definition 3: Uncertain discrete-time singular system (1) is said to be generalized quadratically stabilizable if there exists a linear state feedback control law $u(k) = Kx(k)$, $K \in \mathbb{R}^{m\times n}$, matrices $P > 0$ and Q such that

$$(A_K + \Delta A_K)^T P (A_K + \Delta A_K) - E^T PE + QS^T (A_K + \Delta A_K) + (A_K + \Delta A_K)^T SQ^T < 0 \quad (20)$$

for all admissible uncertainties ΔA and ΔB, where

$$A_K = A + BK \quad \Delta A_K = \Delta A + \Delta BK. \quad (21)$$

It is easy to show that the generalized quadratic stability of system (1) implies the admissibility of the unforced system of (1) for all admissible parameter uncertainties, while the generalized quadratic stabilizability of (1) implies that there exists a linear state feedback control law such that the resulting closed-loop system is admissible. Taking into account this, in the following, attention will be focused on the development of conditions for generalized quadratic stability and generalized quadratic stabilization for the uncertain discrete-time singular system (1), respectively. For this purpose, the following lemmas will be needed.

Lemma 1 [11]: Given matrices Ω, Γ and Ξ of appropriate dimensions and with Ω symmetrical, then

$$\Omega + \Gamma F(\sigma)\Xi + (\Gamma F(\sigma)\Xi)^T < 0$$

for all $F(\sigma)$ satisfying $F(\sigma)F(\sigma)^T \le I$, if and only if there exists a scalar $\epsilon > 0$ such that

$$\Omega + \epsilon\Gamma\Gamma^T + \epsilon^{-1}\Xi^T\Xi < 0.$$

Lemma 2 [15]: Given any matrices $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$ with appropriate dimensions such that $\mathcal{Y} > 0$. Then, we have

$$\mathcal{X}^T\mathcal{Z} + \mathcal{Z}^T\mathcal{X} + \mathcal{X}^T\mathcal{Y}\mathcal{X} \ge -\mathcal{Z}^T\mathcal{Y}^{-1}\mathcal{Z}.$$

Now, we are in a position to present the generalized quadratic stability result.

Theorem 2: The uncertain discrete-time singular system (1) is generalized quadratically stable if and only if there exist a scalar $\epsilon > 0$, matrices $P > 0$ and Q such that the following LMI holds:

$$\begin{bmatrix} QS^TA + A^TSQ^T - E^TPE + \epsilon N_1^TN_1 & A^TP & QS^TM \\ PA & -P & PM \\ M^TSQ^T & M^TP & -\epsilon I \end{bmatrix} < 0 \quad (22)$$

where the matrix $S \in \mathbb{R}^{n\times(n-r)}$ is given in Definition 2.

Proof:

Sufficiency: Suppose that there exist a scalar $\epsilon > 0$, matrices $P > 0$ and Q such that the LMI in (22) holds. Then, by the Schur complement formula, we have

$$\begin{bmatrix} QS^TA + A^TSQ^T - E^TPE & A^TP \\ PA & -P \end{bmatrix} + \epsilon^{-1}\begin{bmatrix} QS^TM \\ PM \end{bmatrix}\begin{bmatrix} QS^TM \\ PM \end{bmatrix}^T + \epsilon\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}^T < 0. \quad (23)$$

Note that for any $F(\sigma)$ satisfying (3) and any scalar $\epsilon > 0$

$$\mathcal{X}F(\sigma)\mathcal{Y} + (\mathcal{X}F(\sigma)\mathcal{Y})^T \le \epsilon^{-1}\mathcal{X}\mathcal{X}^T + \epsilon\mathcal{Y}^T\mathcal{Y}$$

for any matrices $\mathcal{X}$ and $\mathcal{Y}$ with appropriate dimensions. Then, we have

$$\begin{aligned} &\begin{bmatrix} QS^T\Delta A + \Delta A^TSQ^T & \Delta A^TP \\ P\Delta A & 0 \end{bmatrix} \\ &= \begin{bmatrix} QS^TM \\ PM \end{bmatrix} F(\sigma)\begin{bmatrix} N_1 & 0 \end{bmatrix} + \begin{bmatrix} N_1^T \\ 0 \end{bmatrix} \\ &\quad \times F(\sigma)^T\begin{bmatrix} M^TSQ^T & M^TP \end{bmatrix} \\ &\le \epsilon^{-1}\begin{bmatrix} QS^TM \\ PM \end{bmatrix}\begin{bmatrix} QS^TM \\ PM \end{bmatrix}^T + \epsilon\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}^T. \end{aligned}$$

