Roof-Edge Preserving Image Smoothing Based on MRFs

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Abstract— *A novel Markov random field (MRF) model is proposed for roof-edge (as well as step-edge) preserving image smoothing. Image surfaces containing roof-edges are represented by piecewise continuous polynomial functions governed by a few parameters. Piecewise smoothness constraint is imposed on these parameters rather than on the surface heights as is in traditional models for step-edges. In this way, roof edges are preserved without the necessity to deal with instable higher order derivatives.*

Keywords— **Image smoothing, Markov random fields (MRF), maximum *a posteriori* (MAP), roof edge, smoothness.**

I. Introduction

Image smoothing is aimed at removing corrupting noise and restoring true image surfaces. It is performed based on the *smoothness* constraint about image surfaces which assumes that certain physical properties in a neighborhood present some coherence and generally do not change abruptly. The smoothness is imposed on the image surface function by using a Markov random field (MRF) [1], [2], [3] or regularization [4], [5] formulation.

Edges contain important information for image analysis and an important issue in image smoothing is edge preserving. Two major types of edges are steps and roofs. Step-edge preserving smoothing has been well researched and there exist a number of successful models, such as the line process model [1] in the Markov random field (MRF) framework and the weak string and membrane models [5] in the regularization framework; further studies can be found in [6], [7], [8], [9], [10]. These models assume that the underlying surface has zero first order derivatives and are suitable for preserving step-edges but not for roof-edges. Higher order derivatives have to be dealt with for roof-edges, but such algorithms suffer from instability [5].

In this paper, a novel MRF representation is proposed in which first order piecewise smoothness (to explained in the main text) is used for roof-edge (as well as step-edge) preserving smoothing. Image surfaces are assumed to be a piecewise function governed by a few parameters. Roof discontinuities in the image surface function correspond to step discontinuities in some governing parameter functions. So, it suffices to preserve roof edges if the first order piecewise smoothness is imposed on the parameter functions rather than directly on the image surface function. Roof edges in the surface function, *i.e.* step edges in the parameter functions, are preserved as if step edges in the surface are preserved in the line process model. This extends the ability of first order models (such as the line process and weak membrane models) in edge preserving smoothing.

The rest of the paper is organized as follows: Section 2 provides a parametric model for roof-edges, and describes the proposed MRF model for roof-edge preserving smoothing. Experimental results are presented in Section 3.

II. MRF Modeling for Surfaces Containing Roof Edges

A. Smoothness Constraint

Before presenting the new method, a review is given below on how the smoothness constraint can be imposed. Let $f = \{f_1, \ldots, f_m\}$ be the sample points of a (one dimensional, for the moment) image surface. Assuming that f constitutes a Markov random field (MRF) [11], [12], its joint prior distribution is Gibbsian, $p(f) \propto \mathrm{e}^{-U(f)}$ where $U(f)$ is called the prior energy. $p(f)$ encodes prior assumptions on the interested class of MRFs (*e.g.* the expected type of surfaces in the image). When it encodes the smoothness constraint, $U(f)$ can be considered as a measure of the extent to which the smoothness is violated by f.

Let us consider a flat surface of the form $f(x) = a$, where a is any constant. Its first-order derivative is zero, $f'(x) = 0$, and so the smoothness can be imposed by choosing $U(f) = \int [f'(x)]^2 \mathrm{d}x = \sum_i (f_i - f_{i-1})^2$. The energy takes the minimum value of zero when f is absolutely flat, *i.e.* $f_i = f_{i-1}$ for all i. This is referred to as the *first order smoothness* because it involves the first order derivative. Next, let us consider slanted planar surfaces of the form $f(x) = a + bx$, where $b \neq 0$. Such surfaces have zero second-order derivative. Therefore, the smoothness can be imposed by choosing $U(f) = \int [f''(x)]^2 \mathrm{d}x = \sum_i [f_{i+1} - 2f_i + f_{i-1}]^2$. This is referred to as the *second order smoothness*.

