Rogue Wave Modes for the Long Wave-Short Wave Resonance Model

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ABSTRACT

The long wave-short wave resonance model arises physically when the phase velocity of a long wave matches the group velocity of a short wave. It is a system of nonlinear evolution equations solvable by the Hirota bilinear method and also possesses a Lax pair formulation. ‘Rogue wave’ modes, algebraically localized entities in *both* space and time, are constructed from the breathers by a singular limit involving a ‘coalescence’ of wavenumbers in the long wave regime. In contrast with the extensively studied nonlinear Schrödinger case, the frequency of the breather cannot be real and must satisfy a cubic equation with complex coefficients. The same limiting procedure applied to the finite wavenumber regime will yield mixed exponential-algebraic solitary waves, similar to the classical ‘double pole’ solutions of other evolution systems.

KEYWORDS: Breathers, algebraic solitons, rogue waves.

1. Introduction

Resonance often occurs in a nonlinear wave system when special criteria among wavenumbers and frequencies are met. Long wave-short wave resonant interaction is a classic example, where the phase velocity of a long wave matches the group velocity of a short wave. Significant interactions and energy transfer can then occur. A physically important example is the waveguide configuration for a two-layer fluid,[1 – 5] where the short wave envelope on the surface and the long wave on the interface make up an intriguing dynamical system.

Rogue or freak waves are unexpectedly large displacements of a sea surface from an otherwise calm sea state, and have received intensive study.[6 – 12] The focusing nonlinear Schrödinger equation, which governs the evolution of a weakly nonlinear wave packet in deep water, has frequently been employed as a model. A particular localized mode, sometimes known as the 'Peregrine breather' in the literature, relaxes to a plane wave in the far field but possesses a single sharp maximum in amplitude for one localized value in space at one specific instance in time.[13, 14] This algebraic soliton thus serves as a plausible model for rogue waves. Elegant theoretical extensions to incorporate higher order nonlinearities have been performed, and the analytic structures of these solutions have been elucidated.[15, 16] Concurrently, similar entities in the optical context have been investigated and even demonstrated experimentally to some extent.[17, 18]

The objective here is to study 'explode-decay' type wave modes for the long wave-short wave resonant interaction model. This is valuable from the general perspective of nonlinear science, as new families of exact solutions will be obtained in this paper for this nonlinear evolution system. Furthermore, the time scale for such long-short wave resonance is $\varepsilon^{4/3}t$, where t is the time scale of rapid oscillations in the packet and ε is a small, non-dimensional parameter measuring the strength of the wave amplitude. The corresponding time scale for the nonlinear Schrödinger (NLS) equation is $\varepsilon^2 t$ and thus effects of long-short wave resonance can be observed sooner in an asymptotic sense.

Theoretically, a relatively novel analytical technique will be employed. Many studies of rogue waves in the literature utilize the Darboux transformation.[13, 15] Here the rogue wave modes will be derived from multi-soliton or multi-breather obtainable from the Hirota bilinear transformation.[19] Mathematically, this is accomplished by taking a singular limit for solitary modes with nearly identical wavenumbers, augmented by special phase factors, and thus an appropriate name might be a 'coalescence of wavenumbers'.[20, 21] Alternatively these results can also be derived in terms of a 'double pole' (or ‘multiple pole’) solution in the language of the inverse scattering transform,[22, 23] where a double pole in the reflection coefficient also leads to these exponential-algebraic modes. If these procedures are performed in the long wave regime (wavenumber tending to zero), one recovers these rogue waves / purely algebraic modes.

The structure of the paper can now be explained. To illustrate our method, a special ‘coalescence of wavenumbers’ limit will be taken for the pulsating, or ‘breather’, solution of the nonlinear Schrödinger equation to recover the Peregrine soliton in Section 2. In Section 3, we start by presenting the bilinear transform and Lax pair of the long-short interaction system, with a detuning parameter incorporated. A breather is then derived through the bilinear method. Although such a breather can also be obtained by a Bäcklund transformation and the dressing method,[24] the bilinear method is conceptually simpler. Furthermore, the present effort corrects and generalizes previous work by demonstrating that the frequency parameter must be complex and cannot be real.[25] The first contribution here is to calculate the rogue wave (spatiotemporally localized) mode explicitly, by taking a ‘coalescence of wavenumbers’ in the long wave limit. In Section 4, this mechanism is generalized to the finite wavenumber regime and exact exponential-algebraic modes are then obtained accordingly. Such modes will correspond to a ‘double pole' solution arising from an inverse scattering mechanism. Conclusions are drawn in Section 5.

2. The Rogue Wave Solution of the Nonlinear Schrödinger Equation

A breather of the NLS equation,

$$iA_t + A_{xx} + \sigma A^2 A^* = 0,$$

where σ tunes the effect of nonlinear focusing in the system, can be obtained from the bilinear calculations,[26)]

$$A = \alpha \exp(i\sigma\alpha^2 t)[1 + g_1/f] ,$$

$$(iD_t + D_x^2)g_1 . f + D_x^2 f . f = 0, \quad D_x^2 f . f = \sigma\alpha^2[g_1 g_1^* + f(g_1 + g_1^*)] ,$$

$$f = \exp(px) + \exp(-px) + s \exp(i\omega t + i\zeta) + s \exp(-i\omega t - i\zeta),$$

$$g_1 = \lambda \exp(i\omega t + i\zeta) + \mu \exp(-i\omega t - i\zeta),$$

$$s = \left[\frac{1}{1 + \dfrac{p^2}{2\sigma\alpha^2}}\right]^{1/2}, \ \omega = p\sqrt{p^2 + 2\sigma\alpha^2} ,$$

$$\lambda = \frac{sp}{\sigma\alpha^2}\left(p + \sqrt{p^2 + 2\sigma\alpha^2}\right), \ \mu = \frac{sp}{\sigma\alpha^2}\left(p - \sqrt{p^2 + 2\sigma\alpha^2}\right).$$

The parameter ζ, originally arising from the flexibility in choosing the starting point in time, will now be exploited to take on arbitrary values. In particular, on choosing $\exp(i\zeta) = -1$ and taking the limit of p approaching zero, one obtains

$$A = \alpha \exp\left(i\sigma\alpha^2 t\right)\left\{1 - \frac{2\left(1+2i\sigma\alpha^2 t\right)}{\sigma\alpha^2 (x^2 + 2\sigma\alpha^2 t^2 + \frac{1}{2\sigma\alpha^2})}\right\},$$

the familiar Peregrine breather 'rogue wave' solution of the NLS equation.

