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Robust synchronisation of chaotic systems with randomly occurring uncertainties via stochastic sampled-data control

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This article investigates the robust synchronisation problem for uncertain nonlinear chaotic systems. The norm-bounded uncertainties enter into the chaotic systems in random ways, and such randomly occurring uncertainties (ROUs) obey certain Bernoulli distributed white noise sequences. For this synchronisation problem, the sampled-data controller that has randomly varying sampling intervals is considered. In order to fully use the sawtooth structure characteristic of the sampling input delay, a discontinuous Lyapunov functional is proposed based on the extended Wirtinger inequality. By the Lyapunov stability theory and the linear matrix inequality (LMI) framework, the existence condition for the sample-date controller that guarantees the robust mean-square synchronisation of chaotic systems is derived in terms of LMIs. Finally, in order to show the effectiveness of our result, the proposed method is applied to two numerical examples: one is Chua's chaotic systems and the other is the hyperchaotic Rössler system.

Keywords: nonlinear chaotic systems; synchronisation; randomly occurring uncertainties; variable sampling; sampled-data control

1. Introduction

Since the concept of synchronisation was introduced by Pecora and Carroll (1990), chaos synchronisation has been a very hot topic in the nonlinearity community, and has attracted much interest of scientists and engineers due to its potential applications in biology, chemistry, engineering, secure communication and some other nonlinear fields. It should be pointed out that chaotic systems have complex behaviours that possess some special features, such as being extremely sensitive to tiny variations of initial conditions, having bounded trajectories in phase space and so on. To date, various control methods have been applied theoretically and experimentally to the synchronisation of chaotic systems, such as the adaptive feedback control (Xiao and Cao 2009), delayed feedback control (Cao, Li, and Ho 2005), observer-based control (Celikovsky and Chen 2005), backstepping control (Park 2006), impulsive control (Cao, Ho, and Yang 2009; He, Qian, Cao, and Han 2011), $\mathcal{H}_\infty$ control (Lin and Kuo 2011), sliding mode control (Yau 2008), fuzzy control (Lam, Ling, Lu, and Ling 2008), sampled-data control (Lu and Hill 2008) and so on. In recent years, among various control methods, the importance of the sampled-data control scheme has been increasing as the digital hardware and communication technologies are rapidly developing. As most of the controllers are digital controller or networked to the system, these control systems can be modelled by sampled-data systems, whose control signals are kept constant during the sampling period and are allowed to change only at the sampling instant. These discontinuous control signals that have stepwise form cause big trouble to control or analyse the system. In order to effectively deal with the sampled-data control, Astrom and Wittenmark (1989) and Mikheev, Sobolev, and Fridman (1988) introduced a concept that discontinuous-sampled control inputs treat time-varying delayed continuous signals, although applied actual control signals are discontinuous. Since then, the sampled-data control scheme based on the concept of Astrom and Wittenmark (1989) and Mikheev et al. (1988) have been applied to many dynamic systems, such as the complex dynamical networks (Li, Zhang, Hu, and Nie 2011), fuzzy systems (Peng, Han, Yue, and Tian 2011), neural networks (Zhang, He, and Wu 2010) and so on. However, there are only a few works for chaotic systems using the sampled-data control approach (Lu and Hill 2008).

Since the above concept is introduced by Astrom and Wittenmark (1989) and Mikheev et al. (1988), the sampled-data control system can be dealt with the

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time-delayed system. Untill now, so many techniques have been developed to analyse the time-delayed system because time-delay is one of the important facts that causes instability. For example, synchronisation of chaotic neural networks with time-varying delay is investigated in Balasubramaniam, Chandran, and Theesar (2011). In Balasubramaniam, Lakshmanan, and Theesar (2010), state estimation for Markovian jumping recurrent neural networks is studied by considering interval time-varying delays. In Zheng, Zhou, and Wang (2012), stochastic exponential synchronisation of neural networks with mixed delays is discussed. The exponential synchronisation scheme is applied to reaction–diffusion networks with mixed delays using intermittent driving given in Hu, Yu, Jiang, and Teng (2012a).

In sampled-data control systems, selecting proper sampling interval is very important to design suitable controllers. By defining the sampling interval, there are several kinds of sampled-data control methods. When all signals in a given system are sampled at one constant rate, we speak of a single-rate digital control system; when different signals in a given system are sampled at different but constant rates, we speak of a multi-rate digital control system; and when for all signals in a system that are sampled, the sampling rate for each signal is the same, but may be varying from sample to sample, we speak of a digital control system with time-varying sampling intervals. Traditionally, many researches focus more on single-rate digital control systems, however, the time-varying sampling is applied to several practical systems because of its usefulness in recent years. For example, in networked control systems, if a constant sampling period is adopted, the sampling period should be large enough to avoid network congestion when the network is occupied by most users, so network bandwidth cannot be sufficiently used when the network is idle. Therefore, the necessity of the controller with varying sampling interval has strongly come into fore. These have motivated the study of variable sampling and there are a number of papers considering the problem of varying sampling period of control systems. In Tahara, Fujii, and Yokoyama (2007), the variable sampling deadbeat control method for the megawatt (MW) class pulse width modulation (PWM) inverter system is used to overcome the poor control performance that comes from the limitation of hardware. In kilowatt (kW) class PWM inverter system, the output voltage can accurately track the reference voltage under the condition that the carrier frequency is limited to 2 kHz. However, in the MW class PWM inverter system, the carrier frequency cannot be too high due to the performance of switching device. In Ozdemir and Townley (2003), the sampled-data integral control for a large class of infinite-dimensional systems was proposed using a convergent adaptive sampling. In Hu and Michel (2000) and Sala (2005), the stability problem of digital feedback control systems with time-varying sampling periods is discussed. The problem of stochastic stability for networked control systems with both network-induced delay and transmitted data dropout using the time-varying sampling period method, where the number of data packet dropout is driven by a finite state Markov chain, is investigated in Li, Zhang, and Jing (2009). More recently, stochastically-varying sampling intervals are considered, which are said to be the further extended scheme to the case of time-varying sampling intervals. In Gao, Meng, and Chen (2008), mean square stability of the networked control system with stochastically-varying network-induced delay is studied. In Gao, Wu, and Shi (2009), an $\mathcal{H}_\infty$ control for sampled-data control system with probabilistic sampling is also investigated.

On the other hand, in the real-world situation, parameter uncertainties are unavoidable mainly due to the modelling inaccuracies, variations of the operating point, aging of the devices, etc. Therefore, the issue of robustness analysis has been taken into account in all sorts of systems by many researchers (Mohammad-Hoseini, Farrokhi, and Koshkouei 2008; Yang and Ye 2009; Balasubramaniam et al. 2011; Li, Duan, Xie, and Liu 2012; Balasubramaniam, Vembarasan, and Rakkiyappan 2012; Faydasicok and Arik 2012). Very recently, Hu et al. (2012b) have proposed a new type of uncertainties named as randomly occurring uncertainties (ROUs) due to the fact that the uncertainties may be subject to random changes in environmental circumstances, for instance, repairs of components and sudden environmental disturbances, and thus the uncertainties may occur in a probabilistic way with certain types and intensity. To the best of our knowledge, no related results have been established for synchronisation of nonlinear chaotic systems with ROUs.

