Advances in Difference Equations
a SpringerOpen Journal

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Robust stability and stabilization of uncertain switched discrete-time systems

G Rajchakit[1*], T Rojsiraphisal[2,3] and M Rajchakit[1]

*Correspondence: griengkrai@yahoo.com
[1]Major of Mathematics and Statistics, Faculty of Science, Maejo University, Chiangmai, 50290, Thailand
Full list of author information is available at the end of the article

Abstract

This paper is concerned with the robust stability and stabilization for a class of switched discrete-time systems with state parameter uncertainty. Firstly, a new matrix inequality considering uncertainties is introduced and proved. By means of it, a novel sufficient condition for robust stability and stabilization of a class of uncertain switched discrete-time systems is presented. Furthermore, based on the result obtained, the switching law is designed and has been performed well, and some sufficient conditions of robust stability and stabilization have been derived for the uncertain switched discrete-time systems using the Lyapunov stability theorem, block matrix method, and inequality technology. Finally, some examples are exploited to illustrate the effectiveness of the proposed schemes.

Keywords: switching design; uncertain discrete system; robust stability and stabilization; Lyapunov function; linear matrix inequality

1 Introduction

A switched system is a hybrid dynamical system consisting of a finite number of subsystems and a logical rule that manages switching between these subsystems. Switched systems have drawn a great deal of attention in recent years; see [1–37] and references therein. The motivation for studying switched systems comes partly from the fact that switched systems and switched multicontroller systems have numerous applications in control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control, and many other fields. An important qualitative property of switched system is stability [1–3]. The challenge of analyzing the stability of switched system lies partly in the fact that, even if the individual systems are stable, the switched system might be unstable. Using a common quadratic Lyapunov function on all subsystems, the quadratic Lyapunov stability facilitates the analysis and synthesis of switched systems. However, the obtained results within this framework have been recognized to be conservative. In [10], various algorithms both for stability and performance analysis of discrete-time piece-wise affine systems were presented. Different classes of Lyapunov functions were considered, and how to compute them through linear matrix inequalities was also shown. Moreover, the tradeoff between the degree of conservativeness and computational requirements was discussed. The problem of stability analysis and control synthesis of switched systems in the discrete-time domain was addressed in [11]. The approach followed in [11] looked at the existence of a switched quadratic Lyapunov function to check asymptotic stability of the switched system under consideration. Two different linear matrix inequality-based conditions allow to check the existence of such a Lyapunov function. These two conditions have been proved to be equivalent for stability analysis.

There are many methodologies and approaches developed in the switched systems theory: approaches of looking for an appropriate switching strategy to stabilize the system [4], dwell-time and average dwell-time approaches for stability analysis and stabilization problems [5], approaches of studying stability and control problems under a specific class of switching laws [1], or under arbitrary switching sequences [6, 9]. Reference [19] investigated the quadratic stability and linear state feedback and output feedback stabilization of switched delayed linear dynamic systems with, in general, a finite number of noncommensurate constant internal point delays. The results were obtained based on Lyapunov stability analysis *via* appropriate Krasovskii-Lyapunov functionals, and the related stability study was performed to obtain both delay-independent and delay-dependent results. The problem of fault estimation for a class of switched nonlinear systems of neutral type was considered in [20]. Sufficient delay-dependent existence conditions of the H_∞ fault estimator were given in terms of certain matrix inequalities based on the average dwell-time approach. The problem of robust reliable control for a class of uncertain switched neutral systems under asynchronous switching was investigated in [21]. A state feedback controller was proposed to guarantee exponential stability and reliability for switched neutral systems, and the dwelltime approach was utilized for the stability analysis and controller design. The exponential stability for a class of nonlinear hybrid time-delay systems was addressed in [24]. The delay-dependent stability conditions were presented in terms of the solution of algebraic Riccati equations, which allows computing simultaneously the two bounds that characterize the stability rate of the solution.

On another research front line, it has been recognized that parameter uncertainties, which often occur in many physical processes, are main sources of instability and poor performance. Therefore, much attention has been devoted to the study of various systems with uncertainties, and a great number of useful results have been reported in the literature on the issues of robust stability, robust H_∞ control, robust H_∞ filtering, and so on, by considering different classes of parameter uncertainties [12, 14].

Recently, some stability condition and stabilization approaches have been proposed for the switched discrete-time system [15, 18]. In [15], the quadratic stabilization of discrete-time switched linear systems was studied, and quadratic stabilization of switched systems with norm bounded time varying uncertainties was investigated. In [16], the stability property for the switched systems which were composed of a continuous-time LTI subsystem and a discrete time LTI subsystem was studied. There existed a switched quadratic Lyapunov function to check asymptotic stability of the switched discrete-time system in [17].

The objective of this paper is to present novel approaches for the asymptotical stability and stabilization of switched discrete-time system with parametric uncertainties. The parameter uncertainties are time-varying but norm-bounded. Firstly, a new inequality is given. Using the new result, a new sufficient condition for robust stability and stabilization of a class of uncertain switched discrete-time systems is proposed. Furthermore, using the block matrix method, inequality technology, and the Lyapunov stability theorem, some sufficient conditions for robust stability and stabilization have been presented for the uncertain switched discrete-time systems, and the switched law design has been performed. Comparing with [22, 23], the uncertainty in system was not considered in [22, 23], but we consider the uncertainty in systems and the design switching law is simple and easy for application.