3. The Long Wave–Short Wave Resonance Model

The nonlinear evolution systems of the short wave envelope (S) and the induced long wave (L) are derived by multiple scale asymptotic expansion of the underlying fluid dynamics equations. In scaled coordinates, the equations are given by

$$iS_t - S_{xx} = LS, \quad L_t + \Delta L_x = -\sigma(SS^*)_x\,, \tag{1}$$

where Δ is a detuning parameter measuring the deviation from exact resonance. The theoretical formulation will be treated by examining two aspects, namely, the Hirota bilinear transform and the Lax pair.

3.1 The Hirota bilinear transform

Using the dependent variable transformation (f real),

$$S = g/f,\ L = 2(\log f)_{xx}, \tag{2}$$

the Hirota bilinear form of eq. (1) is (C = constant) [2 – 5)]

$$(iD_t - D_x^2)\, g \,.\, f = 0, \qquad (D_xD_t + \Delta D_x^2 - C)\, f \,.\, f = -\,\sigma g g^*. \tag{3}$$

3.2 The Lax pair

Integrable nonlinear systems can often be investigated by converting them into auxiliary linear systems, with the Lax pairs being classic examples. These pairs form the basis for a variety of methods, e.g. the Darboux scheme. They usually take the form:

$R_x = U{\cdot}R,$

$R_t = V{\cdot}R.$

Both the compatibility condition and zero-curvature equation, related by

$R_{xt} = R_{tx}$

⇨ $U_t{\cdot}R + U{\cdot}R_t = V_x{\cdot}R + V{\cdot}R_x$

⇨ $U_t{\cdot}R + U{\cdot}V{\cdot}R = V_x{\cdot}R + V{\cdot}U{\cdot}R$

⇨ $U_t - V_x + [U,V] = 0,$

provide a representation of the nonlinear system through the matrices U and V.

The NLS equation and its conjugate form,

$$iA_t + A_{xx} + \sigma|A|^2A = 0, \qquad -\,iA_t^* + A_{xx}^* + \sigma|A^*|^2A^* = 0,$$

are encapsulated by

$$U=\begin{bmatrix} i\lambda & i\sqrt{\frac{\sigma}{2}}A^* \\ i\sqrt{\frac{\sigma}{2}}A & -i\lambda \end{bmatrix},\quad V=\begin{bmatrix} 2i\lambda^2-\frac{i}{2}\sigma|A|^2 & i\sqrt{2\sigma}\lambda A^*+\sqrt{\frac{\sigma}{2}}A_x^* \\ i\sqrt{2\sigma}\lambda A-\sqrt{\frac{\sigma}{2}}A_x & -2i\lambda^2+\frac{i}{2}\sigma|A|^2 \end{bmatrix}.$$

Likewise, the long wave-short wave resonance model can be written as,

$$iS_t - S_{xx} - LS = 0,\quad -iS_t^* - S_{xx}^* - LS^* = 0,\quad L_t = -\sigma(SS^*)_x,$$

where for simplicity we have concentrated on the case $\Delta = 0$. The above system can be generated from

$$U=\begin{bmatrix} i\lambda & \sqrt{\frac{\sigma}{2}}S & -iL \\ 0 & 0 & -\sqrt{\frac{\sigma}{2}}S^* \\ -i & 0 & -i\lambda \end{bmatrix},\quad V=\begin{bmatrix} \frac{i}{3}\lambda^2 & \sqrt{\frac{\sigma}{2}}\lambda S-i\sqrt{\frac{\sigma}{2}}S_x & \frac{i}{2}\sigma|S|^2 \\ \sqrt{\frac{\sigma}{2}}S^* & -\frac{2i}{3}\lambda^2 & \sqrt{\frac{\sigma}{2}}\lambda S^*-i\sqrt{\frac{\sigma}{2}}S_x^* \\ 0 & -\sqrt{\frac{\sigma}{2}}S & \frac{i}{3}\lambda^2 \end{bmatrix}.$$

Importantly, the spectral problem for the long wave-short wave resonance model is of the third order, whereas the corresponding problem for the NLS equation is of the second order.

3.3 The breather solution for the long wave-short wave resonance model

The breather solution can in principle be generated from either of these formulations. We shall adopt the Hirota approach, as the algebra is slightly simpler. The appropriate expansion is (p real, Ω complex)

$$f = 1 + \exp(ipx - \Omega t + \zeta^{(1)}) + \exp(-ipx - \Omega^* t + \zeta^{(2)})$$

$$+ M \exp(-\Omega t - \Omega^* t + \zeta^{(1)} + \zeta^{(2)}),$$

(ρ_0 = constant = amplitude in the far field)

$$g = \rho_0[1 + a_1\exp(ipx - \Omega t + \zeta^{(1)}) + a_2\exp(-ipx - \Omega^* t + \zeta^{(2)})$$

$$+ Ma_1a_2\exp(-\Omega t - \Omega^* t + \zeta^{(1)} + \zeta^{(2)})]. \tag{4}$$

The major distinction from the corresponding case of the NLS equation is the presence of a complex frequency Ω. $\zeta^{(1)}$, $\zeta^{(2)}$ are arbitrary phase factors. Using the bilinear equations (3), the parameters, a_1, a_2 are given by