In this article, based on a common Lyapunov functional (Yue, Han, and Peng 2004; Naghshtabrizi and Hespanha 2005), we propose a discontinuous Lyapunov functional approach to achieve asymptotic robust synchronisation of uncertain chaotic systems using sampled-data control with stochastically-varying sampling intervals, whose occurrence probabilities are given constants and satisfy the Bernoulli distribution. In order to use the discontinuous Lyapunov functional approach, stochastic variables ($\rho_{ij}(t)$) are defined. The discontinuous Lyapunov functional makes full use of the sawtooth structure characteristic of sampling input delays and thus get less conservative synchronisation criterion for the system. Furthermore, the parameter uncertainties that are time-varying norm bounded and randomly occurred are considered for reality. The derived sufficient condition for the stability is formulated by a linear matrix inequality (LMI) that is easily solvable using various numerical convex optimisation algorithms (Boyd, Ghaoui, Feron, and Balakrishnan 1994). Two numerical examples are included to show the effectiveness of our result. One is a well-known chaotic system and the other is a hyperchaotic system.

Notations: $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ denotes the set of an $m \times n$ real matrix. $X > 0$ (respectively, $X \geq 0$) means that the matrix X is a real symmetric positive definite matrix (respectively, positive semi-definite). I denotes the identity matrix. $0_{m\times n}$ denotes an $m \times n$ zero matrix. $\Phi(i,j)$ denotes the ith row, j-th column element (or block matrix) of matrix Φ. $\star$ in a matrix represents the elements below the main diagonal of a symmetric matrix. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$, respectively, mean the expectation of the stochastic variable x and the expectation of the stochastic variable x conditional on the stochastic variable y. $Pr\{\alpha\}$ means the occurrence probability of the event α. $\{\{A\} \cap \{B\}\}$ means the intersection between set A and B.

2. Problem formulation

Consider the following master (drive) and slave (response) chaotic systems with ROUs:

$$\dot{x}(t) = (A + \delta(t)\Delta A(t))x(t) + (B + \delta(t)\Delta B(t))f(x(t)), \tag{1}$$

$$\dot{y}(t) = (A + \delta(t)\Delta A(t))y(t) + (B + \delta(t)\Delta B(t))f(y(t)) + u(t) \tag{2}$$

where $x(t) = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ and $y(t) = (y_1, y_2, \ldots, y_n)^T \in \mathbb{R}^n$ are state vectors of master and slave systems, respectively, A and B are system matrices with appropriate dimensions, $u(t) = (u_1, u_2, \ldots, u_n)^T \in \mathbb{R}^n$ is the control input and f: $\mathbb{R}^n \to \mathbb{R}^n$ is a continuous nonlinear vector function satisfying the global Lipschitz condition, i.e.

$$\|f(a) - f(b)\| \leq l\,\|a - b\| \quad \forall a, b \in \mathbb{R}^n \tag{3}$$

for a positive scalar l. And, $\Delta A(t)$ and $\Delta B(t)$ are the uncertainties of system matrices of the form

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = DF(t)\begin{bmatrix} E_a & E_b \end{bmatrix} \tag{4}$$

in which D, E_a, E_b are known constant matrices and the time-varying nonlinear function $F(t)$ satisfies

$$F^T(t)F(t) \leq I, \quad \forall t \geq 0. \tag{5}$$

For our synchronisation scheme, let us define an error vector as follows:

$$e(t) = y(t) - x(t). \tag{6}$$

From Equation (6), the error dynamics is given as

$$\dot{e}(t) = (A + \delta(t)\Delta A(t))e(t) + (B + \delta(t)\Delta B(t))\bar{f}(e(t)) + u(t), \tag{7}$$

where $\bar{f}(e(t)) = f(y(t)) - f(x(t))$.

To account for the phenomena of ROUs, we introduce the stochastic variables $\delta(t)$, which are Bernoulli distributed white sequences. Natural assumptions of $\delta(t)$ are as follows:

$$Pr\{\delta(t) = 1\} = \delta, \quad Pr\{\delta(t) = 0\} = 1 - \delta,$$

where $\delta \in [0, 1]$ is a known constant.

Remark 1: Here, there is big probability of the existence of errors between theoretical and practical systems because of unexpected factors, such as sensing error, modelling error, parameter aging and channel strength. Therefore, in order to overcome this phenomenon, it is natural to assume system uncertainties. In addition, the system uncertainties may be the random changes in environmental circumstances, for example, network-induced random failures and repairs of components and sudden environmental disturbances. So, to consider ROUs is worth for reality.

Remark 2: Originally, Bernoulli distributed variables are used to model the presence of the random nonlinearity that mimics the packet dropping scenario in networked world. After introducing the Bernoulli distributed variable to engineering, it has been applied to express many scenarios such as probabilistic actuators or sensors fault (Tian, Yue, and Peng 2010) and random delays (Peng, Yue, Tian, and Gu 2009; Yue, Tian, Wang, and Lam 2009; Hu et al. 2012c). Very recently, Bernoulli distributed variables have been widely used in the concept of random occurring that have various types, such as randomly occurring nonlinearities, randomly occurring delays, randomly occurring sensors saturations (Ding, Wang, Shen, and Shu 2012; Hu et al. 2012b).

Remark 3: The random variable $\delta(t)$ that satisfies $\mathbb{E}\{\delta(t)\} = \delta$ and $\mathbb{E}\{(\delta(t) - \delta)^2\} = \delta(1 - \delta)$, is used to model the probability distribution of the ROUs. Such a description originated from Hu et al. (2012) has never been considered in control problems of chaotic systems.

Then, Equation (7) can be rewritten as

$$\begin{aligned} \dot{e}(t) &= Ae(t) + B\bar{f}(e(t)) + u(t) + \delta(t)Dp(t), \\ p(t) &= F(t)q(t), \\ q(t) &= E_a e(t) + E_b\bar{f}(e(t)). \end{aligned} \tag{8}$$

It should be noted that nowadays, most of the controllers are digital controller or networked to the system. These control systems can be modelled by sampled-data control systems. So the sampled-data control approach is eligible to receive much attention. Hence, this article investigates a design for feedback controller using the sampled-data signal with stochastic sampling such that $\lim_{t\to\infty} y(t) = x(t)$. To do this, the controller takes the following form:

$$u(t) = Ke(t_k), \quad t_k \le t < t_{k+1}, \; k = 0, 1, 2, \ldots, \tag{9}$$

where K is the gain matrix of the feedback controller to be determined later and t_k is the updating instant time of the zero-order-hold (ZOH).

We introduce a stochastic variable $d(t)$ as stochastically-varying sampling interval such that $t_{k+1} - t_k = d(t)$ satisfies the Bernoulli distribution as well, and two stochastic variables, $d(t)$ and $\delta(t)$, are mutually independent. Therefore, it is assumed that we have m sampling intervals, taking values $d_0, d_1, \ldots, d_m$ with $0 = d_0 < d_1 < \cdots < d_m$, and the probability of the occurrence of each is

$$Pr\{d(t) = d_i\} = \beta_i, \quad i = 1, 2, \ldots, m, \tag{10}$$

where $\beta_i \in [0, 1]$ are known constants and $\sum_{i=1}^{m} \beta_i = 1$.

