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Robust stability of the new general 2D model of a class of continuous-discrete linear systems

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Abstract. The problems of asymptotic stability and robust stability of the new general 2D model of scalar linear dynamic continuous-discrete systems, standard and positive, are considered. Simple analytic conditions for asymptotic stability and for robust stability are given. These conditions are expressed in terms of coefficients of the model. The considerations are illustrated by numerical examples. The methods proposed can be generalized to scalar Fornasini-Marchesini and Roesser models of 2D continuous-discrete systems.

Key words: continuous-discrete system, hybrid system, positive system, scalar system, stability, robust stability.

1. Introduction

In continuous-discrete systems both continuous-time and discrete-time components are relevant and interacting and these components can not be separated. Such systems are called the hybrid systems. Examples of hybrid systems can be found in [1–3]. The problems of dynamics and control of hybrid systems have been studied in [3–6].

In this paper we consider the continuous-discrete linear systems whose models have structure similar to the models of 2D discrete-time linear systems. Such models, called the 2D continuous-discrete or 2D hybrid models, have been considered in [7] in the case of positive systems.

The new general model of positive 2D hybrid linear systems has been introduced in [8] for standard and in [9] for fractional systems. The realization and solvability problems of positive 2D hybrid linear systems have been considered in [7, 10, 11] and [12, 13], respectively.

The problems of stability and robust stability of 2D continuous-discrete linear systems have been investigated in [14–20].

The main purpose of this paper is to present simple analytical conditions for stability and for robust stability for the new general 2D model of scalar continuous-discrete linear systems, standard and positive.

The following notation will be used: $\Re$ – the set of real numbers, Z_+- the set of non-negative integers, $\Re_+ = [0, \infty]$; $\Re^{n\times m}$ – the set of $n \times m$ real matrices and $\Re^n_+ = \Re^{n\times 1}_+$.

2. Problem formulation

Consider the new general 2D model of scalar continuous-discrete linear system (for $i \in Z_+$ and $t \in \Re_+$)

$$\dot{x}_1(t,i) = a_{11}x_1(t,i) + a_{12}x_2(t,i) + b_1u(t,i), \tag{1a}$$

$$x_2(t,i+1) = a_{21}x_1(t,i) + a_{22}x_2(t,i) + b_2u(t,i), \tag{1b}$$

$$y(t,i) = c_1x_1(t,i) + c_2x_2(t,i) + du(t,i), \tag{1c}$$

where $\dot{x}_1(t,i) = \partial x_1(t,i)/\partial t$, $x_1(t,i) \in \Re$, $x_2(t,i) \in \Re$, $u(t,i) \in \Re$, $y(t,i) \in \Re$ and a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2, c_1, c_2 and d are constant coefficients.

The general model (1) with $x_1(t,i) \in \Re^{n_1}$, $x_2(t,i) \in \Re^{n_2}$, $u(t,i) \in \Re^m$, $y(t,i) \in \Re^p$ has been introduced in [8].

The boundary conditions for (1a) and (1b) have the form

$$\begin{aligned} x_1(0,i) &= x_1(i), \quad i \in Z_+, \\ x_2(t,0) &= x_2(t), \quad t \in \Re_+. \end{aligned} \tag{2}$$

The model (1) can be written in the form

$$\begin{bmatrix} \dot{x}_1(t,i) \\ x_2(t,i+1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1(t,i) \\ x_2(t,i) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t,i), \tag{3a}$$

$$y(t,i) = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_1(t,i) \\ x_2(t,i) \end{bmatrix} + du(t,i). \tag{3b}$$

The model (1) (or (3), equivalently) will be called the standard new general 2D model of scalar continuous-discrete linear system.

Definition 1. The general model (1) is called positive (internally) if $x_1(t,i) \geq 0$ and $x_2(t,i) \geq 0$ for all boundary conditions $x_1(i) \geq 0$, $i \in Z_+$ and $x_2(t) \geq 0$, $t \in \Re_+$, and all inputs $u(t,i) \geq 0$, $t \in R_+$, $i \in Z_+$.

Theorem 1. The general model (1) is positive (internally) if and only if

$$\begin{aligned} &a_{11} \in \Re, \quad a_{12}, a_{21}, a_{22} \geq 0, \\ &b_1, b_2 \geq 0, \quad c_1,\ c_2 \geq 0, \quad d \geq 0. \end{aligned} \tag{4}$$

Proof. In [8] it was shown that the system (1) with $x_1(t,i) \in \Re^{n_1}$, $x_2(t,i) \in \Re^{n_2}$, $u(t,i) \in \Re^m$, $y(t,i) \in \Re^p$ is positive if

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and only if a_{11} is a Metzler $n_1 \times n_1$ matrix (all of-diagonal entries are nonnegative) and a_{12}, a_{21}, a_{22}, b_1, b_2, c_1, c_2 and d are matrices of appropriate dimensions with nonnegative entries. The condition (4) follows directly from the above for $n_1 = n_2 = m = p = 1$.

