

Robust sustainable development assessment with composite indices aggregating interacting dimensions: the hierarchical-SMAA-Choquet integral approach

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Abstract

The evaluation of sustainable development –and, in particular, rural development– through composite indices requires taking into account a plurality of indicators, which are related to economic, social, and environmental aspects. The points of view evaluated by these indices are naturally interacting: thus, a bonus has to be recognized to units performing well on synergic criteria, whereas a penalisation has to be assigned on redundant criteria. An additional difficulty of the modelization is the elicitation of the parameters for the composite indices, since they are typically affected by some imprecision. In most approaches, all these critical points are usually neglected, which in turn yields an unpleasant degree of approximation in the computation of indices. In this paper we propose a methodology that allows one to simultaneously handle these delicate issues. Specifically, to take into account synergy and redundancy between criteria, we suitably aggregate indicators by means of the Choquet integral. Further, to obtain recommendations that take into account the space of fluctuation related to imprecision in nonadditive weights (capacity of the Choquet integral), we adopt the Robust Ordinal Regression (ROR) and the Stochastic Multicriteria Acceptability Analysis (SMAA). Finally, to study sustainability not only at a comprehensive level (taking into account all criteria) but also at a local level (separately taking into account economic, social, and environmental aspects), we apply the Multiple Criteria Hierarchy Process (MCHP). We illustrate the advantages of our approach in a concrete example, in which we measure the rural sustainability of 51 municipalities in the province of Catania, the largest city of the East Coast of Sicily (Italy).

Keywords: Sustainable development; rural development; composite indices; Choquet integral preference model; Stochastic Multicriteria Acceptability Analysis; Robust Ordinal Regression; Necessary and Possible Preference.

1. Introduction

Sustainable development is a theme that is attracting more and more interest among experts, policymakers, and laymen (see, e.g., [11]). The starting point of the sustainable development discussion is usually fixed in 1987 with the Brundtland Report [18], according to which it concerns

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“development that meets the needs of the present without compromising the ability of future generations to meet their own needs”. However, the concept of sustainable development has a quite rich list of predecessors. In 1972, the publication *The Limits to Growth* [73] used for the first time the term “sustainable” in a modern sense, pointing out the need of “a world system that is: (i) sustainable without sudden and uncontrolled collapse; (ii) capable of satisfying the basic material requirements of all of its people”. Going further back in time at the beginning of the eighteenth century, in his *Sylvicultura Oeconomica* [112], von Carlowitz invited to “act with nature, and not against it”, stating the principle that the “conservation and cultivation of timber should be conducted so as to provide a continuous, persistent and sustaining utilization”. Also many economists have contributed to the current debate on sustainable development (see, e.g., [13]). Just to give some examples, let us mention François Quesnay and the physiocrats [92], who pursued the idea that any wealth comes from Earth; Malthus [66], who studied population’s pressure on limited natural resources; Ricardo [94], who postulated diminishing returns from land use; Pigou [87], who investigated concepts of economic externalities, and proposed a taxation to correct the related inefficiency; Lotka and Volterra [65, 111], who studied the limits of ecological systems through the dynamics of prey-predator models; Georgescu-Roegen [40], who applied the concept of entropy to economics on the basis of the consideration that all natural resources are irreversibly degraded as soon as used in economic activities.

From a political and operational point of view, after being proposed in the Brundtland Report, the concept of sustainable development has been reconsidered in several Summits: the 1992 United Nations Conference on Environment and Development in Rio de Janeiro, the World Summit on Social Development in Copenhagen in 1995, the 2002 World Summit on Sustainable Development in Johannesburg, the 2012 United Nations Conference on Sustainable Development, again in Rio de Janeiro. The sustainable development has been also the subject of several projects of national and international organisations and governments, such as the European Commission’s “Beyond GDP”, the OECD’s “Measuring the Progress of Societies”, the Commission on the Measurement of Economic Performance and Social Progress (CMEPSP), generally referred to as the Stiglitz-Sen-Fitoussi Commission. Recently, on September 25th, 2015, the UN General Assembly voted the 17 Sustainable Development Goals (SDGs) of the 2030 Development Agenda “Transforming our world: the 2030 Agenda for Sustainable Development” [108]. The SDGs have been developed on the basis of the previous Millennium Development Goals (MDGs) stated in Millennium Declaration adopted by the General Assembly of United Nations on September 8th, 2000 [10]. As the MDGs were organized in eight goals with 21 targets and 48 indicators, SDGs associates 169 targets and 232 indicators to the 17 goals. All in all, it is apparent that an increasing attention is being devoted towards data monitoring different aspects of sustainable development.

In fact, beyond the space of the official documents of the United Nations, the idea that indicators are of fundamental importance has been clearly stated in the debate on sustainable development since some time ago. For example, [72] claims that we need many indicators organized hierarchically in an information system and properly integrated to translate ultimate means (solar energy, the biosphere, earth materials, biogeochemical cycles) into ultimate ends (happiness, harmony, identity, fulfillment, self-respect, community) [32]. In an analogous perspective, [20] concludes that the contribution of research, innovation, monitoring, and assessment to sustainability should be directed to manage boundaries between knowledge and action, and has to be coordinated in an integrated knowledge system that supplies pieces of information perceived as salient, legitimate, and credible. Similar concerns are discussed in the postnormal science approach [38], in which the socio-environmental questions are seen as typical post-normal problems characterized by high decision stakes and system uncertainty. These issues have to be handled taking into account data whose imprecision can be described by five aspects: numeral, unit, spread, assessment (expressing salient qualitative information about the data), and pedigree (representing an evaluative description of the mode the information is

produced). The importance of assessing sustainable development by analysing many indicators with the most advanced machine learning and data mining methodologies is also advocated in [85, 98].

From a more applicative point of view, there have been several contributions providing discussions and surveys on the use of data, information, and indicators for sustainable development (see, for example, [31, 71, 82, 100]). In [29] some very interesting reflections on the construction and the use of sustainable development indicators have been proposed. The basic point is that a collection of many diversified indicators to take under control, the so-called “dashboard approach”, is essential but not sufficient. It is instead necessary to combine the dashboard of indicators with a meaningful aggregation in composite indices, thus permitting to measure progress towards the desired goals. In this perspective, construction of composite indices becomes of fundamental importance for sustainable development. In fact, composite indices are more and more adopted in many research and applicative areas, ranging from economic development [12] to innovation [56] and tourism [74]. However, it is in the realm of sustainable development that composite indices have become a central focus of the modelization process [15, 100]. Several critical issues have to be taken into account in using composite indices, both in general [52, 80] and for sustainable development in particular. Let us list the most important questions that need to be addressed.

- (1) *Weighting*: Which weights should be assigned to the single elementary indicators? How to elicit all the parameters necessary to apply the preference model on the basis of the composite index?
- (2) *Aggregation of elementary indicators*: Should we use the classical weighted sum or some more complex aggregation procedure, which may better fit the semantics of the model?
- (3) *Robustness*: How stable are the final results whenever the weights of single indicators and other parameters of the preference model are modified?
- (4) *Structure*: How should the elementary indicators be organized in a hierarchy, according to dimension, goal, targets, and the like?
- (5) *Participation*: How should experts and stakeholders be involved in the construction of the composite indices?

With respect to the above points, there is a widespread consensus that concepts and methodologies developed by MCDA (Multiple Criteria Decision Analysis: for a state-of-the-art survey, see [50]) can give a useful support for applications of composite indices in sustainable development [24, 55, 78, 96]. In this perspective, we propose the application of an advanced MCDA methodology –the hierarchical-SMAA-Choquet integral approach [7]– to construct composite indices. This approach addresses points (1)–(5) according to a logic of adherence to the semantics of the problem at hand.

Specifically, regarding point (2), we shall employ an aggregation procedure that is slightly more complex than the usual weighted sum, namely, the *Choquet integral* [23] (see [47] and [89] for the application of the Choquet integral to MCDA and composite indices, respectively). The rationale of our choice is that the weighted sum is unable to represent synergy and redundancy between elementary indicators, which instead represent important aspects in sustainable development. To describe a simple instance of the relevance of aspects related to synergy and redundancy, suppose we are considering two indicators representing low emissions of two specific pollutants, and one pollutant exacerbates the bad effects of the other. This is a case of synergy between the two indicators, hence it appears natural to give a bonus if there are low emissions for both pollutants. Analogously, one could imagine to have two additional indicators, the first one related again to low emissions of a pollutant, and the second one measuring the funds devoted to research on mitigation of contamination from the same pollutant. In this case, there is a obvious redundancy between the two indicators, since one can expect that research yields a decrease of the emissions for the considered pollutant. Therefore, in this

case, to avoid an over-evaluation it is reasonable reducing the score that would be given by the usual weighted sum. The Choquet integral permits to take into account interactions among criteria, in the form of either synergy or redundancy. This is accomplished by assigning a weight to each subset of indicators –rather than to each single indicator– by means of a “capacity”, which represents an obvious refinement of the weighted sum. The interaction between criteria permits to represent specific attitudes with respect to tolerance or intolerance in compensation of bad evaluations on some indicators with good evaluations on other indicators [69]. This is an important point in constructing composite indices, especially in the domain of sustainability, where, according to [77], a desirable property is “*avoiding the complete compensability*” because “*complete compensability ... is exactly what the concept of sustainability tries to avoid*”. This explains why so frequently the Choquet integral has been used to evaluate different types of sustainability [16, 17, 37, 60, 75, 89, 115, 116].

With respect to points **(1)** and **(5)**, the weights considered by the Choquet integral can be elicited with the simultaneous participation of stakeholders, policymakers, and experts. Indeed, the non-additive ordinal regression [8, 70] applies the basic idea of the ordinal regression [57] to the Choquet integral, thus permitting to elicit weights from some preference information supplied by the stakeholders. This can be expressed in terms of judgments such as “unit a is better than (or indifferent to) unit b ”, or “indicator i_1 is more important than (or as important as) indicator i_2 ”, or “there is a synergy (or a redundancy) between indicators i_3 and i_4 ”, etc.

With respect to point **(3)**, *Robust Ordinal Regression* (ROR [54]) and *Stochastic Multicriteria Acceptability Analysis* (SMAA [62, 63]) can be applied to the Choquet integral (see [9] and [6], respectively). Indeed, point **(3)** refers to the variability of the ranking supplied by the composite index due to a plurality of weights, and, more in general, parameters that can be compatible with the information supplied by stakeholders, policy makers, and experts. To handle these aspects of the procedure, ROR defines the necessary preference relation, which holds for all the sets of compatible weights (and, more in general, parameters), and the possible preference relation, which holds for at least one set of compatible weights: the result is a *necessary and possible preference* [43]. On the other hand, SMAA proposes to consider a probabilistic ranking represented in terms of Rank Acceptability Indices (RAIs) and Pairwise Comparison Indices (PCIs). A RAI gives the probability that, picking randomly one set of compatible weights, a given unit attains a certain rank position, whereas PCI gives the probability that one unit gets a better value than another. The application of the SMAA methodology to composite indices has been proposed in [51].

Finally, with respect to point **(4)**, the idea of a hierarchy of indicators is very rooted in the discussion about sustainable development since its origins. Indeed, following the paradigm proposed by Passet [83] –sustainable development as intersection and relationship between the spheres of environment, economy, and society–, indicators are usually subdivided among these three domains. The above mentioned MDGs and SDGs are also organized in a hierarchy of goals, targets, and indicators. On the point, [29] proposes to harmonize the three spheres of economic, social, and environmental aspects with the SDGs, by assigning the 17 goals as means of the higher level objectives of “efficient allocation”, “fair distribution”, and “sustainable scale”. In MCDA the idea of a hierarchy of criteria has been discussed in [27, 28], where the authors propose an approach called the *Multiple Criteria Hierarchy Process* (MCHP). Applying MCHP to the Choquet integral coupled with ROR and SMAA, one can therefore obtain the *hierarchical-SMAA-Choquet integral approach* [7], which is the methodology here proposed for composite indices in general, and for sustainable development in particular. The obvious advantage of such an approach is that all the above points **(1)**–**(5)** are *simultaneously* addressed, something that has not been done so far in the literature and in practice.

The paper is organized as follows. In Section 2 we deal with methodological aspects, introducing all basic notions relative to the Choquet integral preference model, MCHP, ROR, and SMAA. In Section 3, to show how the proposed methodology works in a real world problem, we apply it to a case study in which 51 municipalities within the province of Catania (Italy) are compared and

ranked according to their level of rural sustainable development. Conclusions and future directions of research are summarized in Section 4.

2. Methodologies

This preliminary section collects all the technical notions and methodologies that are employed in the paper. Specifically, after a brief introduction on terminology and notation (Section 2.1), we recall the preference model based on the Choquet integral (Section 2.2), the robust ordinal regression applied to the Choquet integral (Section 2.3), the stochastic multicriteria acceptability analysis (Section 2.4), and the hierarchical Choquet integral preference model (Section 2.5). These methodologies are then put together in Section 2.6, which describes the approach used for the evaluation of the rural sustainability of municipalities.