In order to design the controller using sampled-data with stochastic sampling, the concept of the time-varying delayed control input which is proposed in Astrom and Wittenmark (1989) and Mikheev et al. (1988) is adopted in this article. Thus, by defining $\tau(t) = t - t_k$, $t_k \le t < t_{k+1}$, the controller (9) can be represented as the following:

$$\begin{aligned} u(t) &= Ke(t_k) \\ &= Ke(t - \tau(t)), \quad t_k \le t \le t_{k+1}. \end{aligned} \tag{11}$$

Here, the time-varying delay $\tau(t)$ satisfies $\dot{\tau}(t) = 1$ and the following probability rule:

- If sampling interval is d_1, then $Pr\{0 \le \tau(t) < d_1\} = 1$
- If sampling interval is d_2, then

$$\begin{cases} Pr\{0 \le \tau(t) < d_1\} = \dfrac{d_1}{d_2} \\ Pr\{d_1 \le \tau(t) < d_2\} = \dfrac{d_2 - d_1}{d_2} \end{cases}$$

$$\vdots$$

- If sampling interval is d_m, then

$$\begin{cases} Pr\{0 \le \tau(t) < d_1\} = \dfrac{d_1}{d_m} \\ Pr\{d_1 \le \tau(t) < d_2\} = \dfrac{d_2 - d_1}{d_m} \\ \qquad \vdots \\ Pr\{d_{m-1} \le \tau(t) < d_m\} = \dfrac{d_m - d_{m-1}}{d_m}. \end{cases} \tag{12}$$

Now, we define the stochastic variables $\alpha_i(t)$ and $\beta_i(t)$ such that

$$\left.\begin{aligned} \alpha_i(t) &= \begin{cases} 1 & d_{i-1} \le \tau(t) < d_i \\ 0 & \text{otherwise} \end{cases} \\ \beta_i(t) &= \begin{cases} 1 & d(t) = d_i \\ 0 & \text{otherwise} \end{cases} \end{aligned}\right., \quad i = 1, \ldots, m \tag{13}$$

with the following probability:

$$\begin{aligned} Pr\{\alpha_i(t) = 1\} &= Pr\{d_{i-1} \le \tau(t) < d_i\} \\ &= \sum_{j=i}^{m} \beta_j \frac{d_i - d_{i-1}}{d_j} = \alpha_i, \end{aligned} \tag{14}$$

$$Pr\{\beta_i(t) = 1\} = Pr\{d(t) = d_i\} = \beta_i, \tag{15}$$

where $i = 1, \ldots, m$, $\sum_{i=1}^{m} \alpha_i = 1$.

It is noted that, stochastic variables $\alpha_i(t)$ and $\beta_i(t)$ satisfy the Bernoulli distribution as well, so we have

$$\begin{aligned} &\mathbb{E}\{\alpha_i(t)\} = \alpha_i, \;\; \mathbb{E}\{(\alpha_i(t) - \alpha_i)^2\} = \alpha_i(1 - \alpha_i) \\ &\mathbb{E}\{\beta_i(t)\} = \beta_i, \;\; \mathbb{E}\{(\beta_i(t) - \beta_i)^2\} = \beta_i(1 - \beta_i) \end{aligned}, \quad i = 1, \ldots, m. \tag{16}$$

Thus the system (8) with m sampling intervals can be expressed as

$$\begin{aligned} &\dot{e}(t) = Ae(t) + B\bar{f}(e(t)) + \sum_{i=1}^{m} \alpha_i(t) Ke(t - \tau_i(t)) + \delta(t) Dp(t), \\ &p(t) = F(t) q(t), \\ &q(t) = E_a e(t) + E_b \bar{f}(e(t)), \end{aligned} \tag{17}$$

where $d_{i-1} \le \tau_i(t) < d_i$.

Definition 2.1 (Gao et al. 2008)**:** The error system (8) is said to be mean square stable if for any $\varepsilon > 0$, there is a $\rho(\varepsilon) > 0$ such that $\mathbb{E}\{\|e(t)\|^2\} < \varepsilon$, $t > 0$, when $\mathbb{E}\{\|e(0)\|^2\} < \rho(\varepsilon)$. In addition, if $\lim_{t\to\infty} \mathbb{E}\{\|e(t)\|^2\} = 0$, for any initial conditions, then the error system (8) is said to be globally mean square asymptotically stable.

Remark 4: The time-varying delay $\tau_i(t)$ in Equation (17) are independent on the stochastic interval. So, by introducing stochastic variables $\alpha_i(t)$, we can remodel system (8)–(17) which is general time-varying delay system. Here, the probability of $\alpha_i(t)$ is indicated in Equation (14), which is originated from Gao et al. (2009). On the other hand, the stochastic sampling interval $d(t)$ can be expressed to stochastic variables $\beta_i(t)$ which are first introduced by Gao et al. (2008), i.e. $d(t) = \sum_{i=1}^{m} \beta_i(t) d_i$.

Remark 5: It should be pointed out that, m stochastic variables $\alpha_i(t)$ and $\beta_i(t)$ are mutually dependent, respectively, but every intersection set of $\alpha_i(t)$ and $\beta_i(t)$, respectively, are zero, i.e.

$$\begin{aligned} &Pr\{\{\alpha_i(t)=1\}\cap\{\alpha_j(t)=1\}\}=0 \\ &Pr\{\{\beta_i(t)=1\}\cap\{\beta_j(t)=1\}\}=0 \end{aligned}, \quad \forall i,j=1,\dots,m,\ i\neq j. \tag{18}$$

3. Main results

In this section, a design problem of the sampled-data feedback controller with stochastic sampling intervals for the synchronisation of chaotic systems will be investigated via a discontinuous Lyapunov functional approach. Before proceeding further, the following lemmas and fact are given.

Lemma 3.1 (Gu, Kharitonov, and Chen 2003)**:** *For any matrix $M>0$, scalars γ_1 and γ_2 satisfying $\gamma_2>\gamma_1$, a vector function x: $[\gamma_1,\gamma_2]\to\mathbb{R}^n$ such that the integrations concerned are well defined, then*

$$\left(\int_{\gamma_1}^{\gamma_2} x(s)\mathrm{d}s\right)^T M\left(\int_{\gamma_1}^{\gamma_2} x(s)\mathrm{d}s\right) \le (\gamma_2-\gamma_1)\int_{\gamma_1}^{\gamma_2} x^T(s)Mx(s)\mathrm{d}s. \tag{19}$$

Lemma 3.2 (Liu, Suplin, and Fridman 2011)**:** *Let $x(t)\in W[a,b)$ and $x(a)=0$. Then for any matrix $R>0$ the following inequality holds*:

$$\int_a^b x(s)^T R x(s)\mathrm{d}s \le \frac{4(b-a)^2}{\pi^2}\int_a^b \dot{x}(s)^T R\dot{x}(s)\mathrm{d}s. \tag{20}$$

Fact 1 (Schur complements)**:** Given constant symmetric matrices Σ_1, Σ_2, Σ_3 where $\Sigma_1=\Sigma_1^T$ and $0<\Sigma_2=\Sigma_2^T$, then $\Sigma_1+\Sigma_3^T\Sigma_2^{-1}\Sigma_3<0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix}<0, \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix}<0.$$

The following is a main result of this article.

