Rogue waves as spatial energy concentrators in arrays of nonlinear waveguides

Yuliy V. Bludov,[1,*] Vladimir V. Konotop,[2,3] and Nail Akhmediev[4]

[1]*Centro de Física, Universiade do Minho, Campus de Gualtar, Braga 4710-057, Portugal*
[2]*Centro de Física Teórica e Computacional, Universidade de Lisboa, Complexo Interdisciplinar, Avenida Professor Gama Pinto 2, Lisboa 1649-003, Portugal*
[3]*Departamento de Física, Faculdade de Ciências, Universidade de Lisboa, Campo Grande, Edifício C8, Piso 6, Lisboa 1749-016, Portugal*
[4]*Optical Sciences Group, Research School of Physics and Engineering, Institute of Advanced Studies, The Australian National University, Canberra ACT 0200, Australia*
Corresponding author: bludov@fisica.uminho.pt

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In an array of nonlinear waveguides, a giant compression of the input beam can be achieved by exciting a rogue wave. Input field almost homogeneously distributed over hundreds of waveguides concentrates practically all the energy into a single waveguide at the output plane of the structure. We determine the required input profile of the electric field to achieve this. We illustrate the phenomenon by modeling the array by direct numerical simulations of the discrete nonlinear Schrödinger equation. © 2009 Optical Society of America

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Rogue waves can be defined as strongly compressed wave packets with high energy content that appear on the otherwise chaotic average wave field. As the chaotic field creates special initial conditions for their appearance randomly, they tend to be unexpected, thus "appearing from nowhere and disappearing without a trace" [1]. The term "rogue waves" comes from oceanography [2], where such waves played a destructive role, and being the most powerful wavelets of ocean waves destroyed many ships over the years. Although in an open ocean rogue waves are undesirable and dangerous, they can be useful in optics [3], opening ways for generating powerful ultrashort pulses with high concentration of energy in the peaks.

One of the main features of the rogue waves until now was their chaotic nature; their appearance in the open ocean is defined by the chaotic nature of the waves in general. Even in optics, their experimental observation is based on detection of random events [3], despite the fact that laboratory conditions would allow us to make experiments more deterministically.

The energy compression into peaks occurs mainly because of the modulation instability (MI) of certain types of nonlinearity [4]. In optics, these are known as "self-focusing types." For MI with an arbitrary period of modulation within the gain bandwidth, the wavelets would be periodically located. For the period approaching infinity, there can be just a single maximum. The latter is usually described as a limiting case and is given by a rational expression [1].

These strong wavelets can also be obtained as nonlinear superpositions of Akhmediev breathers [5], and for integrable systems they can be constructed in analytical forms. When a system loses integrability, analytic solutions also disappear. However, for systems that are still close to being integrable, numerical solutions allow us to reproduce general features of rogue waves as wavelets that compress an appreciable amount of energy into a single highly localized spot. In this Letter, we consider an example of such system.

Namely, we study an array of coupled nonlinear waveguides, described by the discrete nonlinear Schrödinger (DNLS) equation [6],

$$i\dot{q}_n + q_{n+1} + q_{n-1} - 2q_n + \sigma|q_n|^2 q_n = 0. \tag{1}$$

Hereafter $\dot{q}_n \equiv \mathrm{d}q_n/\mathrm{d}\zeta$, ζ is the propagation coordinate (see the figures below), q_n is the dimensionless field amplitude in the nth waveguide, and $\sigma = 1$ and $\sigma = -1$ stand for focusing and defocusing nonlinearities.

We are interested in the process of a controlled formation of a rogue wave, described by Eq. (1). Physically, the process of rogue-wave formation is the inverse of diffraction of the total energy in a single waveguide into neighboring ones. Reversing this process can be done numerically, but on the physical level it involves two main ingredients necessary for observing the phenomenon: the modulational instability and the properly chosen initial conditions. In practical terms, we define the input signal that leads to giant concentration of practically all energy at the output into a single (or a few) waveguide(s). Unlike the conventional ocean and optical rogue waves we consider a wave evolving in space. Thus, our solutions represent rogue waves "frozen" in time.