Hence

$$\begin{aligned} &\begin{bmatrix} QS^T(A+\Delta A) + (A+\Delta A)^TSQ^T - E^TPE & (A+\Delta A)^TP \\ P(A+\Delta A) & -P \end{bmatrix} \\ &\le \begin{bmatrix} QS^TA + A^TSQ^T - E^TPE & A^TP \\ PA & -P \end{bmatrix} \\ &\quad + \epsilon^{-1}\begin{bmatrix} QS^TM \\ PM \end{bmatrix}\begin{bmatrix} QS^TM \\ PM \end{bmatrix}^T + \epsilon\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}^T. \end{aligned}$$

This, together with (23), gives

$$\begin{bmatrix} QS^T(A+\Delta A) + (A+\Delta A)^TSQ^T - E^TPE & (A+\Delta A)^TP \\ P(A+\Delta A) & -P \end{bmatrix} < 0$$

which, by the Schur complement formula again, implies

$$(A+\Delta A)^TP(A+\Delta A) - E^TPE + QS^T(A+\Delta A) + (A+\Delta A)^TSQ^T < 0.$$

Then, by Definition 2, we have that the uncertain discrete-time singular system (Σ) is generalized quadratically stable.

Necessity: Assume that the uncertain singular delay system (Σ) is generalized quadratically stable. It follows from Definition 2 that there exist matrices $P > 0$ and Q such that (19) holds. Thus, for all $F(\sigma)$ satisfying (2) and (3), the following inequality holds:

$$\begin{bmatrix} QS^T(A+\Delta A) + (A+\Delta A)^TSQ^T - E^TPE & (A+\Delta A)^TP \\ P(A+\Delta A) & -P \end{bmatrix} < 0.$$

That is

$$\begin{aligned} &\begin{bmatrix} QS^TA + A^TSQ^T - E^TPE & A^TP \\ PA & -P \end{bmatrix} + \begin{bmatrix} QS^TM \\ PM \end{bmatrix} \\ &\quad \times F(\sigma)\begin{bmatrix} N_1 & 0 \end{bmatrix} + \begin{bmatrix} N_1^T \\ 0 \end{bmatrix}F(\sigma)^T\begin{bmatrix} M^TSQ^T & M^TP \end{bmatrix} < 0 \end{aligned}$$

is satisfied for all $F(\sigma)$ satisfying (2) and (3). By using Lemma 1, we can deduce that there exists a scalar $\epsilon > 0$ such that

$$\begin{aligned} &\begin{bmatrix} QS^TA + A^TSQ^T - E^TPE & A^TP \\ PA & -P \end{bmatrix} + \epsilon^{-1}\begin{bmatrix} QS^TM \\ PM \end{bmatrix} \\ &\quad \times \begin{bmatrix} QS^TM \\ PM \end{bmatrix}^T + \epsilon\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}^T < 0 \end{aligned}$$

which, by the Schur complement formula, gives that the LMI in (22) holds. This completes the proof. □

Remark 3: Theorem 2 provides a necessary and sufficient condition for the generalized quadratic stability of the uncertain discrete-time singular system (1) in terms of a strict LMI in (22), which can be checked numerically very efficiently by using recently developed interior-point methods [2]. It is worth pointing out that such a strict LMI condition for testing the generalized quadratic stability in the context of singular discrete-time systems has not been reported in the literature.

The following theorem gives the result on the generalized quadratic stabilizability.

