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Role of Singlet Gluon Filters in the Colored Quartet Model of Hadrons*)

Hiroshi SUURA**) and Masaaki KURODA***)

Department of Physics, University of Tokyo, Tokyo

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Quark-antiquark annihilation processes in the colored quartet $SU(4)\times SU(3)'$ model, which proceed through $(\mathbf{1},\mathbf{1})$ congregation of gluons——a singlet gluon filter——, are parametrized in such a way as to give a unified description of mass formulas for 0^{-+}, 1^{--} and 2^{++} meson multiplets and suppression factors for production and decays which are mediated by the singlet filters. $\psi(J)$ (3105) will be discussed in the same context.

It is an obvious and appealing thought to consider the recently discovered ψ(3105)****)[1] in the framework of the quartet-quark model proposed by Tarjanne and Teplitz,[2] Maki,[3] Hara[4] and Bjorken and Glashow.[5] Denoting the four quarks by p, n, λ, c (charge 2/3, $-1/3$, $-1/3$, 2/3), ψ would then be the $c\bar{c}$ member of the $1^{--}(J^{PC})$ 16-plet mesons. In view of the not so small partial width for $\phi\to3\pi$, it is a crucial problem in the quartet model how to explain the small hadronic decay width of ψ. In this note we will consider this problem and an associated problem of the boson mass spectrum in the colored quartet model, say, $SU(4)\times SU(3)'$ scheme. Specifically, we will assume that underlying all the strong interactions there is a single basic interaction of the quark with a set of vector gluons, which are members of the $(\mathbf{1},\mathbf{8})$, i.e., $SU(4)$ singlet and color $SU(3)'$ octet. We will further assume that the color symmetry is exact so that all the hadronic ground states are color singlets. The dynamics of non-Abelian gauge fields are not yet well understood, and we propose a simple parametric description of this general idea.

An effective Hamiltonian for a quark-antiquark (q-$\bar{q}$) system in this model may be written as

$$H=V+m_p(U_p+U_n)+m_\lambda U_\lambda+m_c U_c+S\,, \tag{1}$$

where m_p, m_λ, m_c are p-(n-), λ-, c-quark masses respectively. $U_\lambda=\int\bar{\lambda}(x)\lambda(x)d^3x$ with $\lambda(x)$ denoting the λ-quark field, and a similar definition for U_p, U_n and U_c.

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**) Permanent address: Department of Physics and Astronomy, University of Minnesota, Minneapolis, Mn. 55455.

***) Fellow of the Japan Society for the Promotion of Science.

****) We will use the symbol ψ as a temporary convenience, waiting for emergence of an unanimous name for the particle.

V is the q-$\bar{q}$ potential due to gluon exchanges which is supposed to be very large. S comes from the q-$\bar{q}$ annihilation processes which occur only through $(\mathbf{1},\mathbf{1})$ congregation of gluons, which we call 'singlet gluon filter' or simply 'filter', as long as we consider only color-singlet bosons. Since the gluons are $SU(4)$ singlet, the V term in Eq. (1) alone would give a completely degenerate 16-plet $U(4)$ representation for the q-$\bar{q}$ systems. Thus, if $V \gg m_c \gg m_\lambda$, we will have a broken $U(4)$ mass spectrum in the absence of S. The filter term S, which breaks the so-called Okubo-Zweig-Iizuka-Rosner rule,[6] must be very small, i.e., $m_\lambda \gg S$ for 1^{--} mesons because the mass spectrum of the uncharmed nonet obeys the broken $U(3)$ scheme almost exactly. For 0^{-+} mesons, on the other hand, the S term must be fairly large because it is the only term that is responsible for the $SU(3)$ mass scheme for uncharmed mesons. The difference between the two cases may be related to the minimum number of gluons to make a filter $(\mathbf{1},\mathbf{1})$, which is two for C-parity even 0^{-+} mesons and three for C-parity odd 1^{--} mesons. However, we will not be concerned with inner mechanism of the filters in this paper.