Theorem 3.3: *For given positive constants γ, β_i, δ, known matrices E_a, E_b, $D\in\mathbb{R}^{n\times n}$ and the Lipschitz constant $L=l^2I_n$, the feedback controller* (9) *using sampled-data with m sampling intervals guarantees robust synchronisation between master* (1) *and slave* (2) *systems if there exist positive-definite matrices P, Q_i, R_i, $Z_i\in\mathbb{R}^{n\times n}$ and any matrices S_i, G, $H\in\mathbb{R}^{n\times n}$ and positive scalars λ, ϵ satisfying the following LMIs*:

$$\begin{bmatrix} \Gamma & \epsilon\Upsilon^T \\ \star & -\epsilon I \end{bmatrix}<0, \tag{21}$$

$$\begin{bmatrix} R_i & S_i \\ \star & R_i \end{bmatrix}>0, \quad \forall i=1,\dots,m, \tag{22}$$

where

$$\Gamma=\left[\begin{array}{c|c|cccc} \Gamma_1 & \Phi_1 & GB & \Gamma_2 & \delta D \\ \hline \star & \Phi_2 & & \Phi_3 & \\ \hline \star & & -\lambda I & \gamma G^T B^T & 0 \\ \star & \star & \star & \Gamma_3 & \gamma\delta GD \\ \star & & \star & \star & -\epsilon I \end{array}\right],$$

$$\Gamma_1=\alpha_1Q_1-\alpha_1R_1+\lambda L+GA+A^TG^T-\sum_{i=1}^m \bar{Z}_i,$$

$$\Gamma_2=P-G+\gamma A^TG^T,$$

$$\Gamma_3=\sum_{i=1}^m\left(\alpha_ih_i^2R_i+\beta_id_i^2Z_i\right)-\gamma G-\gamma G^T,$$

$$\bar{Z}_i=\sum_{j=i}^m\left(\beta_j\frac{\pi^2h_i}{4d_j}Z_j\right),$$

$$h_i=d_i-d_{i-1},$$

$$\Upsilon=\left[\,E_a \quad 0_{n\times 2nm} \quad E_b \quad 0 \quad 0\,\right],$$

$$\Phi_1=\begin{cases} (1,1)=\alpha_1R_1-\alpha_1S_1+\bar{Z}_1+\alpha_1H \\ (1,2)=\alpha_1S_1 \\ (1,i)=\alpha_{\frac{i+1}{2}}H+\bar{Z}_{\frac{i+1}{2}} \\ \qquad i=3,5,7,\dots,2m-1 \\ \text{otherwise}=0 \end{cases},$$

$$\Phi_2=\begin{cases} (i,i)=\alpha_{\frac{i+1}{2}}(-2R_{\frac{i+1}{2}}+S_{\frac{i+1}{2}}+S^T_{\frac{i+1}{2}})-\bar{Z}_{\frac{i+1}{2}} \\ \qquad i=1,3,5,\dots,2m-1 \\ (i,i)=\alpha_{\frac{i}{2}}(-Q_{\frac{i}{2}}-R_{\frac{i}{2}})+\alpha_{\frac{i}{2}+1}(Q_{\frac{i}{2}+1}-R_{\frac{i}{2}+1}) \\ \qquad i=2,4,6,\dots,2m \\ (i,i+1)=\alpha_{\frac{i+1}{2}}(R_{\frac{i+1}{2}}-S_{\frac{i+1}{2}}) \\ \qquad i=1,3,5,\dots,2m-1 \\ (i,i+1)=\alpha_{\frac{i}{2}+1}(R_{\frac{i}{2}+1}-S_{\frac{i}{2}+1}) \\ \qquad i=2,4,6,\dots,2m-2 \\ (i,i+2)=\alpha_{\frac{i}{2}+1}S_{\frac{i}{2}} \\ \qquad i=2,4,6,\dots,2m-2 \\ \text{otherwise}=0 \end{cases},$$

$$\begin{aligned} &\Phi_3=\left[\,0_{2nm\times n} \quad \Phi_{32} \quad 0_{2nm\times n}\,\right], \\ &\Phi_{32}(i,1)=\alpha_{\frac{i+1}{2}}\gamma H^T \qquad i=1,3,5,\dots,2m-1 \\ &\Phi_{32}(i,1)=0 \qquad i=2,4,6,\dots,2m, \\ &\qquad \alpha_i,Q_i,R_i,\bar{Z}_i=0 \quad i>m. \end{aligned} \tag{23}$$

Also, the desired control gain matrix (9) *is given by $K=G^{-1}H$.*

Proof: Consider the following discontinuous Lyapunov functional for the error system (17):

$$V(e_t) = V_1(e_t) + V_2(e_t) + V_3(e_t), \quad t \in [t_k,\ t_{k+1}), \tag{24}$$

where

$$V_1(e_t) = e^T(t)Pe(t),$$

$$V_2(e_t) = \sum_{i=1}^{m} \alpha_i(t)\left(\int_{t-d_i}^{t-d_{i-1}} e^T(s)Q_i e(s)\mathrm{d}s + h_i \int_{-d_i}^{-d_{i-1}} \int_{t+\theta}^{t} \dot{e}^T(s)R_i\dot{e}(s)\mathrm{d}s\,\mathrm{d}\theta \right),$$

$$V_3(e_t) = \sum_{i=1}^{m} V_{3i}(t),$$

$$V_{3i}(e_t) = \beta_i(t)\left(d_i^2 \int_{t_k}^{t} \dot{e}^T(s)Z_i\dot{e}(s)\mathrm{d}s - \frac{\pi^2}{4}\int_{t_k}^{t}(e(s)-e(t_k))^T Z_i (e(s)-e(t_k))\mathrm{d}s \right).$$

It is noted that it is easy to find that $V_{3i}(t) \geq 0$ from Lemma 2. In addition, it is correct that $\lim_{t\to t_k^-} V(t) \geq V(t_k)$, because $V_{3i}(t)$ will disappear at $t = t_k$.

Define the infinitesimal operator $\mathcal{L}$ of $V(e_t)$ defined as follows:

$$\mathcal{L}V(e_t) = \lim_{h\to 0^+} \frac{1}{h}\{\mathbb{E}\{V(e_{t+h})|e_t\} - V(e_t)\}. \tag{25}$$

Then from (24) and (25), we obtain

$$\mathbb{E}\{\mathcal{L}V_1(t)\} = \mathbb{E}\{2e^T(t)P\dot{e}(t)\}, \tag{26}$$

and

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V_2(t)\} = \mathbb{E}\Bigg\{ \sum_{i=1}^{m} \Bigg(& \alpha_i e^T(t-d_{i-1})Q_i e(t-d_{i-1}) \\ & - \alpha_i e^T(t-d_i)Q_i e(t-d_i) + \alpha_i h_i^2 \dot{e}^T(t)R_i\dot{e}(t) \\ & - \alpha_i h_i \int_{t-\tau_i(t)}^{t-d_{i-1}} \dot{e}^T(s)R_i\dot{e}(s)\mathrm{d}s \\ & - \alpha_i h_i \int_{t-d_i}^{t-\tau_i(t)} \dot{e}^T(s)R_i\dot{e}(s)\mathrm{d}s \Bigg)\Bigg\}. \end{aligned} \tag{27}$$

