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Robust steady-state filtering for systems with deterministic and stochastic uncertainties

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Robust Steady-State Filtering for Systems With Deterministic and Stochastic Uncertainties

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***Abstract*—For uncertain systems containing both deterministic and stochastic uncertainties, we consider two problems of optimal filtering. The first is the design of a linear time-invariant filter that minimizes an upper bound on the mean energy gain between the noise affecting the system and the estimation error. The second is the design of a linear time-invariant filter that minimizes an upper bound on the asymptotic mean square estimation error when the plant is driven by a white noise. We present filtering algorithms that solve each of these problems, with the filter parameters determined via convex optimization based on linear matrix inequalities. We demonstrate the performance of these robust algorithms on a numerical example consisting of the design of equalizers for a communication channel.**

***Index Terms*—Linear matrix inequality, parametric uncertainty, robust filtering.**

I. Introduction

THE problem of signal estimation plays an important role in several engineering applications, especially in signal processing, control, and communications. The basic problem is illustrated in Fig. 1. Here, z is the signal to be estimated, y is the measured signal, and w is a noise input, typically with a stochastic description. The signal estimation problem consists of designing a filter that generates estimates $\hat{z}$ of z that are optimal over realizations of the noise w. The measures for optimality typically involve the estimation error e, which is simply the difference between z and $\hat{z}$.

A number of standard estimation problems in signal processing, control, and communications can be posed in the framework in Fig. 1; see, for example, [1] and [2] and the references therein. Perhaps the best-known estimation problem is the Kalman filtering problem [3]. Here, the plant generating the signal z in Fig. 1 is linear time-invariant (LTI), and the noise input w is a zero-mean unit variance white noise sequence. The Kalman filter implements a recursive algorithm that, at each step, minimizes the mean square value of the estimation error e, i.e., it is an "MMSE filter." This recursive algorithm converges to a steady-state Kalman filter if the plant is stable. Another class of filters for LTI plants are the so-called H_∞-optimal filters [4]. With the quantity $\sum_{k=0}^{\infty}\|w(k)\|^2$ interpreted as the energy of the input signal w, the H_∞-optimal filter minimizes the largest energy gain (or ℓ_2 gain, H_∞ norm of the linear estimation) from input w to the estimation error e. The ideas behind the Kalman filtering algorithms and the H_∞-optimal algorithms can be combined to design the so-called mixed performance filter [5], [6].

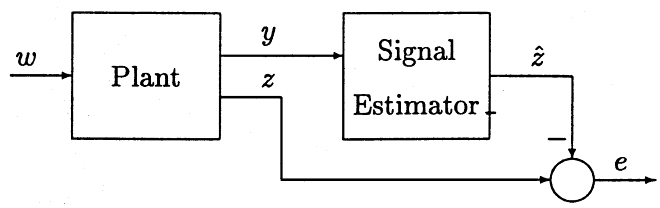

Fig. 1. Block diagram describing the estimation framework.

The specific estimation problems described so far assume that the model of the plant is known exactly. However, in most cases, models of engineering systems are only approximate. This mismatch between the actual plant and its model arises from several factors, such as dynamics and nonlinearities that are either neglected to make the model tractable or are simply unknown, and uncertainties in model parameters and model structure that arise from the limitations of system identification procedures [7]. Therefore, filters must be designed with graceful performance degradation in the presence of modeling errors [8]. This issue of "robust estimation" has been the focus of a number of recent publications; see, for example, [9]–[11].

In the literature, two types of parametric uncertainties have been considered in the context of robust estimation. The first type consists of deterministic uncertainties that are assumed to lie in some bounded set. In this setting, the objective of the designed filter is to optimize the worst-case performance (or an upper bound on the worst-case performance in cases when it is too hard to optimize the worst-case performance directly) of the filtering over all possible values of the uncertainties. In [12], the resulting robust H_∞ estimation problems for such uncertain systems are solved by reformulating them as optimization problems with matrix inequality constraints; these problems are then numerically solved using a heuristic algorithm. In [13], a steady-state robust Kalman filter for systems with norm-bound uncertainties is designed by solving two Riccati equations. In [14], an alternate approach is presented to design robust filters that requires the solution of a single convex optimization problem with linear matrix inequality (LMI) constraints. The filters developed here are all linear time-invariant, and worst-case performance in the presence of uncertainties can be guaranteed by a single quadratic Lyapunov function. To re-

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duce the conservatism by using a single quadratic Lyapunov function for all the possible uncertainties, parameter-dependent Lyapunov functions are applied in robust filter design [15], [16] for the so-called polytopic uncertain systems, where the state-space matrices are affine functions of bounded time-invariant parametric uncertainties.[1] Robust recursive filters for systems with deterministic norm-bound uncertainties can be designed using least-squares approach [17] and set-membership approach [18].

The second kind of parametric uncertainties that have been studied in the literature consist of those with a stochastic description. These uncertainties can also be viewed as multiplicative noise inputs to the system. In this setting, the uncertainties in state-space matrices are random variables with bounded variance. The objective of the designed filter is to minimize the expected squared estimation error (or an upper bound, if necessary) averaged over the uncertainties using the knowledge of their statistics. For such uncertain systems, a robust recursive Kalman filtering algorithm was presented in [19], where at each step, an upper bound on the mean square of the estimation error is minimized using semidefinite programming (SDP). The optimal filtering problem that minimizes the mean energy of the estimation error for systems with stochastic uncertainties remains to be explored.

Existing treatments of the robust estimation problem in the literature consider all the uncertainties to have either a deterministic description or a stochastic description. However, in most realistic situations, systems are best modeled as having a mix of deterministic and stochastic uncertainties. In a typical wireless communication channel, for example, real field experimental data [20], [21] suggest that the channel coefficients are time-varying random variables that depend on uncertainties including propagation path loss, antenna heights, irregular terrain conditions, clutter environments, and local conditions such as buildings, trees, and road intersections. Assuming that the arriving signal is a combination of many reflected signals with random phase, the channel coefficients can be modeled as Gaussian random variables whose mean and variance are themselves uncertain (and thus can be modeled as deterministic uncertainties). This yields a model that has a mixture of deterministic and stochastic uncertainties.