In order to obtain the mass formula and the mixing of q-$\bar{q}$ configurations, we conveniently split the Hamiltonian (1) into three parts; $H_0 = V + m_p(U_p + U_n + U_\lambda + U_c)$, $H_1 = (m_c - m_p)U_c + (m_\lambda - m_p)U_\lambda$ and $H_2 = S$. This is consistent with the condition $V \gg m_c \gg m_\lambda \gg S$. H_0 would give a degenerate squared mass m_0^2 for all the 16 members of a meson multiplet. As a set of basic vectors $\{|\alpha_0\rangle\}$ we choose $|\rho_0\rangle = |p\bar{p} - n\bar{n}\rangle/\sqrt{2}$, $|\omega_0\rangle = |p\bar{p} + n\bar{n}\rangle/\sqrt{2}$, $|\phi_0\rangle = |\lambda\bar{\lambda}\rangle$ and $|\psi_0\rangle = |c\bar{c}\rangle$, where we have listed only central states. A typical state $|\alpha_0\rangle$ represents not only the quark configuration, but also the wave function (the same for all the states) by its subscript 0. The squared mass operator corresponding to S can be defined in terms of $\{|\alpha_0\rangle\}$ by

$$\mathcal{M}_S^2 = m_s^2 \{\sqrt{2}|\omega_0\rangle + |\phi_0\rangle + |\psi_0\rangle\}\{\langle\omega_0|\sqrt{2} + \langle\phi_0| + \langle\psi_0|\}. \tag{2}$$

It should be noted that the filters are exact $SU(4)$ singlet, which is why it is convenient to define $\mathcal{M}_S^2$ in terms of $\{|\alpha_0\rangle\}$. Adding H_1 to H_0, the quark configurations of the basic vectors $\{|\alpha_0\rangle\}$ do not change but the wave functions do. Hence, the eigenstates and eigenvalues of the squared mass operator $\mathcal{M}_0^2 + \mathcal{M}_1^2$ corresponding to $H_0 + H_1$ are given by $|\tilde{\rho}\rangle = |\rho_0\rangle$, $|\tilde{\omega}\rangle = |\omega_0\rangle$, $|\tilde{\phi}\rangle (\neq |\phi_0\rangle)$ and $|\tilde{\psi}\rangle (\neq |\psi_0\rangle)$, with the corresponding squared mass m_0^2, m_0^2, $m_0^2 + m_1^2$ and $m_0^2 + m_2^2$, respectively. We introduce the wave function renormalization

$$Z_1 = \langle\tilde{\phi}|\phi_0\rangle \text{ and } Z_2 = \langle\tilde{\psi}|\psi_0\rangle, \tag{3}$$

where $Z_1 \leq 1$ and $Z_2 \leq 1$. Finally we switch-on the interaction S, which couples the eigenvectors $|\tilde{\alpha}\rangle$ of $\mathcal{M}_0^2 + \mathcal{M}_1^2$. The total mass operator for ω, ϕ, ψ space is given by

$$\mathcal{M}^2 = \begin{pmatrix} m_0^2 + 2m_s^2 & \sqrt{2}Z_1 m_s^2 & \sqrt{2}Z_2 m_s^2 \\ \sqrt{2}Z_1 m_s^2 & m_0^2 + m_1^2 + Z_1^2 m_s^2 & Z_1 Z_2 m_s^2 \\ \sqrt{2}Z_2 m_s^2 & Z_1 Z_2 m_s^2 & m_0^2 + m_2^2 + Z_2^2 m_s^2 \end{pmatrix}. \tag{4}$$

The parameters m_0^2 and m_1^2 can be determined from

$$m_0^2 = m_\rho^2, \quad m_1^2 = 2(m_{K^*}^2 - m_\rho^2). \tag{5}$$

This amounts to using the broken $U(3)$ (or $SU(3)$) scheme. Since $m_s^2 \ll m_1^2, m_0^2$, the off-diagonal elements can be neglected as far as the mass spectrum is concerned. Thus

$$m_\omega^2 = m_0^2 + 2m_s^2, \quad m_\phi^2 = m_0^2 + m_1^2 + Z_1^2 m_s^2, \quad m_\psi^2 = m_0^2 + m_2^2 + Z_2^2 m_s^2. \tag{6}$$