Characteristic function of the model (1) (and (3)) is a polynomial in two independent variables s and z, of the form

$$w(s,z) = \det \begin{bmatrix} s - a_{11} & -a_{12} \\ -a_{21} & z - a_{22} \end{bmatrix} = \\ = sz - sa_{22} - za_{11} + (a_{11}a_{22} - a_{12}a_{21}). \tag{5}$$

Definition 2. The general model (1) is called asymptotically stable (or Hurwitz-Schur stable) if for $u(t,i) \equiv 0$ and bounded boundary conditions (2) the condition $x(t,i) \to 0$ holds for $t, i \to \infty$.

From the papers [14, 20] we have the following theorem.

Theorem 2. The general model (1) is asymptotically stable if and only if

$$w(s,z) \neq 0, \quad \operatorname{Re} s \geq 0, \quad |z| \geq 1. \tag{6}$$

The polynomial (5) satisfying condition (6) is called continuous-discrete stable (C-D stable) or Hurwitz-Schur stable [14].

Now we consider the system (1) with uncertain coefficients a_{11}, a_{12}, a_{21}, a_{22} and assume that

$$a_{ik} \in [a_{ik}^-, \ a_{ik}^+], \quad i,k = 1,2, \tag{7}$$

where a_{ik}^- and a_{ik}^+ with $a_{ik}^- < a_{ik}^+$ $(i,k = 1,2)$ are given real numbers.

By generalization of Definition 2 and Theorems 1 and 2 to the case of systems with uncertain parameters one obtains the following definition and theorems.

Definition 3. The general uncertain model (1) is called robustly stable if for $u(t,i) \equiv 0$ and bounded boundary conditions (2) the condition $x(t,i) \to 0$ holds for $t, i \to \infty$ and for all coefficients a_{ik}, $i,k = 1,2$, satisfying (7).

Theorem 3. The general uncertain model (1), (7) is positive if and only if

$$a_{11} \in [a_{11}^-, \ a_{11}^+] \subset \Re, \quad a_{12}^-, a_{21}^-, a_{22}^- \geq 0, \\ b_1, b_2 \geq 0, \quad c_1, c_2 \geq 0, \quad d \geq 0. \tag{8}$$

Theorem 4. The general uncertain model (1) is robustly stable if and only if condition (6) holds for all coefficients a_{ik}, $i,k = 1,2$, of the polynomial (5) satisfying (7).

The main purpose of this paper is to present simple analytical conditions for stability and for robust stability of general model (1) of continuous-discrete linear systems, standard (i.e. non-positive) and positive.

3. Solution of the problem

3.1. Conditions for stability.

Theorem 5. The general model (1) is asymptotically stable if and only if the following two conditions hold

$$w(s, \exp(j\omega)) \neq 0, \quad \operatorname{Re} s \geq 0, \quad \forall \omega \in [0, \ 2\pi], \tag{9}$$

$$w(jy, z) \neq 0, \quad |z| \geq 1, \quad \forall y \in [0, \ \infty). \tag{10}$$

Proof. From [20] it follows that (6) is equivalent to the conditions

$$w(s,z) \neq 0, \quad \operatorname{Re} s \geq 0, \quad |z| = 1, \tag{11}$$

$$w(s,z) \neq 0, \quad \operatorname{Re} s = 0, \quad |z| \geq 1. \tag{12}$$

It is easy to see that conditions (11) and (12) can be written in the forms (9) and (10), respectively.

Solving the equation $w(s,z) = 0$ for $z = \exp(j\omega)$, where $w(s,z)$ has the form (5), we obtain

$$s(j\omega) = a_{11} + \frac{a_{12}a_{21}}{\exp(j\omega) - a_{22}}. \tag{13}$$

From (13) it follows that $s(j\omega)$ is a discontinuous function in the points $\omega = 0$ and $\omega = \pi$ for $a_{22} = 1$ and $a_{22} = -1$, respectively. Therefore, for excluding this discontinuity, we assume that $a_{22} \neq \pm 1$.