Thus, a model that includes both deterministic and stochastic parametric uncertainties can characterize a larger set of uncertain systems. In this paper, we consider steady-state robust filtering problems for such models; see Fig. 1. Specifically, we design the following.

- *a robust steady-state MMEG filter*, i.e., an LTI filter that minimizes an upper bound on the largest value (over all possible values of the deterministic uncertainties and the statistics of the stochastic uncertainties) of the mean-energy gain (MEG) from the noise input w to the estimation error e;
- *a robust steady-state MAMSE filter*, i.e., an LTI filter that minimizes an upper bound on the largest value (over all possible values of the deterministic uncertainties and the statistics of the stochastic uncertainties) of the asymptotic mean square (AMSE) value of the estimation error e.

In each case, the robust filter design problem is reduced to a convex optimization problem with linear matrix inequality (LMI) constraints. This problem can be numerically solved efficiently using standard SDP algorithms [22]–[24]. The computational burden with SDP increases gracefully (in particular, polynomially) with the problem size. See [25] for a more detailed discussion on the efficiency of SDP algorithms.

The organization of the paper is as follows. In Section II, we describe the mathematical framework underlying our problem and formally pose the two filter design problems. In Sections III and IV, we describe the design of a steady-state robust MMEG filter and MAMSE filter, respectively. In Section V, we present a numerical example, consisting of an equalizer design for a communication channel, that illustrates the performance of the new filters developed in this paper. All the proofs are in the Appendix.

Our notations are standard. $E[\cdot]$ denotes the expectation of a random variable (matrix). $P > 0$ ($P \geq 0$) means that P is a symmetric and positive definite (semidefinite) matrix. $\mathrm{Tr}(\cdot)$ is the trace of a matrix. $\mathbf{Co}\{\cdot\}$ denotes the convex hull. $\|\cdot\|$ is the Euclidean norm. $\mathbf{Z}_+$ is the the set of non negative integers. For discrete time systems considered in the paper, the state-space matrix A is stable if all the eigenvalues of A are strictly inside the unit circle.

II. Preliminaries

Consider the following uncertain system:

$$\begin{aligned} x(k+1) &= A_\Delta(k)x(k) + B_\Delta(k)w(k) \\ y(k) &= C_\Delta(k)x(k) + D_\Delta(k)w(k), \quad z(k) = Lx(k) \end{aligned} \tag{1}$$

where

$$\begin{aligned} A_\Delta(k) &= A_0 + \sum_{t=1}^{n} A_t^d \zeta_t^d(k) + \sum_{j=1}^{m} A_j^s \zeta_j^s(k) \\ B_\Delta(k) &= B_0 + \sum_{t=1}^{n} B_t^d \zeta_t^d(k) + \sum_{j=1}^{m} B_j^s \zeta_j^s(k) \\ C_\Delta(k) &= C_0 + \sum_{t=1}^{n} C_t^d \zeta_t^d(k) + \sum_{j=1}^{m} C_j^s \zeta_j^s(k) \\ D_\Delta(k) &= D_0 + \sum_{t=1}^{n} D_t^d \zeta_t^d(k) + \sum_{j=1}^{m} D_j^s \zeta_j^s(k). \end{aligned}$$

Here, $k = 0, 1, \ldots$, $x(k) \in \mathbf{R}^{n_x}$ is the state, $y(k) \in \mathbf{R}^{n_y}$ is the measured output, $z(k) \in \mathbf{R}^{n_z}$ is the signal we wish to estimate, and $w(k) \in \mathbf{R}^{n_w}$ is the noise input. $\zeta_t^d(k)$ are deterministic uncertain parameters that for each $k \in \mathbf{Z}_+$ satisfy $|\zeta_t^d(k)| \leq 1$, $t = 1, \ldots, n$. ζ_j^s, $j = 1, \ldots, m$ are zero-mean white noise processes that satisfy $E\left[\zeta_i^s(k)\zeta_j^s(l)^T\right] = \delta(i-j)\delta(k-l)$, where $\delta(k)$ is a Dirac Delta function.[2] The initial condition $x(0)$ is a

[1] The uncertain system is called *polytopic* since the set of uncertain state-space matrices is a polytope.

[2] The more general case where ζ_j^s have nonzero means, and/or nonunity variances, can be reduced to the case considered here via simple linear transformations and normalization, followed by absorbing the linear transformation and normalization constants into the state-space matrices.

random vector. The random processes w and ζ_j^s, $j=1,\ldots,m$, and the random vector $x(0)$ are mutually independent.

A closer look at (1) shows that there are product terms between the uncertainties and the noise input $w(k)$. Such terms can be used to accommodate the situation when a system is driven by noise with uncertain variance; see the numerical example in Section V, as well as [11] and [26].

System (1) is said to be *mean square stable* if with $w(k)=0$ and for all $k \in \mathbf{Z}_+$, we have

$$\lim_{k\to\infty} E\left[x(k)x(k)^T\right]=0$$

regardless of the initial condition $x(0)$.

Let

$$\Omega_i = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}, \quad i=1,\ldots,2^n \tag{2}$$

be the vertices of the polytope

$$\Omega = \left\{ \begin{bmatrix} A_0+\sum_{t=1}^n A_t^d\zeta_t^d & B_0+\sum_{t=1}^n B_t^d\zeta_t^d \\ C_0+\sum_{t=1}^n C_t^d\zeta_t^d & D_0+\sum_{t=1}^n D_t^d\zeta_t^d \end{bmatrix} \middle| \, |\zeta_t^d|\le 1 \right\}.$$

The following lemma gives a sufficient condition for the mean square stability of system (1). This lemma is an extension of the corresponding result [27] for systems with stochastic parametric uncertainties to systems with both deterministic and stochastic uncertainties.

Lemma 2.1: System (1) is mean square stable (or second moment asymptotically stable [28]) if there exists a matrix $Q>0$ such that

$$A_i^TQA_i - Q + \sum_{j=1}^m (A_j^s)^TQ(A_j^s) < 0, \quad i=1,\ldots,2^n. \tag{3}$$

Moreover, the quadratic Lyapunov function $V(x(k)) = E\left[x(k)^TQx(k)\right]$ satisfies $V(x(k))\to 0$ as $k\to\infty$.