2.1. Preliminaries

Let A be a finite set of alternatives, and $G = \{g_1, \dots, g_n\}$ a family of evaluation criteria, that is, maps $g_i: A \rightarrow \mathbb{R}$ for all $i \in I = \{1, \dots, n\}$. Of course, in case of composite indicators, criteria have to be identified with elementary indicators. We denote by 2^G the family of all subsets of G (the *powerset* of G). In this paper we assume, without loss of generality, that all criteria are of “gain type”: thus, the larger the evaluation, the better the alternative.

The only “objective” information stemming for the evaluation of the alternatives is the *dominance relation* D on A , denoted by aDb if a is at least as good as b for all criteria, and a is strictly better than b for at least one criterion. Regrettably, the (good) feature of objectiveness that dominance possesses is balanced out by the (bad) feature of its poorness, since many alternatives are deemed incomparable according to it. The latter fact in turn prevents one from obtaining an “effective recommendation” for the problem at hand. Therefore, an aggregation of all evaluations on criteria is necessary to get a more refined – hence more informative – relation on the set of alternatives. To this aim, the literature considers three different families of aggregation methods: (i) value functions [61], (ii) outranking relations [97], and (iii) decisions rules [53].

A *value function* is a map $U: A \rightarrow [0, 1]$, which assigns a real value to each alternative $a \in A$: this number provides an exact estimation of how “good” the alternative a is. The output of this approach is a *total ranking* on A .¹

An *outranking relation* is a binary relation S on the set A of alternatives, where aSb means that alternative a is “at least as good as” alternative b : roughly speaking, there are arguments supporting the fact that a outperforms b , as well as no strong arguments against this conclusion. It follows that the output of the aggregation process using an outranking relation is a *partial ranking* on A , in which both *ex-equo* and incomparability are possible.

Finally, *decision rules* link the comprehensive evaluation of an alternative with its performances on criteria by means of statements of the type “if ..., then ...”. The advantage of this methodology is that the output is easily understandable by the DM.

The approach employed in this paper to aggregate evaluations is of type (i), in fact it uses a value function expressed in terms of the Choquet integral.

¹Technically, a value function may generate a *total preorder* on A , which is a reflexive, transitive, and complete binary relation R on A . (Recall that *reflexive* means aRa for all $a \in A$, *transitive* means that aRb and bRc implies aRc for all $a, b, c \in A$, and *complete* means that aRb or bRa holds for all distinct $a, b \in A$.) The difference between a total ranking and a total preorder is the possibility of *ex-equo*, which is allowed in the latter and not in the former. However, probabilist reasons typically yield a total ranking on the set of alternatives.

2.2. The Choquet integral preference model

A *capacity* on the power set of the family G of criteria is a function $\mu: 2^G \rightarrow [0, 1]$ with the following properties: $\mu(\emptyset) = 0$ (*base constraint*), $\mu(G) = 1$ (*normalization constraint*), and $\mu(R) \leq \mu(S)$ for all $R, S \in 2^G$ such that $R \subseteq S$ (*monotonicity constraint*).

Given a set A of alternatives and a capacity μ on G , the *Choquet integral* [23] is a function $C_\mu: A \rightarrow \mathbb{R}_0^+$, which evaluates each alternative according to μ . The formal definition of the Choquet integral of $a \in A$ (in fact, of the vector $\mathbf{g}(a) = [g_1(a), \dots, g_n(a)]$) is the following:

$$C_\mu(a) = \sum_{i=1}^n [g_{(i)}(a) - g_{(i-1)}(a)] \mu(N_i), \quad (1)$$

where the subscripts (\cdot) of the criteria stand for a permutation of $I = \{1, \dots, n\}$ in a way that $0 = g_{(0)}(a) \leq g_{(1)}(a) \leq \dots \leq g_{(n)}(a)$, and $N_i = \{(i), \dots, (n)\} \subseteq I$ for each $i \in I$.

It is well-known that a capacity μ on G can be equivalently given using its *Möbius representation* [95], which is the (unique) function $m: 2^G \rightarrow \mathbb{R}$ such that the equality

$$\mu(S) = \sum_{R \subseteq S} m(R) \quad (2)$$

holds for each $S \in 2^G$. Then, all constraints can be restated as follows:

- $m(\emptyset) = 0$;
- $\sum_{T \subseteq G} m(T) = 1$;
- $\sum_{T \subseteq S} m(T \cup \{g_i\}) \geq 0$ for all $g_i \in G$ and $S \subseteq G \setminus \{g_i\}$.

A useful feature of the Möbius representation of μ is that the Choquet integral of $a \in A$ can be equivalently written as [46]:

$$C_\mu(a) = \sum_{T \subseteq G} m(T) \min_{g_i \in T} g_i(a).$$

Criteria naturally interact with each other. Thus, the importance of a criterion $g_i \in G$ depends not only on its own relevance, but also on its contribution to all coalitions of criteria. The *Shapley value* [99] of a criterion $g_i \in G$ takes into account these features in the following way:

$$\varphi(\{g_i\}) = \sum_{T \subseteq G \setminus \{g_i\}} \frac{(|G - T| - 1)! |T|!}{|G|!} (\mu(T \cup \{g_i\}) - \mu(T)). \quad (3)$$

Similarly to the Shapley value, we can also evaluate the relationship between two distinct criteria $g_i, g_j \in G$ computing their *interaction index* [79]:

$$\varphi(\{g_i, g_j\}) = \sum_{T \subseteq G \setminus \{g_i, g_j\}} \frac{(|G - T| - 2)! |T|!}{(|G| - 1)!} (\mu(T \cup \{g_i, g_j\}) - \mu(T \cup \{g_i\}) - \mu(T \cup \{g_j\}) + \mu(T)). \quad (4)$$

Again, Möbius representations yield a simplification of formulas (3) and (4) (see [49]):

$$\varphi(\{g_i\}) = \sum_{g_i \in A \subseteq G} \frac{m(A)}{|A|} \quad (5)$$

and

$$\varphi(\{g_i, g_j\}) = \sum_{\{g_i, g_j\} \subseteq A \subseteq G} \frac{m(A)}{|A| - 1}. \quad (6)$$

A direct application of the preference model based on the Choquet integral appears hardly feasible, since it would require the elicitation of $2^{|G|} - 2$ parameters, namely, all the values $\mu(T)$ for $\emptyset \subsetneq T \subsetneq G$ (since $\mu(\emptyset) = 0$ and $\mu(G) = 1$ are fixed, due to the definition of capacity). Computing such a huge number of parameters is almost impossible, even for a rather small set of criteria.

The notion of q -additive capacity, introduced in [48], makes the model better suited for applications, insofar as it requires the elicitation of definitively fewer parameters. Recall that a capacity is q -additive if $m(T) = 0$ for all $T \subseteq G$ such that $|T| > q$. Said differently, given a suitable integer q such that $1 \leq q < n$, the q -capacity $\mu(T)$ of any family $T \subseteq G$ of criteria is obtained, according to formula (2), by summing up the Möbius value $m(R)$ of all its subfamilies $R \subseteq T$ having cardinality bounded by q . Luckily, in the majority of real world applications, a value of $q = 2$ appears to be unrestrictive. The obvious advantage of a 2-additive capacity is that it involves the elicitation of a rather limited number of parameters, namely, $n + \binom{n}{2}$: a value $m(\{g_i\})$ for each singleton $g_i \in G$, and a value $m(\{g_i, g_j\})$ for each unordered pair $\{g_i, g_j\} \subseteq G$.

Using the Möbius representation (measure) m of a 2-additive capacity μ , the three constraints (base, normalization, and monotonicity) assume the following form:

$$(C.1) \text{ (base) } m(\emptyset) = 0;$$

$$(C.2) \text{ (normalization) } \sum_{g_i \in G} m(\{g_i\}) + \sum_{\{g_i, g_j\} \subseteq G} m(\{g_i, g_j\}) = 1;$$

$$(C.3) \text{ (monotonicity) for all } g_i \in G \text{ and } \emptyset \neq T \subseteq G \setminus \{g_i\}, \begin{cases} m(\{g_i\}) \geq 0 \\ m(\{g_i\}) + \sum_{g_j \in T} m(\{g_i, g_j\}) \geq 0. \end{cases}$$

As a consequence, the Choquet integral of $a \in A$ can be computed in a simpler way as follows:

$$C_\mu(a) = \sum_{g_i \in G} m(\{g_i\}) g_i(a) + \sum_{\{g_i, g_j\} \subseteq G} m(\{g_i, g_j\}) \min\{g_i(a), g_j(a)\}. \quad (7)$$

In this context, the two equations (5) and (6) – respectively expressing the Shapley value of a criterion and the interaction index of a pair of criteria – can be further simplified as follows:

$$\varphi(\{g_i\}) = m(\{g_i\}) + \sum_{g_j \in G \setminus \{g_i\}} \frac{m(\{g_i, g_j\})}{2}, \quad (8)$$

and

$$\varphi(\{g_i, g_j\}) = m(\{g_i, g_j\}). \quad (9)$$

2.3. Non Additive Robust Ordinal Regression (NAROR)

NAROR (*Non Additive Robust Ordinal Regression*, see [9]) belongs to the family of ROR methods (*Robust Ordinal Regression*, see [26, 54]). As it happens in all ROR methods, in NAROR the DM is asked to provide preference information related to a subset $A^* \subseteq A$ of *reference alternatives*, which (s)he supposed to know quite well. This type of information is given at two (somehow complementary) levels of complexity by (i) comparing alternatives to each other, and (ii) comparing intensities of preferences to each other.

Specifically, for each $a, b, c, d \in A^* \subseteq A$, the DM may provide the following pieces of information:

- a is *preferred* to b , denoted by $a \succ b$ (which yields the constraint $C_\mu(a) \geq C_\mu(b) + \varepsilon$);
- a is *indifferent* to b , denoted by $a \sim b$ (which yields $C_\mu(a) = C_\mu(b)$);
- a is preferred to b *more than* c is preferred to d , denoted by $(a, b) \succ^* (c, d)$ (which yields $C_\mu(a) - C_\mu(b) \geq C_\mu(c) - C_\mu(d) + \varepsilon$ and $C_\mu(c) \geq C_\mu(d) + \varepsilon$);
- the intensity of preference of a over b is the *same as* the intensity of preference of c over d , denoted by $(a, b) \sim^* (c, d)$ (which yields $C_\mu(a) - C_\mu(b) = C_\mu(c) - C_\mu(d)$ and $C_\mu(c) \geq C_\mu(d) + \varepsilon$).

Furthermore –differently from other ROR methods– in NAROR the DM may also provide some preference information on criteria $g_i, g_j, g_l, g_k \in G$, typically:

- criterion g_i is *more important* than criterion g_j , denoted by $g_i \succ g_j$ (which yields the constraint $\varphi(\{g_i\}) \geq \varphi(\{g_j\}) + \varepsilon$);
- criteria g_i and g_j are *indifferent*, denoted by $g_i \sim g_j$ (which yields $\varphi(\{g_i\}) = \varphi(\{g_j\})$);
- criteria g_i and g_j are *positively (negatively) interacting* (which yields $\varphi(\{g_i, g_j\}) \geq \varepsilon$ ($\leq -\varepsilon$));
- criterion g_i is preferred to criterion g_j *more than* criterion g_l is preferred to criterion g_k , denoted by $(g_i, g_j) \succ^* (g_l, g_k)$ (hence $\varphi(\{g_i\}) - \varphi(\{g_j\}) \geq \varphi(\{g_l\}) - \varphi(\{g_k\}) + \varepsilon$, $\varphi(\{g_l\}) \geq \varphi(\{g_k\}) + \varepsilon$);
- the difference of importance between g_i and g_j is the *same as* the difference of importance between g_l and g_k , denoted by $(g_i, g_j) \sim^* (g_l, g_k)$ (hence $\varphi(\{g_i\}) - \varphi(\{g_j\}) = \varphi(\{g_l\}) - \varphi(\{g_k\})$).

Notice that, in the above constraints, ε is an *auxiliary variable* being a nonnegative small number used to convert strict inequalities into weak inequalities: for example, the weak inequality $C_\mu(a) \geq C_\mu(b) + \varepsilon$ codifies the strict inequality $C_\mu(a) > C_\mu(b)$. The output of NAROR consists of a pair (\succsim^N, \succsim^P) of preference relations on the set A of alternatives: this is called a *necessary and possible preference* (NaP-preference, see [3, 4, 22, 41, 42, 43, 44, 45]).² The *necessary preference* \succsim^N and the *possible preference* \succsim^P are defined as follows for each $a, b \in A$:

$$\begin{aligned} a \succsim^N b &\stackrel{\text{def}}{\iff} C_\mu(a) \geq C_\mu(b) \quad \text{for all compatible capacities,} \\ a \succsim^P b &\stackrel{\text{def}}{\iff} C_\mu(a) \geq C_\mu(b) \quad \text{for at least one compatible capacity,} \end{aligned}$$

where a *compatible capacity* is a set of Möbius measures (satisfying base, normalization and monotonicity properties) for which the preference information provided by the DM is restored.