For edge-preserving smoothing, *piecewise continuous* surfaces are considered. For piecewise flat surfaces containing step edges, the following *first order piecewise smoothness* can be used

$$U(f) = \int g(f')\mathrm{d}x = \sum_i g(f_i - f_{i-1}) \qquad (1)$$

$g(\eta)$ should satisfy several conditions [10] of which a necessary condition, $\lim_{\eta \to \infty} g'(\eta) = C$ ($C \geq 0$ is a constant), is the key for the edge-preserving capability. When g is also a function of η^2, then $g'(\eta) = 2\eta h(\eta)$ where $h(\eta)$ is some function which determines interaction between neighboring points.

When applied to the surface heights (pixel values), the first order piecewise smoothness such as encoded by the line-process [1] and the weak string and membrane [5] do not care about roof edges; they allow the surface to crease without increasing the energy [5]. As such, they always smooth out roof edges and thus are unsuitable for detecting roof edges [5].

One might suggest to use higher order piecewise smoothness, such as weak rod and plate [5]. However, such models are usually not recommended because of their instability [5]. In the following, a roof-edge model is defined by using a piecewise polynomial function, and a novel MRF model encoding the first order piecewise smoothness is proposed to perform roof-edge preserving smoothing.

B. Modeling Roof-Edges

An *ideal roof edge* can be modeled as a joining point of two planar surfaces. Fig.1 shows a few examples of roof edges, noting that they are not necessarily local extrema in surface height. A planar surface takes the form $z(x) = a + bx$ if it is on a 1D domain, or $z(x, y) = a + bx + cy$ if on 2D, where a, b and c are some constant parameters. Let the parameters be denoted collectively by $f = \{a, b, c\}$ and a parameterized plane be denoted by $z = z_f(x, y) = a + bx + cy$. Each distinct set of constants $\{a, b, c\}$ represents a distinct plane. For a slanted plane, at least one of b and c is nonzero.

Let us look at Fig.2 to see what happens when the line process model is used to smooth $z(x)$: the discontinuity in b (also in a) is smoothed out and the information about the roof is lost. However, imposing the first order piecewise smoothness on these parameters can solve this problem; see below.

A *general roof* can be modeled as a joining point of two non-linear surfaces represented by $z = z_f(x, y) = a(x, y) + b(x, y)x +$

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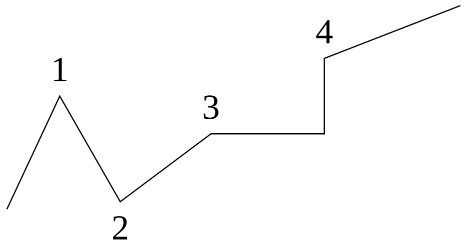

Fig. 1. Examples of roof edges in 1D. Corner points 1, 2 and 3 are roof edge points. No.4 is a roof to the right but a step to the left.

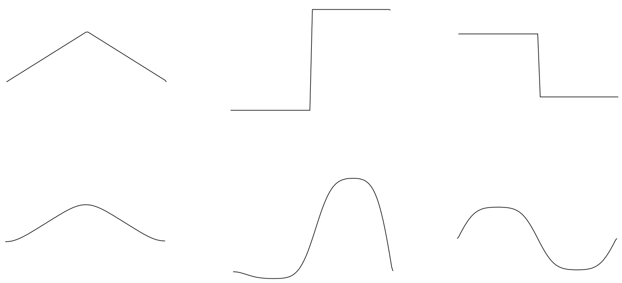

Fig. 2. A clean ideal roof in 1D before (top) and after (bottom) being smoothed by the line-process model. From left to right are $z(x) = a(x)+b(x)x$, $a(x)$, and $b(x)$ functions, respectively. The smoothing is performed on the $z(x)$. The $a(x)$ and $b(x)$ in the figure are computed from the respective $z(x)$ functions. The discontinuities in a and b, and hence the roof edge, are smoothed out.