By using Lemma 1 and Theorem 1 in Park, Ko, and Jeong (2011), the integral terms of $\mathcal{L}V_2(t)$ can be bounded as

$$\begin{aligned} & -\alpha_i h_i \int_{t-\tau_i(t)}^{t-d_{i-1}} \dot{e}^T(t)R_i\dot{e}(s)\mathrm{d}s - \alpha_i h_i \int_{t-d_i}^{t-\tau_i(t)} \dot{e}^T(s)R_i\dot{e}(s)\mathrm{d}s \\ & \leq -\alpha_i \begin{bmatrix} \eta_{1i}(t) \\ \eta_{2i}(t) \end{bmatrix}^T \begin{bmatrix} \frac{1}{1-\kappa_i}R_i & 0 \\ \star & \frac{1}{\kappa_i}R_i \end{bmatrix} \begin{bmatrix} \eta_{1i}(t) \\ \eta_{2i}(t) \end{bmatrix} \\ & \leq -\alpha_i \begin{bmatrix} \eta_{1i}(t) \\ \eta_{2i}(t) \end{bmatrix}^T \begin{bmatrix} R_i & S_i \\ \star & R_i \end{bmatrix} \begin{bmatrix} \eta_{1i}(t) \\ \eta_{2i}(t) \end{bmatrix}, \end{aligned} \tag{28}$$

where $\eta_{1i}(t) = \int_{t-\tau_i(t)}^{t-d_{i-1}} \dot{e}(s)\mathrm{d}s$, $\eta_{2i}(t) = \int_{t-d_i}^{t-\tau_i(t)} \dot{e}(s)\mathrm{d}s$, $\kappa_i = (d_i - \tau_i(t))(d_i - d_{i-1})^{-1}$.

It is noted that the discontinuous Lyapunov functional $V_3(t)$, which originates from Liu and Fridman (2012), makes full use of the sawtooth structure characteristic of sampling input delays. However, in this article, the interval of integration $V_3(t)$, $[t_k, t]$, stochastically occur because of the definition of $\tau(t) = t - t_k$. If the sampling interval $d(t) = d_2$, then t_k exists in two intervals such that $[t - \tau_1(t), t]$ and $[t - \tau_2(t),\ t - d_1]$ with probabilies $\frac{d_1}{d_2}$ and $\frac{d_2 - d_1}{d_2}$, respectively. Therefore in order to completely use the information of sawtooth structure delay, $\tau(t)$, we introduce stochastic variables $\rho_{\mathrm{ij}}(t)$ such that

$$\rho_{ij}(t) = \begin{cases} 1, & \beta_i(t)\alpha_j(t) = 1 \\ 0, & \text{otherwise} \end{cases} \quad j \leq i = 1, \ldots, m \tag{29}$$

with the following probability:

$$Pr\{\rho_{ij}(t) = 1\} = \beta_i \frac{d_j - d_{j-1}}{d_i} = \rho_{ij}. \tag{30}$$

where $\sum_{i=1}^{m}\sum_{j=1}^{i}\rho_{ij} = 1$.

Then, the Lyapunov functional $V_{3i}(t)$ can be rewritten as

$$\begin{aligned} V_{3i}(t) = {} & \beta_i(t) d_i^2 \int_{t-\tau(t)}^{t} \dot{e}^T(s)Z_i\dot{e}(s)\mathrm{d}s \\ & - \sum_{j=1}^{i}\left(\rho_{ij}(t)\frac{\pi^2}{4}\int_{t-\tau_j(t)}^{t}(e(s) - e(t-\tau_j(t)))^T \times Z_i(e(s) - e(t-\tau_j(t)))\mathrm{d}s \right), \end{aligned}$$

and its expectation is

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V_{3i}(t)\} = \mathbb{E}\Bigg\{ & \beta_i d_i^2 \dot{e}^T(t)Z_i\dot{e}(t) \\ & - \sum_{j=1}^{i}\left(\rho_{ij}\frac{\pi^2}{4} \begin{bmatrix} e(t) \\ e(t-\tau_j(t)) \end{bmatrix}^T \begin{bmatrix} Z_i & -Z_i \\ \star & Z_i \end{bmatrix} \times \begin{bmatrix} e(t) \\ e(t-\tau_j(t)) \end{bmatrix}\right)\Bigg\} \quad i = 1, 2, \ldots, m. \end{aligned} \tag{31}$$

From the property of the nonlinear function $f(\cdot)$, we can obtain the following equation:

$$\lambda\bar{f}(e(t))^T\bar{f}(e(t)) \leq \lambda e^T(t)Le(t). \tag{32}$$

According to the error system (17), for any appropriately dimensioned matrix G, the following equation holds:

$$\mathbb{E}\bigg\{2\big[e^T(t)G+\gamma\dot{e}^T(t)G\big] \times\bigg[-\dot{e}(t)+Ae(t)+BF(t) +\sum_{i=1}^{m}\alpha_i Ke(t-\tau_i(t))+\delta Dp(t)\bigg]\bigg\}=0, \qquad (33)$$

where we let $H=GK$.

Also, from (4) and (5), we have

$$p^T(t)p(t)\le q^T(t)q(t). \qquad (34)$$

Then, there exists a positive constant, ϵ, satisfying the following equation:

$$\epsilon\big[\zeta^T(t)\Upsilon^T\Upsilon\zeta(t)-p^T(t)p(t)\big]\ge 0, \qquad (35)$$

where Υ are defined in (23) and $\zeta(t)$ is

$$\zeta(t)=\big[e^T(t)\quad e_m^T(t)\quad \bar{f}^T(e(t))\quad \dot{e}^T(t))\quad p^T(t)\big]^T,$$

with

$$e_m(t)=\big[e^T(t-\tau_1(t))\quad e^T(t-d_1)\quad e^T(t-\tau_2(t))\quad e^T(t-d_2)\quad\cdots\quad e^T(t-\tau_m(t))\quad e^T(t-d_m)\big]^T.$$

By using the S-procedure and adding left sides of (32), (33) and (35) to $\mathbb{E}\{\mathcal{L}V(t)\}$, we can obtain the following new upper bound of $\mathbb{E}\{\mathcal{L}V(t)\}$

$$\mathbb{E}\{\mathcal{L}V(t)\}\le\mathbb{E}\Big\{\zeta^T(t)\big(\Gamma+\epsilon\Upsilon^T\Upsilon\big)\zeta(t)\Big\}. \qquad (36)$$

Then, by Fact 1, the negativeness of the matrix $\Gamma+\epsilon\Upsilon^T\Upsilon$ in Equation (36) is equivalent to the LMI (21). Therefore, if LMI (21) holds, then, by Definition 1, the error system is mean square stable. This implies that the synchronisation between the master system (1) and the slave system (2) is achieved by the designed controller (9). This completes the proof. □

If the term $V_3(t)$ is neglected, then the discontinuous Lyapunov functional (24) becomes the continuous Lyapunov functional. If Z_i $(i=1,\dots,m)$ in (24) choose $Z_i=0$ as zero matrices, then we can obtain the following result.