The rest of this paper is organized as follows. The problem is formulated in Section 2. Section 3 deals with robust stability and stabilization criteria for a class of discrete-time switched system with uncertainty. Numerical examples are provided to illustrate the theoretical results in Section 4, and the conclusions are drawn in Section 5.

2 Preliminaries

The following notations will be used throughout this paper. R^+ denotes the set of all real nonnegative numbers; R^n denotes the n-dimensional space with the scalar product of two vectors $\langle x, y\rangle$ or $x^T y$; $R^{n\times r}$ denotes the space of all matrices of $(n \times r)$-dimension. A^T denotes the transpose of A; a matrix A is symmetric if $A = A^T$.

Matrix A is semipositive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A \geq B$ means $A - B \geq 0$. $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\min}(A) = \min\{\operatorname{Re}\lambda : \lambda \in \lambda(A)\}$.

Consider uncertain discrete systems with interval time-varying delay of the form

$$\begin{aligned} &x(k+1) = \big(A_\gamma + \Delta A_\gamma(k)\big)x(k) + \big(B_\gamma + \Delta B_\gamma(k)\big)x\big(k - d(k)\big), \quad k = 0, 1, 2, \dots, \\ &x(k) = v_k, \quad k = -d_2, -d_2 + 1, \dots, 0, \end{aligned} \tag{2.1}$$

where $x(k) \in R^n$ is the state, $\gamma(\cdot) : R^n \to \mathcal{N} := \{1, 2, \dots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(k)) = i$ implies that the system realization is chosen as the ith system, $i = 1, 2, \dots, N$. It is seen that the system (2.1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(k)$ hits predefined boundaries. A_i, B_i, $i = 1, 2, \dots, N$ are given constant matrices and the time-varying uncertain matrices $\Delta A_i(k)$ and $\Delta B_i(k)$ are defined by

$$\Delta A_i(k) = E_{ia}F_{ia}(k)H_{ia}, \Delta B_i(k) = E_{ib}F_{ib}(k)H_{ib}, \tag{2.2}$$

where E_{ia}, E_{ib}, H_{ia}, H_{ib} are known constant real matrices with appropriate dimensions.

$F_{ia}(k)$, $F_{ib}(k)$ are unknown uncertain matrices satisfying

$$F_{ia}^T(k)F_{ia}(k) \leq I, \qquad F_{ib}^T(k)F_{ib}(k) \leq I, \quad k = 0, 1, 2, \dots. \tag{2.3}$$

The time-varying function $d(k)$ satisfies the following condition:

$$0 < d_1 \leq d(k) \leq d_2, \quad \forall k = 0, 1, 2, \dots.$$

Remark 2.1 It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero.

Definition 2.1 The uncertain switched system (2.1) is robustly stable if there exists a switching function $\gamma(\cdot)$ such that the zero solution of the uncertain switched system is asymptotically stable for all uncertainties which satisfy (2.2) and (2.3).

Definition 2.2 The system of matrices $\{J_i\}$, $i = 1, 2, \ldots, N$, is said to be strictly complete if for every $x \in R^n \setminus \{0\}$ there is $i \in \{1, 2, \ldots, N\}$ such that $x^T J_i x < 0$.

It is easy to see that the system $\{J_i\}$ is strictly complete if and only if

$$\bigcup_{i=1}^{N} \alpha_i = R^n \setminus \{0\},$$

where

$$\alpha_i = \{x \in R^n : x^T J_i x < 0\}, \quad i = 1, 2, \ldots, N.$$

Proposition 2.1 ([38]) *The system $\{J_i\}$, $i = 1, 2, \ldots, N$, is strictly complete if there exist $\delta_i \geq 0$, $i = 1, 2, \ldots, N$, $\sum_{i=1}^{N} \delta_i > 0$ such that*

$$\sum_{i=1}^{N} \delta_i J_i < 0.$$

If $N = 2$ then the above condition is also necessary for the strict completeness.

Proposition 2.2 (Cauchy inequality) *For any symmetric positive definite matrix $N \in M^{n\times n}$ and $a, b \in R^n$ we have*

$$\pm a^T b \leq a^T N a + b^T N^{-1} b.$$

Proposition 2.3 ([38]) *Let E, H and F be any constant matrices of appropriate dimensions and $F^T F \leq I$. For any $\epsilon > 0$, we have*

$$EFH + H^T F^T E^T \leq \epsilon E E^T + \epsilon^{-1} H^T H.$$

3 Main results

3.1 Stability

Let us set

$$W_i(S_1, S_2, P, Q) = \begin{bmatrix} W_{i11} & W_{i12} & W_{i13} \\ * & W_{i22} & W_{i23} \\ * & * & W_{i33} \end{bmatrix},$$

where

$$\begin{aligned}
W_{i11} &= Q - P + 2H_{ia}^T H_{ia} + H_{ib}^T H_{ib}, \\
W_{i12} &= S_1 - A_i^T S_1^T, \\
W_{i13} &= -S_1 B_i - A_i^T S_2^T, \\
W_{i22} &= P + S_1 + S_1^T + S_1 E_{ia} E_{ia}^T S_1^T + S_2 E_{ia} E_{ia}^T S_2^T + H_{ib}^T H_{ib}, \\
W_{i23} &= S_2 - S_1 B_i, \qquad (3.1)
\end{aligned}$$