The solutions of Eq. (1) with $\sigma = \pm 1$ are linked by the staggered transformation [7]. If $q_n(\zeta)$ is a solution of Eq. (1) for $\sigma = 1$, then $(-1)^n \bar{q}_n(\zeta)\exp(-4i\zeta)$ (where an overbar stands for the complex conjugation) is the solution of the same equation, but for $\sigma = -1$. Therefore the following consideration is limited to the case $\sigma = 1$.

In the low-amplitude limit, smooth solutions of Eq. (1) can be approximated by the respective solutions of the nonlinear Schrödinger equation (NLSE). This

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equation is obtained using the ansatz $q_n = \epsilon\psi(x,z)$, where ϵ is a small parameter, $x=\epsilon n$, and $z=\epsilon^2\zeta$. For $\sigma = 1$, we have $i\psi_z+\psi_{xx}+|\psi|^2\psi=0$. The NLSE has an exact solution in the form of a rogue wave [1]. Of course, we do not expect validity of this solution for the whole spatial interval for our discrete model. However, strong localization can occur in the case of discrete equations as well. Namely, for the input conditions whose profiles over the waveguides are smooth enough, one can expect the formation of rogue waves similar to those in the continuous model.

Following this strategy and assuming that the length of each waveguide in the array is L, we solve Eq. (1) subject to the initial condition

$$q_n(0) = Q_n \equiv \epsilon\left(1 - 4\frac{1-2i\epsilon^2 L}{1+2\epsilon^2 n^2 + 4\epsilon^4 L^2}\right)e^{-i\epsilon^2 L} \quad (2)$$

and look for the output profile at $\zeta=L$, i.e. $q_n(L)$. In the initial condition (2) $\epsilon \ll 1$ is the control parameter, which can be interpreted as the background input intensity. It also controls the discreteness effect, as in the limit $\epsilon \to 0$ Eq. (2) approximates the initial condition of the corresponding exact solution for the NLSE [1].

To study spatial evolution of solutions of Eq. (1) with the initial condition (2) we have performed numerical simulations. In the limiting case of a homogeneous distribution with the intensity ϵ^2 one would observe the well-known MI scenario (see, e.g., [8]). However, even smooth modulation of the initial amplitude and phase, results in a very different dynamics. The respective results are summarized in Fig. 1.

Preliminary simulations (Fig. 1a) indeed show strong localization of the almost homogeneous input beam. Only 30 central waveguides are shown in all figures. This solution clearly corresponds to the rogue wave of the respective NLSE. However, even an

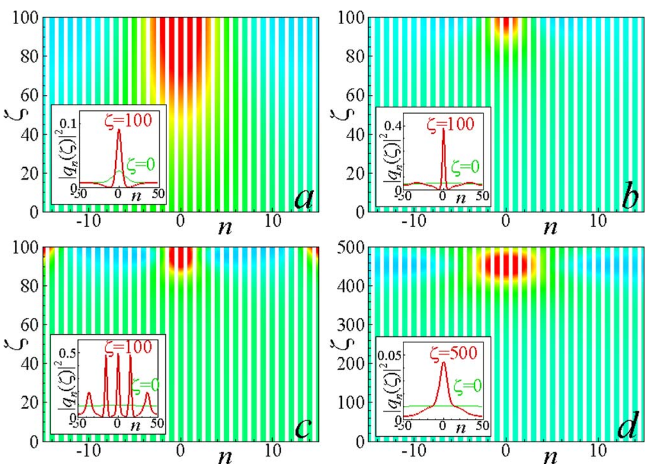

Fig. 1. (Color online) Contour plot of intensity $|q_n|^2$ on the plane (n,ζ). We used periodic boundary condition $q_n=q_{n+N}$ with $N=101$ and initial condition (2). Parameters are a, $\epsilon = 0.1$, $L=100$; b, $\epsilon = 0.2$, $L=100$; c, $\epsilon = 0.3$, $L=100$; d, $\epsilon = 0.1$, $L=500$. The insets show input and output field profiles.

NLSE allows for a much higher degree of compression when a higher-order rational solution is chosen [1].

Thus the above result is not optimized from the viewpoint of energy concentration. We can pose a few questions. In particular, is it possible to enhance the effect by increasing the background intensity ϵ? Or, are the waveguide lengths accurately chosen to give maximal concentration exactly at the output, i.e., at $\zeta=L$, as this happen for the exact NLSE solution corresponding to Eq. (2)? To answer these questions we have performed simulations for various values of ϵ (panels b and c) and for different L (panel d).