Corollary 3.4: *For given positive constants γ, β_i, δ, known matrices E_a, E_b, $D\in\mathbb{R}^{n\times n}$ and the Lipschitz constant $L=lI_n$, if there exist positive-definite matrices P, Q_i, R_i, $Z_i\in\mathbb{R}^{n\times n}$ and matrices S_i, G, $H\in\mathbb{R}^{n\times n}$ and positive scalar λ, ϵ satisfying the LMIs* (21) *such that* $(21)|_{Z_i=0}$, $(\forall i=1,\dots m)$, *and* (22), *then there exists a sampled feedback controller* (9) *with m sampling intervals which guarantees synchronisation between the master* (1) *and slave* (2) *chaotic systems. Moreover, the desired control gain matrix in* (9) *can be given by* $K=G^{-1}H$.

Remark 6: The synchronisation criteria for chaotic systems via the discontinuous and continuous Lyapunov functional come from Theorem 1 and Corollary 1, respectively. The main difference of two Lyapunov functionals is the existence of $V_3(t)$, which makes full use of the sawtooth structure characteristic of sampling input delays. Thus, theoretically the conservatism of Theorem 1 is less than Corollary 1, which will be validated by numerical examples in the next section.

Remark 7 : In this article, $\beta_i(t)$ and $\rho_{ij}(t)$ are used for utilisation of discontinuous Lyapunov functional approach, which makes full use of the sawtooth structure characteristic of sampling input delay $\tau(t)$. It is the key idea of this article. To the best of our knowledge, the discontinuous Lyapunov functional approach has never been tackled in the synchronisation problem of chaotic systems using the sampled-data control system with stochastic sampling intervals.

4. Numerical examples

In order to show the effectiveness of the proposed control scheme and derived results, two numerical examples, which are very well-known chaotic systems with ROUs via the sampled-data control having multi sampling intervals, are presented. In both the examples, MATLAB, YALMIP 3.0 and SeDuMi 1.3 are used to solve LMI problems and the parameter γ is chosen as 0.01.

4.1 *Example 1: Chua's circuit*

The first example is about the synchronisation of Chua's circuits (Chua, Komuro, and Matsumoto 2000), and its chaotic behaviour is displayed in Figure 1. The Chua's circuit is described by the following parameters:

$$A=\begin{bmatrix}-am_1 & a & 0\\ 1 & -1 & 1\\ 0 & -b & 0\end{bmatrix},\quad B=\begin{bmatrix}-a(m_0-m_1) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$f(x_{ik}(t))=\frac{1}{2}(|x_{ik}(t)+c|-|x_{ik}(t)-c|),\quad k=1,\dots,n$$

with the parameters are $a=9$, $b=14.28$, $c=1$, $m_0=-1/7$, $m_1=2/7$ and the nonlinear function $f(\cdot)$ satisfies the Lipschitz condition with $l=1$.

The parameters associated with system uncertainties are given $D=I_n$, $E_a=0.3I_n$, $E_b=0.4I_n$,

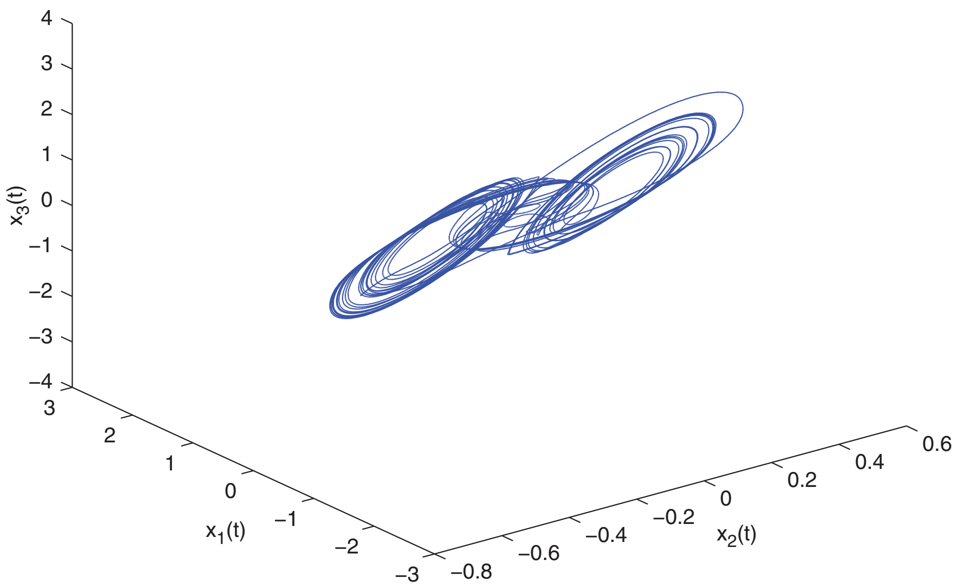

Figure 1. The chaotic behaviour of Chua's circuit.

Table 1. The maximum value of d_2 for different d_1 ($\beta_1 = 0.8$).

d_1		0.01	0.02	0.04	0.06	0.08	0.1	0.15
d_2	Theorem 1	1.113	1.078	1.008	0.935	0.864	0.791	0.602
	Corollary 1	1.086	1.028	0.906	0.782	0.654	0.521	×

Table 2. The maximum value of d_2 for different β_1 ($d_1 = 0.1$).

β_1		0.9	0.8	0.7	0.6	0.5	0.4	0.3
d_2	Theorem 1	1.597	0.791	0.573	0.462	0.395	0.350	0.317
	Corollary 1	0.980	0.521	0.369	0.294	0.257	0.235	0.221

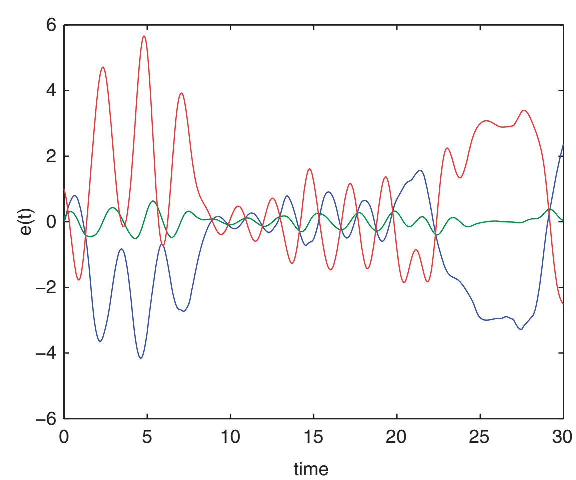

Figure 2. The uncontrolled error signals of Example 1.

$F(t) = 0.4 + 0.2 \sin t$, $\delta = 0.4$ and initial conditions are chosen as $x(0) = [-0.1 \ \ -0.5 \ \ -0.7]$, $y(0) = [-0.1 \ -0.4 \ 0.3]$.

In this example, we consider two sampling intervals d_1, d_2. By Theorem 1 and Corollary 1, we can obtain the maximum sampling interval d_2 for various cases, as given in Tables 1 and 2. It shows the maximum values of d_2 with respect to different d_1 and β_1, respectively. It can be seen from Tables 1 and 2 that Theorem 1 gives the more improved results than Corollary 1, as mentioned in Remark 4.