Lemma 2.1 implies that for any $X(k)=E\left[x(k)x(k)^T\right]\ge 0$ and satisfying the recursion

$$\begin{aligned} X(k+1) =& E\left[A_\Delta(k)x(k)x(k)^TA_\Delta(k)^T\right] \\ =& A(k)X(k)A(k)^T + \sum_{j=1}^m A_j^sX(k)(A_j^s)^T \end{aligned} \tag{4}$$

where $A(k)=A_0+\sum_{t=1}^n A_t^d\zeta_t^d(k)$, (3) guarantees the convergence of $X(k)$ as $k\to\infty$. Indeed, this conclusion can be generalized that for any $X(k)$ ($X(k)$ can be indefinite) satisfying the recursion (4), (3) guarantees the convergence of $X(k)$. This conclusion follows from the fact that $X(k)$ can be represented as $X_1(k)-X_2(k)$, where $X_1(k)\ge 0$ and $X_2(k)\ge 0$ satisfy the recursion (4) separately. Lemma 2.1 will be applied in the proof of Lemma 4.1.

Lemma 2.1 gives a sufficient condition for the mean square stability of system (1). If there is no deterministic uncertainty, i.e., $\zeta_t^d(k)=0$ for all t and k, (3) is also necessary for the system to be mean square stable [27], [28]. However, for more general $\zeta_t^d(k)$, (3) is not necessary for the mean square stability since it requires the use of a *single* quadratic Lyapunov function to prove the mean square stability of system (1) for all possible choices of the state-space matrices $A(k)$. If there exists a matrix $Q>0$ such that (3) holds, system (1) is said to be *mean square quadratically stable*.

Our objective in this paper is to design a steady-state linear time-invariant filter

$$x_f(k+1)=A_fx_f(k)+B_fy(k), \quad \hat{z}(k)=C_fx_f(k) \tag{5}$$

where $x_f(k)\in\mathbf{R}^{n_{x_f}}$. We may write down a state-space realization for the interconnection in Fig. 1:

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ x_f(k+1) \end{bmatrix} &= \begin{bmatrix} A_\Delta(k) & 0 \\ B_fC_\Delta(k) & A_f \end{bmatrix} \begin{bmatrix} x(k) \\ x_f(k) \end{bmatrix} \\ &\quad + \begin{bmatrix} B_\Delta(k) \\ B_fD_\Delta(k) \end{bmatrix} w(k) \\ e(k) &= [\,L \quad -C_f\,] \begin{bmatrix} x(k) \\ x_f(k) \end{bmatrix}. \end{aligned} \tag{6}$$

We will focus on two design objectives:

Robust MMEG filter design

If $x(0)=0$ almost surely, and the input signal w has bounded mean energy, i.e., $\sum_{k=0}^\infty E\left[\|w(k)\|^2\right]\le 1$, we wish to determine the filter parameters $\{A_f,B_f,C_f\}$ by solving the following problem:

$$\begin{aligned} &\texttt{Minimize}:\ \gamma_\infty \\ &\texttt{Subject to}: \sum_{k=0}^\infty E\left[\|e(k)\|^2\right] < \gamma_\infty. \end{aligned} \tag{7}$$

Condition (7) has the interpretation of minimizing an upper bound on the largest value (over all possible values of the deterministic uncertainties and the statistics of the stochastic uncertainties) of the mean-energy gain from the noise input w to the estimation error e.

Robust MAMSE filter design

We wish to determine the filter parameters $\{A_f, B_f, C_f\}$ to solve the following problem:

$$\begin{aligned} &\texttt{Minimize}:\ \gamma_2 \\ &\texttt{Subject to}: \exists N \text{ s.t. } E\left[\|e(k)\|^2\right] < \gamma_2,\ \forall k\ge N \end{aligned} \tag{8}$$

where the input signal w is a zero-mean unit variance white noise. Condition (8) has the interpretation of minimizing an upper bound on the largest (over all possible values of the deterministic uncertainties and the statistics of the stochastic uncertainties) asymptotic mean square value of the estimation error e.

III. Robust MMEG Filter

In this section, we consider the robust MMEG filter design problem. For system (1), with $x(0)=0$ almost surely, the quantity

$$\sup_{\left\{\sum_{k=0}^\infty E[\|w(k)\|^2]\le 1,\zeta^d\right\}} \sum_{k=0}^\infty E\left[\|y(k)\|^2\right] \tag{9}$$

will be referred to as the worst-case mean energy gain (worst-case MEG). The following lemma gives a sufficient condition for the worst-case MEG to be less than a level γ_∞.

Lemma 3.1: The worst-case MEG of system (1) is less than γ_∞ if there exists a matrix $P > 0$ such that

$$\begin{bmatrix} \mathcal{P} & \mathcal{A}_i P & \mathcal{B}_i & 0 \\ P\mathcal{A}_i^T & P & 0 & P\mathcal{C}_i^T \\ \mathcal{B}_i^T & 0 & \gamma_\infty I & \mathcal{D}_i^T \\ 0 & \mathcal{C}_i P & \mathcal{D}_i & I \end{bmatrix} > 0, \quad i = 1, \ldots, 2^n \tag{10}$$

where

$$\begin{aligned} \mathcal{P} =& \mathbf{diag}\,(P, \ldots, P) \\ \mathcal{A}_i =& [(A_i)^T, (A_1^s)^T, \ldots, (A_m^s)^T]^T \\ \mathcal{B}_i =& [(B_i)^T, (B_1^s)^T, \ldots, (B_m^s)^T]^T \\ \mathcal{C}_i =& [(C_i)^T, (C_1^s)^T, \ldots, (C_m^s)^T]^T \\ \mathcal{D}_i =& [(D_i)^T, (D_1^s)^T, \ldots, (D_m^s)^T]^T. \end{aligned}$$

The upper bound on the worst-case MEG given by (10) can be conservative, in general. However, if there exist no deterministic uncertainties, it turns out that the upper bound is tight, i.e., it equals the worst-case MEG. We also note that if the worst-case MEG is bounded, then the system must be mean square quadratically stable. This simply follows from the fact that (10) implies that

$$\begin{bmatrix} \mathcal{P} & \mathcal{A}_i P \\ P\mathcal{A}_i^T & P \end{bmatrix} > 0, \quad i = 1, \ldots, 2^n$$

which is equivalent to (3) with $Q = P^{-1}$.