$$W_{i33} = -Q - S_2B_i - B_i^T S_2^T + S_2E_{ib}E_{ib}^T S_2^T + 2S_1E_{ib}E_{ib}^T S_1^T + H_{ib}^T H_{ib},$$

$$J_i(S_1, Q) = (d_2 - d_1)Q - S_1A_i - A_i^T S_1^T + S_1E_{ia}E_{ia}^T S_1^T + H_{ia}^T H_{ia},$$

$$\alpha_i = \{x \in R^n : x^T J_i(S_1, Q)x < 0\}, \quad i = 1, 2, \ldots, N,$$

$$\bar{\alpha}_1 = \alpha_1, \qquad \bar{\alpha}_i = \alpha_i \setminus \bigcup_{j=1}^{i-1} \bar{\alpha}_j, \quad i = 2, 3, \ldots, N.$$

The main result of this paper is summarized in the following theorem.

Theorem 3.1 *The uncertain switched system (2.1) is robustly stable if there exist symmetric positive definite matrices $P > 0$, $Q > 0$ and matrices S_1, S_2 satisfying the following conditions*

(i) $\exists \delta_i \geq 0$, $i = 1, 2, \ldots, N$, $\sum_{i=1}^N \delta_i > 0 : \sum_{i=1}^N \delta_i J_i(S_1, Q) < 0$,

(ii) $W_i(S_1, S_2, P, Q) < 0$, $i = 1, 2, \ldots, N$.

The switching rule is chosen as $\gamma(x(k)) = i$, whenever $x(k) \in \bar{\alpha}_i$.

Proof Consider the following Lyapunov-Krasovskii functional for any ith system (2.1)

$$V(k) = V_1(k) + V_2(k) + V_3(k),$$

where

$$V_1(k) = x^T(k)Px(k), \qquad V_2(k) = \sum_{i=k-d(k)}^{k-1} x^T(i)Qx(i),$$

$$V_3(k) = \sum_{j=-d_2+2}^{-d_1+1} \sum_{l=k+j}^{k-1} x^T(l)Qx(l).$$

We can verify that

$$\lambda_1 \|x(k)\|^2 \leq V(k). \tag{3.2}$$

Let us set $\xi(k) = [x^T(k)x^T(k+1)x^T(k-d(k))]^T$, and

$$H = \begin{pmatrix} 0 & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad G = \begin{pmatrix} P & 0 & 0 \\ I & I & 0 \\ 0 & 0 & I \end{pmatrix}.$$

Then, the difference of $V_1(k)$ along the solution of the system is given by

$$\begin{aligned} \Delta V_1(k) &= x^T(k+1)Px(k+1) - x^T(k)Px(k) \\ &= \xi^T(k)H\xi(k) - 2\xi^T(k)G^T \begin{pmatrix} 0.5x(k) \\ 0 \\ 0 \end{pmatrix}, \end{aligned} \tag{3.3}$$

because of

$$\xi^T(k)H\xi(k) = x(k+1)Px(k+1).$$

Using the expression of system (2.1)

$$0 = -S_1x(k+1) + S_1(A_i + E_{ia}F_{ia}H_{ia})x(k) + S_1(B_i + E_{ib}F_{ib}H_{ib})x\big(k-d(k)\big),$$
$$0 = -S_2x(k+1) + S_2(A_i + E_{ia}F_{ia}H_{ia})x(k) + S_2(B_i + E_{ib}F_{ib}H_{ib})x\big(k-d(k)\big),$$

we have

$$\begin{aligned}
&-2\xi^T(k)G^T\begin{pmatrix} 0.5x(k) \\ -S_1x(k+1) + S_1(A_i + E_{ia}F_{ia}H_{ia})x(k) + S_1(B_i + E_{ib}F_{ib}H_{ib})x(k-d(k)) \\ -S_2x(k+1) + S_2(A_i + E_{ia}F_{ia}H_{ia})x(k) + S_2(B_i + E_{ib}F_{ib}H_{ib})x(k-d(k)) \end{pmatrix} \\
&= -\xi^T(k)G^T\begin{pmatrix} 0.5I & 0 & 0 \\ S_1A_i + S_1E_{ia}F_{ia}H_{ia} & -S_1 & S_1B_i + S_1E_{ib}F_{ib}H_{ib} \\ S_2A_i + S_2E_{ia}F_{ia}H_{ia} & -S_2 & S_2B_i + S_2E_{ib}F_{ib}H_{ib} \end{pmatrix}\xi(k) \\
&\quad - \xi^T(k)\begin{pmatrix} 0.5I & (S_1A_i + S_1E_{ia}F_{ia}H_{ia})^T & (S_2A_i + S_2E_{ia}F_{ia}H_{ia})^T \\ 0 & -S_1^T & -S_2^T \\ 0 & (S_1B_i + S_1E_{ib}F_{ib}H_{ib})^T & (S_2B_i + S_2E_{ib}F_{ib}H_{ib})^T \end{pmatrix}G\xi(k).
\end{aligned}$$