By solving LMI problems given in (21) and (22) of Theorem 1 with $d_1 = 0.1$, $d_2 = 0.2$ and $\beta_1 = 0.3$, we can obtain the following control gain:

$$K = \begin{bmatrix} -4.9521 & -3.4626 & 0.4270 \\ -0.9048 & -1.9505 & -0.1152 \\ 1.1345 & 5.1330 & -6.7120 \end{bmatrix}. \tag{37}$$

In order to show effectiveness of the proposed control scheme, the uncontrolled error signals are depicted in Figure 2. Under the given control gain, K,

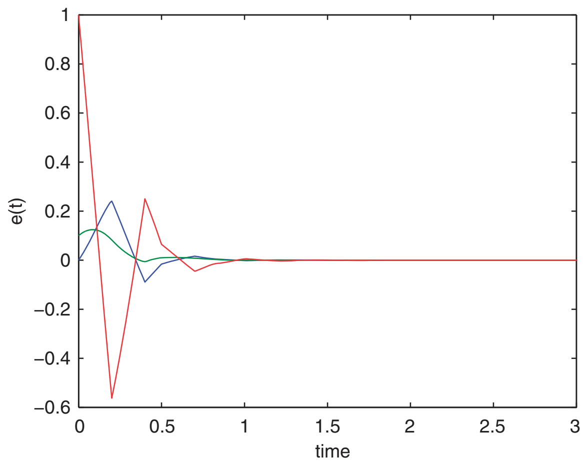

Figure 3. The controlled error signals of Example 1.

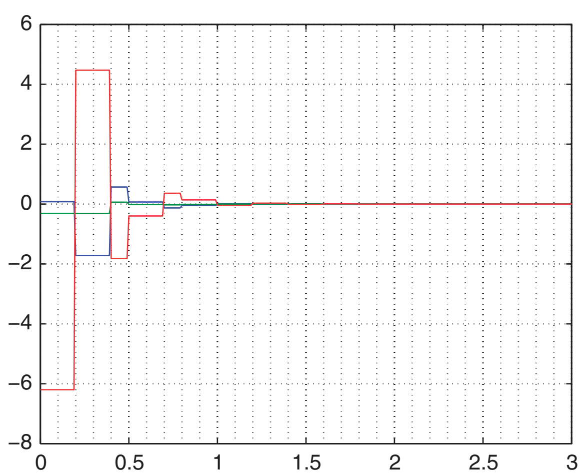

Figure 4. The sampled-data control input with two sampling intervals in Example 1.

the simulation result of the controlled error signals and the sampled control inputs are presented in Figures 3 and 4, respectively. As seen in Figure 3, the trajectories of error signals are indeed well stabilised. It means that the slave system (2) is synchronised up to the master system (1) by control inputs, which are seen in Figure 4. Finally, Figure 5 displays the stochastic parameters, $\delta(t)$ and $d(t)$.

4.2 *Example 2. Hyperchaotic Rössler system*

Next example is about the synchronisation of hypechaotic (Rossler 1979) which are described by

$$A=\begin{bmatrix}0 & -1 & -1 & 0\\ 1 & 0.25 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & -0.5 & 0.05\end{bmatrix},\quad B=I_n,$$

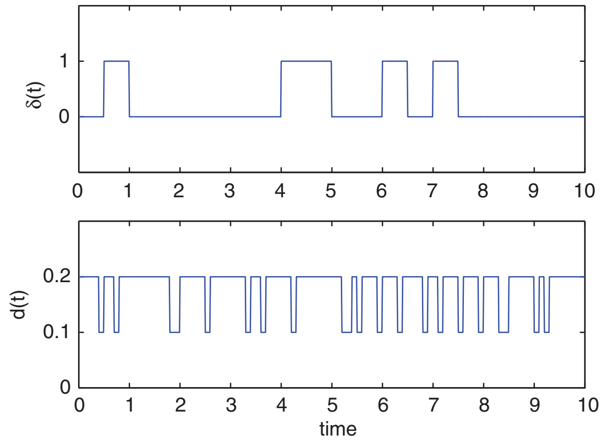

Figure 5. The stochastic parameters, $\delta(t)$ and $d(t)$, of Example 1.

$$f(x(t))=\begin{bmatrix}0\\ 0\\ 3+x_1(t)x_3(t)\\ 0\end{bmatrix},$$

and its chaotic behaviour is displayed in Figure 6.

The Lipschitz constant of nonlinear function $f(\cdot)$ is $l=10$ and initial conditions of each system are chosen as $x(0)=[-10\ -6\ 0\ 10]$, $y(0)=[-5\ -6\ 5\ 10]$.

The parameters for system uncertainties are given: $D=I_n$, $E_a=0.5I_n$, $E_b=0.2I_n$, $F(t)=0.4+0.2\sin t$ and $\delta=0.3$. It is assumed that this example has three sampling intervals d_1, d_2, d_3.

Under the above simulation setting, the values of the maximum sampling interval d_3 specified by Tables 3 and 4 can be obtained by Theorem 1 and Corollary 1.

When the sampling intervals are $d_1=0.04$, $d_2=0.08$, $d_3=0.12$ and the probability of each sampling interval is $\beta_1=0.3$, $\beta_2=0.3$, $\beta_3=0.4$, respectively, the control gain matrix calculated by Theorem 1 is given by

$$K=\begin{bmatrix}-14.4064 & 1.0790 & 1.1050 & 0.0981\\ -0.8802 & -14.8533 & 0.1179 & -1.1901\\ 0.1175 & 0.1290 & -14.0431 & 0.0237\\ 0.1139 & -0.2160 & 0.5304 & -14.2046\end{bmatrix}. \tag{38}$$

The uncontrolled and controlled error signals are presented in Figures 7 and 8, respectively. Comparing with Figures 7 and 8, controlled error signals become zero as time goes to infinity. However, in the case of the uncontrolled system, error signals do not approach to zero as expected. It implies that our proposed controller achieves the synchronisation of hyperchaotic

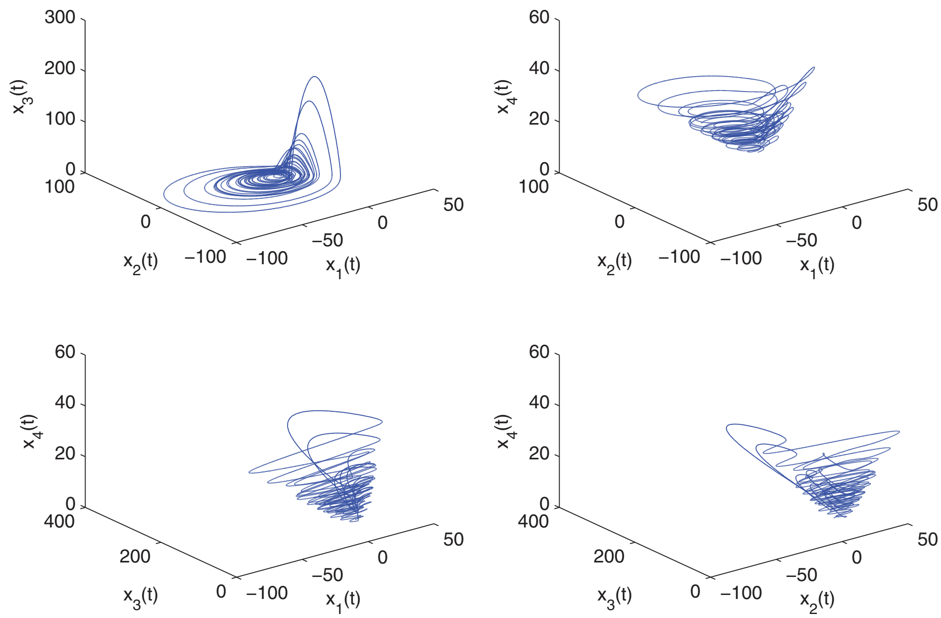

Figure 6. The chaotic behaviour of the hyperchaotic Rössler system.