$$a_1 = -(p^2 + i\Omega)/(p^2 - i\Omega), \qquad a_2 = -(p^2 + i\Omega^*)/(p^2 - i\Omega^*), \tag{5}$$

and thus the relations $a_1^* = 1/a_2$, $a_2^* = 1/a_1$ follow. The dispersion relation is

$$(\Omega - \Delta pi)(\Omega^2 + p^4) = 2i\sigma\rho_0^2p^3, \tag{6}$$

and hence for small p, the asymptotic expansion for Ω is

$$\Omega = p[\Omega_0 + \Omega_2p^2 + O(p^4)],$$

where the leading order frequency Ω_0 satisfies

$$(\Omega_0 - \Delta i)\Omega_0^2 = 2i\sigma\rho_0^2. \tag{7}$$

The coefficient M is given by

$$M = 1 + 4p^4/[(\Omega + \Omega^*)^2].$$

To obtain a rogue wave mode, we perform a long wave limit expansion similar to that done in Section 2. On choosing $\exp(\zeta^{(1)}) = \exp(\zeta^{(2)}) = -1$, we obtain rogue wave modes where the short wave component is

$$S = \rho_0\left\{1 + \frac{-4 + 2x\left(\Omega_0 - \Omega_0^*\right) + 2it\left[\Omega_0^2 + \left(\Omega_0^*\right)^2\right]}{\left|\Omega_0\right|^2\left[\left(x + it\dfrac{\Omega_0 - \Omega_0^*}{2}\right)^2 + \dfrac{\left(\Omega_0 + \Omega_0^*\right)^2}{4}t^2 + \dfrac{4}{\left(\Omega_0 + \Omega_0^*\right)^2}\right]}\right\}, \qquad (8)$$

while the long wave component is given by

$$L = \frac{4}{\left[\left(x + it\dfrac{\Omega_0 - \Omega_0^*}{2}\right)^2 + \dfrac{\left(\Omega_0 + \Omega_0^*\right)^2}{4}t^2 + \dfrac{4}{\left(\Omega_0 + \Omega_0^*\right)^2}\right]} - \frac{2\left[2x + it\left(\Omega_0 - \Omega_0^*\right)\right]^2}{\left[\left(x + it\dfrac{\Omega_0 - \Omega_0^*}{2}\right)^2 + \dfrac{\left(\Omega_0 + \Omega_0^*\right)^2}{4}t^2 + \dfrac{4}{\left(\Omega_0 + \Omega_0^*\right)^2}\right]^2}. \qquad (9)$$

Ω_0 is given by eq. (7). The denominators in eqs. (8, 9) are clearly nonsingular. This set of solutions constitutes purely algebraic modes exhibiting 'explode-decay' behavior (Figs. 1, 2).

4. The Double Pole Solution

Theoretically this 'coalescence of wavenumbers' approach can also be applied to the case where the pre-existing wave is of a *finite* wavelength. The nonlinear Schrödinger equation

$$iA_t + A_{xx} + |A|^2 A = 0$$

provides an instructive perspective. In addition to a purely algebraic / rogue wave mode described in Section 2, there exists another exact, mixed exponential-algebraic mode given by

$$A = \frac{4\sqrt{2}r^2 \exp(ir^2 t)\left[\left(2rt - \frac{i}{r}\right)\cosh(rx) + ix\sinh(rx)\right]}{\cosh(2rx) + 8r^4t^2 + 2r^2x^2 + 1}.$$

This solution was investigated in the 1960s and 1970s using various methods, e.g. in terms of merging eigenvalues in the inverse scattering transform and also by the dressing methods (reference [20] and works cited therein). Physically this and related solutions can be interpreted as weakly bounded groups of solitons. For the present discussion, such exponential-algebraic modes can be derived by a coalescence of nearly identical wavenumbers at a *finite* wavenumber in a multi-soliton expression, rather than the long wave (zero wavenumber) limit of a breather in Section 2.[20)] Physically, another difference between the exponential-algebraic and the rogue wave modes for the present system is that the former actually decays in the far field (x or t going to infinity), while the latter goes to a constant state.

For system (1), a straightforward calculation starting from a localized 2-soliton expression will yield a singularity, as a few coefficients in the Hirota expansion become indefinitely large in the 'coalescence of wavenumbers' process. An analytical way to avoid this singularity, and to gain a more physically realistic picture, can be obtained by incorporating another horizontal spatial dimension (y). To simplify the algebraic complexity, we shall now ignore the effect of detuning and restrict our attention to a 2-soliton of the long wave-short wave model, where the governing equations and an exact expression are known from earlier works,[3)] (F real)

$$iS_t + iS_y - S_{xx} - LS = 0, \quad L_t = -\sigma(SS^*)_x, \tag{10}$$

$$S = G/F, \quad L = 2(\log F)_{xx},$$

$$G = \exp(\eta_1) + \exp(\eta_2)$$

$$+ a(1, 2, 1^*) \exp(\eta_1 + \eta_2 + \eta_1^*) + a(1, 2, 2^*) \exp(\eta_1 + \eta_2 + \eta_2^*),$$

$$F = 1 + a(1, 1^*)\exp(\eta_1 + \eta_1^*) + a(1, 2^*)\exp(\eta_1 + \eta_2^*) + a(2, 1^*)\exp(\eta_2 + \eta_1^*)$$

$$+ a(2, 2^*)\exp(\eta_2 + \eta_2^*) + a(1, 2, 1^*, 2^*) \exp(\eta_1 + \eta_2 + \eta_1^* + \eta_2^*),$$

$$\eta_n = p_n x + q_n y - \Omega_n t + \eta_n^{(0)}, \ \Omega_n = ip_n^2 + q_n, \ n = 1, 2,$$

$$a(i, j^*) = [(p_i + p_j^*)(\Omega_i + \Omega_j^*)]^{-1}, \ a(i, j) = (p_i - p_j)(\Omega_i - \Omega_j),$$

$$a(i^*, j^*) = [a(i, j)]^*, \ a(i, j, k^*) = a(i, j)\, a(i, k^*)\, a(j, k^*),$$

$$a(i, j, k^*, l^*) = a(i, j)\, a(i, k^*)\, a(i, l^*)\, a(j, k^*)\, a(j, l^*)\, a(k^*, l^*).$$