Substituting $\omega = 0$ and $\omega = \pi$ in (13) we obtain, respectively,

$$s_0 = s(j0) = a_{11} + \frac{a_{12}a_{21}}{1 - a_{22}}, \tag{14}$$

$$s_\pi = s(j\pi) = a_{11} - \frac{a_{12}a_{21}}{1 + a_{22}}. \tag{15}$$

Let $s(j\omega) = u(\omega) + jv(\omega)$, where $u(\omega) = \operatorname{Re} s(j\omega)$, $v(\omega) = \operatorname{Im} s(j\omega)$. It is easy to check that $[u(\omega) - s_c]^2 + v^2(\omega) = r^2$, where $s_c = 0.5(s_0 + s_\pi)$, $r = |s_0 - s_c|$. This means that the plot of $s(j\omega)$, $\omega \in [0, \ 2\pi]$, is a circle with the center s_c and radius r. Hence, the condition $\operatorname{Re} s(j\omega) < 0$ holds for all $\omega \in [0, \ 2\pi]$ if and only if

$$\max \left\{ a_{11} - \frac{a_{12}a_{21}}{a_{22} - 1}, \ a_{11} - \frac{a_{12}a_{21}}{1 + a_{22}} \right\} < 0. \tag{16}$$

From the above we have the following lemma.

Lemma 1. For the general model (1) the condition (9) is equivalent to (16).

Now we consider the condition (10).

Lemma 2. For the general model (1) the condition (10) is equivalent to

$$-1 < a_{22} < 1 \quad \text{and} \quad a_{11}^2 - (a_{11}a_{22} - a_{12}a_{21})^2 > 0. \tag{17}$$

Proof. From (5) for $s = jy$ we have that the root of the equation $w(jy, z) = 0$ has the form

$$z(jy) = \frac{jya_{22} - (a_{11}a_{22} - a_{12}a_{21})}{jy - a_{11}}. \tag{18}$$

The condition (10) holds if and only if $|z(jy)| < 1, \forall y \in \Re$, i.e.

$$y^2(1 - a_{22}^2) + a_{11}^2 - (a_{11}a_{22} - a_{12}a_{21})^2 > 0, \quad \forall y \in \Re. \tag{19}$$

It is easy to see that (19) is equivalent to (17).

Theorem 6. The general model (1) is asymptotically stable if and only if

$$-1 < a_{22} < 1 \tag{20}$$

and (16) is satisfied, or equivalently, one of the following conditions holds:

$$a_{12}a_{21} \geq 0, \quad a_{11} < \frac{a_{12}a_{21}}{a_{22}-1}. \tag{21}$$

$$a_{12}a_{21} < 0, \quad a_{11} < \frac{a_{12}a_{21}}{1+a_{22}}. \tag{22}$$

Proof. It follows directly from Theorem 5 and Lemmas 1 and 2.

Example 1. Consider the model (1) with $a_{12} = -1$ and $a_{21} = 1$. Check stability of the model for $a_{22} = -0.5$ and $a_{22} = 0.5$.

In this case $a_{12}a_{21} = -1 < 0$ and the necessary condition (20) holds. From (22) it follows that the model is asymptotically stable if and only if:

- $a_{11} < -2$ for $a_{22} = -0.5$,
- $a_{11} < -2/3$ for $a_{22} = 0.5$.

In the case of positive general model (1), from Theorems 1 and 6 one obtains the following theorem.

Theorem 7. The positive general model (1) is asymptotically stable if and only if

$$\begin{gathered} a_{12}a_{21} \geq 0, \quad 0 < a_{22} < 1, \\ a_{11} < \frac{a_{12}a_{21}}{a_{22}-1}. \end{gathered} \tag{23}$$

Example 2. Let us consider positive model (1) with $a_{12} = a_{21} = 1$. Check stability of the model for $a_{22} = 0$ and $a_{22} = 0.5$.

From Theorem 7 we have that the model is positive and asymptotically stable if and only if:

- $a_{11} < -1$ for $a_{22} = 0$,
- $a_{11} < -2$ for $a_{22} = 0.5$.

3.2. Conditions for robust stability.

Let us consider two real interval numbers $A = [a^-,\ a^+]$, $a^- < a^+$ and $B = [b^-,\ b^+]$, $b^- < b^+$.

Recall, that real interval number $X = [x^-,\ x^+]$ is the set of real numbers x such that $x^- \leq x \leq x^+$.