We are now ready to state our main result on the robust MMEG filtering: that a robust MMEG filter for system (1) can be designed by solving a semidefinite programming problem with linear matrix inequality constraints.

Theorem 3.2: For the uncertain system (1), there exists a full order steady-state filter (5) such that the worst-case MEG from the noise input w to the estimation error e is less than γ_∞ if there exist $Z = Z^T \in \mathbf{R}^{n_x \times n_x}$, $Y = Y^T \in \mathbf{R}^{n_x \times n_x}$, $H \in \mathbf{R}^{n_x \times n_x}$, $F \in \mathbf{R}^{n_x \times n_y}$, and $G \in \mathbf{R}^{n_z \times n_x}$ such that

$$\begin{bmatrix} M_{11} & M_{12,i} & M_{13,i} & 0 \\ M_{12,i}^T & \mathcal{Z} & 0 & M_{24} \\ M_{13,i}^T & 0 & \gamma_\infty I & 0 \\ 0 & M_{24}^T & 0 & I \end{bmatrix} > 0, \quad i = 1, \ldots, 2^n \tag{11}$$

where

$$M_{11} = \begin{bmatrix} \mathcal{Z} & & \\ & \ddots & \\ & & \mathcal{Z} \end{bmatrix}, \quad M_{12,i} = \begin{bmatrix} \mathcal{A}_{f,0}^i \\ \mathcal{A}_{f,1} \\ \vdots \\ \mathcal{A}_{f,m} \end{bmatrix}$$

$$M_{13,i} = \begin{bmatrix} \mathcal{B}_{f,0}^i \\ \mathcal{B}_{f,1} \\ \vdots \\ \mathcal{B}_{f,m} \end{bmatrix}, \quad M_{24} = \begin{bmatrix} L^T - G^T \\ L^T \end{bmatrix}$$

$$\mathcal{A}_{f,0}^i = \begin{bmatrix} ZA_i & ZA_i \\ YA_i + FC_i + H & YA_i + FC_i \end{bmatrix}$$

$$\mathcal{A}_{f,j} = \begin{bmatrix} ZA_j^s & ZA_j^s \\ YA_j^s + FC_j^s & YA_j^s + FC_j^s \end{bmatrix}$$

$$\mathcal{B}_{f,0}^i = \begin{bmatrix} ZB_i \\ YB_i + FD_i \end{bmatrix}, \ \mathcal{B}_{f,j} = \begin{bmatrix} ZB_j^s \\ YB_j^s + FD_j^s \end{bmatrix}$$

and $$\mathcal{Z} = \begin{bmatrix} Z & Z \\ Z & Y \end{bmatrix}. \tag{12}$$

If there is no deterministic uncertainty, the smallest γ_∞ such that (11) holds is the exact value of MEG.

Condition (11) is a sufficient condition for the existence of a linear time-invariant filter such that the worst-case MEG from w to e of system (1) is less than γ_∞. For system (1) with no parametric uncertainties, γ_∞ corresponds to the H_∞ norm constraint on the linear estimation.

By minimizing γ_∞, we can design an optimal robust MMEG filter (problem (7)). This can be represented as the following semidefinite programming problem:

$$\texttt{Minimize}: \gamma_\infty, \quad \texttt{Subject to}: (11). \tag{13}$$

We summarize the various steps comprising the construction of a robust MMEG filter using a feasible solution of LMI (11).

Robust MMEG Filtering Algorithm

1) Solve the semidefinite programming problem **(13)** using an efficient SDP algorithm **[22]–[24]**, and find the optimal values of Z, Y, H, F, and G.

2) Define $V = (Y - Z)^{1/2}$ and $U = -Z^{-1}V$. Then, P and P^{-1} can be represented as

$$P = \begin{bmatrix} Z^{-1} & -Z^{-1}(Y-Z)^{1/2} \\ -(Y-Z)^{1/2}Z^{-1} & (Y-Z)^{1/2}Z^{-1}(Y-Z)^{1/2} + I \end{bmatrix} \tag{14a}$$

and

$$P^{-1} = \begin{bmatrix} Y & (Y-Z)^{1/2} \\ (Y-Z)^{1/2} & I \end{bmatrix}. \tag{14b}$$

Note that **(11)** implies $Y > Z$ and that V is nonsingular.

3) According to **(31)**, construct the state-space matrices of the optimal MMEG filter

$$\begin{bmatrix} A_f & B_f \\ C_f & 0 \end{bmatrix} = \begin{bmatrix} V^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} H & F \\ G & 0 \end{bmatrix} \begin{bmatrix} -V^{-1} & 0 \\ 0 & I \end{bmatrix}. \tag{15}$$

For the steady-state filter we constructed above, the quadratic Lyapunov function $E\left[x(k)Qx(k)\right]$, where $Q = P^{-1}$ defined in (14), guarantees that irrespective of the uncertainties ζ^d and ζ^s, we have

$$\frac{\sum_{k=0}^{\infty} E\left[\|e(k)\|^2\right]}{\sum_{k=0}^{\infty} E\left[\|w(k)\|^2\right]} < \gamma_\infty.$$

IV. Robust MAMSE Filter

Suppose the input w is a zero-mean, unit variance white noise random process. We now consider problem (8) of designing a steady-state filter (5) that minimizes an upper bound on the largest (over all possible values of the deterministic uncertainties and the statistics of the stochastic uncertainties) asymptotic mean square value of the estimation error e. Here, we note that for uncertain bilinear stochastic systems, where the norm-bound deterministic uncertainties appear in the state and the measurement metrics (A, C), robust filter with estimation error variance constraints was designed in [29].