First, comparing panels b and c we observe that the growing effect of the discreteness (i.e., large ϵ) leads to a shift of the maximum of the intensity toward the input. This effect becomes even more pronounced, for larger lengths of the waveguides (in the continuum case, an increase of L would lead to the maximal intensity approaching $\zeta=L$). Then the energy concentration occurs inside the structure rather than at the output plane $\zeta=L$ (see Fig. 1d). We also observe that by increasing ϵ one indeed can increase the peak intensity. The increase of ϵ twice results in the peak intensity four times larger (c.f. insets in panels a and b). This happens below a threshold, above which the appearance of MI peaks gradually hampers the existence of the rogue wave (see Fig. 1c). For the parameters of Fig. 1 this threshold is around $\epsilon \approx 0.3$. In the case of big ϵ, the rogue wave can be even indistinguishable from MI background. The fact that we indeed observed MI follows from the direct estimate of the "most unstable" wavenumber $q_0 \approx \epsilon$, which corresponds to the wavelength $\lambda_0 \approx 2\pi/\epsilon$, equal to $\approx$20 waveguides separating the nearest peaks.

Is it possible to enhance the focusing effect using larger number of waveguides? The results of the numerical study answering this question are shown in Fig. 2 (c.f. Fig. 1d). As one can see from Fig. 2b, the maximal intensity grows with an increase of the number of waveguides in array, reaching the absolute maximum at $N \approx 70$, and saturates at the level of $\approx$200 waveguides. This number corresponds to the natural limit $n \gg \epsilon^{-1}$.

So far, we considered the focusing effect based on initial conditions creating the rogue-wave solution of the continuous NLSE. The natural question arises: Is it possible to numerically generate the solution of the DNLS equation that has exactly the properties of the continuous rogue wave that "appear from nowhere

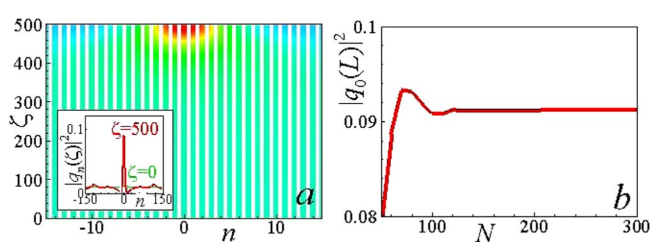

Fig. 2. (Color online) a, Contour plot of intensity $|q_n(\zeta)|^2$ obtained for $N=301$, $\epsilon = 0.1$, and $L=500$. We used periodic boundary conditions and the initial profile (2). b, Output intensity in the central waveguide versus N.

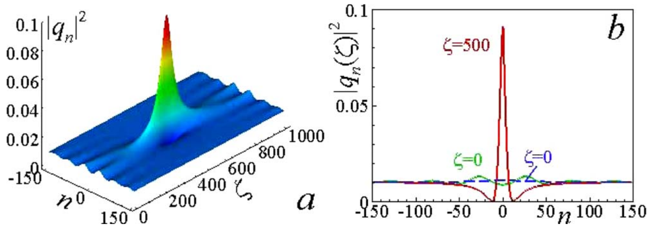

Fig. 3. (Color online) a, Plot of intensity $|q_n|^2$ on the (n,ζ) plane for $\epsilon = 0.1$, $N=301$, $L=500$, obtained from Eq. (1) subject to the periodic boundary and fitted initial conditions. b, Fitted initial condition at $\zeta = 0$ (solid green curve), input intensity (2) (dashed blue line) and the field profile at the middle line $\zeta = 500$ (solid red curve).

and disappear without a trace" [1]. Answering this question would be a justification of the terminology that we use classifying the concentrator effect as a rogue-wave generation.