From Eqs. (5) and (6), we can in principle determine the values of m_0^2, m_1^2, m_s^2 and Z_1^2. However, this is not a practical way, because m_s^2 and Z_1 are extremely sensitive to the values of m_ρ^2 and $m_K{}^{*2}$. Instead, using

$$2m_\phi^2 - Z_1^2 m_\omega^2 - (2 - Z_1^2) m_K{}^{*2} = \left(1 + \frac{Z_1^2}{2}\right) m_1^2, \tag{7}$$

we determine m_1^2 as a function of Z_1 near 1. (We take the $K_0{}^*$ mass, i.e., $m_K{}^{*2} = 0.803$.) Then from Eqs. (5) and (6) we determine $m_0^2 = m_\rho^2$ and m_s^2. For $Z_1 = 1 \sim 0.8$ the values of m_1^2, m_0^2 and m_s^2 are quite stable: $m_1^2 = 0.442 \sim 0.451$, $m_\rho^2 = 0.582 \sim 0.577$ and $m_s^2 = 0.016 \sim 0.018$. We find below that the value of m_s^2 in this range is compatible with $\Gamma(\phi \to 3\pi)$. In the last equation of (6), we can safely neglect $Z_2^2 m_s^2$, and we obtain $m_2^2 = 9.05$. The state vectors can be obtained readily neglecting terms involving m_s^4. For example,

$$|\phi\rangle = \frac{\sqrt{2} Z_1 m_s^2}{m_1^2} |\tilde{\omega}\rangle + |\tilde{\phi}\rangle - \frac{Z_1 Z_2 m_s^2}{m_2^2 - m_1^2} |\tilde{\psi}\rangle,$$

$$|\psi\rangle = \frac{\sqrt{2} Z_2 m_s^2}{m_2^2} |\tilde{\omega}\rangle + \frac{Z_1 Z_2 m_s^2}{m_2^2 - m_1^2} |\tilde{\phi}\rangle + |\tilde{\psi}\rangle. \tag{8}$$

The value of m_s^2 can be tested by evaluating $\Gamma(\phi \to 3\pi)$. The 3π decay of ϕ and ψ mesons can occur only through filters, while decays like $\omega \to 3\pi$ and $\phi \to K^+K^-$ occur without going through filters. The coupling of the filter with hadrons (3π) must somehow be suppressed strongly in the case of ψ, because otherwise $\Gamma(\psi \to 3\pi)$ would be too large. In order to estimate such rates we make a postulate stating that the interaction between a filter and hadrons can occur only through q-$\bar{q}$ resonant states. According to this vector-dominance-like postulate, $\phi(\psi) \to 3\pi$ occurs like $\phi(\psi) \to \text{filters} \to \omega \to 3\pi$. Since the same filter process gives rise to the mixing of $\tilde{\omega}$, $\tilde{\phi}$ and $\tilde{\psi}$ configurations, we have only to know the $\tilde{\omega}$ content of $\phi(\psi)$ to calculate $\Gamma(\phi \to 3\pi)$ ($\Gamma(\psi \to 3\pi)$). Hence we have $\Gamma(\phi \to 3\pi) = P(\tilde{\omega}/\phi) \cdot \Gamma(\tilde{\omega} \to 3\pi)$, where $P(\tilde{\omega}/\phi)$ is the fraction of $\tilde{\omega}$ in ϕ and from Eq. (8) is given by $P(\tilde{\omega}/\phi) = (\sqrt{2} Z_1 m_s^2 / m_1^2)^2$. Using the vector-dominance-model we estimate $\Gamma(\tilde{\omega}(1019) \to 3\pi) \sim 200$ MeV, which gives $P(\tilde{\omega}/\phi) \sim 3 \times 10^{-3}$. Another estimate of $P(\tilde{\omega}/\phi)$ is given by the ratio of production cross section of ϕ to that of ω in hadron-hadron collisions, where ϕ or ψ will be produced only through filters. According to a πp collision data by Ayres et al.,[7] $P(\tilde{\omega}/\phi) = (3.5 \pm 1.0) \times 10^{-3}$. On the other hand, our formula gives $P(\tilde{\omega}/\phi)=(2.8\sim2.0)\times10^{-3}$ for $Z_1=1\sim0.8$ in good agreement with the decay and the production data.