Lemma 4.1: Consider system (1). Suppose there exists a matrix $P > 0$ such that for $i = 1, \ldots, 2^n$

$$A_i P A_i^T - P + B_i B_i^T + \sum_{j=1}^{m} \left((A_j^s) P (A_j^s)^T + B_j^s (B_j^s)^T \right) < 0. \quad (16)$$

Then, there exists $N > 0$ such that if $k \geq N$, then $E\left[x(k)x(k)^T\right] < P$. Moreover, if $E\left[x(0)x(0)^T\right] < P$, then $E\left[x(k)x(k)^T\right] < P$ for every $k \geq 0$.

Lemma 4.1 provides a solution to the MAMSE filtering problem, which is summarized in the following theorem.

Theorem 4.2: For system (1), there exists a full order steady-state filter (5) such that $E\left[\|e(k)\|^2\right] < \gamma_2$ when k is large enough if there exist $Z = Z^T \in \mathbf{R}^{n_x \times n_x}$, $Y = Y^T \in \mathbf{R}^{n_x \times n_x}$, $H \in \mathbf{R}^{n_x \times n_x}$, $F \in \mathbf{R}^{n_x \times n_y}$, and $G \in \mathbf{R}^{n_z \times n_x}$ such that

$$\begin{aligned} &\mathrm{Tr}\,(W) < \gamma_2 \\ &\begin{bmatrix} Z & Z & L^T - G^T \\ Z & Y & L^T \\ L - G & L & W \end{bmatrix} > 0 \\ &\begin{bmatrix} \mathcal{Z} & M_{12,i} & M_{13,i} \\ M_{12,i}^T & M_{22} & 0 \\ M_{13,i}^T & 0 & I \end{bmatrix} > 0, \quad i = 1, \ldots, 2^n \end{aligned} \quad (17)$$

where

$$\begin{aligned} M_{22} &= \begin{bmatrix} \mathcal{Z} & & \\ & \ddots & \\ & & \mathcal{Z} \end{bmatrix} \\ M_{12,i} &= [\mathcal{A}_{f,0}^i \quad \mathcal{A}_{f,1} \quad \cdots \quad \mathcal{A}_{f,m}] \\ M_{13,i} &= [\mathcal{B}_{f,0}^i \quad \mathcal{B}_{f,1} \quad \cdots \quad \mathcal{B}_{f,m}] \end{aligned}$$

with $\mathcal{Z}, \mathcal{A}_{f,0}^i, \mathcal{A}_{f,1}, \ldots, \mathcal{A}_{f,m}, \mathcal{B}_{f,0}^i, \mathcal{B}_{f,1}, \ldots, \mathcal{B}_{f,m}$ defined in (12).

Here, we note that for system (1) with no parametric uncertainties, γ_2 corresponds to the H_2 norm constraint on the linear estimation. The following algorithm constructs a robust MAMSE filter for system (1) based on Theorem 4.2.

Robust MAMSE Filtering Algorithm

1) Solve the semidefinite programming problem

$$\texttt{Minimize}: \gamma_2, \quad \texttt{Subject to}: (17) \quad (18)$$

using an efficient SDP algorithm [22]–[24], and find the optimal values of Z, Y, H, F, and G.

2) Define $V = (Y - Z)^{1/2}$ and $U = -Z^{-1}V$.

3. Construct the desired MAMSE filter by

$$\begin{bmatrix} A_f & B_f \\ C_f & 0 \end{bmatrix} = \begin{bmatrix} V^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} H & F \\ G & 0 \end{bmatrix} \begin{bmatrix} -V^{-1} & 0 \\ 0 & I \end{bmatrix}. \quad (19)$$

In addition, the Lyapunov function $E[x(k)Qx(k)]$, where $Q = P^{-1}$ defined in (14), guarantees that $E\left[\|e(k)\|^2\right] < \gamma_2$ when k is large enough.

Our robust steady-state filtering algorithm extends the results in [9] and [14] by incorporating stochastic uncertainties in the system model. In addition, our filtering algorithm also has the advantage that the so-called "mixed performance" filtering problem [5], [6] can be solved easily by combining the corresponding LMI conditions. This will be illustrated with the numerical example that we describe next.

V. Numerical Example: Equalizer for Communication Channels

We present an application of the robust MMEG and MAMSE filtering techniques proposed in this paper on designing a linear time-invariant equalizer for a communication channel. With s denoting the signal (modeled as white noise) that is transmitted through the channel and w denoting an additive white receiver noise, the system is modeled by the state equations

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.1 & 0.5 \\ 0 & 0.1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} s(k) \\ y(k) &= \begin{bmatrix} 1+\zeta(k) \\ 1+\zeta(k) \end{bmatrix}^T x(k) + (5+\zeta(k))s(k) + 0.3w(k). \end{aligned} \quad (20)$$

ζ denotes the uncertainty, assumed to satisfy $\zeta(k) = 0.1\zeta_d(k) + 0.1\zeta_s(k)$, where $\zeta_d(k)$ is deterministic and satisfies $|\zeta_d(k)| < 1$ for all k; ζ_s is a zero-mean white noise process with a unit variance and is independent of w and s.

System (20) is a model of a wireless communication channel estimated using a training sequence or a pilot signal. $\zeta_d(k)$ is the deterministic parametric uncertainty that represents the mean of the channel identification error, whose bounds depend on the coherence time of the channel; ζ_s is the normalized stochastic parametric uncertainty that accounts for the variance of the channel identification error, which depends on the received reference signal power, the noise density, and the channel estimator bandwidth [30].

In general, the coefficients of a wireless communication channel are complex random variables, where the phase information represents the delay of the sine waveform. The model considered here has real coefficients and can be thought of as representing an equivalent channel for either the in-phase signal or the quadrature signal.

The nonideal channel frequency response causes successively transmitted symbols to interfere with each other [this is just the familiar intersymbol interference (ISI)]. We will therefore design a linear time-invariant equalizer for the channel (20) in order to estimate the transmitted signal $s(k)$ (see Fig. 2), with the performance criteria presented in Sections II–IV. We will allow ourselves a unit delay in equalization.