On using $p_1 = p + i\varepsilon$, $p_2 = p - i\varepsilon$, $q_1 = q + im\varepsilon$, $q_2 = q - im\varepsilon$, $\exp(\eta_1^{(0)}) = i/\varepsilon$, $\exp(\eta_2^{(0)}) = -i/\varepsilon$, and by letting $\varepsilon \to 0$ we obtain

$$G = \exp\left(-ip^2 t\right)$$

$$\left\{ \begin{aligned} & 2\left[\left(-x - my + mt\right) + 2pit\right]\exp\left(px + qy - qt\right) \\ & + \frac{-2p + im}{2p^2q^2}\left[\left(\frac{2p}{q} + 2pt\right) + i\left(\frac{1}{p} + \frac{m}{q} - x - my + mt\right)\right]\exp\left[3\left(px + qy - qt\right)\right] \end{aligned} \right\}$$

and

$$F = 1 + \frac{1}{2pq}\exp\left[2\left(px + qy - qt\right)\right]\left\{ \begin{aligned} & 2\left[\left(x + my - mt - \frac{\frac{1}{p} + \frac{m}{q}}{2}\right)^2 + 4p^2\left(t + \frac{1}{2q}\right)^2\right] \\ & + \left(\frac{2p^2}{q^2} + \frac{1}{2p^2} + \frac{m^2}{2q^2}\right) \end{aligned} \right\}$$

$$+ \frac{4p^2 + m^2}{16p^4q^4}\exp\left[4\left(px + qy - qt\right)\right] \tag{11}$$

and these solutions also provide an analytical perspective for the failure of a calculation using only one spatial dimension. The solution is singular as the spanwise wavenumber (q) tends to zero.

System (10, 11) will be termed a ‘double pole’ solution, a terminology borrowed from the inverse scattering transformation (IST) mechanism. In IST, the unknown wave profile is usually determined by studying the reflection and transmission properties of incident waves. In conventional calculations of solitons using the Gel'fand-Levitan integral equations, the poles of the reflection coefficient are usually of the first order.

When the pole is of a higher order, special exact solutions are obtained. [23, 27, 28] A 'double pole' solution usually arises from a singular limit of two solitons with opposite signs of amplitudes, and analytically displays both exponential and positive definite polynomial functions. For the classical nonlinear Schrödinger equation, such double pole solution can also be obtained from the multi-soliton expression by a 'coalescence of wavenumbers' approach, i.e. a singular limit of two nearly equal wavenumbers.[20] Consequently, even though the merging of poles formulations have not yet been fully worked out for eq. (1), we shall still loosely term such expressions as the 'double pole' solution.

The dynamics of this double pole solution is illustrated in Fig. 3 for some typical parameter values. The taller soliton catches up with the shorter one, and they exchange identity without actual merging. The analytic structure of $|S|$ resembles an ordinary 2-soliton and a heuristic explanation can be offered. The long-short system, just like the NLS equation, admits a 2-soliton breather,[29] where two solitary pulses with nearly equal frequencies pulsate periodically. This 'coalescence of wavenumbers' destroys this beating behavior and the oscillatory character of the amplitudes disappears, as confirmed in Fig. 3. The analytical structure of eq. (11) is similar to those double pole solutions of the NLS and the modified Korteweg-de Vries equations, but details of the inverse scattering mechanism applied to eq. (10) and a comparison with the present result will be left for future investigations.

5. Conclusions

The long wave-short wave resonance model arises as a simplified model of certain circumstances in the dynamics of the upper ocean, as well as other physical contexts. Theoretically it admits both the Hirota bilinear transform and Lax pair formulations. In this paper, both purely algebraic and exponential-algebraic modes are derived. The critical difference between the nonlinear Schrödinger equation and the present long-short wave system is the nature of the dispersion relation, namely, the breather solution of the latter must have a complex frequency solvable from a cubic polynomial. Purely algebraic modes are obtained from a singular limit in the long wave regime. The exponential-algebraic modes are derived from a similar procedure applied in the realm of finite wavenumber.

There are many directions for future works from both the theoretical and practical perspectives. A higher order breather solution can be constructed, and from there analytic structures of multiple rogue wave modes can be studied.

The effects of the detuning parameter on the evolution and stability of wave trains can be investigated numerically, and will provide useful insight into the dynamics of the upper ocean. Furthermore, a breather periodic in time and spatially localized in space can be calculated, similar to the situation for the NLS equation, but technical details remain to be worked out. Finally, this whole mechanism can in principle be applied to other integrable nonlinear evolution equations, generating exact solutions and information for further research in nonlinear evolution equations.

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Figures Captions

(1) Fig. 1: Intensity of the short wave envelope $|S|^2$ (eq. (8)) of the rogue wave mode versus x and t: $\Delta = 0.1$, $\sigma = 1$, $\rho_0 = 1$, (a) three dimensional view; (b) top view.

(2) Fig. 2: The long wave L (eq. (9)) of the rogue wave mode versus x and t: $\Delta = 0.1$, $\sigma = 1$, $\rho_0 = 1$, (a) three dimensional view; (b) top view.

(3) Fig. 3: The interaction of solitary pulses in a 'double pole' solution (eq. (11)), $|S|^2 = |G/F|^2$, $p = 1$, $q = 0.1$, $m = 1$: (a) $t = -20$ (just before the interaction), (b) $t = 0$ (just after the interaction).