Denoted by \mathcal{E}^{DM} the set of all the above constraints translating the DM's preference information (including constraints (C.1), (C.2), and (C.3), see Section 2.1), the existence of a compatible capacity can be checked by solving the following linear programming problem:

$$\varepsilon^* = \max \varepsilon, \quad \text{subject to } \mathcal{E}^{DM}.$$

If \mathcal{E}^{DM} is feasible and $\varepsilon^* > 0$, then there exists at least one compatible capacity. Otherwise, there are inconsistencies in the preferences provided by the DM: these inconsistencies can be identified (and removed) by means of one of the methods presented in [76].

In case that at least one compatible capacity exists, the necessary and possible preferences relative to each pair $a, b \in A$ of alternatives can be computed by using the following set of constraints:

²Technically, a NaP-preference is a pair (\succsim^N, \succsim^P) of binary relations on A satisfying the following properties: (*core transitivity*) \succsim^N is a preorder; (*soft extension*) \succsim^P extends \succsim^N ; (*transitive coherence*) for all $a, b, c \in A$, $a \succsim^N b \succsim^P c$ implies $a \succsim^P c$, and $a \succsim^P b \succsim^N c$ implies $a \succsim^P c$; (*mixed completeness*) for all $a, b \in A$, $a \succsim^N b$ or $b \succsim^P a$.

Under the *Axiom of Choice*, a NaP-preference can be characterized by the existence of a nonempty family \mathcal{T} of total preorders on A such that $\succsim^N = \bigcap \mathcal{T}$ and $\succsim^P = \bigcup \mathcal{T}$ (see [43] for details).

$\mathcal{E}^N(a, b) : C_\mu(b) \geq C_\mu(a) + \varepsilon$ and \mathcal{E}^{DM} ;

$\mathcal{E}^P(a, b) : C_\mu(a) \geq C_\mu(b)$ and \mathcal{E}^{DM} .

Specifically, a is *necessarily preferred* to b ($a \succsim^N b$) if either $\mathcal{E}^N(a, b)$ is infeasible or $\varepsilon^N \leq 0$, where $\varepsilon^N = \max \varepsilon$, subject to $\mathcal{E}^N(a, b)$. Similarly, a is *possibly preferred* to b ($a \succsim^P b$) if $\mathcal{E}^P(a, b)$ is feasible and $\varepsilon^P > 0$, where $\varepsilon^P = \max \varepsilon$, subject to $\mathcal{E}^P(a, b)$.

2.4. Stochastic Multiocriteria Acceptability Analysis (SMAA)

As already summarized in Section 2.1, the aggregation of the evaluations of alternatives on the considered criteria can be performed by either (i) a value function, or (ii) an outranking relation, or (iii) decision rules. In the first two approaches, several types of parameters need to be determined, namely: (1) a capacity on G for the Choquet integral preference model; (2) indifference, preference and veto thresholds for the outranking approach [97].

Since in most cases the evaluations of the alternatives as well as the preference model parameters are not known with certainty, SMAA (*Stochastic Multicriteria Acceptability Analysis*, see [62, 63]) methodology turn out to be extremely useful. In fact, SMAA is a family of MCDA methods, which take into account uncertainty and/or imprecision on both the preference parameters and the alternative evaluations: their goal is to provide a “robust recommendation” for the solution of the problem at hand. (For some recent applications of the SMAA methodology to sustainability and similar subjects see [36, 64, 103].)

Notice that SMAA methods have been already applied to choice, ranking and sorting problems by considering value functions and outranking relations as a preference model [106]. In what follows, we briefly describe one of these methods, called SMAA-2 [63], which deals with ranking problems, and has a value function as its underlying preference model. We shall employ SMAA-2 in our approach. The rationale for this choice is related to the goal of *ranking* the 51 municipalities in the province of Catania with respect to their environmental sustainability, since this method has features that fit quite well our setting.

To make notation more compact, henceforth we shall denote alternatives by a_1, a_2, \dots (instead of a, b, \dots as before). The value function of each alternative is the weighted average of its numerical evaluations by all criteria. Thus, for each $a_k \in A$, we have

$$U(a_k, w) = \sum_{i=1}^n w_i g_i(a_k) \quad (10)$$

where $w \in W = \{(w_1, \dots, w_n) \in \mathbb{R}^n : w_i \geq 0 \text{ and } \sum_{i=1}^n w_i = 1\}$ is the vector of weights of criteria.

In SMAA methods, the preference information is represented by two probability distributions –denoted by f_χ and f_W – defined, respectively, on the evaluation space $\chi = [g_i(a_k)]_{i,k}$ (which is composed of all the “performance matrices” of alternatives) and on the weight space W . Then for each performance matrix $\xi \in \chi$ and each weight vector $w \in W$, equation (10) yields a complete ranking of the alternatives. As a consequence, the rank position of alternative $a_k \in A$ can be computed by the following *rank function*:

$$rank(k, \xi, w) = 1 + \sum_{h \neq k} \rho(U(\xi_h, w) > U(\xi_k, w)),$$

where $\rho(\text{false}) = 0$ and $\rho(\text{true}) = 1$.

Given $\xi \in \chi$, SMAA-2 also computes the set of weights of criteria for which alternative a_k assumes rank s . Formally, for each $s \in \{1, 2, \dots, |A|\}$, let

$$W_k^s(\xi) = \{w \in W : rank(k, \xi, w) = s\}.$$

In order to take into account the plurality of possible rankings and get robust recommendations, in SMAA-2 additional indices are computed, as described below.

- *Rank acceptability index* $b^s(a_k)$: this represents the probability that alternative a_k has rank s . More precisely, $b^s(a_k)$ provides a measure of the set of parameters (compatible with the DM's preference information) that assigns rank s to the alternative a_k . Formally, it is defined by

$$b^s(a_k) = \int_{\xi \in \mathcal{X}} f_{\mathcal{X}}(\xi) \int_{w \in W_k^s(\xi)} f_W(w) dw d\xi.$$

- *Pairwise winning index* $p(a_h, a_k)$: this represents the probability that alternative a_h is preferred to alternative a_k . More precisely, $p(a_h, a_k)$ provides a measure of the set of parameters (compatible with the DM's preference information) for which a_h is strictly better than a_k . Formally, it is defined by

$$p(a_h, a_k) = \int_{w \in W} f_W(w) \int_{\xi \in \mathcal{X}: U(\xi_h, w) > U(\xi_k, w)} f_{\mathcal{X}}(\xi) d\xi dw.$$

From a computational point of view, the multidimensional integrals defining the above indices are estimated by using a Monte Carlo methodology.

2.5. Multiple Criteria Hierarchy Process (MCHP) and the Choquet integral preference model

In MCHP (*Multiple Criteria Hierarchy Process*, see [27]), the evaluation criteria are not all located at the same level, instead they are hierarchically structured. Thus, there is a *root criterion* (the “comprehensive objective”) at level zero, a set of subcriteria of the root criterion at level one, etc. The criteria at the lowest level of the hierarchy (which are the “leaves” of the associated tree of criteria) are called *elementary*. See Figure 1.

To make terminology more suggestive, we shall be using the following notation:

- G is the comprehensive set of criteria (at all levels of the hierarchy), and g_0 is the root criterion;
- I_G is the set of indices of the criteria in G ;
- $E_G \subseteq I_G$ is the set of indices of elementary criteria;
- $g_{\mathbf{r}}$ is a generic non-root criterion (where \mathbf{r} is a vector with length equal to the level of the criterion);
- $g_{(\mathbf{r},1)}, \dots, g_{(\mathbf{r},n(\mathbf{r}))}$ are the immediate subcriteria of criterion $g_{\mathbf{r}}$ (located at the level below $g_{\mathbf{r}}$);
- $E(g_{\mathbf{r}})$ is the set of indices of all the elementary criteria descending from $g_{\mathbf{r}}$;
- $E(F)$ is the set of indices of the elementary criteria descending from at least one criterion in the subfamily $F \subseteq G$ (that is, $E(F) = \bigcup_{g_{\mathbf{r}} \in F} E(g_{\mathbf{r}})$);
- $G_{\mathbf{r}}^l$ is the set of subcriteria of $g_{\mathbf{r}}$ located at level l in the hierarchy (below $g_{\mathbf{r}}$).

For instance, in Figure 1, $E_G = \{(1, 1, 1), \dots, (2, 3, 2)\}$, $g_{(1,1)}$ and $g_{(1,2)}$ are the immediate subcriteria of $g_{\mathbf{1}}$, $E(g_{\mathbf{2}}) = \{(2, 1, 1), \dots, (2, 3, 2)\}$, $G_{\mathbf{2}}^2 = \{g_{(2,1)}, g_{(2,2)}, g_{(2,3)}\}$, $G_{\mathbf{2}}^3 = \{g_{(2,1,1)}, \dots, g_{(2,3,2)}\}$, etc.

Let $a \in A$ be an alternative, $g_{\mathbf{r}} \in G$ a non-elementary criterion (that is, $\mathbf{r} \in I_G \setminus E_G$), and μ a capacity defined on the powerset of the set $\{g_{\mathbf{t}} : \mathbf{t} \in E_G\}$ of elementary criteria. The computation of the Choquet integral of a on $g_{\mathbf{r}}$ is based on a capacity $\mu_{\mathbf{r}}$, which is derived from the original

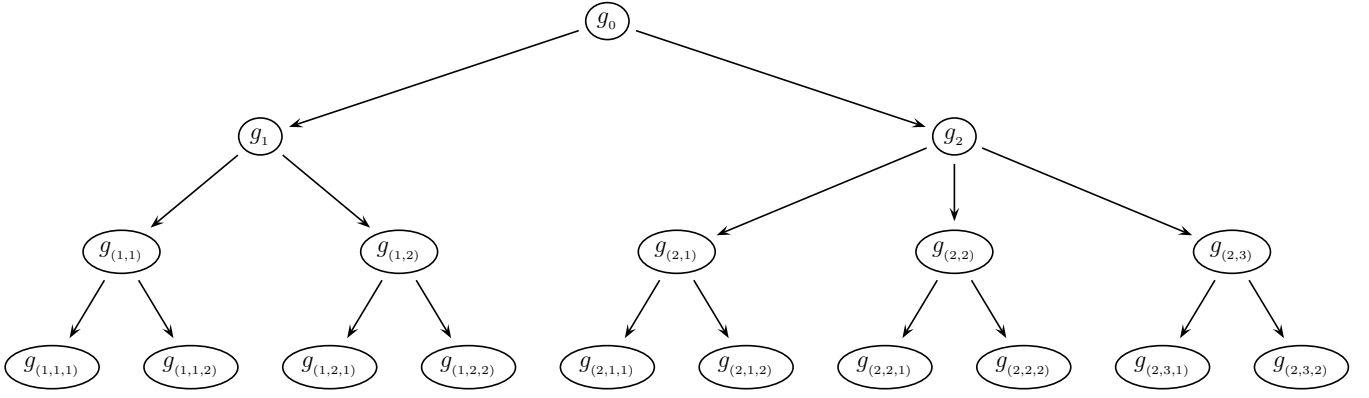


Figure 1: A hierarchy of criteria, displayed in three levels. The root criterion is g_0 (at level zero), and has two subcriteria g_1 and g_2 (at level one); there are ten elementary criteria at the last (third) level.

capacity μ and is defined on the powerset of the set of the immediate subcriteria of $g_{\mathbf{r}}$, that is, $\{g_{(\mathbf{r},1)}, \dots, g_{(\mathbf{r},n(\mathbf{r}))}\}$. Formally, the capacity $\mu_{\mathbf{r}}$ is defined as follows for each $F \subseteq \{g_{(\mathbf{r},1)}, \dots, g_{(\mathbf{r},n(\mathbf{r}))}\}$:

$$\mu_{\mathbf{r}}(F) = \frac{\mu(\{g_{\mathbf{t}} : \mathbf{t} \in E(F)\})}{\mu(\{g_{\mathbf{t}} : \mathbf{t} \in E(g_{\mathbf{r}})\})}$$

where $\mu(\{g_{\mathbf{t}} : \mathbf{t} \in E(g_{\mathbf{r}})\}, \emptyset) \neq 0$ because, on the contrary, criteria from $E(g_{\mathbf{r}})$ would have null importance and would not be meaningful.

Then the Choquet integral of a on $g_{\mathbf{r}}$ is computed by the formula

$$C_{\mu_{\mathbf{r}}}(a) = \frac{C_{\mu}(a_{\mathbf{r}})}{\mu(\{g_{\mathbf{t}} : \mathbf{t} \in E(g_{\mathbf{r}})\})} \quad (11)$$

where $a_{\mathbf{r}}$ is a “fictitious” alternative having the same evaluations as a on elementary criteria in $E(g_{\mathbf{r}})$, and null evaluation on elementary criteria outside $E(g_{\mathbf{r}})$ (that is, $g_{\mathbf{t}}(a_{\mathbf{r}}) = g_{\mathbf{t}}(a)$ if $\mathbf{t} \in E(g_{\mathbf{r}})$, and $g_{\mathbf{t}}(a_{\mathbf{r}}) = 0$ otherwise).