In order to employ the techniques proposed in the paper, we need to add one more state variable in (20). The unit delay in

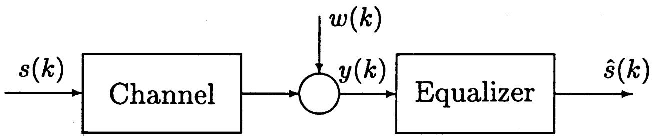

Fig. 2. Communication channel with an equalizer.

equalization adds another state variable, resulting in the following modified channel model:

$$x(k+1) = \begin{bmatrix} 0.1 & 0.5 & 1 & 0 \\ 0 & 0.1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} s(k+1)$$
$$y(k) = \begin{bmatrix} 1+\zeta(k) \\ 1+\zeta(k) \\ 5+\zeta(k) \\ 0 \end{bmatrix}^T x(k) + 0.3w(k)$$
$$z(k) = [0 \quad 0 \quad 0 \quad 1]\, x(k). \tag{21}$$

Using the techniques presented in Sections II–IV, we design a filter to estimate z (i.e., the transmitted signal s with a unit-delay) to minimize an upper bound of the worst-case AMSE of the estimation, subject to an additional MEG constraint [5], [6]. (The MEG constraint can be thought as a safeguard against the possibility that the statistics of w or s are not actually white [31].) This mixed performance filtering problem—that of designing an optimal steady-state MAMSE filter subject to an MEG bound of γ_∞—is

$$\begin{aligned} &\texttt{minimize:} \quad \gamma_2 \\ &\texttt{Subject to:} \text{ (11) and (17).} \end{aligned} \tag{22}$$

Solving the optimization problem (22) for a series of values of γ_∞, we obtain a "tradeoff" curve, shown in a solid line in Fig. 3. Every point on the tradeoff curve represents a linear time-invariant equalizer that is guaranteed to yield i) a worst-case MEG from w and s to e that is less than γ_∞ and ii) with unit variance white noise processes w and s, a worst-case AMSE of the estimation that is less than γ_2. The dashed line in Fig. 3 shows the best bound on the MAMSE of the estimation with no MEG constraint. This is simply the optimal answer from the semidefinite programming problem (18).

In Fig. 3, we also show the performance of the "zero filter." We require the equalizer coefficients $A_f = B_f = C_f = 0$. This filter leads to a trivial equalizing result of $\hat{z}(k) = 0$. Not surprisingly, this filter yields AMSE = MEG = 1, which is verified by the H_2 and H_∞ norms of (21) with $z(k)$ as the system output.

We next consider one specific filter, constructed from the solution of the optimization problem (22) with $\gamma_\infty = 1$. From Fig. 3, it can be seen that the corresponding value of γ_2 is about 0.19. Thus, this filter guarantees that the worst-case MEG from w and s to e is less than one for all the uncertainties ζ_s and ζ_d and that the worst-case AMSE of the estimation is less than 0.19 or −7.2 dB. For this filter, we simulate two quantities:

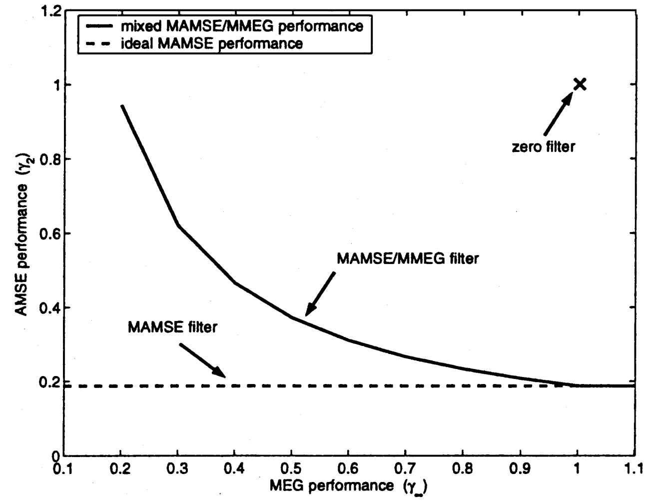

Fig. 3. Tradeoff between the optimized MEG and the AMSE performance constraints.

- the averaged mean square value of the estimation error at time step k, computed as
$$\text{MSE}(k) = E\left[\|e(k)\|^2\right] \approx \frac{1}{N}\sum_{i=1}^{N} |e_i(k)|^2 \tag{23}$$
where $e_i(k)$ is the time-course of the estimation error over the ith Monte Carlo run;
- the averaged energy gain of the filter from s to e at time step k, computed as
$$\text{MEG}(k) = \frac{\sum_{j=0}^{k} E\left[\|e(j)\|^2\right]}{\sum_{j=0}^{k} E\left[\|s(j)\|^2\right]} \approx \frac{\sum_{j=0}^{k}\sum_{i=1}^{N} |e_i(j)|^2}{\sum_{j=0}^{k}\sum_{i=1}^{N} |s_i(j)|^2} \tag{24}$$
where $e_i(k)$ and $s_i(k)$ are the time-courses of the estimation error and the transmitted signal over the ith Monte Carlo run.

Two sets of simulations are shown in Fig. 4. The first case, which is shown in Fig. 4(a), corresponds to the deterministic uncertainty ζ_d being held constant at 0.9999, i.e., $\zeta_d(k) = 0.9999$. The second case, which is shown in Fig. 4(b), corresponds to $\zeta_d(k)$, assuming random values in (−1, 1). The transmitted signal s and the additive noise w have zero mean and unit variance. The estimates of the MSE and MEG were computed by averaging over 1000 runs [i.e., $N = 1000$ in (23) and (24)].

It is evident from Fig. 4 that the MSE in both cases is bounded by $\gamma_2 = 0.19$, as guaranteed by our design. It is also clear that the MEG in both cases is bounded by $\gamma_\infty = 1$; indeed, in this case, the MEG is bounded by γ_2. (It can be shown from (24) that when the input signal is white, it is always true that MEG$(k) \leq$ AMSE for large enough k. For more general (colored) input signals, the worst-case MEG of the estimation error is upper bounded by γ_∞.)