$\Gamma(\psi\to3\pi)$ can be treated similarly, and we obtain $\Gamma(\psi\to3\pi)=(Z_2m_1^2/Z_1m_2^2)^2\cdot(k_\psi/k_\phi)^3\xi\ \Gamma(\phi\to3\pi)$. The first factor is equal to $P(\tilde{\omega}/\psi)/P(\tilde{\omega}/\phi)$ according to Eq. (8). k_ψ and k_ϕ are decay momentum of ψ and ϕ, respectively. ξ is an expected, but unknown suppression factor representing high momentum cutoff. Inserting all the known numbers and putting $Z_1\sim1$, we find $\Gamma(\psi\to3\pi)=Z_2^2\xi\times450$ keV. An idea about the magnitude of Z_2 may be obtained by using the non-relativistic positronium wave functions for $|\psi_0\rangle$ and $|\tilde{\psi}\rangle$, which gives $Z_2=8(m_pm_c)^{3/2}\times(m_p+m_c)^3$. If we set $m_p/m_c\sim m_\omega/m_\psi$, we have $Z_2\sim0.5$. An experimental estimate of Z_2 may be made from the production cross section of ψ, which is $\sim10^{-34}$ cm^2 according to the Brookhaven group.[1] Neglecting kinematical differences we expect that $\sigma(\psi)/\sigma(\omega)=P(\tilde{\omega}/\psi)=(\sqrt{2}Z_2m_s^2/m_2^2)^2\sim0.8\times10^{-5}Z_2^2$ according to Eq. (8). Using an experimental value of $\sigma(\omega)=0.16$ mb at $p_L=10$ GeV/c in pp collision by Almedia et al.[8] as a guide, we can make a rough estimate that $\sigma(\psi)\sim Z_2^2\times10^{-33}$ cm^2. Thus we obtain $Z_2^2\gtrsim10^{-1}$, which leads to a reasonable value of $\Gamma(\psi\to3\pi)$ with ξ of the order of a few tenths.

The same mass formula can be applied to the pseudoscalar 16-plet. It should be noted that if H_0+H_2 were diagonalized first ($V>S\gg m_c, m_\lambda$), then we would have broken $SU(4)$ scheme. A $U(4)$ type scheme was used by Maki et al.[9] and it was found there that with $Z_1=Z_2=1$, charmed meson masses came out too low. The difficulty can be avoided if $E(1420)$ is chosen as the ninth member of the multiplet instead of $\eta'(958)$. In this note, we will take conventionally η' as the member. Instead of Eq. (6) we have

$$m_0^2=m_\pi^2=0.018,\quad m_1^2=2(m_K^2-m_\pi^2)=0.46\,. \tag{9}$$

To determine the mass of predominant $c\bar{c}$ configuration, we must know the value of m_2^2. One way to evaluate it will be to assume that the ratio m_2^2/m_1^2 is common for both 1^{--} and 0^{-+} multiplets. In $U(4)$ or $SU(4)$ scheme this is justified group-theoretically, but one can certainly doubt its validity when the charmed quark mass is so large. This assumption gives $m_2^2=9.29$. With this large value of m_2^2, the mass operator (4) practically decouples for $\tilde{\omega}$-$\tilde{\phi}$ sector and $\tilde{\psi}$ sector. (We will keep these notations just to indicate the $q\bar{q}$ configuration.) Diagonalizing the mass operator and using the mass of η and η' as input, we obtain $m_s^2=0.28$ and $Z_1=0.78$, which are insensitive to the value of Z_2. Note that the value of m_s^2 is by a factor 10 larger than the corresponding value for 1^{--} mesons. The mass of η'', the counterpart of ψ, is given by $m_{\eta''}=3.05\sim3.1$ for $Z_2=0\sim1$. The state vectors are given by