As a consequence, we can naturally associate a preference relation $\succsim_{\mathbf{r}}$ to each non-elementary node $g_{\mathbf{r}}$ as follows:

$$a \succsim_{\mathbf{r}} b \quad \stackrel{\text{def}}{\iff} \quad C_{\mu_{\mathbf{r}}}(a) \geq C_{\mu_{\mathbf{r}}}(b)$$

where $a, b \in A$.

As previously observed, 2-additive capacities are in general sufficient for the majority of practical cases. The Shapley value (8) and the interaction index (9) – which are classically defined for a “flat” structure of criteria, that is, when no hierarchy is involved – can be suitably computed in terms of the Möbius representation also for a hierarchical structure of criteria. For instance, given a non-elementary criterion $g_{\mathbf{r}}$ and three of its subcriteria placed at the level l of the hierarchy – say, $g_{(\mathbf{r},w)}, g_{(\mathbf{r},w_1)}, g_{(\mathbf{r},w_2)} \in G_{\mathbf{r}}^l$ – we have:

$$\varphi_{\mathbf{r}}^l(\{g_{(\mathbf{r},w)}\}) = \left(\sum_{\mathbf{t} \in E(g_{(\mathbf{r},w)})} m(\{g_{\mathbf{t}}\}) + \sum_{\mathbf{t}_1, \mathbf{t}_2 \in E(g_{(\mathbf{r},w)})} m(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\}) + \sum_{\substack{\mathbf{t}_1 \in E(g_{(\mathbf{r},w)}) \\ \mathbf{t}_2 \in E(G_{\mathbf{r}}^l \setminus \{g_{(\mathbf{r},w)}\})}} \frac{m(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\})}{2} \right) \frac{1}{\mu(\{g_{\mathbf{t}} : \mathbf{t} \in E(g_{\mathbf{r}})\})}, \quad (12)$$

and

$$\varphi_{\mathbf{r}}^l(\{g_{(\mathbf{r},w_1)}, g_{(\mathbf{r},w_2)}\}) = \sum_{\substack{\mathbf{t}_1 \in E(g_{(\mathbf{r},w_1)}) \\ \mathbf{t}_2 \in E(g_{(\mathbf{r},w_2)})}} m(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\}) \frac{1}{\mu(\{g_{\mathbf{t}} : \mathbf{t} \in E(g_{\mathbf{r}})\})}. \quad (13)$$

We refer the interested reader to [7] for a detailed description of the extension of the Choquet integral preference model to the setting of hierarchically structured criteria.

2.6. ROR and SMAA applied to the hierarchical Choquet integral preference model

In order to apply the hierarchical Choquet integral preference model, we need to determine the Möbius representation of a capacity defined on the powerset of the set of elementary criteria. In particular, for the special case of a 2-additive capacity, we have to determine $m(\{g_t\})$ for each elementary criterion g_t , and $m(\{g_{t_1}, g_{t_2}\})$ for each unordered pair of elementary criteria $\{g_{t_1}, g_{t_2}\}$. To elicit these values, we shall employ an “ordinal regression” technique, which is based on the provision of suitable pieces of indirect preference information. Below we describe this technique.

Given a non-elementary criterion g_r , the DM may provide the following type of preference information for some alternatives $a_h, a_k, a_v, a_z \in A$ (cf. Section 2.3):

- a_h is preferred to a_k on g_r , denoted by $a_h \succ_r a_k$ (which is translated into the constraint $C_{\mu_r}(a_h) \geq C_{\mu_r}(a_k) + \varepsilon$);
- a_h is indifferent to a_k on g_r , denoted by $a_h \sim_r a_k$ (that is, $C_{\mu_r}(a_h) = C_{\mu_r}(a_k)$);
- on g_r , a_h is preferred to a_k more than a_v is preferred to a_z , denoted by $(a_h, a_k) \succ_r^* (a_v, a_z)$ (that is, $C_{\mu_r}(a_h) - C_{\mu_r}(a_k) \geq C_{\mu_r}(a_v) - C_{\mu_r}(a_z) + \varepsilon$ and $C_{\mu_r}(a_v) \geq C_{\mu_r}(a_z) + \varepsilon$);
- on g_r , the intensity of preference of a_h over a_k is the same as the intensity of preference of a_v over a_z , denoted by $(a_h, a_k) \sim_r^* (a_v, a_z)$ (that is, $C_{\mu_r}(a_h) - C_{\mu_r}(a_k) = C_{\mu_r}(a_v) - C_{\mu_r}(a_z)$ and $C_{\mu_r}(a_v) \geq C_{\mu_r}(a_z) + \varepsilon$).

Furthermore, considering criteria $g_{r_1}, g_{r_2}, g_{r_3}, g_{r_4} \in G_r^l$, the DM may provide the following type of preference information:

- g_{r_1} is more important than g_{r_2} , denoted by $g_{r_1} \succ g_{r_2}$ (which is translated into $\varphi_r^l(\{g_{r_1}\}) \geq \varphi_r^l(\{g_{r_2}\}) + \varepsilon$);
- g_{r_1} and g_{r_2} are equally important, denoted by $g_{r_1} \sim g_{r_2}$ (that is, $\varphi_r^l(\{g_{r_1}\}) = \varphi_r^l(\{g_{r_2}\})$);
- g_{r_1} and g_{r_2} are positively interacting (that is, $\varphi_r^l(\{g_{r_1}, g_{r_2}\}) \geq \varepsilon$);
- g_{r_1} and g_{r_2} are negatively interacting (that is, $\varphi_r^l(\{g_{r_1}, g_{r_2}\}) \leq -\varepsilon$);
- the interaction between g_{r_1} and g_{r_2} is greater than the interaction between g_{r_3} and g_{r_4} , which is codified differently according to the following subcases:
 - if there is positive interaction between both pairs of criteria, then the constraints translating this preference are $\varphi_r^l(\{g_{r_1}, g_{r_2}\}) - \varphi_r^l(\{g_{r_3}, g_{r_4}\}) \geq \varepsilon$ and $\varphi_r^l(\{g_{r_3}, g_{r_4}\}) \geq \varepsilon$;
 - if there is negative interaction between both pairs of criteria, then the constraints translating this preference are $\varphi_r^l(\{g_{r_1}, g_{r_2}\}) - \varphi_r^l(\{g_{r_3}, g_{r_4}\}) \leq -\varepsilon$ and $\varphi_r^l(\{g_{r_3}, g_{r_4}\}) \leq -\varepsilon$;
- the difference of importance between g_{r_1} and g_{r_2} is greater than the difference of importance between g_{r_3} and g_{r_4} , denoted by $(g_{r_1}, g_{r_2}) \succ_r^* (g_{r_3}, g_{r_4})$ (that is, $\varphi_r^l(\{g_{r_1}\}) - \varphi_r^l(\{g_{r_2}\}) \geq \varphi_r^l(\{g_{r_3}\}) - \varphi_r^l(\{g_{r_4}\}) + \varepsilon$ and $\varphi_r^l(\{g_{r_3}\}) - \varphi_r^l(\{g_{r_4}\}) \geq \varepsilon$);
- the difference of importance between g_{r_1} and g_{r_2} is the same of the difference of importance between g_{r_3} and g_{r_4} , denoted by $(g_{r_1}, g_{r_2}) \sim_r^* (g_{r_3}, g_{r_4})$ (that is, $\varphi_r^l(\{g_{r_1}\}) - \varphi_r^l(\{g_{r_2}\}) = \varphi_r^l(\{g_{r_3}\}) - \varphi_r^l(\{g_{r_4}\})$).

As explained in Section 2.3, ε is an auxiliary variable used to convert the strict inequalities into weak ones. Similarly, we denote by \mathcal{E}^{DM} the set of constraints translating the DM's preference information along with the base, monotonicity and normalization constraints. All in all, to check if there exists at least one compatible capacity, one has to solve the following linear programming problem:

$$\varepsilon^* = \max \varepsilon, \quad \text{subject to } \mathcal{E}^{DM}.$$

If \mathcal{E}^{DM} is feasible and $\varepsilon^* > 0$, then there exists at least one compatible capacity; otherwise, there are inconsistencies, which need to be identified [76].

As in Section 2.3, the output of this analysis is a NaP-preference, whose computation is described below. Let $g_{\mathbf{r}}$ be a non-elementary criterion. Consider the following two sets of constraints:

$$\mathcal{E}_{\mathbf{r}}^N(a_h, a_k) : \quad C_{\mu_{\mathbf{r}}}(a_k) \geq C_{\mu_{\mathbf{r}}}(a_h) + \varepsilon \quad \text{and} \quad \mathcal{E}^{DM};$$

$$\mathcal{E}_{\mathbf{r}}^P(a_h, a_k) : \quad C_{\mu_{\mathbf{r}}}(a_h) \geq C_{\mu_{\mathbf{r}}}(a_k) \quad \text{and} \quad \mathcal{E}^{DM}.$$

Then, a_h is *necessarily preferred* to a_k with respect to criterion $g_{\mathbf{r}}$ (denoted by $a_h \succ_{\mathbf{r}}^N a_k$) if either $\mathcal{E}_{\mathbf{r}}^N(a_h, a_k)$ is infeasible, or $\varepsilon_{\mathbf{r}}^N \leq 0$, where $\varepsilon_{\mathbf{r}}^N = \max \varepsilon$, subject to $\mathcal{E}_{\mathbf{r}}^N(a_h, a_k)$. Analogously, a_h is *possibly preferred* to a_k with respect to criterion $g_{\mathbf{r}}$ (denoted by $a_h \succ_{\mathbf{r}}^P a_k$) if both $\mathcal{E}_{\mathbf{r}}^P(a_h, a_k)$ is feasible and $\varepsilon_{\mathbf{r}}^P > 0$, where $\varepsilon_{\mathbf{r}}^P = \max \varepsilon$, subject to $\mathcal{E}_{\mathbf{r}}^P(a_h, a_k)$.

In concrete scenarios, it is likely that simultaneously a_h is possibly preferred to a_k , and a_k is possibly preferred to a_h . However, in these cases it may also happen that the set of compatible capacities for which a_h is preferred to a_k is in some form larger than the set of compatible capacities for which a_k is preferred to a_h . In other words, the necessary and possible preferences are invariant with respect to the extent of the sets of compatible capacities for which one alternative is preferred to the other. This is the point where the SMAA methodology turns to be useful, insofar as it allows one to estimate how good an alternative is in comparison to others, and how often it is preferred over the others. Below we explain how to apply SMAA to the hierarchical Choquet integral preference model.

The set of linear constraints in \mathcal{E}^{DM} defines a convex set of Möbius parameters. Then we may employ the *Hit-And-Run* (HAR) method [102, 107, 109] to explore this set of parameters. Specifically, HAR iteratively samples a set of capacities expressed in terms of Möbius parameters satisfying \mathcal{E}^{DM} , until a stopping condition is met. For each sampled set of Möbius parameters and each given criterion $g_{\mathbf{r}}$, one can compute values of the Choquet integral for all alternatives. The output is a ranking of the alternatives with respect to criterion $g_{\mathbf{r}}$. Having many rankings as samples, one can then compute all indices that are typical of the SMAA methodology (see Section 2.3), namely:

- the *rank acceptability index* $b_{\mathbf{r}}^s(a_k)$, which is the frequency of having alternative a_k at position s in the ranking obtained w.r.t. criterion $g_{\mathbf{r}}$;
- the *pairwise winning index* $p_{\mathbf{r}}(a_h, a_k)$, which is the frequency of having a preference of a_h over a_k w.r.t. criterion $g_{\mathbf{r}}$.

Furthermore, using the rank acceptability indices, two additional indices can be computed as follows [5, 58]:

- the *downward cumulative rank acceptability index* $b_{\mathbf{r}}^{\leq s}(a_k)$, which is the frequency of having alternative a_k at a position not greater than s w.r.t. criterion $g_{\mathbf{r}}$, that is,

$$b_{\mathbf{r}}^{\leq s}(a_k) = \sum_{q=1}^s b_{\mathbf{r}}^q(a_k);$$

- the upward cumulative rank acceptability index $b_{\mathbf{r}}^{\geq s}(a_k)$, which is the frequency of having alternative a_k at a position not lower than s w.r.t. criterion $g_{\mathbf{r}}$, that is,

$$b_{\mathbf{r}}^{\geq s}(a_k) = \sum_{q=s}^{|A|} b_{\mathbf{r}}^q(a_k).$$

Finally, observe that the employed methodology allows us to get both NaP-preferences and SMAA indices at a comprehensive level as well (that is, with respect to the root criterion $g_{\mathbf{0}}$).

3. The case study

To show how the hierarchical-SMAA-Choquet integral works, we apply the methodology described in the previous section to compare and rank with respect to their level of rural sustainable development 51 municipalities within the province of Catania, a city on the East Coast of the Italian region of Sicily. Before presenting, step by step, the application of the procedure in the case study, we give a brief sketch of the overall context in which the results of our analysis need to be interpreted, that is, the European Union (EU) policy with respect to the rural sustainability development. After, we present the application of the proposed approach showing the support that it supplies to stakeholders, experts and policy makers.