Theorem 3: The uncertain discrete-time singular system (1) is generalized quadratically stabilizable if and only if there exist scalars $\epsilon > 0$, $\delta > 0$, matrices $P > 0$ and Q such that

$$\Gamma := P^{-1} - \epsilon^{-1} M M^T > 0 \tag{24}$$

and

$$\begin{aligned} &QS^TA + A^TSQ^T - E^TPE + \epsilon N_1^T N_1 + \epsilon^{-1} QS^T M M^T SQ^T \\ &+ \left(A^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} \left(A^T + \epsilon^{-1} QS^T M M^T\right)^T \\ &- \Psi \Phi^{-1} \Psi^T < 0 \end{aligned} \tag{25}$$

where the matrix $S \in \mathbb{R}^{n \times (n-r)}$ is given in Definition 2, and

$$\begin{aligned} \Phi &= B^T \Gamma^{-1} B + \epsilon N_2^T N_2 + \delta I \\ \Psi &= QS^T B + \left(A^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} B + \epsilon N_1^T N_2. \end{aligned}$$

In this case, a robustly stabilizing state feedback control law can be chosen by

$$u(k) = -\Phi^{-1} \Psi^T x(k). \tag{26}$$

Proof:

Sufficiency: Applying the state feedback controller (26) into (1) results in the following closed-loop system:

$$Ex(k+1) = \left[A_c + MF(\sigma) N_c\right] x(k) \tag{27}$$

where

$$A_c = A - B\Phi^{-1}\Psi^T \quad N_c = N_1 - N_2 \Phi^{-1} \Psi^T.$$

Note $B^T \Gamma^{-1} B + \epsilon N_2^T N_2 < \Phi$. Then, by a routine calculation, it can be verified that

$$\begin{aligned} &QS^TA_c + A_c^TSQ^T + \epsilon N_c^T N_c \\ &\quad + \left(A_c^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} \left(A_c^T + \epsilon^{-1} QS^T M M^T\right)^T \\ &\le QS^TA_c + A_c^TSQ^T + \epsilon N_1^T N_1 \\ &\quad - \epsilon N_1^T N_2 \Phi^{-1} \Psi^T - \epsilon \Psi \Phi^{-1} N_2^T N_1 \\ &\quad + \left(A^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} \left(A^T + \epsilon^{-1} QS^T M M^T\right)^T \\ &\quad - \left(A^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} B \Phi^{-1} \Psi^T \\ &\quad - \Psi \Phi^{-1} B^T \Gamma^{-1} \left(A^T + \epsilon^{-1} QS^T M M^T\right)^T + \Psi \Phi^{-1} \Psi^T \\ &= QS^TA + A^TSQ^T + \epsilon N_1^T N_1 \\ &\quad + \left(A^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} \left(A^T + \epsilon^{-1} QS^T M M^T\right)^T \\ &\quad - \Psi \Phi^{-1} \Psi^T. \end{aligned}$$

This, together with (25), gives

$$\begin{aligned} &QS^TA_c + A_c^TSQ^T - E^TPE + \epsilon N_c^T N_c + \epsilon^{-1} QS^T M M^T SQ^T \\ &+ \left(A_c^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} \left(A_c^T + \epsilon^{-1} QS^T M M^T\right)^T < 0 \end{aligned}$$

which, by the Schur complement formula, implies

$$\begin{bmatrix} QS^TA_c + A_c^TSQ^T - E^TPE + \epsilon N_c^T N_c & A_c^TP & QS^TM \\ PA_c & -P & PM \\ M^TSQ^T & M^TP & -\epsilon I \end{bmatrix} < 0.$$

Thus, by using Theorem 2, we have that the closed-loop system (27) is generalized quadratically stable, which implies that the uncertain discrete-time singular system (1) is generalized quadratically stabilizable.