Therefore, from (3.3), it follows that

$$\begin{aligned}
\Delta V_1(k) = {}& x^T(k)\big[-P - S_1(A_i + E_{ia}F_{ia}H_{ia}) - (A_i + E_{ia}F_{ia}H_{ia})^TS_1^T\big]x(k) \\
&+ 2x^T(k)\big[S_1 - A_i^TS_1^T - (E_{ia}F_{ia}H_{ia})^TS_1^T\big]x(k+1) \\
&+ 2x^T(k)\big[-S_1B_i - S_1(E_{ib}F_{ib}H_{ib}) - A_i^TS_2^T - (E_{ia}F_{ia}H_{ia})^TS_2^T\big]x\big(k-h(k)\big) \\
&+ x(k+1)\big[P + S_1 + S_1^T\big]x(k+1) \\
&+ 2x(k+1)\big[S_2 - S_1B_i - S_1(E_{ib}F_{ib}H_{ib})\big]x\big(k-h(k)\big) \\
&+ x\big(k-h(k)\big)\big[-S_2(B_i + E_{ib}F_{ib}H_{ib}) - (B_i + E_{ib}F_{ib}H_{ib})^TS_2^T\big]x\big(k-h(k)\big).
\end{aligned}$$

Applying Proposition 2.2, Proposition 2.3 and condition (2.2), the following estimations hold

$$\begin{aligned}
&-E_{ia}F_{ia}(k)H_{ia}^TS_1^T - S_1E_{ia}F_{ia}(k)H_{ia} \le S_1E_{ia}E_{ia}^TS_1^T + H_{ia}^TH_{ia}, \\
&-E_{ib}F_{ib}(k)H_{ib}^TS_2^T - S_2E_{ib}F_{ib}(k)H_{ib} \le S_2E_{ib}E_{ib}^TS_2^T + H_{ib}^TH_{ib}, \\
&-2x^T(k)S_1E_{ia}F_{ia}(k)H_{ia}x(k+1) \le x(k+1)S_1E_{ia}E_{ia}^TS_1^Tx(k+1) + x^T(k)H_{ia}^TH_{ia}x(k), \\
&-2x^T(k)S_1E_{ib}F_{ib}(k)H_{ib}x\big(k-h(k)\big) \\
&\quad \le x\big(k-h(k)\big)S_1E_{ib}E_{ib}^TS_1^Tx\big(k-h(k)\big) + x^T(k)H_{ib}^TH_{ib}x(k), \\
&-2x^T(k)S_2E_{ia}F_{ia}(k)H_{ia}x(k+1) \\
&\quad \le x(k+1)S_2E_{ia}E_{ia}^TS_2^Tx(k+1) + x^T(k)H_{ia}^TH_{ia}x(k), \\
&-2x^T(k+1)S_1E_{ib}F_{ib}(k)H_{ib}x\big(k-h(k)\big) \\
&\quad \le x\big(k-h(k)\big)S_1E_{ib}E_{ib}^TS_1^Tx\big(k-h(k)\big) + x^T(k+1)H_{ib}^TH_{ib}x(k+1).
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\Delta V_1(k) \leq{}& x^T(k)\big[-P - S_1A_i - A_i^T S_1^T + S_1E_{ia}E_{ia}^T S_1^T + 3H_{ia}^T H_{ia} + H_{ib}^T H_{ib}\big]x(k) \\
&+ 2x^T(k)\big[S_1 - A_i^T S_1^T\big]x(k+1) + 2x^T(k)\big[-S_1B_i - A_i^T S_2^T\big]x\big(k-h(k)\big) \\
&+ x(k+1)\big[P + S_1 + S_1^T + S_1E_{ia}E_{ia}^T S_1^T + S_2E_{ia}E_{ia}^T S_2^T + H_{ib}^T H_{ib}\big]x(k+1) \\
&+ 2x(k+1)[S_2 - S_1B_i]x\big(k-h(k)\big) \\
&+ x\big(k-h(k)\big)\big[-S_2B_i - B_i^T S_2^T + S_2E_{ib}E_{ib}^T S_2^T \\
&+ 2S_1E_{ib}E_{ib}^T S_1^T + H_{ib}^T H_{ib}\big]x\big(k-h(k)\big).
\end{aligned} \tag{3.4}$$

The difference of $V_2(k)$ is given by

$$\begin{aligned}
\Delta V_2(k) &= \sum_{i=k+1-d(k+1)}^{k} x^T(i)Qx(i) - \sum_{i=k-d(k)}^{k-1} x^T(i)Qx(i) \\
&= \sum_{i=k+1-d(k+1)}^{k-d_1} x^T(i)Qx(i) + x^T(k)Qx(k) - x^T\big(k-d(k)\big)Qx\big(k-d(k)\big) \\
&\quad + \sum_{i=k+1-d_1}^{k-1} x^T(i)Qx(i) - \sum_{i=k+1-d(k)}^{k-1} x^T(i)Qx(i).
\end{aligned} \tag{3.5}$$

Since $d(k) \geq d_1$, we have

$$\sum_{i=k+1-d_1}^{k-1} x^T(i)Qx(i) - \sum_{i=k+1-d(k)}^{k-1} x^T(i)Qx(i) \leq 0,$$

and hence from (3.5) we have

$$\Delta V_2(k) \leq \sum_{i=k+1-d(k+1)}^{k-d_1} x^T(i)Qx(i) + x^T(k)Qx(k) - x^T\big(k-d(k)\big)Qx\big(k-d(k)\big). \tag{3.6}$$