3.1. The rural sustainable development in the European Union policy

The structure of the Common Agricultural Policy (CAP), that is, the agricultural policy of the European Union (EU), is based on two different pillars. The first one is characterized by a specific *sectoral policy*, in fact it is the pillar of the globalized agricultural market for commodities. The second one – probably the most important and current – is characterized by a *territorial approach*, and it is useful to support the social cohesion and the integrated sustainable development of all rural areas [34]. With respect to this latter point, nowadays, it is apparent that the rural development policy is inspired by the strategic approach of EUROPA 2020 [25], which is structured to foster a smart, sustainable and inclusive development. As a matter of fact, sustainability has become the key factor in motivating all European policy actions aimed at sustaining any kind of human activity [19]. These actions are financially supported by both the EU and the national governments of its members, according to a co-financing principle. As a consequence, resources are distributed at a national and a regional level, according to specific criteria and priorities of intervention, and based on an estimation of the effectiveness of the employed policies [81]. Consequently, rural development depends on all those policies that have as a goal the creation of a new balance between urban and territorial space, as well as the defence of the quality of rural life, including young farm generations and a long term sustainable environmental quality [93]. Indeed, the conditions under which rural sustainability is defined, looks at a long time view through the logic of safeguarding and enhancing the resources of the territory, the environment and human capacities, including culture, the tradition of the territory and the complex of the intangible and relational resources [21]. Therefore, the actual understanding of the concept of agriculture is determinate by the abandon of the vision of agriculture as a sector and has moved towards a territorial dimension characterized by a form of cross-sectoral rurality with reciprocal implications to other economic, social and environmental dimensions. In this context, the rural dimension looks at the role and the functions of agriculture. They emerge from the necessity to protecting the environment and the landscape, capitalizing on natural and environmental resources through tourism and culture, technological innovation, social cohesion and the development of quality of life in rural centers outside strictly urban areas [90].

This strategic policy framework also motivates the interest of the EU in controlling climate changes, mainly induced by a wrong productive system that badly affects the environment. In view of

an effective financial strategy of support, it appears then inevitable to undertake a careful comparison of different rural areas with respect to several indicators that are naturally linked to sustainability [105]. Measuring sustainability has been already the goal of *green accounting*, outperformed by evaluating several and different sustainability indices [77]. However, there is not a unique system to combine such different types of indices in a way immediately useful for policy makers. The sustainability indices are typically of two categories: monetary and physical. For example, the Index of Sustainable Economic Welfare (ISEW)[33], Weak Sustainability Index [84], and the so-called El Serafy approach [114] belong to the first category of indices, whereas instances of the second category are Human Appropriation of Net Primary Production [110], and the Ecological Footprint [113]. All these indicators belong to the family of composite indicators, so that the approach we are proposing of a new generation methodology to construct composite indices seems well appropriate.

3.2. *The application and the results of the hierarchical-SMAA-Choquet integral approach*

The specific example that we are going to analyze regards the evaluation of the rural sustainability of 51 municipalities within the province of Catania, which is the second largest city in the region of Sicily (Italy). To overcome the difficulties connected to the very definition of “rural area”, we have identified these municipalities using the classification in the 2007–2013 program of rural development of Sicily. The evaluation is based on a set of indicators that has been already used in [14] to assess the sustainable rural development of the municipalities in Umbria, another Italian region. In the latter case study, the evaluation has been performed using another methodology, namely, the Dominance-based Rough Set Approach (DRSA) [53].

The reasons for which we choose a *provincial* scale rather than a *regional* scale –which is the one typically employed in this type of analysis– are connected to the possibility to directly interpret the results of the decision model in view of a deep knowledge and understanding of the territory. Technically, this change of scale is legitimated by the fact that the province of Catania has a level of complexity which is definitively comparable to that of a region, in terms of quality and variety of resources as well as on the level of social relationship.

As confirmed by some previous research concerning the Eastern Coast of Sicily, the province of Catania offers a privileged position of observation for an assessment of rural sustainability, due to the contiguity of remarkably different socio-economic situations: residual but well-rooted industrial districts, tiny but highly developed technological areas, locations exclusively dedicated to touristic activity, agro-naturalistic settlements on the slopes of the Etna volcano, wineries that make the pride of the Sicilian island and so on. On the other hand, several areas of the province of Catania are characterized by a clear and marked backwardness, in particular rural areas with serious problems of underdevelopment, a fragile eco-environmental condition and a low level of anthropic dimension.

In view of all these highly contrasting and variegated features of the surroundings of the province of Catania, our analysis –even if implemented at a provincial level– has no methodological shortcomings.

Our original task was to apply the described methodology to evaluate the rural sustainable development of *all* the 58 municipalities within the province of Catania (see OECD [2] and EUROSTAT [1]). However, 7 out of these 58 municipalities were to be excluded from our analysis, since they are not classified as “rural” according to the 2007-2013 program of the rural development of Sicily. The remaining 51 municipalities, considered in the presented study, are listed in Table 1.

The data set of the municipalities is collected from ISTAT (the Italian Institute of Statistics, www.istat.it), considering the census data relative to 2010 (on agriculture) and 2011 (on population). The municipalities are evaluated on the basis of the criteria described in [14]. Specifically, three macro-criteria are considered: Social (So), Economic (Ec), and Environmental (En). These criteria have been further decomposed into more detailed sub-criteria as follows:

Social Sustainability. In order to measure social sustainability in rural areas, three simple indicators were taken into consideration, since data used to compute them are very easy to be

Table 1: List of the municipalities of Catania

Alternative	Municipality	Alternative	Municipality	Alternative	Municipality
a_1	Aci Bonaccorsi	a_{18}	Linguaglossa	a_{35}	Ramacca
a_2	Aci Catena	a_{19}	Maletto	a_{36}	Randazzo
a_3	Aci Sant'Antonio	a_{20}	Maniace	a_{37}	Riposto
a_4	Acireale	a_{21}	Mascali	a_{38}	San Cono
a_5	Adrano	a_{22}	Mascalucia	a_{39}	San Giovanni la Punta
a_6	Belpasso	a_{23}	Mazzarrone	a_{40}	San Michele di Ganzaria
a_7	Biancavilla	a_{24}	Militello in Val di Catania	a_{41}	San Pietro Clarenza
a_8	Bronte	a_{25}	Milo	a_{42}	Santa Maria di Licodia
a_9	Calatabiano	a_{26}	Mineo	a_{43}	Santa Venerina
a_{10}	Caltagirone	a_{27}	Mirabella Imbaccari	a_{44}	Sant'Alfio
a_{11}	Camporotondo Etneo	a_{28}	Nicolosi	a_{45}	Scordia
a_{12}	Castel di Iudica	a_{29}	Palagonia	a_{46}	Trecastagni
a_{13}	Castiglione di Sicilia	a_{30}	Paternó	a_{47}	Tremestieri Etneo
a_{14}	Fiumefreddo di Sicilia	a_{31}	Pedara	a_{48}	Valverde
a_{15}	Giarre	a_{32}	Piedimonte Etneo	a_{49}	Viagrande
a_{16}	Grammichele	a_{33}	Raddusa	a_{50}	Vizzini
a_{17}	Licodia Eubea	a_{34}	Ragalna	a_{51}	Zafferana Etnea

collected. These indicators are useful to represent the presence of human settlements capable of maintaining an equilibrium between the original identity of the territory and its anthropic size. Notice that the large size of settlements has typically a negative impact on the rural dimension, due to an increased usage of resources and a consequent impoverishment of the territory [39].

1. *Population Scattering Index (PSI)*: it is the ratio between the population living in individual homes, villages, small towns, and the total population. An increase of this indicator signals an improvement of the social dimension for the sustainability of rural development. The relevance of PSI in the assessment of rural development lies in the fact that it is based on human settlements in terms of both territorial distribution and absorption of resources. The presence of habitual settlements on a rural territory –which is by definition scarcely inhabited– reveals the vitality of the rural area under consideration.
2. *Concentrated Population/km² (CP)*: This indicator measures the degree of housing density in the considered rural area. However, contrary to PSI, its increase highlights a worsening of the rural development, due to the motivations described for PSI.
3. *Total Residents (R)*: Also for this indicator an inverse direction is observed, that is, the higher its level, and the lower the performance. Indeed, with respect to social sustainability, if the resident population increases, the level of rural development decreases, due to a depletion of the resources of the territory.

Economic Sustainability. Among the sustainability indicators of rural development related to the economic dimension, the following ones have been considered:

1. *Owned Homes/Total Homes (OH)*: According to the current literature [59], an increase of this index is correlated to a worsening of the rural development. Indeed, an enlargement of property ownership by residents is a condition of fragility for rural sustainability, because the growing number of people living in the area induces the necessity of new houses. A value of the index close to one means that the trend to build new houses is booming.
2. *Agricultural Age Structure (AA)*: This indicator is directly linked to the European strategy “Europe 2020”. Its increase represents a tendency for a greater entrepreneurial engagement capacity in a specific territory.
3. *Number of Bed Spaces in Rural Tourism Accommodation/km² (BS)*: With respect to this indicator, the areas with a higher number of farmhouses are more oriented to an economic development.

Environmental Sustainability. Among several possible indicators, we have selected those that somehow take into account –even in a hybrid way– the fact that economy and environment have to be considered together. Specifically, we consider:

1. *Typical Products Rate (TPR)*: It is ratio of the number of typical products of the considered municipality over the total number of typical products in Sicily. A typical product characterizes a municipality since its production is related to its specific environmental conditions [88, 91].
2. *Irrigation Water Use/UAA (IWU)*.
3. *Livestock Standard units/UAA (LS)*.
4. *Organic UAA/UAA (OU)*.

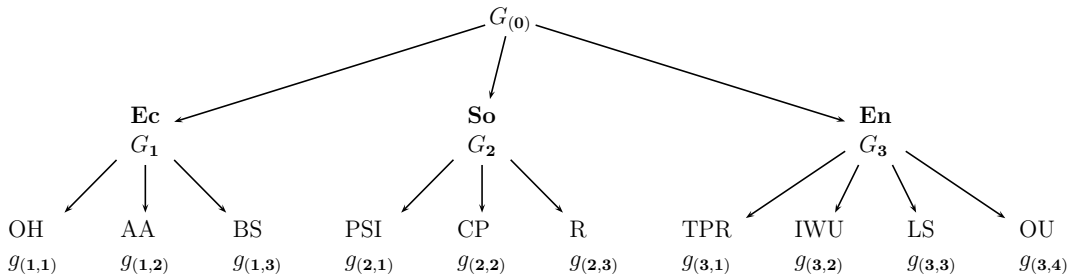
The classical indicators 2, 3 and 4 are related to the resource absorption of the surface devoted to agriculture [86]. As a measurement of this territorial dimension, we employed the Utilized Agricultural Area (UAA), according to what is typical for this purpose.

The elementary subcriteria are described in Table 2, with their different scale of evaluation, and a description of whether they have to be maximized (denoted by \uparrow) or minimized (denoted by \downarrow). The whole hierarchy of criteria is displayed in Figure 2.

Table 2: Description of the elementary subcriteria

Elementary subriterion	Description	Unit	Preference
Owned Homes/Total Homes (OH)	ratio between the number of total homes owned by the residents and the total number of homes	[0, 1]	\downarrow
Agricultural Age Structure (AA)	ratio between farms conducted by young farmers and the total number of farms	[0, 1]	\uparrow
Bed Spaces in Rural Tourism Accommodation/km ² (BS)	number of bed spaces in rural tourism accommodation per unit of area	1/km ²	\uparrow
Population Scattering Index (PSI)	ratio between the population living in individual homes, villages and small towns, and the total population	[0, 1]	\uparrow
Concentrated Population/km ² (CP)	concentrated population per unit of area	1/km ²	\downarrow
Total Residents (R)	total number of residents		\downarrow
Typical Products Rate (TPR)	ratio between the number of typical products from agriculture in the municipality and the total number of typical products of Sicily	[0, 1]	\uparrow
Irrigation Water Use/UAA (IWU)	amount of water used for irrigation per unit (Hectare) of Utilized Agricultural Area (UAA)	m ³ /Hectare	\downarrow
Livestock Standard units/UAA (LS)	number of animals breeding converted into Livestock Standard Units per unit of UAA	LSU/UAA	\downarrow
Organic UAA/UAA (OU)	ratio between organic UAA and total UAA	[0, 1]	\uparrow

Figure 2: Hierarchical structure of criteria considered in the case study



Now assume that the DM –e.g., a policy maker who has a preference structure oriented towards rural sustainability– provides the following preference information on the considered elementary criteria and on the macro-criteria (notice that each statement is translated into a constraint, which is written in round brackets):

- (i) **So** is more important than **En**, which, in turn, is more important than **Ec** ($\varphi_0(\text{So}) \geq \varphi_0(\text{En}) + \varepsilon$ and $\varphi_0(\text{En}) \geq \varphi_0(\text{Ec}) + \varepsilon$);
- (ii) at a comprehensive level, AA is more important than TRP ($\varphi_0(\text{AA}) \geq \varphi_0(\text{TRP}) + \varepsilon$);
- (iii) with respect to **Ec**, OU is more important than LS ($\varphi_1^2(\text{OU}) \geq \varphi_1^2(\text{LS}) + \varepsilon$);

- (iv) with respect to **So**, PSI is more important than CP ($\varphi_2^2(\text{PSI}) \geq \varphi_2^2(\text{CP}) + \varepsilon$);
- (v) at a comprehensive level, CP is more important than OH ($\varphi_0^2(\text{CP}) \geq \varphi_0^2(\text{OH}) + \varepsilon$);
- (vi) with respect to **En**, LS and OU are positively interacting ($\varphi_3^2(\text{LS}, \text{OU}) \geq \varepsilon$);
- (vii) BS and TPR are positively interacting ($\varphi_0^2(\text{BS}, \text{TPR}) \geq \varepsilon$);
- (viii) OH and CP are negatively interacting ($\varphi_0^2(\text{OH}, \text{CP}) \leq -\varepsilon$).

