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Robust trajectory optimisation of a TSTO spaceplane using uncertainty-based aerodynamic models

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This paper presents a multi-objective trajectory optimisation under uncertainty for the ascent and descent flight paths for the first stage of a two-stage, semi-reusable space launch system. Using Orbital Access' Orbital 500-R launcher as a test case, within the preliminary design phase, robust multi-disciplinary design optimisation is being used to analyse the trade-offs with both the vehicle and system design and operation. An area of focus is on the predicted aerodynamic performance, and its impact on the design and planned mission scenarios. The model uncertainties are quantified for the atmospheric model and aerodynamic surrogate models and integrated into the optimal control solver MODHOC extended by the addition of an unscented transformation to handle the uncertainties. A multi-objective optimisation under uncertainty is run, examining the Pareto-optimal sets for the ascent and descent trajectories of the first stage spaceplane.

I. Introduction

Uncertainty quantification (UQ) is the science of quantifying the uncertainties within the predicted performance of a system. At system level, UQ analysis can translate into the assessment of the whole system reliability or the reliability of one or more components. An uncertainty quantification analysis is, therefore, a fundamental step towards de-risking any technological solution as it provides a quantification of the variation in performance and probability of recoverable or unrecoverable system failures, given existing information.

Integrating UQ into the optimisation process leads to, generally, two types of multi-disciplinary design optimisation: reliability-based design optimisation (RBDO) which finding an optimal design with low probability of failure, and robust design optimisation (RDO) which aims to reduce the variability of the system performance. [1, 2]

In this paper, an RDO is applied to the Orbital 500-R, an air launched, two-stage-to-orbit semi-reusable launch system under design by Orbital Access Ltd. [3–5] Within the current conceptual design phase of the Orbital-500R, one of the areas of study is the uncertainty related to the aerodynamic modelling of the re-usable spaceplane. The goal is, therefore, to assess the robustness of the guidance trajectory design considering the uncertainty related to the atmospheric and aerodynamic models. Furthermore, given the early stage of the vehicle design and the wide scope of use required by the commercial side, a number of mission scenarios are possible, each of which is affected differently by the same sets of uncertainties.

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II. Robust design optimisation

MODHOC, Multi-Objective Direct Hybrid Optimal Control solver [6, 7], is based on a Direct Finite Elements Transcription (DFET) of the optimal control problem [8] and a solution of the transcribed problem with a multi-agent, multi-objective optimisation algorithm (MACS) [9]. By combining MACS and DFET, MODHOC has the ability to perform a global exploration of the solution space and to converge locally to optimal solutions. It works for arbitrarily connected multi-phase problems with general dynamics, and ensures an even spread set of solutions within the Pareto-optimal set. The software has been successfully used for the trajectory and design optimisation of vertical and horizontal launch systems [10, 11], deployment of constellations [12], interplanetary exploration missions [13] and the design of multi-debris removal missions.

Recently, MODHOC was extended to include an Square Root Unscented Transformation in the formulation of the optimal control problem.

Unscented Transformations capture the first statistical moments, mean and covariance, of the distributions of the states of a system subject to uncertainty and undergoing arbitrary non-linear transformations by propagating a small number of sigma points [14]. If the system depends on n uncertain variables, whose mean and covariances are known, the unscented transformation requires the propagation of $2n + 1$ samples. The first sigma point takes the mean value for all the uncertain variables, while the others assume the mean plus (or minus) the square root of the matrix of the covariances of the uncertain variables. All the sigma points are propagated simultaneously with the mean and covariance of the final states computed as a weighted combination of the final states of each sigma point.

A known problem of the Unscented Transformation is that it can generate covariance matrices that are not semidefinite positive. To address this, the Square Root Unscented Transformation [15] was implemented. Algorithmically it is very similar to the standard UT, but differs in the way the samples are generated and has the advantage that the resulting covariance matrices are guaranteed to be semi-definite positive (up to machine precision).

III. Launch system models

The Orbital-500R system is composed of a first stage reusable spaceplane, capable of rocket-powered ascent and an unpowered, glided descent, and an expendable, rocket-based upper stage (see Fig. 1).