The difference of $V_3(k)$ is given by

$$\begin{aligned}
\Delta V_3(k) &= \sum_{j=-d_2+2}^{-d_1+1} \sum_{l=k+j+1}^{k} x^T(l)Qx(l) - \sum_{j=-d_2+2}^{-d_1+1} \sum_{l=k+j}^{k-1} x^T(l)Qx(l) \\
&= \sum_{j=-d_2+2}^{-d_1+1} \Bigg[\sum_{l=k+j}^{k-1} x^T(l)Qx(l) + x^T(k)Q(\xi)x(k) \\
&\quad - \sum_{l=k+j}^{k-1} x^T(l)Qx(l) - x^T(k+j-1)Qx(k+j-1)\Bigg] \\
&= \sum_{j=-d_2+2}^{-d_1+1} \big[x^T(k)Qx(k) - x^T(k+j-1)Qx(k+j-1)\big] \\
&= (d_2 - d_1)x^T(k)Qx(k) - \sum_{j=k+1-d_2}^{k-d_1} x^T(j)Qx(j).
\end{aligned} \tag{3.7}$$

Since $d(k) \leq d_2$, and

$$\sum_{i=k=1-d(k+1)}^{k-d_1} x^T(i)Qx(i) - \sum_{i=k+1-d_2}^{k-d_1} x^T(i)Qx(i) \leq 0,$$

we obtain from (3.6) and (3.7) that

$$\Delta V_2(k) + \Delta V_3(k) \leq (d_2 - d_1 + 1)x^T(k)Qx(k) - x^T\bigl(k - d(k)\bigr)Qx\bigl(k - d(k)\bigr). \tag{3.8}$$

Therefore, combining the inequalities (3.4), (3.8) gives

$$\Delta V(k) \leq x^T(k)J_i(S_1, Q)x(k) + \xi^T(k)W_i(S_1, S_2, P, Q)\xi(k), \tag{3.9}$$

where

$$W_i(S_1, S_2, P, Q) = \begin{bmatrix} W_{i11} & W_{i12} & W_{i13} \\ * & W_{i22} & W_{i23} \\ * & * & W_{i33} \end{bmatrix},$$

$$W_{i11} = Q - P + 2H_{ia}^T H_{ia} + H_{ib}^T H_{ib},$$

$$W_{i12} = S_1 - A_i^T S_1^T,$$

$$W_{i13} = -S_1 B_i - A_i^T S_2^T,$$

$$W_{i22} = P + S_1 + S_1^T + S_1 E_{ia} E_{ia}^T S_1^T + S_2 E_{ia} E_{ia}^T S_2^T + H_{ib}^T H_{ib},$$

$$W_{i23} = S_2 - S_1 B_i,$$

$$W_{i33} = -Q - S_2 B_i - B_i^T S_2^T + S_2 E_{ib} E_{ib}^T S_2^T + 2S_1 E_{ib} E_{ib}^T S_1^T + H_{ib}^T H_{ib},$$

$$J_i(S_1, Q) = (d_2 - d_1)Q - S_1 A_i - A_i^T S_1^T + S_1 E_{ia} E_{ia}^T S_1^T + H_{ia}^T H_{ia}.$$

Therefore, we finally obtain from (3.9) and the condition (ii) that

$$\Delta V(k) < x^T(k)J_i(S_1, Q)x(k), \quad \forall i = 1, 2, \ldots, N, k = 0, 1, 2, \ldots.$$

We now apply the condition (i) and Proposition 2.1, the system $J_i(S_1, Q)$ is strictly complete, and the sets α_i and $\bar{\alpha}_i$ by (3.1) are well defined such that

$$\bigcup_{i=1}^{N} \alpha_i = R^n \setminus \{0\},$$

$$\bigcup_{i=1}^{N} \bar{\alpha}_i = R^n \setminus \{0\}, \quad \bar{\alpha}_i \cap \bar{\alpha}_j = \emptyset, i \neq j.$$

Therefore, for any $x(k) \in R^n$, $k = 1, 2, \ldots$, there exists $i \in \{1, 2, \ldots, N\}$ such that $x(k) \in \bar{\alpha}_i$. By choosing switching rule as $\gamma(x(k)) = i$ whenever $x(k) \in \bar{\alpha}_i$, from the condition (3.9) we have

$$\Delta V(k) \leq x^T(k)J_i(S_1, Q)x(k) < 0, \quad k = 1, 2, \ldots,$$

which, combining the condition (3.2) and the Lyapunov stability theorem [39], concludes the proof of the theorem. □

3.2 Stabilization

Consider uncertain control discrete-time systems with interval time-varying delay of the form

$$\begin{aligned} &x(k+1) = \big(A_\gamma + \Delta A_\gamma(k)\big)x(k) + \big(D_\gamma + \Delta D_\gamma(k)\big)u(k), \quad k = 0,1,2,\ldots, \\ &x(k) = v_k, \quad k = -d_2, -d_2+1, \ldots, 0, \end{aligned} \tag{3.10}$$

where $x(k) \in R^n$ is the state, $u(k) \in R^m$, $m \le n$, is the control input, $\gamma(\cdot) : R^n \to \mathcal{N} := \{1,2,\ldots,N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(k)) = i$ implies that the system realization is chosen as the ith system, $i = 1,2,\ldots,N$. It is seen that the system (2.1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(k)$ hits predefined boundaries. We consider a delayed feedback control law