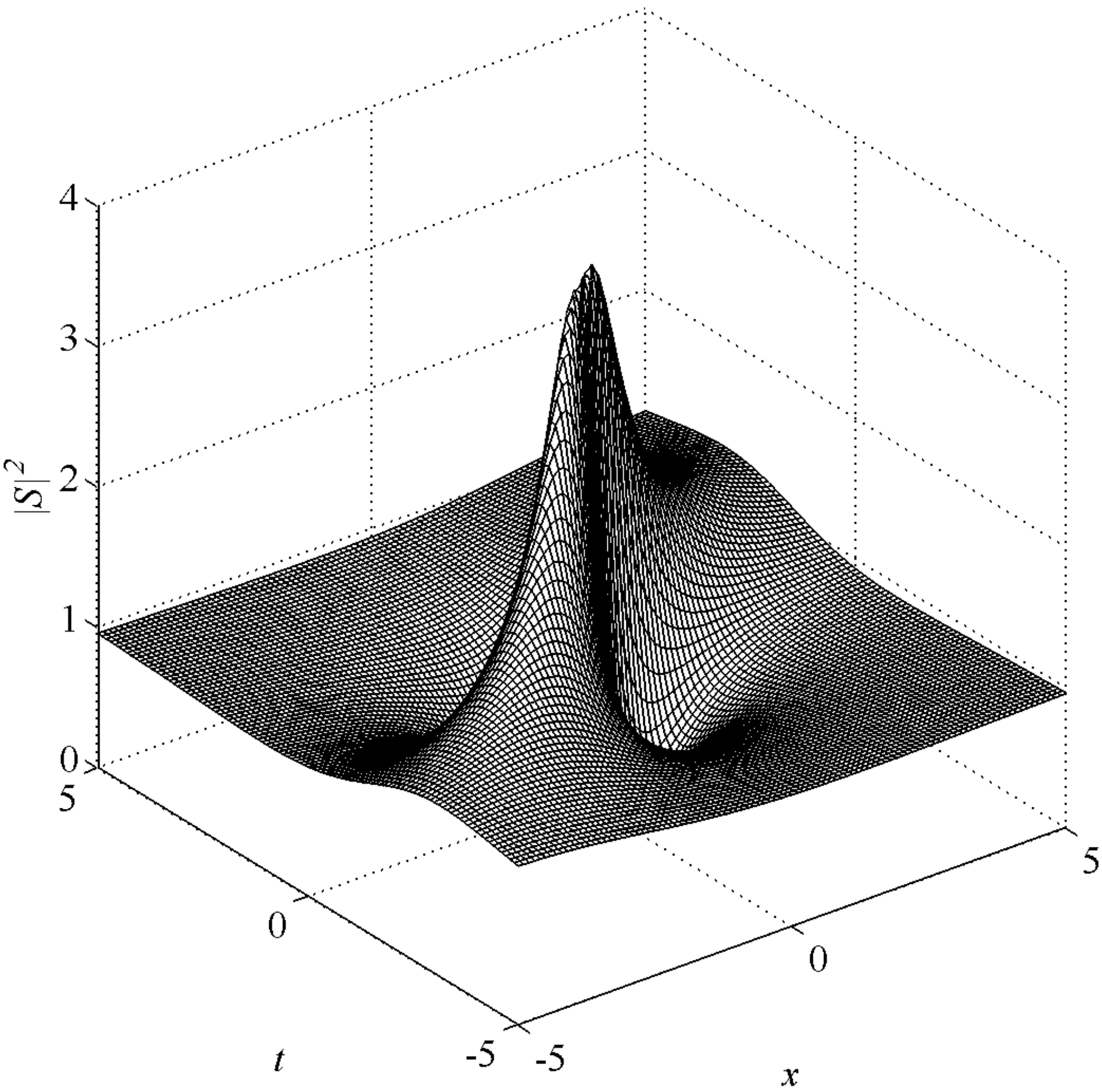


Fig. 1(a)