The exact solution for this problem can hardly be found analytically. Therefore we again employ numerics. The first requirement is obtaining initial conditions (we call them *fitted*) that would obey the equality $q_n(0)=\overline{q_n}(2L)$. The second requirement is that the maximal peak of the rogue wave should appear at $z=L$ in the central waveguide. These are the same properties that characterize the exact rogue-wave solution of the NLSE. Figure 3a shows the numerical solution of this problem, where we show the spatial density distribution of the *discrete rogue wave* having a single maximum with high intensity at the very center of the (n,ζ) plane and symmetric decay of the profile in each direction. The solution was obtained using forward–backward propagation technique with profile adjustments at each cycle. Qualitatively, this solution is similar to the conventional time-dependent rogue wave of the NLSE [1]. To stress this similarity, we plot the 3-D profile [Fig. 3a] along with the transverse intensity distribution at the middle line [Fig. 3b].

Finally, we consider the effect of boundary conditions on the discrete rogue waves. To do this, we performed numerical integration of Eq. (1) subject to the fitted initial conditions with either zero or the "traveling wave" boundary conditions. The latter represents the analytical limit $|n|\rightarrow\infty$ of Eq. (2). The results are presented in Fig. 4. As one can see from Fig. 4a, the zero boundary condition results in a shift of the rogue-wave peak to the values below $\zeta = 500$. Thus the peak appears inside the waveguide array rather than at the output plane. Increasing the number of waveguides can reduce this shift. On the other hand, using traveling wave boundary conditions with numerically fitted initial condition leaves the maximum of the rogue wave at the output plane of waveguide array (Fig. 4b).

The representative experiment could be done by starting with all the energy in a single waveguide

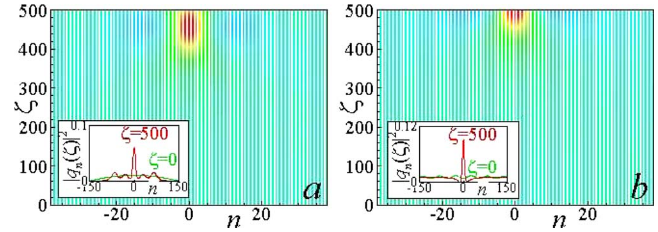

Fig. 4. (Color online) a, Contour plot of intensity $|q_n|^2$ in the (n,ζ) plane. We used the following: a, zero boundary condition $q_{-(N+1)/2}=q_{(N+1)/2}=0$ and the initial condition $q_n(0)=\sqrt{1-4n^2/(N+2)^2}Q_n$ with Q_n given by (2); b, traveling wave boundary conditions $q_{-(N+1)/2}=q_{(N+1)/2}=\epsilon\exp(i\epsilon^2\zeta)$ and the fitted initial condition. Other parameters are $\epsilon=0.1$, $N=301$, and $L=500$. The insets show the input and output field profiles.

and using phase-conjugated and amplified output launched into another similar array. Clearly, it will be focused back into the central waveguide. Our initial condition suggests a close alternative to this solution. It may be not yet the most efficient in terms of concentration, but it provides a profile (Lorentzian) that roughly has the same effect.

To conclude, we found a possibility of observing the discrete rogue waves in arrays of nonlinear optical waveguides. Such rogue waves can be used to construct optical energy concentrators. Giant energy concentration at the output of the array is achieved when initial conditions are properly adjusted to the length of the system and to the number of waveguides. Concentration occurs despite the fact that the input energy has nearly homogeneous distribution.

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References

1. N. Akhmediev, A. Ankiewicz, and M. Taki, Phys. Lett. A **373**, 675 (2009).
2. P. Müller, Ch. Garrett, and A. Osborne, Oceanogr. **18**, 185 (2005).
3. D. R. Solli, C. Ropers, P. Koonath, and B. Jalali, Nature **450**, 1054 (2007).
4. D. H. Peregrine, J. Austral. Math. Soc **25**, 16 (1983).
5. N. Akhmediev, J. M. Soto-Crespo, and A. Ankiewicz, Phys. Lett. A **373**, 2137 (2009).
6. D. N. Christodoulides and R. I. Joseph, Opt. Lett. **13**, 794 (1988).
7. G. L. Alfimov, V. A. Brazhnyi, and V. V. Konotop, Physica D **194**, 127 (2004).
8. F. Kh. Abdullaev, S. A. Darmanyan, and J. Garnier, in *Progress in Optics*, E. Wolf, ed. (Elsevier Science, 2002), Vol. 44, pp. 303–365.