$$|\eta\rangle=0.97|\tilde{8}\rangle+0.23|\tilde{0}\rangle-0.02Z_2|\tilde{\psi}\rangle\,,$$
$$|\eta'\rangle=0.97|\tilde{0}\rangle-0.23|\tilde{8}\rangle-0.05Z_2|\tilde{\psi}\rangle\,,$$
$$|\eta''\rangle=|\tilde{\psi}\rangle+Z_2[0.04|\tilde{\omega}\rangle+0.03|\tilde{\phi}\rangle], \tag{10}$$

where

$$|\tilde{0}\rangle=\frac{1}{\sqrt{3}}[\sqrt{2}|\omega\rangle+|\phi\rangle] \quad \text{and} \quad |\tilde{8}\rangle=\frac{1}{\sqrt{3}}[|\tilde{\omega}\rangle-\sqrt{2}|\tilde{\phi}\rangle].$$

The mixing angle is $-13°$. The mass formula for the 2^{++} multiplet can be treated exactly as in the case of 1^{--} mesons. We determine the parameters as $m_1{}^2=0.63$, $m_s{}^2=-0.05$ and $m_0{}^2=m_{A_2}^2=1.70$ for $Z_1=1\sim0.8$. Again assuming that $m_2{}^2/m_1{}^2$ is common for all the 16-plets, we obtain $m_2{}^2=12.8$, which determines the mass of the $J=2$ counterpart of ψ, say f'', as 3.8.

The radiative decay of ψ can be evaluated in terms of $\Gamma(\phi\to\eta\gamma)$, using the $\tilde{\psi}$ component in η and η' given in Eq. (10).

$$\Gamma(\psi\to\eta\gamma)=4\frac{P(\tilde{\psi}/\eta)}{P(\tilde{\phi}/\eta)}\left(\frac{k_\psi}{k_\phi}\right)^3\left(\frac{m_\lambda}{m_c}\right)^2\Gamma(\phi\to\eta\gamma)=Z_2{}^2\left(\frac{m_\lambda}{m_c}\right)^2\times 20\,\text{keV}\,.$$

The factor $(m_\lambda/m_c)^2$ appears assuming that the decay is due to the magnetic dipole transition. Similarly, $\Gamma(\psi\to\eta'\gamma)=Z_2{}^2(m_\lambda/m_c)^2\times130\,\text{keV}$. Taking $(m_\lambda/m_c)^2\sim1/10$ and $Z_2{}^2\lesssim0.5$, these decay rates are quite reasonable. $\Gamma(\psi\to\eta''\gamma)$ should be small because of the small Q value.

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Note added:

We chose ω-like wave functions $|\alpha_0\rangle$ to define the singlet projection operator (2). However, there are no apriori reasons for such a particular choice. We should write instead of (2),

$$\mathcal{M}_S^2=m_s'^2(\sqrt{2}|\bar{\omega}\rangle+|\bar{\phi}\rangle+|\bar{\psi}\rangle)(\sqrt{2}\langle\bar{\omega}|+\langle\bar{\phi}|+\langle\bar{\psi}|),$$

where the wave functions $|\bar{\alpha}\rangle$ represents an average spatial state into which S projects. The matrix elements of $\mathcal{M}_S^2$ involve now three parameters $Z_\omega=\langle\tilde{\omega}|\bar{\omega}\rangle$, $Z_\phi=\langle\tilde{\phi}|\bar{\phi}\rangle$ and $Z_\psi=\langle\tilde{\psi}|\bar{\psi}\rangle$, which are all not greater than 1. Z_ω can be absorbed into m_s^2, so that

$$m_s^2=m_s'^2Z_\omega\,.$$

Thus, instead of Eq. (3), we have

$$Z_1=\langle\tilde{\phi}|\bar{\phi}\rangle/Z_\omega\,,\qquad Z_2=\langle\tilde{\psi}|\bar{\psi}\rangle/Z_\omega\,.$$

Now Z_1 and Z_2 are completely arbitrary parameters which should be determined from experimental data. The analysis made in this paper shows that both Z_1 and Z_2 are less than 1, indicating that the state $|\bar{\alpha}\rangle$ is in fact close to $|\alpha_0\rangle$.

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