Necessity: Assume that the uncertain discrete-time singular system (1) is generalized quadratically stabilizable. Then, by Definition 3, it follows that there exist a linear state feedback control law $u(k) = Kx(k)$, $K \in \mathbb{R}^{m \times n}$, matrices $P > 0$ and Q such that (20) is satisfied for all admissible uncertainties ΔA and ΔB. Thus, by using Theorem 2, we have that there exists a scalar $\epsilon > 0$ such that

$$\begin{bmatrix} QS^TA_K + A_K^TSQ^T - E^TPE + \epsilon N_K^T N_K & A_K^TP & QS^TM \\ PA_K & -P & PM \\ M^TSQ^T & M^TP & -\epsilon I \end{bmatrix} < 0. \tag{28}$$

where A_K is given in (21) and

$$N_K = N_1 + N_2 K. \tag{29}$$

By the Schur complement formula, it follows from (28) that

$$\begin{aligned} &QS^TA_K + A_K^TSQ^T - E^TPE + \epsilon N_K^T N_K + \epsilon^{-1} QS^T M M^T SQ^T \\ &+ \left(A_K^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} \left(A_K^T + \epsilon^{-1} QS^T M M^T\right)^T < 0. \end{aligned} \tag{30}$$

Substituting the expressions of A_K and N_K into (30) yields

$$\begin{aligned} &QS^TA + A^TSQ^T - E^TPE + \epsilon N_1^T N_1 + \epsilon^{-1} QS^T M M^T SQ^T \\ &+ \left(A^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} \left(A^T + \epsilon^{-1} QS^T M M^T\right)^T \\ &+ \Psi K + K^T \Psi^T + K^T \left[B^T \Gamma^{-1} B + \epsilon N_2^T N_2\right] K < 0. \end{aligned} \tag{31}$$

From this inequality, it is easy to show that there exists a scalar $\delta > 0$ such that

$$\begin{aligned} &QS^TA + A^TSQ^T - E^TPE + \epsilon N_1^T N_1 + \epsilon^{-1} \\ &QS^TMM^TSQ^T + \left(A^T + \epsilon^{-1} QS^T M M^T\right) \Gamma^{-1} \\ &\times \left(A^T + \epsilon^{-1} QS^T M M^T\right)^T + \Psi K + K^T \Psi^T \\ &+ K^T \left[B^T \Gamma^{-1} B + \epsilon N_2^T N_2 + \delta I\right] K < 0. \end{aligned} \tag{32}$$

By Lemma 2, it follows that

$$\begin{aligned} &\Psi K + K^T \Psi^T + K^T \left[B^T \Gamma^{-1} B + \epsilon N_2^T N_2 + \delta I\right] \\ &\qquad K \ge -\Psi \Phi^{-1} \Psi^T. \end{aligned}$$

This together with (32) implies that (25) holds, which completes the proof. □

Remark 4: It is noted that Theorem 2 in [22] also provides a necessary and sufficient condition for generalized quadratic stabilizability for uncertain discrete-time systems. However, [22, Th. 2] is obtained under the assumption that $\operatorname{rank}(E, M) = \operatorname{rank} E$. Also, [22, Th. 2] can only be applicable to the case when there is not any parameter uncertainty in the input matrix; that is, $\Delta B = 0$, while Theorem 3 in this note is obtained without any assumptions on the system matrices; furthermore, Theorem 3 can be applicable to systems with parameter uncertainties in both state and input matrices. In view of this, Theorem 3 is more general and elegant than [22, Th. 2].

In the case when there is no parameter uncertainties in the uncertain discrete-time singular system (1); that is, (1) reduces to

$$Ex(k+1) = Ax(k) + Bu(k) \tag{33}$$

then, by Theorem 3, we have the following stabilization result.