Before implementing the proposed methodology, we perform a normalization procedure, because applications of the Choquet integral always require that all evaluations are expressed on the same scale. Specifically, if criterion g_i has a decreasing direction of preference, then the following expression is used:

$$\overline{g}_i(a) = \frac{\max_i - g_i(a)}{\max_i - \min_i},$$

where $g_i(a)$ is the evaluation of a on criterion g_i , \max_i and \min_i are the maximum and the minimum evaluations of alternatives on g_i , and $\overline{g}_i(a)$ is the normalized value. On the other hand, whenever criterion g_i has an increasing direction of preference, the following dual formula is adopted:

$$\overline{g}_i(a) = \frac{g_i(a) - \min_i}{\max_i - \min_i}.$$

Notice that the normalization chosen for this application is the most used among the different methods available in the literature on sustainability [100].

Regrettably, the application of NAROR at the comprehensive level and at each macro-criterion (social, economic, and environmental) yields results that are insufficient to get a good insight into our decision problem. Indeed, beyond the relation of weak dominance, the application of NAROR only adds few new pairs to the necessary preference relations at the comprehensive level, as well as on the macro-criteria. Specifically, at the comprehensive level, NAROR provides the following additional necessary relations:

$$a_{35} \succsim^N a_{45}, \quad a_{44} \succsim^N a_{41}, \quad a_9 \succsim^N a_{39}, \quad a_{32} \succsim^N a_{39}, \quad a_{51} \succsim^N a_{39}, \quad a_{21} \succsim^N a_{39}, \quad a_{36} \succsim^N a_{39}, \quad a_{36} \succsim^N a_{15}.$$

Further, with respect to the macro-criterion **En**, NAROR only provides the additional necessary preference relation $a_{11} \succsim_3^N a_{15}$.

Thus, in order to get a more refined understanding of the case study under examination, we implement SMAA. To start, we compute the best and the worst position reachable from each municipality: to that end, we consider the whole set of capacities compatible with the preferences provided by the DM, as well as the three rank positions presenting the highest rank acceptability indices (hence the three most frequent positions taken by that municipality).

In order to compare municipalities, we shall give much attention to the reference ranking on **So**, since this macro-criterion is the most important according to the DM's evaluation (cf. preference information (i) provided by the DM). Specifically, we rank the municipalities with respect to their highest rank acceptability index placing. Therefore, for instance, in the first positions one finds those municipalities that present the highest rank acceptability index for the first positions: see Table 3(c), where we report the three highest rank acceptabilities indices, as well as the best and worst positions of the first seven and the last three municipalities in the considered ranking. For the sake of completeness, in Table 3(a)-3(b)-3(d), the same computations are performed at a comprehensive level, on macro-criterion **Ec**, and on macro-criterion **En**, respectively.

Let us first analyze Table 3(c), which, according to the importance given by the DM to macro-criterion **So** may be considered the most relevant of all four. As a matter of fact, we employ the

results obtained for **So** as a reference for the analysis of all other aspects. More specifically, we shall restrict our analysis to (i) the top seven alternatives w.r.t. **So**, and (ii) the last three alternatives w.r.t. **So**.

- (i) a_{25} (Milo) is surely the best municipality, since its rank acceptability index for the first position is equal to 100%. From a social point of view, also a_{20} (Maniace), a_{34} (Ragalna) and a_{44} (Sant'Alfio) appear quite good, since they have high rank acceptability indices for the first rank positions, and take always a position in the intervals $[2, 8]$, $[4, 9]$, and $[2, 7]$, respectively (see Table 3(c)). At the same time, even if a_{12} (Castel di Iudica) and a_{26} (Mineo) have high rank acceptabilities for the third and fourth places, respectively, their rank position varies in the intervals $[3, 13]$ for Castel di Iudica, and $[2, 15]$ for Mineo.
- (ii) The last three municipalities present the highest rank acceptability indices for the lowest positions. In particular, a_4 (Acireale) is very frequently in the 49th position; it is the last ranked municipality in almost one third of the cases, and its best position is the 43rd. Furthermore, a_{47} (Tremestieri Etneo) presents the highest rank acceptability index for the last but one place in the ranking (49.54%), and it may reach at its best the 41st position, even if with a negligible frequency. Finally, a_2 (Aci Catena) is the municipality presenting the highest rank acceptability for the last position (63.64%), and its second and third highest rank acceptability indices are for the 50th and the 49th positions. The best position it reaches is the 46th, even if the rank acceptability index for this position is basically zero (0.02%).

In what follows, we provide some comments on the rankings obtained in the other three cases, and compare the relative performances with those displayed in the reference rankings on macro-criterion **So**. We start with the rankings given in Table 3(b), computed according to macro-criterion **Ec**.

- (i) Differently from what happens with respect to macro-criterion **So**, a_{25} (Milo) appears to be quite unstable with respect to macro-criterion **Ec**: indeed, its highest rank acceptability index, corresponding to the 27th position, is very low (17.17%), and the interval of variation for its positions is quite large, ranging from the 18th to the 43rd place. Instead, a_{20} (Maniace) is quite good on economic aspects, since its three highest rank acceptability indices are those corresponding to the positions going from the 2nd to the 4th and it is always placed between the 1st and the 18th position. Furthermore, a_{12} (Castel di Iudica) is evaluated quite well on **Ec**, since it presents its three highest rank acceptability indices for the positions going from the 8th to the 10th; however, its range of positions is rather large, being within the interval $[6, 25]$. Finally, it is interesting that municipalities a_{34} (Raddusa) and a_{13} (Fiumefreddo di Sicilia), which showed good performances on **So**, always have their three highest rank acceptability indices for positions at the bottom of the ranking with respect to macro-criterion **Ec**.
- (ii) The last three municipalities on **So** – namely, a_4 , a_{47} , a_2 – improve their positions with respect to macro-criterion **Ec**, even if they are still placed at the bottom of the ranking. In particular, a_4 and a_{47} can reach a position in the first half of the ranking, but this happens with very marginal frequencies. Moreover, a_4 has a frequency of the 0.01% of being in the 6th position, whereas a_{47} has the 19th as its best rank-position, even if with an almost null frequency.

Analyzing the same municipalities with respect to macro-criterion **En** (see Table 3(d)), the following observations can be done.

- (i) a_{34} is the best among the first seven considered municipalities, insofar as it has the highest rank acceptability indices for the positions going from the 7th to the 9th. Moreover, this

municipality can fill the fourth place in the ranking even if with a negligible frequency, and its last position is the 18th again with a frequency very close to zero. The other two municipalities presenting quite stable results are a_{13} and a_{44} , since they have the highest rank acceptability for positions not lower than 12th, and they can never reach a position lower than the 20th.

- (ii) Among the three last municipalities – namely, a_4 , a_{47} , a_2 – only a_{47} improves its position, since its most frequent position is the 26th even if with a low rank acceptability index (10.46%). Moreover, even if a_4 and a_2 can reach the 22nd and the 18th as their highest positions, they are very frequently in the last part of the ranking. Indeed, a_4 presents its highest rank acceptability indices for positions 42nd, 41st and 43rd, whereas a_2 has the 44th, 43rd and 45th as the positions for which it has the highest rank acceptability indices.

Finally, examining Table 3(a), the following remarks can be done at a comprehensive level.

- (i) a_{20} and a_{25} can be considered the best municipalities, since they have the three highest rank acceptability indices for the first three positions. Moreover, a_{20} can be placed at the top of this ranking, because its worst rank-position is the 7th against the 13th of a_{25} . Furthermore, a_{12} can be ranked just after the first two, since its most frequent positions are the 4th, the 5th and the 3rd with frequencies 34.63%, 25.32% and 20.46%, respectively.
- (ii) The three last municipalities do not improve the position they had in the **So** ranking. Indeed, all of them present their highest rank acceptability indices for the lowest positions in the ranking. Moreover, a_2 takes the last position (51st) with the first highest rank acceptability equal to 76.33%, and the other two municipalities can reach the last position, too (in particular, a_4 with a frequency of 6.24%).

In order to get a ranking of the considered municipalities with respect to **Ec**, **So**, **En** and at the comprehensive level, we have calculated the *barycenter* of the Möbius representation of capacities compatible with the preferences provided by the DM. Their values are shown in Table 4.

From this table, one can notice that, without considering interaction between indicators, the first three more important elementary criteria are the ones relative to the macro-criterion **So**, i.e., **PSI**, **R** and **CP**, with $m(\{PSI\}) > m(\{R\}) > m(\{CP\})$. Some positive and negative interactions between criteria – apart from the preference information explicitly given by the DM – can be deduced from the same table: for instance, the negative and positive interaction, respectively, between **PSI** and **R** on one hand, and **LS** and **OU** on the other hand).

Computing the Choquet integral value for each municipality with the barycenter of the Möbius representation of all compatible capacities, we obtain four complete rankings of municipalities at the comprehensive level as well as at the macro-criteria level. In Tables 5(a), 5(b), 5(c) and 5(d), we show the places taken by the municipalities under consideration by using the capacity in terms of Möbius representation displayed in Table 4.

As expected, the position taken by a municipality depends on the criterion we are considering. For example, with respect to macro-criteria **Ec** and **En**, Mineo takes positions 27 and 24, respectively, whereas its position becomes the 12th at a comprehensive level. Similarly, Milo is respectively ranked 28th and 18th on macro-criteria **Ec** and **En**; instead, at a comprehensive level it takes the 2nd position.

The obtained results highlight the features of the employed approach, which provides insights into evaluation of sustainable development from two different perspectives: for each node of the hierarchy of the criteria, and at a global level. The interested reader can download the file containing complete results by clicking on the following link: supplementary data.

3.3. A comparison with other methods and merits of the proposed approach

Inspired by [89], in this section we shall provide a comparison between (i) the results obtained by the application of our approach (hereafter denoted by **HCh**), and (ii) the results obtained by applying three other methodologies, which are described below.

1. The *arithmetic mean (AM)*: In this case, all the elementary criteria are equally weighted.
2. The *EWAt operator (EWAt)*: In this case, the weight is equally distributed among the criteria, starting from the top of the hierarchy. Inside the same node, again, all criteria have exactly the same weight. Therefore, looking at the hierarchy of criteria considered in our case study, Economic, Social, and Environmental macro-criteria have all a weight of 1/3. This weight has to be equally distributed among the elementary criteria descending from **Ec**, as well as between the elementary criteria descending from **So**. Thus, these six elementary criteria have all a weight of 1/9. Analogously, the weight 1/3 of the environmental macro-criterion **En** has to be equally split among the four elementary criteria descending from it, assigning a weight of 1/12 to each of them.
3. The *Shapley integral (ShVa)* [67]: This is a weighted mean, where the weights assigned to the elementary criteria are the Shapley values of the same elementary criteria, that is

$$Sh_{\mu}(a) = \sum_{g_i \in G} \varphi(\{g_i\}) \cdot g_i(a). \quad (14)$$

The reader may consult [35, 101, 104] for different methods used to evaluate sustainability, as well as [30] for a paper where the methodologies 1–3 are presented in a more detailed way. Below we summarize the main differences between these three methodologies and our proposal.

- In methodologies 1–3, all criteria are put together to obtain a comprehensive evaluation of the alternatives at hand. On the other hand, **HCh** is able to give information not only at comprehensive level, but also considering some macro-criterion corresponding to a particular aspect of the problem (Economic, Social, or Environmental).
- **AM** and **EWAt** are unable to consider possible interactions between criteria. On the contrary, by using the Choquet integral to aggregate performances on criteria, **HCh** can naturally take into account these interactions.
- As observed in [68], considering a 2-additive measure, the Choquet integral $C_{\mu}(a)$ of an alternative a and the Shapley integral $Sh_{\mu}(a)$ of the same alternative are linked by the equality

$$C_{\mu}(a) = Sh_{\mu}(a) - \frac{1}{2} \sum_{\{g_i, g_j\} \subseteq G} m(\{g_i, g_j\}) |g_i(a) - g_j(a)|. \quad (15)$$

In fact, as explained in [68], eq. (15) says that the value assigned to a by the Choquet integral is composed of two parts: a linear component $Sh_{\mu}(a)$, and a non-linear one. The semantics of (15) is natural: the Choquet integral “corrects” the Shapley integral by taking into account the interactions between all possible pairs of criteria. Let us underline that, even if eq. (15) links the Choquet integral and the Shapley integral, they are conceptually different in the way they aggregate the evaluations of the alternatives on the criteria at hand. Indeed, the Shapley integral is a weighted sum where the weights are given by the Shapley values. Therefore, it does not take into account in an explicit way the possible interactions between criteria but they are only used to obtain the weights of the criteria. The Choquet integral, instead, takes explicitly into account the interactions between criteria in the aggregation phase.