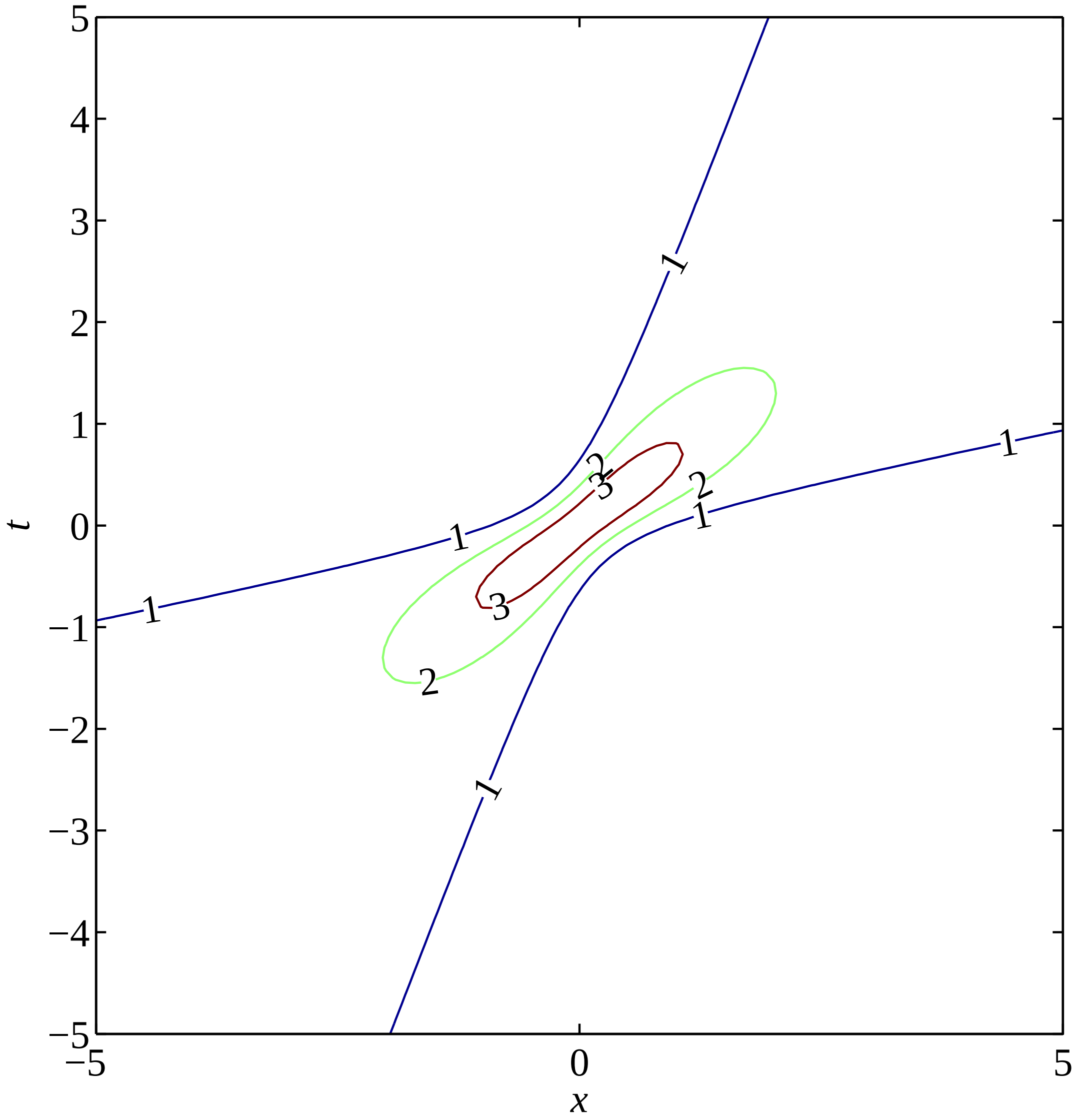


Fig. 1(b)

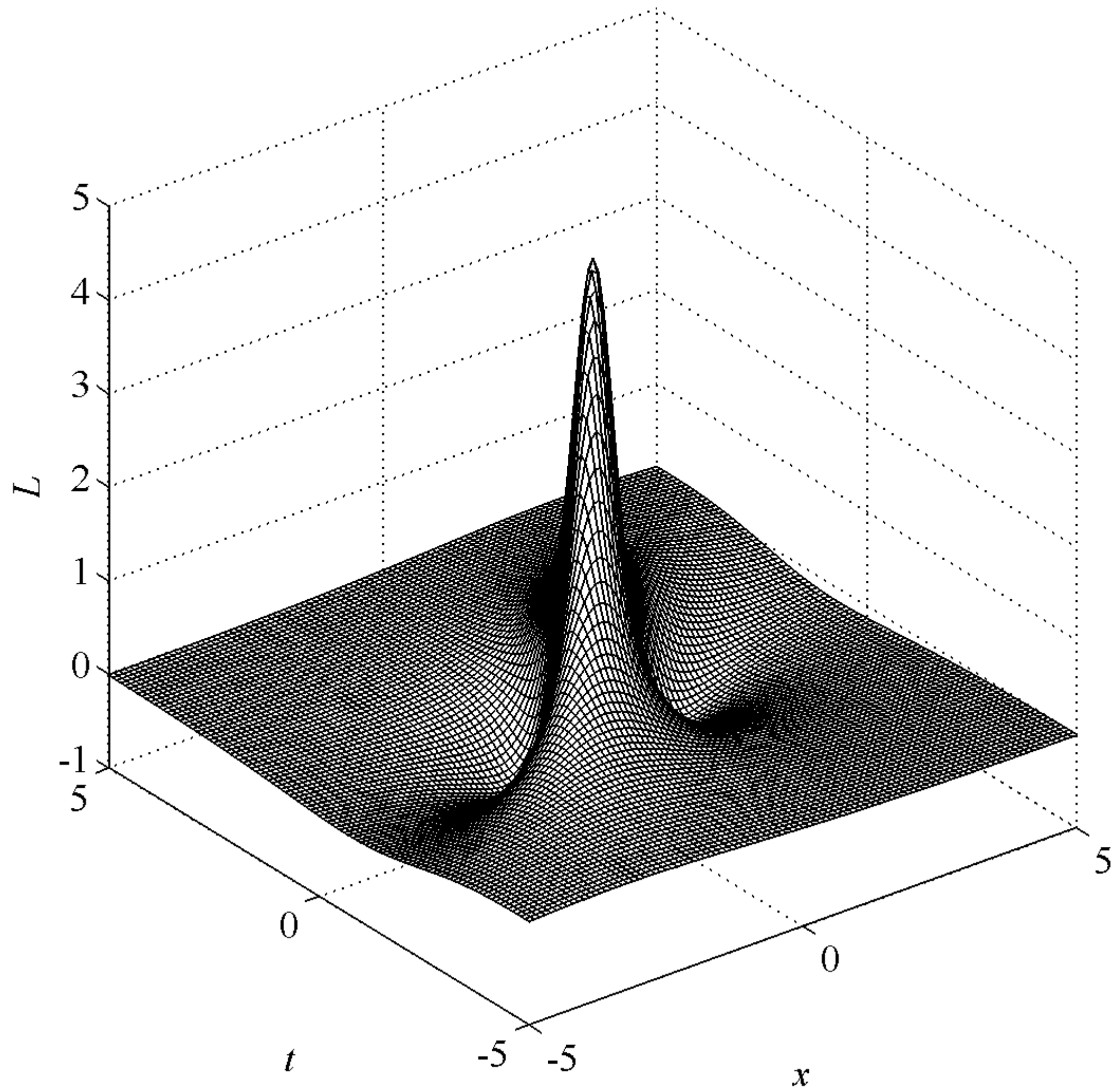
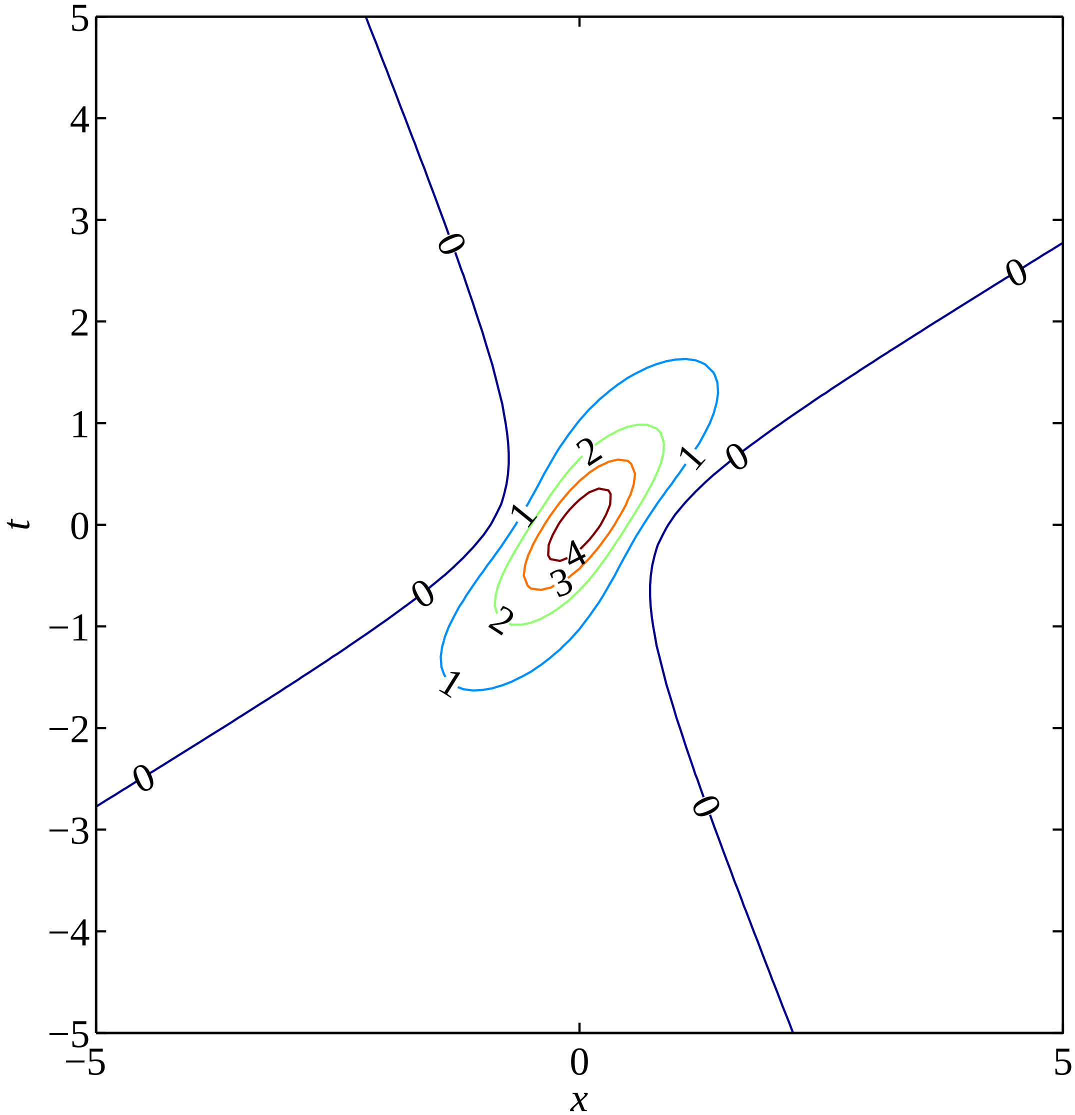