Corollary 1: There exists a state feedback controller for the discrete-time singular system (33) such that the closed-loop system is admissible if and only if there exist a scalar $\delta > 0$, matrices $P > 0$ and Q such that

$$QS^TA + A^TSQ^T - E^TPE + A^TPA - \left(QS^T + A^TP\right)B\left(B^TPB + \delta I\right)^{-1}B^T\left(QS^T + A^TP\right)^T < 0. \tag{34}$$

In this case, a robustly stabilizing state feedback control law is given by

$$u(k) = -\left(B^TPB + \delta I\right)^{-1}B^T\left(QS^T + A^TP\right)^T x(k).$$

Remark 5: It is worth noting that by Corollary 1, stabilizing state feedback controllers for the discrete-time singular system (33) can be obtained directly when solutions to the matrix inequality (34) are found, and no decomposition of matrices is involved, which is in contrast with the design method proposed in [22, Cor. 1], where some decomposition is needed when designing a desired state feedback controller. Such a decomposition may result in certain numerical problems and, thus, makes the design procedure relatively complicated.

IV. Numerical Example

In this section, we provide an example to demonstrate the applicability of the proposed method.

Consider an uncertain discrete-time singular system described in (1) with parameters as follows:

$$E = \begin{bmatrix} 1 & 0 & 0.5 \\ 2 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 2.4 & 0.2 & 1.2 \\ 4 & 1.5 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad M = \begin{bmatrix} 0.1 \\ 0.2 \\ -0.1 \end{bmatrix}$$
$$N_1 = \begin{bmatrix} 0.35 & 0.4 & 0.7 \end{bmatrix} \quad N_2 = \begin{bmatrix} 0.1 & 0.2 & 0 \end{bmatrix}.$$

In this example, we assume the uncertain matrix $F(\sigma) = \sin(\sigma)$. It is easy to see that the nominal discrete-singular system is not regular, noncausal, and unstable. The purpose of this example is the design of a state feedback control law such that the resulting closed-loop system is admissible for all admissible uncertainties. To this end, we choose

$$S = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T.$$

Then, it can be verified that

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix} \quad Q = \begin{bmatrix} 0.7 \\ -4.4 \\ -8.4 \end{bmatrix} \quad \delta = 0.01 \quad \epsilon = 1.2$$

satisfy the matrix inequality in (24) and (25). Therefore, by Theorem 3, a robustly stabilizing state feedback control law can be chosen as

$$u(k) = \begin{bmatrix} -3.5469 & -2.2978 & -3.5197 \\ 4.7353 & 5.0297 & 9.5332 \\ -6.5998 & -5.9272 & -12.0526 \end{bmatrix} x(k).$$

V. Conclusion

The problems of robust stability and robust stabilization for uncertain discrete-time singular systems with parameter uncertainties in both the state and input matrices have been studied. In terms of a strict LMI, a new necessary and sufficient condition for a discrete-time singular system to be regular, causal and stable has been proposed. The concepts of generalized quadratic stability and generalized quadratic stabilization have been proposed. Necessary and sufficient conditions for generalized quadratic stability and generalized quadratic stabilization have been proposed in terms of a strict LMI and a set of strict matrix inequalities, respectively. Based on these, the robust stability and robust stabilization problems have been solved. An explicit construction of a desired state feedback control law has also been given, which can be obtained without decomposition of any matrices. The derived results in this note are of theoretical elegance in comparison with the existing results on generalized quadratic stability and generalized quadratic stabilization in the literature.

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Quadratic Stability of a Class of Switched Nonlinear Systems

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***Abstract*—Quadratic stability of a class of switched nonlinear systems is studied in this note. We first transform quadratic stability problem into an equivalent nonlinear programming problem. Then, we derive a necessary and sufficient condition for quadratic stability of this class of switched systems by using Karush–Kuhn–Tucker condition for nonlinear programming problems. The necessary and sufficient condition is given in terms of the strict completeness of a certain set of functions on a subset of the state space, which is much easier to check.**

***Index Terms*—Completeness, Karush–Kuhn–Tucker (KKT) condition, quadratic stability, switched systems.**

I. Introduction

Switched systems have attracted much research attention in control theory field during recent years. Typically, a switched system consists of a number of subsystems, either continuous-time or discrete-time ordinary dynamic systems, and a switching law, which defines a specific subsystem being activated during a certain interval of time. Switched systems arise in many engineering applications. Some typical examples of switched systems can be found, for example, in power systems [19], computer disk drives [8], transmission and stepper motors [4], constrained robotics [1], automated highways [18], and the cart-pendulum control [21], just to name a few.