It is well known from the interval analysis that (see [21, 22], for example)

$$\begin{gathered} A - B = \{a - b:\ a \in A,\ b \in B\} = \\ = [a^- - b^+,\ \ a^+ - b^-], \end{gathered} \tag{24}$$

$$A \cdot B = \{a \cdot b:\ a \in A,\ b \in B\} = [\alpha,\ \ \beta], \tag{25}$$

where

$$\alpha = \min(a^-b^-,\ a^-b^+,\ a^+b^-,\ \ a^+b^+), \tag{26}$$

$$\beta = \max(a^-b^-, a^-b^+, a^+b^-,\ \ a^+b^+) \tag{27}$$

and

$$\begin{gathered} A/B = \{a/b:\ a \in A,\ b \in B\} = \\ = [a^-,\ \ a^+] \cdot [1/b^+,\ \ 1/b^-], \quad 0 \notin B. \end{gathered} \tag{28}$$

Hence, for any fixed $a_{12} \in [a_{12}^-,\ a_{12}^+]$ and $a_{21} \in [a_{21}^-,\ a_{21}^+]$ we have $a_{12}a_{21} \in [\alpha_{12}^-,\ \alpha_{12}^+]$, where

$$\alpha_{12}^- = \min(a_{12}^-a_{21}^-,\ a_{12}^-a_{21}^+,\ a_{12}^+a_{21}^-,\ a_{12}^+a_{21}^+), \tag{29a}$$

$$\alpha_{12}^+ = \max(a_{12}^-a_{21}^-,\ a_{12}^-a_{21}^+,\ a_{12}^+a_{21}^-,\ a_{12}^+a_{21}^+). \tag{29b}$$

From (20) and (7) it follows that the necessary condition for robust stability has the form

$$-1 < a_{22}^- < \ a_{22}^+ < 1. \tag{30}$$

Using the rules (25), (28) we obtain the following:

$$\frac{a_{12}a_{21}}{a_{22}-1} \in [\alpha_1,\ \beta_1], \tag{31}$$

where

$$\alpha_1 = \min\left(\frac{\alpha_{12}^-}{a_{22}^+-1},\ \frac{\alpha_{12}^-}{a_{22}^--1},\ \frac{\alpha_{12}^+}{a_{22}^+-1},\ \frac{\alpha_{12}^+}{a_{22}^--1}\right), \tag{32}$$

$$\beta_1 = \max\left(\frac{\alpha_{12}^-}{a_{22}^+-1},\ \frac{\alpha_{12}^-}{a_{22}^--1},\ \frac{\alpha_{12}^+}{a_{22}^+-1},\ \frac{\alpha_{12}^+}{a_{22}^--1}\right) \tag{33}$$

and

$$\frac{a_{12}a_{21}}{a_{22}+1} \in [\alpha_2,\ \beta_2], \tag{34}$$

where

$$\alpha_2 = \min\left(\frac{\alpha_{12}^-}{a_{22}^++1},\ \frac{\alpha_{12}^-}{a_{22}^-+1},\ \frac{\alpha_{12}^+}{a_{22}^++1},\ \frac{\alpha_{12}^+}{a_{22}^-+1}\right), \tag{35}$$

$$\beta_2 = \max\left(\frac{\alpha_{12}^-}{a_{22}^++1},\ \frac{\alpha_{12}^-}{a_{22}^-+1},\ \frac{\alpha_{12}^+}{a_{22}^++1},\ \frac{\alpha_{12}^+}{a_{22}^-+1}\right). \tag{36}$$

Theorem 8. The general uncertain model (1), (7) is robustly stable if and only if the necessary condition (30) is satisfied and

$$a_{11}^+ < \min(\alpha_1,\ \alpha_2). \tag{37}$$

Proof. Using the rule (24) and (7) with $i = k = 1$, (31), (34), from (16) we obtain the condition $\max\{a_{11}^+ - \alpha_1,\ \ a_{11}^+ - \alpha_2\} < 0$, which can be written in the form (37). The proof follows from Theorem 6.

From the above considerations the following algorithm for robust stability analysis of the standard uncertain general model (1), (7) follows.

Algorithm 1.

Step 1. Compute α_{12}^-, α_{12}^+ from (29) and α_1, α_2 from (32) and (35), respectively.

Step 2. Check satisfaction of the conditions of Theorem 8.

Example 3. Find values of coefficient a_{11} for which the uncertain general model (1) with $a_{12} \in [-1,\ 2]$, $a_{21} \in [2,\ 3]$ and $a_{22} \in [-0.5,\ 0.5]$ is robustly stable.

According to Algorithm 1 we have:

Step 1. From (29) and (32), (35) one obtains: $\alpha_{12}^- = -3$, $\alpha_{12}^+ = 6$, $\alpha_1 = -12$, $\alpha_2 = -6$.