$$u(k) = \big(C_i + \Delta C_i(k)\big)x\big(k - d(k)\big), \quad k = -h_2, \ldots, 0, \tag{3.11}$$

and C_i, $i = 1,2,\ldots,N$ is the controller gain to be determined. A_i, D_i, $i = 1,2,\ldots,N$ are given constant matrices and the time-varying uncertain matrices $\Delta A_i(k)$, $\Delta D_i(k)$, and $\Delta C_i(k)$ are defined by: $\Delta A_i(k) = E_{ia}F_{ia}(k)H_{ia}$, $\Delta D_i(k) = E_{id}F_{id}(k)H_{id}$, and $\Delta C_i(k) = E_{ic}F_{ic}(k)H_{ic}$ where E_{ia}, E_{id}, E_{ic}, H_{ia}, H_{id}, H_{ic} are known constant real matrices with appropriate dimensions. $F_{ia}(k)$, $F_{id}(k)$, $F_{ic}(k)$ are unknown uncertain matrices satisfying

$$F_{ia}^T(k)F_{ia}(k) \le I, \qquad F_{id}^T(k)F_{id}(k) \le I, \qquad F_{ic}^T(k)F_{ic}(k) \le I, \quad k = 0,1,2,\ldots. \tag{3.12}$$

The time-varying function $d(k)$ satisfies the following condition:

$$0 < d_1 \le d(k) \le d_2, \quad \forall k = 0,1,2,\ldots.$$

Remark 3.1 It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero.

Applying the feedback controller (3.11) to the system (3.10), the closed-loop discrete time-delay system is

$$x(k+1) = (A_i + \Delta A_i x(k) + (D_i + \Delta D_i)(C_i + \Delta C_i)x\big(k - d(k)\big), \quad k = 0,1,2,\ldots. \tag{3.13}$$

Definition 3.1 The uncertain switched control system (3.10) is robustly stabilizable if there is a delayed feedback control (3.11) such that the switched system (3.13) is robustly stable.

Let us set

$$W_i = \begin{bmatrix} W_{i11} & W_{i12} & W_{i13} \\ * & W_{i22} & W_{i23} \\ * & * & W_{i33} \end{bmatrix},$$

where

$$\begin{aligned}
&W_{i11} = Q - P + 2H_{ia}^T H_{ia},\\
&W_{i12} = S_1 - A_i^T S_1^T,\\
&W_{i13} = -S_1 - A_i^T S_2^T,\\
&W_{i22} = P + S_1 + S_1^T + S_1 E_{ia} E_{ia}^T S_1^T + S_2 E_{ia} E_{ia}^T S_2^T,\\
&W_{i23} = S_2 - S_1,\\
&W_{i33} = -Q - S_2 - S_2^T,\\
&J_i(S_1, Q) = (d_2 - d_1)Q - S_1 A_i - A_i^T S_1^T + S_1 E_{ia} E_{ia}^T S_1^T + H_{ia}^T H_{ia},\\
&\alpha_i = \{x \in R^n : x^T J_i(S_1, Q)x < 0\}, \quad i = 1, 2, \ldots, N,\\
&\bar{\alpha}_1 = \alpha_1, \qquad \bar{\alpha}_i = \alpha_i \setminus \bigcup_{j=1}^{i-1} \bar{\alpha}_j, \quad i = 2, 3, \ldots, N.
\end{aligned} \tag{3.14}$$

Theorem 3.2 *The switched control system (3.10) is robustly stabilizable by the delayed feedback control (3.11), where*

$$(C_i + \Delta C_i) = (D_i + \Delta D_i)^T \big[(D_i + \Delta D_i)(D_i + \Delta D_i)^T\big]^{-1}, \quad i = 1, 2, \ldots, N,$$

if there exist symmetric matrices $P > 0$, $Q > 0$ and matrices S_1, S_2 satisfying the following conditions

(i) $\exists \delta_i \geq 0$, $i = 1, 2, \ldots, N$, $\sum_{i=1}^N \delta_i > 0 : \sum_{i=1}^N \delta_i J_i(S_1, Q) < 0$,

(ii) $W_i(S_1, S_2, P, Q) < 0$, $i = 1, 2, \ldots, N$.

The switching rule is chosen as $\gamma(x(k)) = i$, whenever $x(k) \in \bar{\alpha}_i$.

Proof Using the feedback control (3.11), the closed-loop system leads to the system (2.1), where

$$\begin{aligned}
(B_i + \Delta B_i) &= (D_i + \Delta D_i)(C_i + \Delta C_i)\\
&= (D_i + \Delta D_i)(D_i + \Delta D_i)^T \big[(D_i + \Delta D_i)(D_i + \Delta D_i)^T\big]^{-1} = I.
\end{aligned}$$

Since $S_1(B_i + \Delta B_i) = S_1$, $(B_i + \Delta B_i)^T S_1^T = S_1^T$, $S_2(B_i + \Delta B_i) = S_2$, $(B_i + \Delta B_i)^T S_2^T = S_2^T$, the stability condition of the closed-loop system (3.13), by Theorem 3.1, is immediately derived. □

Remark 3.2 Note that the results proposed in [40–42] for switching systems to be asymptotically stable under an arbitrary switching rule. The asymptotic stability for switching linear discrete-time delay systems studied in [12] was limited to constant delays. In [43], a class of switching signals has been identified for the considered switched discrete-time delay systems to be stable under the averaged well time scheme.