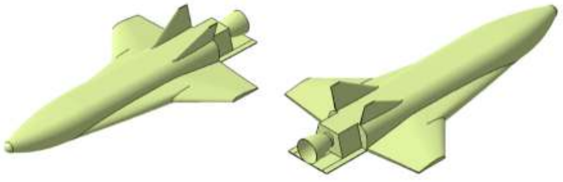


Fig. 1 Orbital 500-R spaceplane

The primary mission of the Orbital 500-R system is to deliver payloads up to 500 kg to a 650 km circular orbit at an inclination of 88.2 degrees. The secondary extended mission is to deliver payloads to a maximum altitude of 1200 km with payloads up to 150 kg. These mission parameters derive from investigations during the previous Future Small Payload Launcher UK study, with the goal of establishing the Orbital 500-R as a commercial logistics system for in-orbit delivery. [5, 16]

The flight dynamics are modelled as a variable-mass point with three degrees of freedom in the Earth-centered Earth-fixed reference frame, subject to gravitational, aerodynamic lift and drag forces. The state vector contains the translational position and velocity $\mathbf{x}(t, \mathbf{u}) = [h,\ \lambda,\ \theta,\ v,\ \gamma,\ \chi, m]$ where h is the altitude, (λ, θ) are the geodetic latitude and longitude, v is the magnitude of the relative velocity vector directed by the flight path angle γ and the flight heading angle χ, and $m(t)$ is the time-varying vehicle mass. [17] The vehicle is controlled through an open-loop control vector $\mathbf{u} = [\tau, \alpha, \mu]$ where τ is a percent control on the applied thrust, the angle of attack α, and bank angle μ of the vehicle.

The aerodynamic lift and drag coefficients were modelled using an artificial neural network with given an aerodynamic database for the training data coming from numerical simulations of differing fidelity, e.g., panel methods, CFD solvers such as SU2, ANITA and ANSYS Fluent.

IV. Intended results

The full paper will include results of trajectory optimisation of the trajectory starting from the moment the engines are ignited after release from the carrier aircraft, up to the stage separation point, and descending down to a nominal point on the landing approach vector. Different sets of design objectives will be analysed, including minimising the mass of propellant required, minimising the induced loads on the vehicle, and looking at down- and crossrange capabilities. This will be done against maximising the robustness by minimising uncertainty; in this case, the sum of the sum of the square of all the entries of the covariance matrix.

The quantification of the uncertainty on the aerodynamic and atmospheric models will also be discussed.

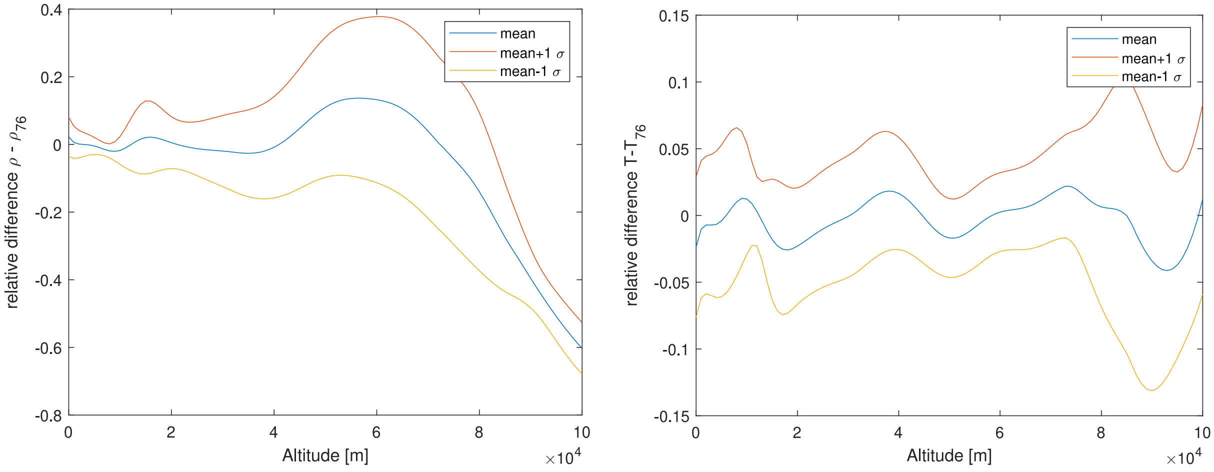
The next subsection gives some initial results looking only at the re-entry trajectory optimisation accounting for atmospheric model uncertainty.

A. Initial results

A study has been performed looking at the optimisation of the unpowered descent trajectory of the spaceplane, from a fixed initial point, accounting for uncertainty in the atmospheric model only.

Analysis to date on the design of the Orbital 500-R has employed the US-76 Standard atmospheric model [18]. The US-76 is a global static standard model, giving the atmospheric pressure p, temperature T and density ρ as function of altitude up to 1000 km. In order to assess the robustness of the design against uncertainties in the atmospheric model, it was necessary to develop a model for its uncertainty. Higher fidelity local atmospheric models were used for comparison to derive the required statistical models.