For the purpose of comparison, we also show the averaged MSE of the zero filter in Fig. 4. The estimation error of the zero filter is the same as the transmitted signal.

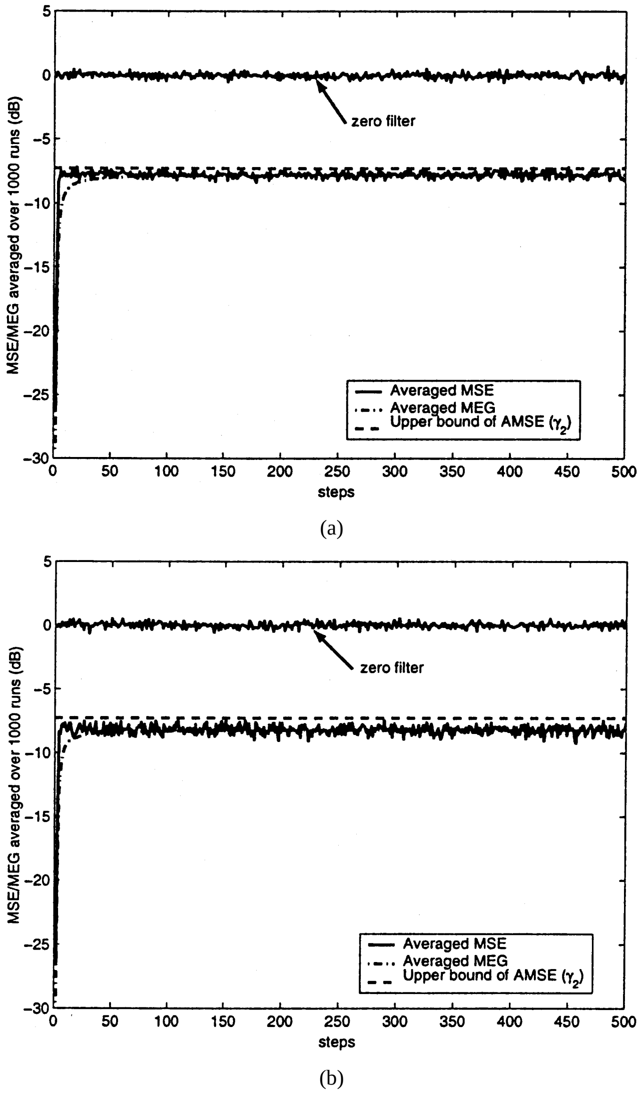

Fig. 4. Averaged mean square and mean energy gain of the estimation error.

VI. Conclusion

We have developed robust minimum mean-energy gain (MMEG) and minimum asymptotic mean-square error (MAMSE) filtering algorithms for systems with both deterministic and stochastic uncertainties. We have illustrated, via a numerical example, the application of these filtering techniques toward designing equalizers for communication channels.

Appendix

A. Proof of Lemma 2.1

Suppose that there exists a matrix $Q > 0$ such that (3) holds. By convexity, (3) implies that

$$A(k)^T Q A(k) - Q + \sum_{j=1}^{m} (A_j^s)^T Q (A_j^s) < 0, \quad k = 0, 1, 2, \ldots. \tag{25}$$

With $w(k) = 0, k = 0, 1, \ldots$, the correlation of the state satisfies the recursion (4) and $X(k) \geq 0$. Define a Lyapunov function $V(x(k)) = E\left[x(k)^T Q x(k)\right] = \mathrm{Tr}\,(QX(k))$. Then, $V(x(k)) \geq 0$, with equality holds if and only if $X(k) = 0$. Then

$$V(x(k+1)) - V(x(k)) = \mathrm{Tr}\left(X(k)\left(A(k)^T Q A(k) - Q + \sum_{j=1}^{m} (A_j^s)^T Q (A_j^s)\right)\right).$$

From (25), we get that $V(x(k+1)) - V(x(k)) \leq 0$ with equality holds if and only if $X(k) = 0$. Therefore, $V(x(k))$ is monotically decreasing and approaches zero as $k \to \infty$. Thus, we have $\lim_{k\to\infty} X(k) = 0$.

B. Proof of Lemma 3.1

First, let us define a Lyapunov function $V(x(k)) = E\left[x(k)^T Q x(k)\right]$ with $Q > 0$. If

$$V(x(k+1)) - V(x(k)) < \gamma_\infty E\left[\|w(k)\|^2\right] - E\left[\|y(k)\|^2\right] \tag{26}$$

we then have

$$\sum_{k=0}^{\infty} E\left[\|y(k)\|^2\right] < \gamma_\infty \sum_{k=0}^{\infty} E\left[\|w(k)\|^2\right].$$

Following a similar argument as in the proof of Lemma 2.1, it can be checked that (26) is equivalent to LMI conditions

$$\begin{bmatrix} Q & 0 \\ 0 & \gamma_\infty I \end{bmatrix} - \begin{bmatrix} \mathcal{A}_i & \mathcal{B}_i \\ \mathcal{C}_i & \mathcal{D}_i \end{bmatrix}^T \begin{bmatrix} \mathcal{Q} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \mathcal{A}_i & \mathcal{B}_i \\ \mathcal{C}_i & \mathcal{D}_i \end{bmatrix} > 0 \tag{27}$$

for $i = 1, \ldots, 2^n$, where $\mathcal{Q} = \mathbf{diag}\,(Q, \ldots, Q)$. With a change of variable $P = Q^{-1}$ and standard matrix manipulations, it follows that (27) is equivalent to (10).