4 Numerical examples

Example 4.1 (Stability) Consider the uncertain switched discrete-time system (2.1), where the delay function $d(k)$ is given by

$$d(k) = 1 + 4\sin^2\frac{k\pi}{2}, \quad k = 0,1,2,\dots,$$

$$(A_1, B_1) = \left(\begin{bmatrix} -0.1 & 0.01 \\ 0.02 & -0.2 \end{bmatrix}, \begin{bmatrix} -0.7 & 0.01 \\ 0.02 & 0.3 \end{bmatrix}\right),$$

$$(A_2, B_2) = \left(\begin{bmatrix} -0.2 & 0.02 \\ 0.03 & -0.3 \end{bmatrix}, \begin{bmatrix} -0.5 & 0.02 \\ 0.04 & 0.12 \end{bmatrix}\right),$$

$$(H_{1a}, H_{1b}) = \left(\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}\right),$$

$$(H_{2a}, H_{2b}) = \left(\begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}\right),$$

$$(E_{1a}, E_{1b}) = \left(\begin{bmatrix} 5.3 & 0 \\ 0 & 3.4 \end{bmatrix}, \begin{bmatrix} 3.2 & 0 \\ 0 & 5.5 \end{bmatrix}\right),$$

$$(E_{2a}, E_{2b}) = \left(\begin{bmatrix} 3.5 & 0 \\ 0 & 3.3 \end{bmatrix}, \begin{bmatrix} 2.2 & 0 \\ 0 & 4.3 \end{bmatrix}\right),$$

$$(F_{1a}, F_{1b}) = \left(\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}\right),$$

$$(F_{2a}, F_{2b}) = \left(\begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}\right).$$

By LMI toolbox of Matlab, we find that the conditions (i), (ii) of Theorem 3.1 are satisfied with $d_1 = 1$, $d_2 = 5$, $\delta_1 = 1$, $\delta_2 = 1$, and

$$P = \begin{bmatrix} 1.1329 & -0.0010 \\ -0.0010 & 1.7289 \end{bmatrix}, \qquad Q = \begin{bmatrix} 0.0506 & -0.0011 \\ -0.0011 & 0.4454 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} -0.0169 & 0.0002 \\ 0 & -0.0798 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} -0.0230 & 0 \\ 0 & -0.0067 \end{bmatrix}.$$

In this case, we have

$$\big(J_1(S_1, Q), J_2(S_1, Q)\big) = \left(\begin{bmatrix} -0.2170 & -0.0026 \\ -0.0026 & -1.8633 \end{bmatrix}, \begin{bmatrix} -0.3591 & -0.0016 \\ -0.0016 & -2.0531 \end{bmatrix}\right).$$

Moreover, the sum

$$\delta_1 J_1(R, Q) + \delta_2 J_2(R, Q) = \begin{bmatrix} -0.5761 & -0.0042 \\ -0.0042 & -3.9164 \end{bmatrix},$$

is negative definite, *i.e.*, the first entry in the first row and the first column $-0.5761 < 0$ is negative and the determinant of the matrix is positive. The sets α_1 and α_2 are given as

$$\alpha_1 = \{(x_1, x_2) : -0.2170x_1^2 - 0.0052x_1x_2 - 1.8633x_2^2 < 0\},$$
$$\alpha_2 = \{(x_1, x_2) : 0.3591x_1^2 + 0.0032x_1x_2 + 2.0531x_2^2 > 0\}.$$

Obviously, the union of these sets is equal to $R^2 \setminus \{0\}$. The switching regions are defined as

$$\overline{\alpha}_1 = \{(x_1, x_2) : -0.2170x_1^2 - 0.0052x_1x_2 - 1.8633x_2^2 < 0\},$$
$$\overline{\alpha}_2 = \alpha_2 \setminus \overline{\alpha}_1.$$

By Theorem 3.1 the uncertain system is robustly stable and the switching rule is chosen as $\gamma(x(k)) = i$ whenever $x(k) \in \bar{\alpha}_i$.

Example 4.2 (Stabilization) Consider the uncertain switched discrete-time control system (3.10), where the delay function $d(k)$ is given by

$$d(k) = 1 + 7\sin^2\frac{k\pi}{2}, \quad k = 0, 1, 2, \ldots,$$
$$(A_1, B_1, C_1) = \left(\begin{bmatrix} -0.3 & 0.02 \\ 0.03 & -0.2 \end{bmatrix}, \begin{bmatrix} -0.4 & 0.02 \\ 0.04 & 0.6 \end{bmatrix}, \begin{bmatrix} -0.1 & 0.03 \\ 0.02 & -0.5 \end{bmatrix}\right),$$
$$(A_2, B_2, C_2) = \left(\begin{bmatrix} -0.5 & 0.01 \\ 0.02 & -0.4 \end{bmatrix}, \begin{bmatrix} -0.2 & 0.03 \\ 0.05 & 0.3 \end{bmatrix}, \begin{bmatrix} -0.2 & 0.01 \\ 0.04 & -0.3 \end{bmatrix}\right),$$
$$(H_{1a}, H_{1b}, H_{1c}) = \left(\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \begin{bmatrix} 0.1 & 0 \\ 0 & 0.4 \end{bmatrix}\right),$$
$$(H_{2a}, H_{2b}, H_{2c}) = \left(\begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}\right),$$
$$(E_{1a}, E_{1b}, E_{1c}) = \left(\begin{bmatrix} 1.3 & 0 \\ 0 & 1.4 \end{bmatrix}, \begin{bmatrix} 1.2 & 0 \\ 0 & 1.5 \end{bmatrix}, \begin{bmatrix} 1.5 & 0 \\ 0 & 1.6 \end{bmatrix}\right),$$
$$(E_{2a}, E_{2b}, E_{2c}) = \left(\begin{bmatrix} 1.5 & 0 \\ 0 & 1.3 \end{bmatrix}, \begin{bmatrix} 1.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \begin{bmatrix} 1.2 & 0 \\ 0 & 1.3 \end{bmatrix}\right),$$
$$(F_{1a}, F_{1b}, F_{1c}) = \left(\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \begin{bmatrix} 0.1 & 0 \\ 0 & 0.4 \end{bmatrix}\right),$$
$$(F_{2a}, F_{2b}, F_{2c}) = \left(\begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}\right).$$