Step 2. In this case the necessary condition (30) holds and from (37) we have $a_{11}^+ < \min(\alpha_1,\ \alpha_2) = -12$. This means that the model is robustly stable if and only if $a_{11} \in (-\infty,\ -12)$.

Now we consider the following special cases:

- $[\alpha_{12}^-,\ \alpha_{12}^+] \subset [0,\ \infty) \Leftrightarrow \quad \alpha_{12}^- \geq 0$,
- $[\alpha_{12}^-,\ \alpha_{12}^+] \subset (-\infty,\ 0] \Leftrightarrow \quad \alpha_{12}^+ \leq 0$,

where α_{12}^- and α_{12}^+ are computed form (29).

Assume that the necessary condition (30) holds. From (32) and (35) we obtain the following:

- if $\alpha_{12}^- \geq 0$ then

$$\alpha_1 = \frac{\alpha_{12}^+}{a_{22}^+ - 1} < 0, \qquad \alpha_2 = \frac{\alpha_{12}^-}{a_{22}^+ + 1} > 0, \tag{38}$$

- if $\alpha_{12}^+ \leq 0$ then

$$\alpha_1 = \frac{\alpha_{12}^+}{a_{22}^- - 1} > 0, \qquad \alpha_2 = \frac{\alpha_{12}^-}{a_{22}^- + 1} < 0. \tag{39}$$

Hence, from Theorem 8 we have the following lemmas.

Lemma 3. The standard uncertain general model (1), (7) with $\alpha_{12}^- \geq 0$ is robustly stable if and only if

$$-1 < a_{22}^- < \ a_{22}^+ < 1 \quad \text{and} \quad a_{11}^+ < \frac{\alpha_{12}^+}{a_{22}^+ - 1}. \tag{40}$$

Lemma 4. The standard uncertain general model (1), (7) with $\alpha_{12}^+ \leq 0$ is robustly stable if and only if

$$-1 < a_{22}^- < \ a_{22}^+ < 1 \quad \text{and} \quad a_{11}^+ < \frac{\alpha_{12}^-}{a_{22}^- + 1}. \tag{41}$$

In the case of positive uncertain model (1), (7) the conditions (8) holds. In this case $\alpha_{12}^- \geq 0$. From (8) and Lemma 3 we have the following theorem.

Theorem 9. The uncertain general model (1), (7) is positive and robustly stable if and only if

$$0 \leq a_{22}^- < \ a_{22}^+ < 1 \quad \text{and} \quad a_{11}^+ < \frac{\alpha_{12}^+}{a_{22}^+ - 1}. \tag{42}$$

Example 4. Consider the general uncertain model (1) with $a_{12} \in [-5,\ -1]$, $a_{21} \in [2,\ 4]$, $a_{22} \in [-0.6,\ 0.6]$. Find values of the coefficient a_{11} for which the model is robustly stable.

In this case from (29) we have $\alpha_{12}^- = -20$, $\alpha_{12}^+ = -2$. Because $\alpha_{12}^+ < 0$ and (30) holds, we apply condition (41) of Lemma 4. From this condition we have that the model is robustly stable if and only if $a_{11}^+ < \alpha_{12}^-/(a_{22}^- + 1) = -20/0.4 = -50$. The same result one obtains from Algorithm 1.

Example 5. Find values of the coefficient a_{11} for which is robustly stable the positive uncertain general model (1) with $a_{12} \in [1,\ 4]$, $a_{21} \in [2,\ 6]$ and $a_{22} \in [0,\ 0.5]$.

From (29) and Theorem 9 we obtain $\alpha_{12}^- = 2$, $\quad \alpha_{12}^+ = 24$ and $a_{11}^+ < -24/0.5 = -48$. This means that the positive model is robustly stable if and only if $a_{11} \in (-\infty,\ -48)$. The same result one obtains from Algorithm 1.

4. Concluding remarks

Simple analytical conditions for stability and for robust stability of the new general 2D model (1) of scalar continuous-discrete linear systems, standard and positive, have been given. These conditions are expressed in terms of the coefficients of the model.

In particular it has been shown that:

- the general model (1) is asymptotically stable if and only if the conditions of Theorem 6 hold,
- the general model (1) is positive and asymptotically stable if and only if the conditions (23) hold (Theorem 7),
- the general uncertain standard model (1), (7) is robustly stable if and only if the condition (37) holds (Theorem 8),
- the general uncertain model (1), (7) is positive and robustly stable if and only if the conditions (42) hold (Theorem 9).

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