$c(x, y)y$ where a, b and c are now some functions of x and y. A step edge is a step (C^0) discontinuity in $z(x, y)$; a roof is signaled by a step (C^0) discontinuity in either $b(x, y)$ or $c(x, y)$. In addition, there is usually also a discontinuity in $a(x, y)$ at either a roof or a step discontinuity. If $a(x, y)$, $b(x, y)$ and $c(x, y)$ are (piecewise) constant, then $z_f(x, y)$ degenerates to be (piecewise) planar. If they are (piecewise) C^0 continuous, then $z_f(x, y)$ consists of (piecewise) nonlinear surface(s).

For notational convenience, let the points on the x-y grid be indexed by a single index $i \in \mathcal{S} = \{1, \ldots, m\}$ where $\mathcal{S}$ is called the set of *sites* in MRF literature. When all the $z_i = z_f(x_i, y_i)$ belong to a single planar surface of the form $z(x, y) = a+bx+cy$, the problem of finding the underlying a, b and c parameter constants can be well solved, for example, by using the least squares method. However, when they are due to piecewise surfaces and the segmentation of the data into separate surfaces is unknown, finding the underlying parameters $f = \{a, b, c\}$ as piecewise functions is a non-trivial problem.

When the segmentation is unknown, we may consider the parameters are some functions of (x, y): $a_i = a(x_i, y_i)$, $b_i = b(x_i, y_i)$ and $c_i = c(x_i, y_i)$, collectively denoted by $f_i = [a_i, b_i, c_i]^T$. As such, the surface heights is reconstructed as $z_i = z_{f_i}(x_i, y_i) = a_i + b_i x_i + c_i y_i$. Consider a_i, b_i and c_i as random variables, f_i a random vector, and $f = \{f_1, \ldots, f_m\}$ a random field. We have the following remarks about relationship between the a, b, c functions and the local configuration of f:

- In the case of a single planar surface, the parameter functions $a(x, y)$, $b(x, y)$ and $c(x, y)$ are constant over the x-y domain. Therefore, there must be $f_i = f_{i'}$ for all $i \neq i'$.
- In the case of a piecewise planar surface, the functions are piecewise constant over the x-y domain. Therefore, $f_i = f_{i'}$ holds only when both z_i and $z_{i'}$ belong to the same plane, and $f_i \neq f_{i'}$ otherwise. Discontinuities occur at boundary locations where two surfaces meet.
- In the case of a single nonlinear surface, the functions $a(x, y)$, $b(x, y)$ and $c(x, y)$ are C^0 continuous over the x-y domain. Therefore, there should be no discontinuity between f_i and $f_{i'}$ when i and i' are neighboring sites.
- In the case of a piecewise nonlinear surface containing roofs, *i.e.* C^1 discontinuities in z, the parameter functions are piecewise C^0 continuous over the x-y domain. Therefore, two neighboring parameter vectors f_i and $f_{i'}$ should be similar when z_i and $z_{i'}$ belong to a continuous part of the surface; otherwise, there should be a discontinuity between f_i and $f_{i'}$.

C. MAP-MRF Solution

Firstly, let us define the prior for the parameter vector field $f = \{f_1, \ldots, f_m\}$ which is assumed to be Markovian. The $a(x, y)$, $b(x, y)$ and $c(x, y)$ are piecewise constant functions when $z(x, y)$ consists of piecewise planar surfaces, and therefore their partial derivatives with respect to x and y should be zero at non-edge locations. These constraints may be imposed by using the following "prior energy"

$$U(f) = \sum_i \sum_{i' \in \mathcal{N}_i} g(\rho(f_i, f_{i'})) \tag{2}$$

where $\mathcal{N}_i$ is the set of neighbors of i (*e.g.* the 4 or 8 directly adjacent points), g is an adaptive potential function satisfying certain conditions [10] including the necessary condition mentioned earlier (*e.g.* $g(\eta) = 1/(1 + \eta^2)$ used for experiments in this paper), and

$$[\rho(f_i, f_{i'})]^2 = w_a[a_i - a_{i'}]^2 + w_b[b_i - b_{i'}]^2 + w_c[c_i - c_{i'}]^2 \tag{3}$$

is a weighted distance between f_i and $f_{i'}$ where $w > 0$ are the weights.