By LMI toolbox of Matlab, we find that the conditions (i), (ii) of Theorem 3.2 are satisfied with $d_1 = 1$, $d_2 = 8$, $\delta_1 = 2$, $\delta_2 = 1$, and

$$P = \begin{bmatrix} 1.5097 & 0.0002 \\ 0.0002 & 1.5904 \end{bmatrix}, \qquad Q = \begin{bmatrix} 0.0012 & -0.0003 \\ -0.0003 & 0.2180 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} -0.3434 & -0.0029 \\ -0.0014 & -0.3647 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 0.0036 & 0.0031 \\ 0.0011 & -0.0064 \end{bmatrix}.$$

In this case, we have

$$\bigl(J_1(S_1, Q), J_2(S_1, Q)\bigr) = \left(\begin{bmatrix} -1.5519 & -0.0176 \\ -0.0176 & -1.6807 \end{bmatrix}, \begin{bmatrix} -1.6306 & -0.0097 \\ -0.0097 & -1.7089 \end{bmatrix} \right).$$

Moreover, the sum

$$\delta_1 J_1(R, Q) + \delta_2 J_2(R, Q) = \begin{bmatrix} -4.7344 & -0.0449 \\ -0.0449 & -5.0703 \end{bmatrix},$$

is negative definite, *i.e.*, the first entry in the first row and the first column $-4.7344 < 0$ is negative and the determinant of the matrix is positive. The sets α_1 and α_2 are given as

$$\alpha_1 = \bigl\{(x_1, x_2) : -1.5519x_1^2 - 0.0352x_1x_2 - 1.6807x_2^2 < 0\bigr\},$$
$$\alpha_2 = \bigl\{(x_1, x_2) : 1.6306x_1^2 + 0.0194x_1x_2 + 1.7089x_2^2 > 0\bigr\}.$$

Obviously, the union of these sets is equal to $R^2 \setminus \{0\}$. The switching regions are defined as

$$\overline{\alpha}_1 = \bigl\{(x_1, x_2) : -1.5519x_1^2 - 0.0352x_1x_2 - 1.6807x_2^2 < 0\bigr\},$$
$$\overline{\alpha}_2 = \alpha_2 \setminus \overline{\alpha}_1.$$

By Theorem 3.2, the control system is robustly stabilizable and the switching rule is $\sigma(x(k)) = i$ whenever $x(k) \in \bar{\Omega}_i$, the delayed feedback control is

$$u_1(k) = \begin{bmatrix} -0.0850x_1^1(k - d(k)) + 0.0300x_1^2(k - d(k)) \\ 0.0200x_1^1(k - d(k)) - 0.2440x_1^2(k - d(k)) \end{bmatrix},$$
$$u_2(k) = \begin{bmatrix} -0.1520x_2^1(k - d(k)) + 0.0100x_2^2(k - d(k)) \\ +0.0400x_2^1(k - d(k)) - 0.1830x_2^2(k - d(k)) \end{bmatrix}.$$

5 Conclusion

This paper has proposed a switching design for the robust stability and stabilization of uncertain switched linear discrete-time systems with interval time-varying delays. Based on the discrete Lyapunov functional, a switching rule for the robust stability and stabilization for the uncertain system is designed *via* linear matrix inequalities.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally and significantly in writing this paper. The authors read and approved the final manuscript.

Author details
[1]Major of Mathematics and Statistics, Faculty of Science, Maejo University, Chiangmai, 50290, Thailand. [2]Department of Mathematics, Faculty of Science, Chiangmai University, Chiangmai, 50000, Thailand. [3]Centre of Excellence in Mathematics, CHE, Si Ayutthaya Rd., Bangkok, 10400, Thailand.

Acknowledgements
This work is partially supported by the Thai Research Fund Grant, the Higher Education Commission and Faculty of Science, Maejo University, Thailand (for the first and third authors) and also by Centre of Excellence in Mathematics, the Commission on Higher Education, Thailand (for the second author). The authors thank anonymous reviewers for valuable comments and suggestions, which allowed us to improve the paper.

Received: 1 January 2012 Accepted: 25 July 2012 Published: 6 August 2012

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doi:10.1186/1687-1847-2012-134
Cite this article as: Rajchakit et al.: **Robust stability and stabilization of uncertain switched discrete-time systems.** *Advances in Difference Equations* 2012 **2012**:134.