To stress this point, we shall provide a small didactic example. Let us suppose that students a and b are evaluated on Mathematics (M) and Physics (P) as shown in Table 6. In general, M and P are negatively interacting since a student good in M is good in P too and, consequently, the importance of both subjects together should be lower than the sum of the importance of the two subjects alone. For this reason, let us suppose that the students can be evaluated considering the capacity in Table 7. Following eq. (3), we have that the Shapley values of the two criteria are the same: $\varphi(\{M\}) = \varphi(\{P\}) = 0.5$; aggregating the evaluations of the two students by using the Shapley integral and the Choquet integral we get the following:

– by applying eq. (14),

$$Sh_{\mu}(a) = 18 \cdot 0.5 + 30 \cdot 0.5 = (18 + 30) \cdot 0.5 = 24,$$

and

$$Sh_{\mu}(b) = 24 \cdot 0.5 + 24 \cdot 0.5 = 24,$$

– by applying eq. (1)

$$Ch_{\mu}(a) = 18 \cdot \mu(\{M, P\}) + 12 \cdot \mu(\{P\}) = 18 \cdot 1 + 12 \cdot 0.7 = 26.4$$

and

$$Ch_{\mu}(b) = 24 \cdot \mu(\{M, P\}) = 24 \cdot 1 = 24.$$

It is therefore evident that the two procedures are aggregating the evaluations in a different way. Indeed, the Shapley integral is reduced to an arithmetic mean that is not able to take into account the interaction between the two subjects and, consequently, makes indifferent a and b . The Choquet integral, instead, is able to consider this interaction and, therefore, a is preferred to b .

- In methodologies 1–3, only a single weight vector is used to get the final recommendations on the problem at hand. In HCh, we use instead several weight vectors compatible with the preferences provided by the DM. Each of these weight vectors represents a different point of view, which typically yields a more faithful modelization. At any rate, to make fair the comparison between our method and those taken into account, we shall compute the Choquet integral of each municipality at global level as well as at partial one considering the barycenter of the capacities compatible with the preferences provided by the DM. In this way, we can compare the single rankings obtained in correspondence of the analyzed methods.

In Tables 8-11, we reported the rank-positions taken by the municipalities in correspondence of the four considered methods (HCh, AM, EWAt, ShVa) at a comprehensive level as well as on the three considered macro-criteria.

In order to further compare the four methods, in Table 12 we reported the Kendall-Tau coefficient between each pair of methods not only at comprehensive level but also at the partial level. The following considerations are in order.

- The Kendall-Tau between the rankings obtained by means of HCh and ShVa are very high (not lower than 0.932). One can therefore wonder why using the Choquet integral instead of the Shapley integral. Anyway, as observed above, even if the Shapley integral has the form of a weighted sum, the weights of criteria are computed by considering the interactions between criteria. Nevertheless, we underlined that the Choquet integral takes into account the interactions between criteria in a more complete and explicit way and, consequently, some differences in the rankings obtained by the two methods can be observed (see, for example, the positions got by a_{10} and a_{51} , as shown in Table 8).

- The lowest Kendall-Tau coefficient at comprehensive level is obtained in correspondence of (HCh,AM) and (HCh,EWA_t). This confirms the difference between the Choquet integral on one hand, and AM and EWA_t on the other hand. (Notice that AM and EWA_t are the only two methods that, among the four considered, take into account neither importance nor interaction between criteria elicited from preference information provided by the DM.) This shows the importance of using the Choquet integral instead of AM and EWA_t whenever one wants to base the aggregation of elementary indicators on preferences of stakeholders and policy makers.

To conclude this section, let us summarize the merits of the proposed approach.

1. Aggregating the performances of the municipalities on the considered criteria by means of the Choquet integral permits to take into account not only the importance of the single aspects alone, but also the possible positive and negative interactions observable between them.
2. The use of indirect preference information involves less cognitive effort from the part of the DM, who is not obliged to provide exact values for the parameters of the model, but can provide preferences in a simpler way in terms of comparison between criteria or alternatives, as well as in terms of positive or negative interactions between criteria.
3. Robustness concerns are dealt by using the NAROR and SMAA methodologies. Indeed, while in classical sustainability studies recommendations are given by considering a single weight vector (in fact, in our application, interactive weight vectors expressed by capacities), in our approach we explore the plurality of weight vectors compatible with the indirect preference information provided by the DM. In this way, we are simultaneously taking into account several preferential attitudes represented by each one of the considered weight vectors, providing necessary and possible preferences (supplied by the NAROR) and probabilistic preferences (supplied by the SMAA). More precisely, on one hand, the necessary and possible preference relations given by NAROR display those preferences that are true for all or for at least one compatible weight vector; on the other hand, the SMAA computes the frequency with which a municipality fills a certain rank position, as well as the frequency with which a municipality is preferred or indifferent to another one.
4. A hierarchy of criteria provides a deeper analysis of the problem at hand, since the stakeholders can get information not only at comprehensive level (considering all aspects at once), but also at a partial level, thus considering each single aspect a time. From a sustainability point of view, this procedure allows stakeholders and policymakers to identify and evaluate the strong and weak points of each municipality: this provides important information on which efficient policies can be defined and implemented in order to reinforce strong points and reduce weak points.

4. Conclusions

We have proposed a methodology to construct composite indices, taking into account the following relevant points:

- interaction of elementary indicators;
- hierarchical organization of considered indicators;
- participation of experts, stakeholders, and policy makers to the construction of composite indices;
- consideration of robustness concerns, related to the stability of the results with respect to the variability of the weights assigned to considered indicators.

We have suggested to simultaneously handle these points by applying a multiple criteria decision analysis methodology, which has been recently proposed in literature: the hierarchical-SMAA-Choquet integral. This approach has the following features:

- aggregates the indicators by means of the Choquet integral preference model, which takes into account possible interactions between them;
- applies the Robust Ordinal Regression (ROR) and the Stochastic Multicriteria Acceptability Analysis (SMAA) to handle robustness concerns related to the consideration of the whole set of weights assigned to composite indicators and compatible with preferences expressed by experts, stakeholders and policy makers;
- adopts Multiple Criteria Hierarchy Process (MCHP) to aggregate indicators not only at the comprehensive level, but also at all the intermediate levels of the hierarchy.

We have discussed how this approach appears to be well suited for composite indices related to sustainable development assessment, and shown how it works on a case study related to rural sustainable development.

We believe that the methodology presented here possesses most of the features necessary to be applied to all domains in which composite indices are adopted or can be adopted, with a specific relevance to sustainable development. Consequently, we plan to apply this methodology to construct composite indices in different areas of interest. We envisage also to apply other MCDA methodologies to the same case study. In this respect, a comparison of the obtained results would highlight the different features of the employed methods.

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Table 3: The three highest rank acceptability indices and the best and worst positions for the municipalities ranked (a) at a comprehensive level, (b) according to macro-criterion **Ec**, (c) according to macro-criterion **So**, and (d) according to macro-criterion **En**.

(a) Comprehensive level

Municipality	$high_1$	$b_0^{high_1}(\cdot)$ (%)	$high_2$	$b_0^{high_2}(\cdot)$ (%)	$high_3$	$b_0^{high_3}(\cdot)$ (%)	Best	$b_0^{Best}(\cdot)$ (%)	Worst	$b_0^{Worst}(\cdot)$ (%)
a_{25}	2	46.44	1	32.41	3	14.36	1	32.41	13	0.00
a_{20}	1	58.01	2	36.58	3	4.83	1	58.01	7	0.00
a_{12}	4	34.63	5	25.32	3	20.46	2	0.37	14	0.00
a_{26}	12	18.04	11	17.47	10	14.48	4	0.07	27	0.00
a_{34}	8	18.70	10	18.19	9	16.31	3	0.01	20	0.00
a_{44}	5	18.13	4	15.22	7	13.35	1	0.05	27	0.00
a_{13}	13	21.42	14	18.46	15	14.52	5	0.00	32	0.00
...										...
a_4	49	25.26	48	23.54	50	15.20	42	0.00	51	6.24
a_{47}	46	25.26	47	19.25	48	15.90	29	0.00	51	0.10
a_2	51	76.33	50	19.32	49	3.93	46	0.00	51	76.33

(b) Economic (**Ec**)

Municipality	$high_1$	$b_1^{high_1}(\cdot)$ (%)	$high_2$	$b_1^{high_2}(\cdot)$ (%)	$high_3$	$b_1^{high_3}(\cdot)$ (%)	Best	$b_1^{Best}(\cdot)$ (%)	Worst	$b_1^{Worst}(\cdot)$ (%)
a_{25}	27	17.17	28	14.96	26	14.62	18	0.01	43	0.00
a_{20}	2	26.11	3	18.08	4	17.60	1	14.76	18	0.00
a_{12}	9	27.81	10	27.41	8	18.82	6	2.19	25	0.00
a_{26}	28	11.01	27	8.70	29	8.26	7	0.00	45	0.01
a_{34}	36	26.39	37	23.81	33	9.64	22	0.00	43	0.00
a_{44}	13	8.60	6	6.60	14	5.05	3	1.26	48	0.00
a_{13}	46	27.72	47	25.83	48	24.41	23	0.00	49	3.30
...										...
a_4	31	12.48	33	10.86	36	10.54	6	0.01	41	0.01
a_{47}	33	13.26	31	13.25	29	10.53	19	0.00	47	0.00
a_2	43	21.13	40	16.45	39	14.19	33	0.00	49	0.02

(c) Social (**So**)

Municipality	$high_1$	$b_2^{high_1}(\cdot)$ (%)	$high_2$	$b_2^{high_2}(\cdot)$ (%)	$high_3$	$b_2^{high_3}(\cdot)$ (%)	Best	$b_2^{Best}(\cdot)$ (%)	Worst	$b_2^{Worst}(\cdot)$ (%)
a_{25}	1	100.00					1	100.00	1	100.00
a_{20}	2	99.55	3	0.38	5	0.06	2	99.55	8	0.00
a_{12}	3	96.40	4	2.69	5	0.43	3	96.40	13	0.00
a_{26}	4	92.87	6	3.87	5	2.30	2	0.00	15	0.00
a_{34}	5	94.76	6	3.86	4	1.18	4	1.18	9	0.00
a_{44}	6	91.52	3	3.12	4	2.88	2	0.45	7	0.00
a_{13}	7	79.62	8	18.21	9	0.74	2	0.00	11	0.01
...										...
a_4	49	36.25	51	30.30	47	14.17	43	0.00	51	30.30
a_{47}	50	49.54	49	29.39	48	7.91	41	0.00	51	6.03
a_2	51	63.64	50	31.44	49	4.45	46	0.02	51	63.64

(d) Environmental (**En**)

Municipality	$high_1$	$b_3^{high_1}(\cdot)$ (%)	$high_2$	$b_3^{high_2}(\cdot)$ (%)	$high_3$	$b_3^{high_3}(\cdot)$ (%)	Best	$b_3^{Best}(\cdot)$ (%)	Worst	$b_3^{Worst}(\cdot)$ (%)
a_{25}	17	29.95	18	29.50	19	12.15	14	0.55	30	0.00
a_{20}	14	9.95	4	9.16	13	7.98	4	9.16	33	0.06
a_{12}	15	10.53	16	8.78	14	6.95	5	0.28	34	0.12
a_{26}	26	22.17	25	21.84	27	19.68	15	0.00	31	0.00
a_{34}	8	33.61	9	28.38	7	14.36	4	0.00	18	0.01
a_{44}	12	17.83	6	15.35	11	13.15	3	0.00	18	0.09
a_{13}	9	28.79	10	20.82	8	17.62	3	0.00	20	0.00
...										...
a_4	42	45.58	41	13.16	43	12.04	22	0.00	47	0.00
a_{47}	26	10.46	27	9.40	37	7.93	9	0.00	50	0.00
a_2	44	30.55	43	30.28	45	12.84	18	0.00	50	0.04