Fig. 2(a)



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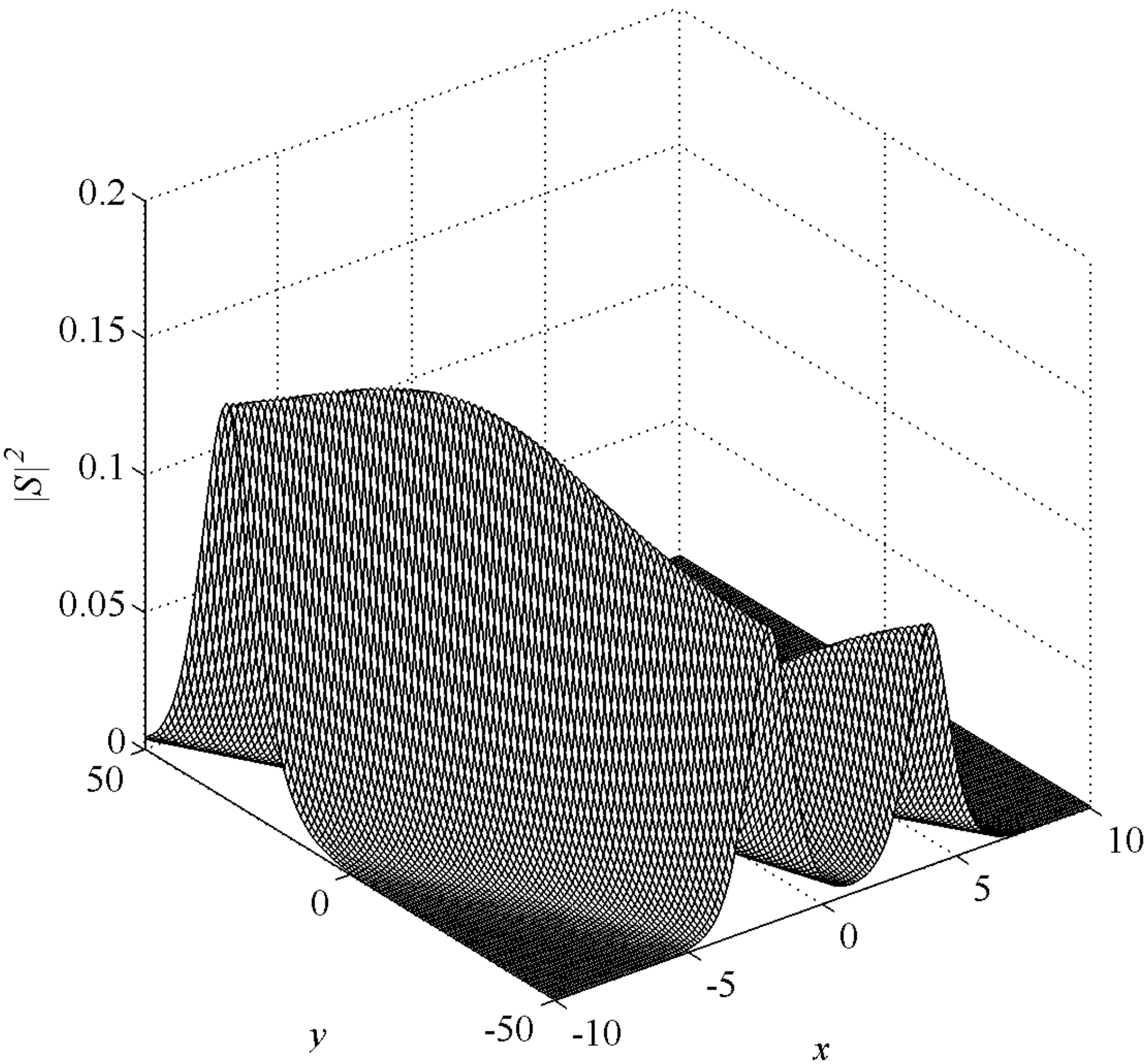