Also, switched systems arise in the application of multiple controllers, which has been widely used in adaptive control, where a higher level, logic-based supervisor provides switching between a family of candidate controllers so as to achieve desired performance of the overall system in closed loop. Switched controller systems, as a special type of switched systems, often provide satisfactory control solution that guarantees stability and good performance when no single controller is effective [11]–[13].

The central issue in the study of switched systems is stability due to the hybrid nature of their operation. A number of works in this direction [3], [10], [9], [13], [15], [17], [20] have appeared recently. The existence of a common *Lyapunov* function for all subsystems has been found to be a necessary and sufficient condition for a switched system to be asymptotically stable under arbitrary switching law [13]. A considerable number of techniques to construct such a *Lyapunov* function exist (see [15], for instance). Most of realistic switched systems, however, do not possess a common *Lyapunov* function, yet they still may be asymptotically stable under some properly chosen switching law. Single and multiple Lypunov function techniques are effective tools for finding such a switching law [3], [10], [9].

By and large, quadratic stability is a preferable system property. Unlike ordinary linear systems, for which quadratic stability is equivalent to asymptotic stability, many switched systems that are asymptotically stable are not necessarily quadratic stable too even in cases of switched linear systems. Hence specific tools are needed to deal with the problem of quadratic stability of switched systems. In earlier works (see [9], for instance) quadratic stability of piece-wise linear systems was studied. Some sufficient conditions have been summarized recently in [5] and [13]. However, so far very few results on necessary and sufficient conditions for quadratic stability of switched nonlinear systems have been reported. In [16] and [17], quadratic stability is shown to be equivalent to the strict completeness of a certain set of functions generated by a positive definite matrix. Nonetheless, it is well known that strict completeness is extremely difficult to verify in general.

In this note, we propose a solution to the problem of quadratic stability of a class of switched nonlinear dynamic systems in a different way. We first transform quadratic stability problem into an equivalent nonlinear programming problem. Then, with the help of Karush–Kuhn–Tucker (KKT) condition for nonlinear programming problems, we derive a necessary and sufficient condition for these switched nonlinear systems to be quadratically stable. This condition appears in terms of the strict completeness of a certain set of functions on a subset of the state space rather than on the entire state space. This subset is contained in a submanifold whose dimension is two lower than the system's dimension. Therefore, it is much easier to check for the strict completeness on such a subset of the state–space.

II. Preliminaries

In the study of stability of switched systems, the concept of completeness is useful.

Definition 2.1 [16], [17]: A set of continuous functions $\{v_1, v_2, \ldots, v_k\}$, where $v_j : \Omega \subset R^n \to R$, is called complete on Ω if for any $x \in \Omega$, there exists an $i \in \{1, 2, \ldots, k\}$ such that $v_i(x) \leq 0$. In addition, the set $\{v_1, v_2, \ldots, v_k\}$ is called strictly complete on Ω if for any $x \in \Omega$, $x \neq 0$, there exists an $i \in \{1, 2, \ldots, k\}$, such that $v_i(x) < 0$.

The concept of (strict) completeness of functions is a generalization of the concept of (negative) nonpositive definite functions.

Consider the switched nonlinear system

$$\dot{x} = f_i(x) \tag{1}$$

where $x \in R^n$, $\{f_q : q = 1, 2, \ldots, k\}$ is a family of smooth vector fields defined on R^n, and i is a switching signal taking values in $\{1, 2, \ldots, k\}$, which may depend on time t or $x(t)$ or both.

Definition 2.2: System(1) is said to be asymptotically stable if there exists a switching law with the "state feedback" form $i = i(x)$ such that the corresponding switched system is asymptotically stable.

As a straightforward consequence of the direct *Lyapunov* method, asymptotic stability of System (1) follows from the strict completeness of a properly chosen set of functions.

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