C. Proof of Theorem 3.2

From Lemma 3.1, we have

$$\frac{\sum_{k=0}^{\infty} E\left[\|e(k)\|^2\right]}{\sum_{k=0}^{\infty} E\left[\|w(k)\|^2\right]} < \gamma_\infty$$

if there exists a matrix $P > 0$ such that

$$\begin{bmatrix} \mathcal{P} & \bar{\mathcal{A}}_i P & \bar{\mathcal{B}}_i & 0 \\ P\bar{\mathcal{A}}_i^T & P & 0 & P\bar{\mathcal{C}}_i^T \\ \bar{\mathcal{B}}_i^T & 0 & \gamma_\infty I & 0 \\ 0 & \bar{\mathcal{C}}_i P & 0 & I \end{bmatrix} > 0, \quad i = 1, \ldots, 2^n \tag{28}$$

where

$$\bar{\mathcal{A}}_i = \left[\begin{array}{cc} A_i & 0 \\ B_f C_i & A_f \\ \hline A_1^s & 0 \\ B_f C_1^s & 0 \\ \hline \vdots & \vdots \\ \hline A_m^s & 0 \\ B_f C_m^s & 0 \end{array}\right], \quad \bar{\mathcal{B}}_i = \left[\begin{array}{c} B_i \\ B_f D_i \\ \hline B_1^s \\ B_f D_1^s \\ \hline \vdots \\ \hline B_m^s \\ B_f D_m^s \end{array}\right]$$

and $\bar{\mathcal{C}}_i = [\, L \quad -C_f \,]$. Let P be partitioned as

$$P = \begin{bmatrix} X & U \\ U^T & \star \end{bmatrix} \quad \text{and} \quad P^{-1} = \begin{bmatrix} Y & V \\ V^T & \star \end{bmatrix} \tag{29}$$

where $X, Y \in \mathbf{R}^{n_x \times n_x}$, $U, V \in \mathbf{R}^{n_x \times n_{x_f}}$. According to the matrix completion lemma (see, for example, [32]), by requiring $n_{x_f} = n_x$, such a decomposition is feasible for some fixed X and Y if and only if X and Y satisfy

$$X \geq Y^{-1} > 0. \tag{30}$$

Define new variables [14]

$$H = VA_f U^T X^{-1}, \quad F = VB_f, \quad G = C_f U^T X^{-1} \tag{31}$$

and $Z = X^{-1}$. Note that we may always require V to be nonsingular in (29). If V is singular, we may add some perturbations on P to enforce this requirement. Multiplying by $\mathbf{diag}\left(T^T, \ldots, T^T, I, I\right)$ and $\mathbf{diag}\,(T, \ldots, T, I, I)$ from the left and right sides of (28), where

$$T = \begin{bmatrix} Z & Y \\ 0 & V^T \end{bmatrix}$$

we get the linear matrix inequality (11). Since T is nonsingular, (11) and (28) are equivalent. Finally, we note that by Schur's complement lemma, (30) is implied by the LMI (11). This completes the proof.

D. Proof of Lemma 4.1

Suppose that there exists $P > 0$ such that (16) holds. Let $A(k) = A_0 + \sum_{t=1}^{n} A_t^d \zeta_t^d(k)$ and $B(k) = B_0 + \sum_{t=1}^{n} B_t^d \zeta_t^d(k)$. Since $\{A(k), B(k)\} \in \mathbf{Co}\,\{A_i, B_i\}$, $i = 1, \ldots, 2^n$, where A_i, B_i are defined in (2), we have

$$A(k)PA(k)^T - P + B(k)B(k)^T + \sum_{j=1}^{m} \left((A_j^s)P(A_j^s)^T + B_j^s(B_j^s)^T\right) < -\epsilon I$$

for some $\epsilon > 0$.

By (1), $X(k) = E\left[x(k)x(k)^T\right]$ satisfies the recursion

$$X(k+1) = A(k)X(k)A(k)^T + B(k)B(k)^T + \sum_{j=1}^{m} \left(A_j^s X(k)(A_j^s)^T + B_j^s(B_j^s)^T\right).$$

We then have

$$P - X(k+1) > A(k)(P - X(k))A(k)^T + \sum_{j=1}^{m} A_j^s(P - X(k))(A_j^s)^T + \epsilon I.$$

It can be verified by recursion that

$$P - X(k+1) > M(k+1) + \epsilon I \tag{32}$$

where

$$M(k+1) = A(k)M(k)A(k)^T + \sum_{j=1}^{m} A_j^s M(k)(A_j^s)^T$$

and $M(0) = P - X(0)$. Condition (16) implies that

$$A_i P A_i^T - P + \sum_{j=1}^{m} (A_j^s)P(A_j^s)^T < 0, \quad i = 1, \ldots, 2^n.$$

Then, from Lemma 2.1, we have $\lim_{k\to\infty} M(k) = 0$. Therefore, there exists N such that if $k \geq N$, then $P > X(k)$. To prove the second claim, suppose that $P > X(0)$. It can be immediately verified from (32) that $P > X(k)$ for $k \geq 0$.

E. Proof of Theorem 4.2

The proof of Theorem 4.2 is similar to the proof of Theorem 3.2.

First, from Lemma 4.1, there exists a filter (5) such that $E\left[\|e(k)\|^2\right] < \gamma_2$ when k is large enough, if

$$\mathrm{Tr}\,(W) < \gamma_2, \quad [\,L \quad -C_f\,]\,P \begin{bmatrix} L^T \\ -C_f^T \end{bmatrix} < W$$
$$\begin{bmatrix} P & \bar{\mathcal{A}}_i \mathcal{P} & \bar{\mathcal{B}}_i \\ \mathcal{P}\bar{\mathcal{A}}_i^T & \mathcal{P} & 0 \\ \bar{\mathcal{B}}_i^T & 0 & I \end{bmatrix} > 0, \quad i = 1, \ldots, 2^n \tag{33}$$

where $\mathcal{P} = \mathbf{diag}\,(P, \ldots, P)$, and

$$\bar{\mathcal{A}}_i = \left[\begin{array}{cc|cc|c|cc} A_i & 0 & A_1^s & 0 & \cdots & A_m^s & 0 \\ B_f C_i & A_f & B_f C_1^s & 0 & \cdots & B_f C_m^s & 0 \end{array}\right]$$

and

$$\bar{\mathcal{B}}_i = \left[B_i \quad B_f D_i \;\middle|\; B_1^s \quad B_f D_1^s \;\middle|\cdots\middle|\; B_m^s \quad B_f D_m^s\right].$$

Following similar steps as in the proof of Theorem 3.2, the claim made in the theorem follows.

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