NRLMSISE-00 [19] is a local atmospheric model that accounts for daily and seasonal effects, solar and magnetic activity and geographical dependencies. A statistical analysis of the difference of the models was performed treating those the 8 input parameters as uncertain. Using a low discrepancy Halton sequence, 10^5 samples were taken and the corresponding values for T and ρ were computed using the NRLMSISE-00 model for all altitudes in the range between 0 and 100 km. To account for the possible differences between the models, the relative average difference between the values of the two model was computed for all the thermodynamic quantities. These relative errors were then treated as random fluctuations, for which averages and covariances were computed as a function of altitude. These relative errors and covariances deviations are plotted in Fig. 2.



(a) Density

(b) Temperature

Fig. 2 Relative difference between the uncertainty statistical mean and covariance for the atmospheric parameters and the nominal US-76 values

The first objective J_1 is the integral of the expected value of the square of the total accelerations $\mathbf{a}$, which should guarantee that the accelerations will be minimised on average along all the trajectory. The second objective J_2 is the sum of the square of all the entries of the covariance matrix of the final states, and will reduce the uncertainty of the final state.

The equations were discretised employing the DFET transcription using 3 elements of order 7 for all the states and the controls. All the sigma points are propagated simultaneously giving 30 states and 2 controls. In order to have a good

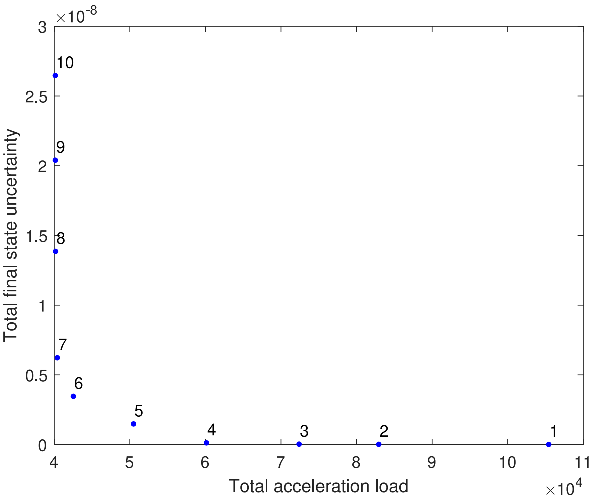


Fig. 3 Pareto front

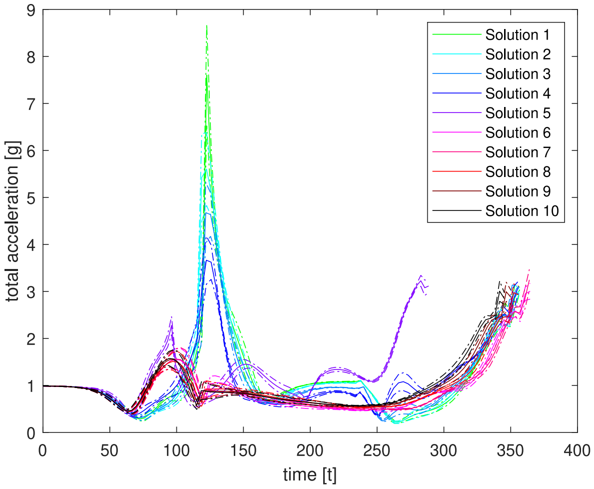
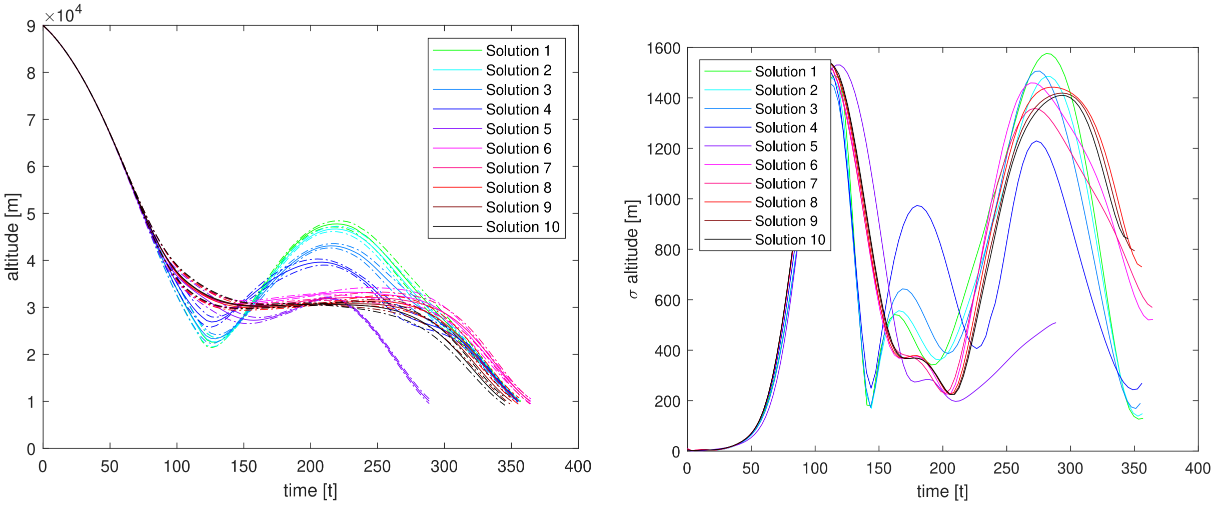