Table 3. The maximum value of d_3 for different d_1 and d_2 ($\beta_1 = 0.5$, $\beta_2 = 0.3$).

d_2 \ d_1		0.01	0.02	0.03	0.04	0.05	0.06
0.04	Theorem 1	0.298	0.274	0.246			
	Corollary 1	0.248	0.209	0.155			
0.06	Theorem 1	0.274	0.251	0.225	0.197	0.167	
	Corollary 1	0.205	0.176	0.136	0.090	×	
0.08	Theorem 1	0.252	0.227	0.200	0.172	0.142	0.110
	Corollary 1	0.147	0.125	0.090	×	×	×
0.1	Theorem 1	0.226	0.202	0.174	0.145	0.155	×
	Corollary 1	×	×	×	×	×	×

Table 4. The maximum value of d_3 for different β_1 and β_2 ($d_1 = 0.02$, $d_2 = 0.05$).

β_2 \ β_1		0.4	0.5	0.6	0.7	0.8
0.5	Theorem 1	0.470				
	Corollary 1	0.328				
0.4	Theorem 1	0.250	0.502			
	Corollary 1	0.178	0.364			
0.3	Theorem 1	0.186	0.263	0.534		
	Corollary 1	0.131	0.195	0.399		
0.2	Theorem 1	0.153	0.194	0.276	0.560	
	Corollary 1	0.112	0.141	0.212	0.434	
0.1	Theorem 1	0.133	0.160	0.203	0.289	0.598
	Corollary 1	0.101	0.118	0.151	0.229	0.470

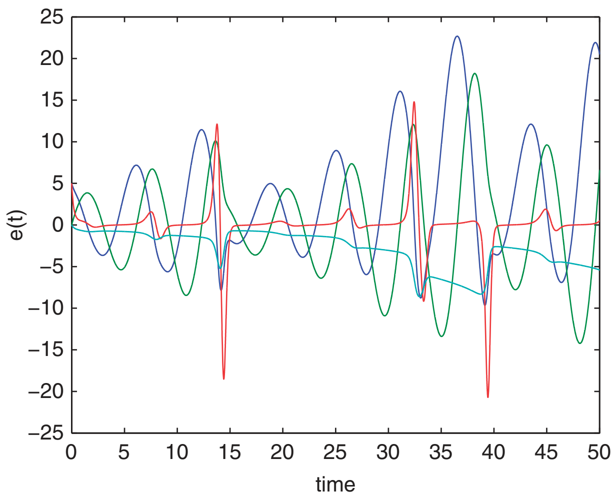

Figure 7. The uncontrolled error signals of Example 2.

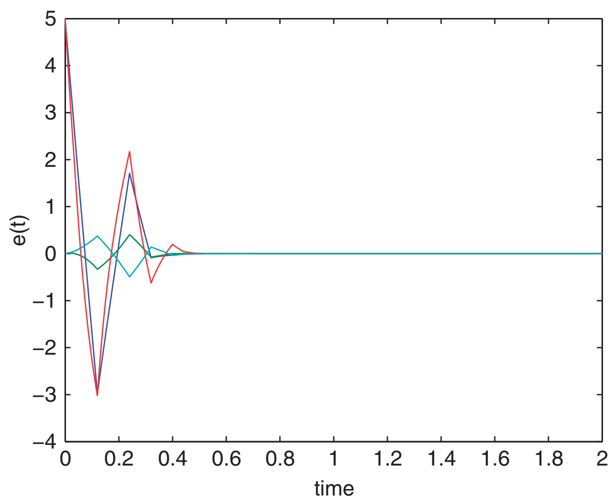

Figure 8. The controlled error signals of Example 2.

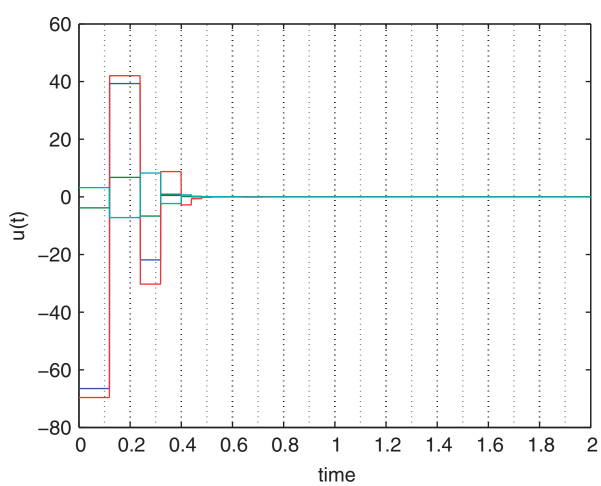

Figure 9. The sampled-data control input with three sampling intervals in Example 2.

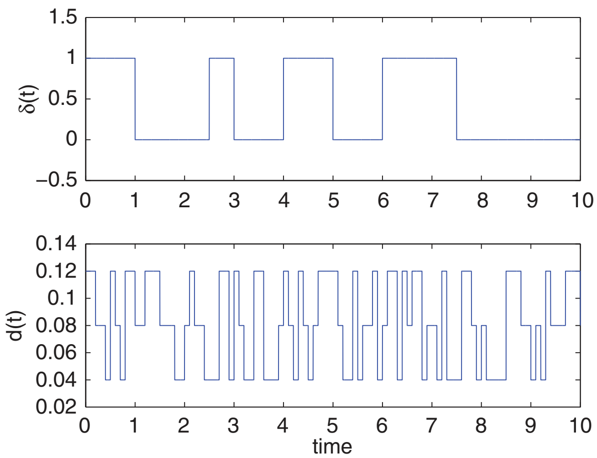

Figure 10. The stochastic parameters, $\delta(t)$ and $d(t)$, of Example 2.

Rössler systems. The applied control inputs which consisted of sampled signals are displayed in Figure 9 and the stochastic parameters, $\delta(t)$, $d(t)$, are presented in Figure 10.

5. Conclusions

In this article, the robust synchronisation problem of chaotic systems via sampled-data control with stochastic sampling interval has been studied. The sampled-data control system has been remodelled to a delay system with stochastic variables using the input-delay approach. In addition, ROU was considered in the sense of reality. In order to use full information of sawtooth structure characteristic of the sampling delay, the discontinuous Lyapunov functional has been proposed by introducing new stochastic parameters $\beta_i(t)$ and $\rho_{ij}(t)$. The results show that the use of the discontinuous Lyapunov functional gets less conservatism than the use of the continuous Lyapunov functional. Then, the criteria for designing synchronisation controllers are expressed by terms of LMIs. Two numerical examples have been illustrated to show the performance of the proposed controller.

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