Table 4: Barycenter of the Möbius representations of compatible capacities

$m(\{OH\})$ 0.0871	$m(\{AA\})$ 0.1103	$m(\{BS\})$ 0.0869	$m(\{PSI\})$ 0.1508	$m(\{CP\})$ 0.1187	$m(\{R\})$ 0.1218	$m(\{TPR\})$ 0.0695	$m(\{IWU\})$ 0.0882	$m(\{LS\})$ 0.0649	$m(\{OU\})$ 0.0948	$m(\{OH, AA\})$ -0.0001
$m(\{OH, BS\})$ -0.0003	$m(\{OH, PSI\})$ -0.0002	$m(\{OH, CP\})$ -0.0163	$m(\{OH, R\})$ -0.0001	$m(\{OH, TPR\})$ 0.0004	$m(\{OH, IWU\})$ -0.0002	$m(\{OH, LS\})$ 0.0000	$m(\{OH, OU\})$ 0.0006	$m(\{AA, BS\})$ -0.0006	$m(\{AA, PSI\})$ -0.0044	$m(\{AA, CP\})$ 0.0006
$m(\{AA, R\})$ 0.0005	$m(\{AA, PR\})$ -0.0005	$m(\{AA, IWU\})$ -0.0007	$m(\{AA, LS\})$ 0.0000	$m(\{AA, OU\})$ 0.0005	$m(\{BS, PSI\})$ -0.0004	$m(\{BS, CP\})$ 0.0000	$m(\{BS, R\})$ -0.0014	$m(\{BS, TPR\})$ 0.0133	$m(\{BS, IWU\})$ -0.0005	$m(\{BS, LS\})$ -0.0003
$m(\{BS, OU\})$ -0.0001	$m(\{PSI, CP\})$ 0.0002	$m(\{PSI, R\})$ -0.0008	$m(\{PSI, TPR\})$ -0.0010	$m(\{PSI, IWU\})$ 0.0011	$m(\{PSI, LS\})$ 0.0005	$m(\{PSI, OU\})$ 0.0010	$m(\{CP, R\})$ -0.0001	$m(\{CP, TPR\})$ 0.0003	$m(\{CP, IWU\})$ -0.0002	$m(\{CP, LS\})$ 0.0001
$m(\{CP, OU\})$ 0.0003	$m(\{R, TPR\})$ -0.0002	$m(\{R, IWU\})$ 0.0004	$m(\{R, LS\})$ -0.0002	$m(\{R, OU\})$ -0.0003	$m(\{TPR, IWU\})$ 0.0001	$m(\{TPR, LS\})$ 0.0000	$m(\{TPR, OU\})$ -0.0002	$m(\{IWU, LS\})$ 0.0004	$m(\{IWU, OU\})$ 0.0006	$m(\{LS, OU\})$ 0.0144

Table 5: Positions taken by the municipalities in the rankings obtained by using the Möbius representation shown in Table 4 and corresponding Choquet value

(a) Comprehensive level		(b) Economic (Ec)	
Municipality	Position (Choquet value)	Municipality	Position (Choquet value)
Milo	2 (0.64)	Milo	28 (0.27)
Maniace	1 (0.66)	Maniace	3 (0.54)
Castel di Iudica	4 (0.58)	Castel di Iudica	8 (0.39)
Mineo	12 (0.53)	Mineo	27 (0.27)
Ragalna	10 (0.54)	Ragalna	36 (0.22)
Sant'Alfio	5 (0.56)	Sant'Alfio	14 (0.32)
Castiglione di Sicilia	13 (0.51)	Castiglione di Sicilia	47 (0.16)
...
Acireale	49 (0.26)	Acireale	32 (0.24)
Tremestieri Etneo	46 (0.30)	Tremestieri Etneo	31 (0.24)
Aci Catena	51 (0.22)	Aci Catena	41 (0.19)
(c) Social (So)		(d) Environmental (En)	
Municipality	Position (Choquet value)	Municipality	Position (Choquet value)
Milo	1 (1.00)	Milo	18 (0.55)
Maniace	2 (0.82)	Maniace	13 (0.57)
Castel di Iudica	3 (0.76)	Castel di Iudica	16 (0.56)
Mineo	4 (0.75)	Mineo	24 (0.50)
Ragalna	5 (0.73)	Ragalna	8 (0.60)
Sant'Alfio	6 (0.71)	Sant'Alfio	11 (0.59)
Castiglione di Sicilia	7 (0.68)	Castiglione di Sicilia	9 (0.60)
...
Acireale	49 (0.21)	Acireale	42 (0.34)
Tremestieri Etneo	50 (0.20)	Tremestieri Etneo	33 (0.46)
Aci Catena	51 (0.16)	Aci Catena	43 (0.30)

Table 6: Evaluations of the students on the two subjects

	M	P
a	18	30
b	24	24

Table 7: Capacity used to evaluate the two students

	$\{M\}$	$\{P\}$	$\{M, P\}$
μ	0.7	0.7	1

Table 8: Comparison between the four methods: Positions taken by each municipality at comprehensive level

Municipality	HCh-AM-EWAt-ShVa	Municipality	HCh-AM-EWAt-ShVa	Municipality	HCh-AM-EWAt-ShVa
a_1	48-50-49-49	a_{18}	15-18-20-16	a_{35}	9-6-5-8
a_2	51-51-51-51	a_{19}	28-31-35-28	a_{36}	3-2-2-3
a_3	40-39-40-40	a_{20}	1-1-1-1	a_{37}	35-36-32-35
a_4	49-48-48-48	a_{21}	30-34-33-31	a_{38}	14-13-13-14
a_5	34-29-31-34	a_{22}	47-47-47-47	a_{39}	50-49-50-50
a_6	27-25-25-27	a_{23}	8-4-4-6	a_{40}	15-19-19-19
a_7	24-17-21-22	a_{24}	21-21-18-21	a_{41}	41-41-42-41
a_8	20-22-24-20	a_{25}	2-3-3-2	a_{42}	19-16-15-17
a_9	29-32-29-30	a_{26}	12-12-12-12	a_{43}	38-40-39-39
a_{10}	31-27-30-29	a_{27}	32-33-34-33	a_{44}	5-7-7-5
a_{11}	25-28-27-25	a_{28}	26-26-26-26	a_{45}	43-42-41-43
a_{12}	4-5-6-4	a_{29}	37-37-36-37	a_{46}	23-23-22-23
a_{13}	13-14-16-13	a_{30}	39-35-37-38	a_{47}	46-46-46-46
a_{14}	42-43-43-42	a_{31}	7-8-10-9	a_{48}	45-45-45-45
a_{15}	44-44-44-44	a_{32}	18-20-17-18	a_{49}	38-38-38-36
a_{16}	33-30-28-32	a_{33}	16-15-14-15	a_{50}	6-10-9-7
a_{17}	11-9-8-11	a_{34}	10-11-11-10	a_{51}	22-24-23-24

Table 9: Comparison between the four methods: Positions taken by each municipality on Economic aspects

Municipality	HCh-AM-EWAt-ShVa	Municipality	HCh-AM-EWAt-ShVa	Municipality	HCh-AM-EWAt-ShVa
a_1	49-49-49-49	a_{18}	48-48-48-48	a_{35}	1-1-1-1
a_2	41-43-43-41	a_{19}	51-51-51-51	a_{36}	2-2-2-2
a_3	35-34-34-35	a_{20}	3-5-5-3	a_{37}	5-3-3-4
a_4	32-31-31-31	a_{21}	29-27-27-25	a_{38}	9-11-11-9
a_5	26-23-23-27	a_{22}	20-25-25-20	a_{39}	44-44-44-44
a_6	16-15-15-16	a_{23}	4-4-4-5	a_{40}	37-37-37-38
a_7	21-24-24-21	a_{24}	10-8-8-10	a_{41}	46-45-45-46
a_8	38-39-39-37	a_{25}	28-28-28-29	a_{42}	25-21-21-26
a_9	17-17-17-17	a_{26}	27-29-29-28	a_{43}	45-46-46-45
a_{10}	33-33-33-33	a_{27}	42-42-42-42	a_{44}	14-13-13-13
a_{11}	50-50-50-50	a_{28}	39-38-38-39	a_{45}	12-12-12-12
a_{12}	8-9-9-8	a_{29}	6-6-6-6	a_{46}	19-16-16-19
a_{13}	47-47-47-47	a_{30}	7-7-7-7	a_{47}	31-32-32-32
a_{14}	30-30-30-30	a_{31}	18-22-22-18	a_{48}	43-41-41-43
a_{15}	34-35-35-34	a_{32}	15-14-14-15	a_{49}	40-40-40-40
a_{16}	22-20-20-23	a_{33}	11-10-10-11	a_{50}	13-18-18-14
a_{17}	23-26-26-22	a_{34}	36-36-36-36	a_{51}	24-19-19-24

Table 10: Comparison between the four methods: Positions taken by each municipality on Social aspects

Municipality	HCh-AM-EWAt-ShVa	Municipality	HCh-AM-EWAt-ShVa	Municipality	HCh-AM-EWAt-ShVa
a_1	42-42-42-42	a_{18}	16-15-15-16	a_{35}	19-20-20-20
a_2	51-51-51-51	a_{19}	13-13-13-13	a_{36}	15-17-17-15
a_3	41-41-41-40	a_{20}	2-2-2-2	a_{37}	39-40-40-39
a_4	49-49-49-49	a_{21}	23-24-24-23	a_{38}	20-19-19-19
a_5	44-44-44-44	a_{22}	48-48-48-48	a_{39}	47-47-47-47
a_6	31-32-32-31	a_{23}	18-16-16-18	a_{40}	11-11-11-12
a_7	40-38-38-41	a_{24}	26-26-26-26	a_{41}	35-35-35-35
a_8	28-28-28-29	a_{25}	1-1-1-1	a_{42}	12-12-12-11
a_9	22-22-22-22	a_{26}	4-4-4-4	a_{43}	21-21-21-21
a_{10}	43-43-43-43	a_{27}	24-23-23-24	a_{44}	6-6-6-6
a_{11}	14-18-18-14	a_{28}	25-25-25-25	a_{45}	37-37-37-37
a_{12}	3-3-3-3	a_{29}	33-33-33-34	a_{46}	29-29-29-28
a_{13}	7-7-7-7	a_{30}	46-46-46-46	a_{47}	50-50-50-50
a_{14}	34-34-34-33	a_{31}	36-36-36-36	a_{48}	38-39-39-38
a_{15}	45-45-45-45	a_{32}	10-10-10-10	a_{49}	30-30-30-30
a_{16}	32-31-31-32	a_{33}	17-14-14-17	a_{50}	8-8-8-8
a_{17}	9-9-9-9	a_{34}	5-5-5-5	a_{51}	27-27-27-27

Table 11: Comparison between the four methods: Positions taken by each municipality on Environmental aspects

Municipality	HCh-AM-EWAt-ShVa	Municipality	HCh-AM-EWAt-ShVa	Municipality	HCh-AM-EWAt-ShVa
a_1	51-51-51-51	a_{18}	2-4-4-2	a_{35}	34-34-34-34
a_2	43-44-44-43	a_{19}	20-22-22-20	a_{36}	6-6-6-6
a_3	35-35-35-35	a_{20}	13-17-17-14	a_{37}	47-47-47-47
a_4	42-42-42-42	a_{21}	38-38-38-38	a_{38}	26-26-26-26
a_5	12-8-8-12	a_{22}	44-43-43-44	a_{39}	50-50-50-50
a_6	23-23-23-23	a_{23}	19-14-14-19	a_{40}	17-16-16-17
a_7	5-2-2-3	a_{24}	32-32-32-32	a_{41}	37-37-37-37
a_8	4-5-5-4	a_{25}	18-18-18-18	a_{42}	25-24-24-24
a_9	39-39-39-39	a_{26}	24-25-25-25	a_{43}	40-41-41-41
a_{10}	10-7-7-10	a_{27}	27-30-30-27	a_{44}	11-12-12-11
a_{11}	14-15-15-13	a_{28}	15-13-13-15	a_{45}	49-49-49-49
a_{12}	16-19-19-16	a_{29}	46-46-46-46	a_{46}	22-20-20-21
a_{13}	9-10-10-9	a_{30}	31-28-28-31	a_{47}	33-33-33-33
a_{14}	45-45-45-45	a_{31}	1-1-1-1	a_{48}	48-48-48-48
a_{15}	41-40-40-40	a_{32}	29-29-29-29	a_{49}	36-36-36-36
a_{16}	30-27-27-30	a_{33}	28-31-31-28	a_{50}	3-11-11-7
a_{17}	7-3-3-5	a_{34}	8-9-9-8	a_{51}	21-21-21-22

Table 12: Kendall-Tau of the rankings obtained by the four methods at global level as well as on the three considered macro-criteria

	<i>Comprehensive</i>				<i>Economic</i>				<i>Social</i>				<i>Environmental</i>								
	HCh	AM	EWAt	ShVa	HCh	AM	EWAt	ShVa	HCh	AM	EWAt	ShVa	HCh	AM	EWAt	ShVa					
HCh	1	0.92	0.912	0.976	HCh	1	0.938	0.938	0.985	HCh	1	0.976	0.979	0.992	HCh	1	0.93	0.932	0.984		
AM		1	0.948	0.940	AM		1	0.940	AM		1	0.974	AM		1	1	0.984	AM		0.984	
EWAt			1	0.932	EWAt			1	0.940	EWAt			1	0.974	EWAt			1	0.948	EWAt	
ShVa				1	ShVa				1	ShVa				1	ShVa				1	ShVa	