Fig. 4 Total accelerations of each solution in the Pareto-optimal set

approximation of the Pareto Front, MODHOC was run for a total of 30000 function evaluations, keeping 10 solutions in the archive. The computed Pareto front is shown in Fig. 3, which confirms a trade-off between the expected total acceleration load and the total uncertainty of the final state. Figure 4 shows the accelerations over the trajectory.

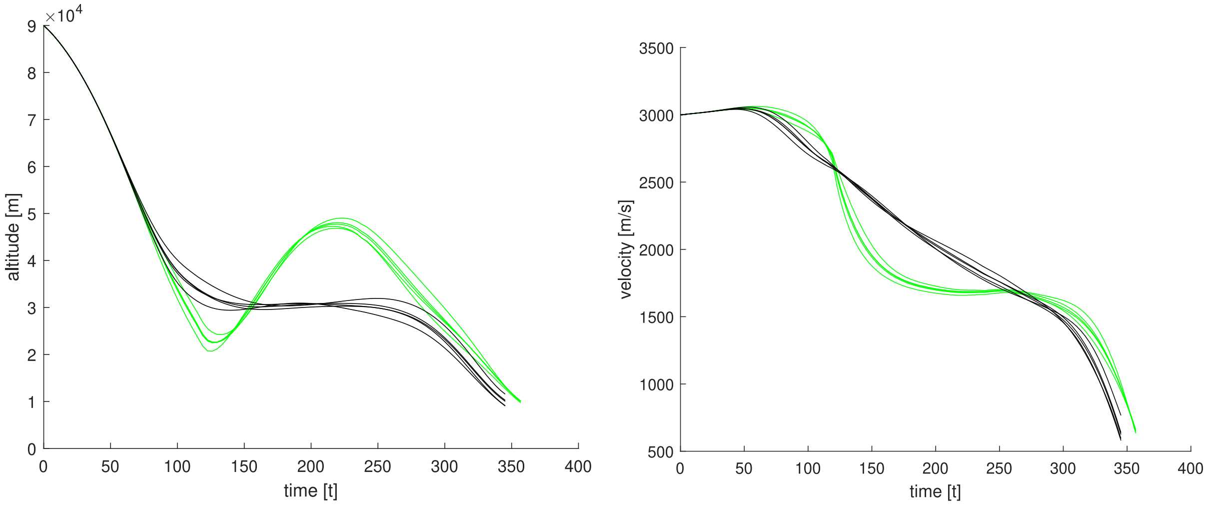
Figure 5 show the time history of the altitude for all the Pareto-optimal solutions, with the dashed and dotted lines indicating the 1σ uncertainty. Figure x shows the total accelerations as a function of time.

Figures 6 show the time history of altitude and velocity of all sigma points for the two extrema of the Pareto front (Solutions 1 and 10). As expected, the green lines have a much lower scattering at the final time than the black lines, indicating that Solution 1 (green) is subject to less uncertainty than solution 10 (black). This figures also give an idea of the complexity of the problem tackled by this approach, where the same control law is applied to multiple independent sigma points (lines with the same colour) and is able to steer the system to a given expected final state while also reducing the uncertainty associated to the final state, or reducing the expected acceleration load.



(a) (b)

Fig. 5 Trajectory altitude and standard deviation over time



(a) (b)

Fig. 6 Altitude and velocity for all sigma points for the extrema of the Pareto front (Solutions 1 and 10)

Acknowledgements

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