Fig. 3(a)

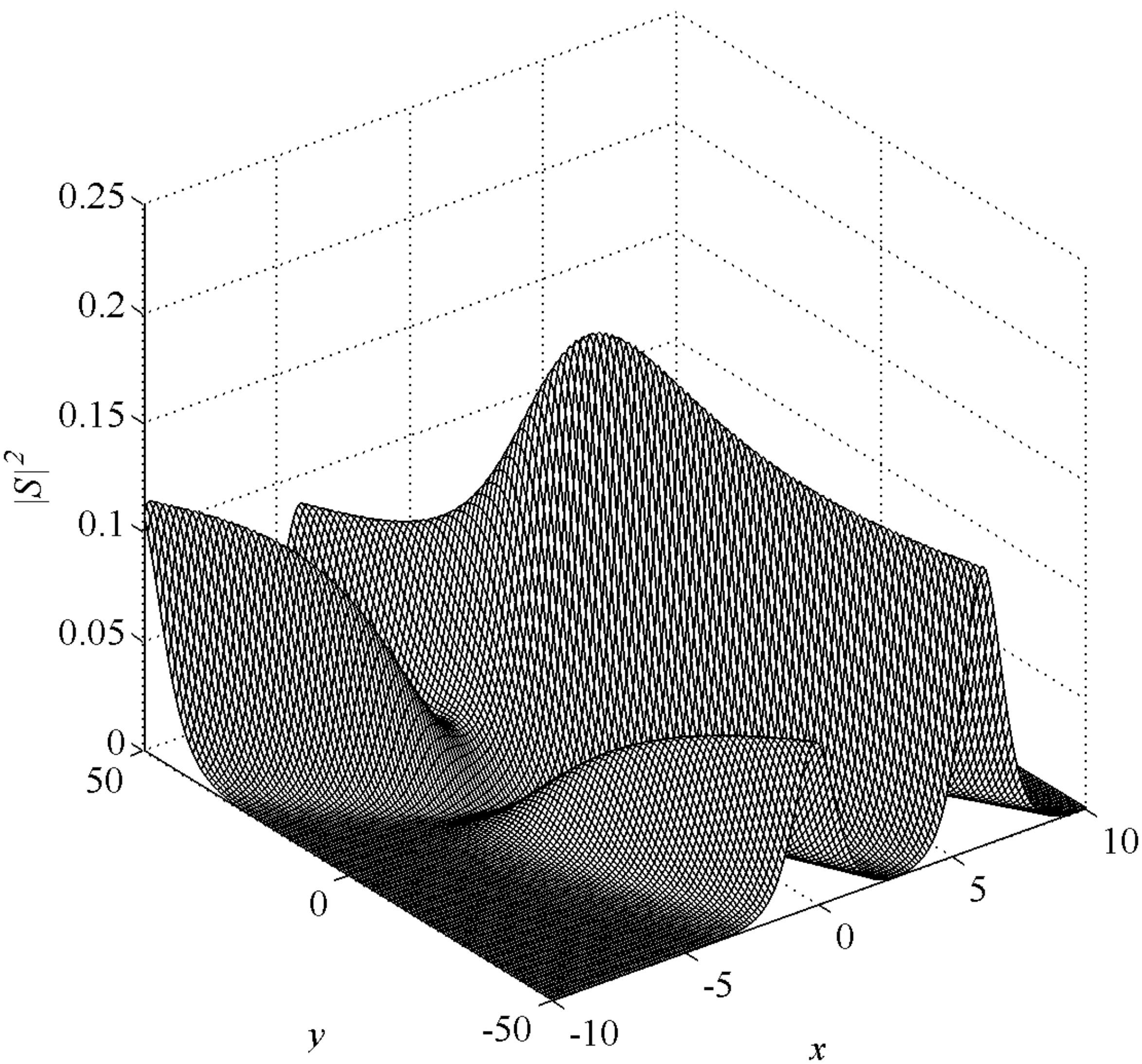

Fig. 3(b)