Then, let us define the observation model. Assume that the data $d = \{d_1, \ldots, d_m\}$, $d_i = z_{f_i}(x_i, y_i) + e_i$, is the true surface height z_i corrupted by independently and identically distributed (i.i.d.) Gaussian noise e_i. Then the conditional density of data $p(d \mid f)$ is a Gibbs distribution with the following "likelihood energy"

$$U(d \mid f) = \sum_i [z_{f_i}(x_i, y_i) - d_i]^2 = \sum_i (a_i + b_i x_i + c_i y_i - d_i)^2 \tag{4}$$

This imposes the constraint from the data.

Now, the posterior $p(f \mid d)$ is a Gibbs distribution with the "posterior energy" function $U(f \mid d) = U(d \mid f) + U(f)$. For $z = a + bx + cy$, the energy can be written as

$$U(f \mid d) = \sum_i (a_i + b_i x_i + c_i y_i - d_i)^2 + \sum_i \sum_{i' \in \mathcal{N}_i} g(\sqrt{w_a[a_i - a_{i'}]^2 + w_b[b_i - b_{i'}]^2 + w_c[c_i - c_{i'}]^2}) \tag{5}$$

The MAP estimate is defined as $f^* = \arg\min_f U(f \mid d)$.

The proposed MAP-MRF model generalizes the line process model of [1]. The latter is a special case of the former with $b_i = c_i = 0$: When $b_i = c_i = 0$, then the surface function is reduced to $z_i = a_i$ and the two models become equivalent.

D. Energy Minimization

The posterior energy $E(f) = U(f \mid d)$ may be minimized by using a deterministic gradient descent algorithm to achieve $\frac{\partial E(f)}{\partial a_i} = \frac{\partial E(f)}{\partial b_i} = 0$ for all i. Annealing can be incorporated into the iterative gradient descent process to help escape from local minima. For example, introduce a "temperature" parameter T

into the h function so that it becomes $h_T(\eta) = \frac{1}{1+\eta^2/T}$. At the beginning, T is set to a high value $T^{(0)}$. As the iteration continues, it is decreased towards the target value of 1 according to, *e.g.* $T^{(t+1)} \leftarrow 0.9T^{(t)}$. This is shown to be effective.

Randomization can also be incorporated into the gradient descent to help escaping from local minima. In a simple method called randomized gradient descent, a random weight λ is applied between the two terms in every updating for every i and n. It is more efficient than random sampling methods such as Metropolis algorithm [13] and Gibbs sampler [1] because the acceptance probability of any update is always one. A randomized neighborhood system [14] also works well in avoiding local minima.

III. Experimental Results

Range images are used because in such data there are geometrically well defined roof and step edges. The 8-adjacency is used for defining the MRF neighborhood system. The surfaces are assumed to be of the form $z(x_i, y_i) = a_i + b_i x_i + c_i y_i$. The a_i, b_i and c_i parameters are initially estimated from the data by local bilinear fit. The MAP estimates a^*, b^* and c^* are computed by minimizing $U(f \mid d)$ (using an iterative gradient descent combined with annealing). The smoothed surface is reconstructed as $z_i^* = a_i^* + b_i^* x_i + c_i^* y_i$. Roof edges are detected by thresholding the directional derivatives of b^* and c^*, and step edges are detected by thresholding z^*; this is followed by a local maximum selection operation to get the final edge map.

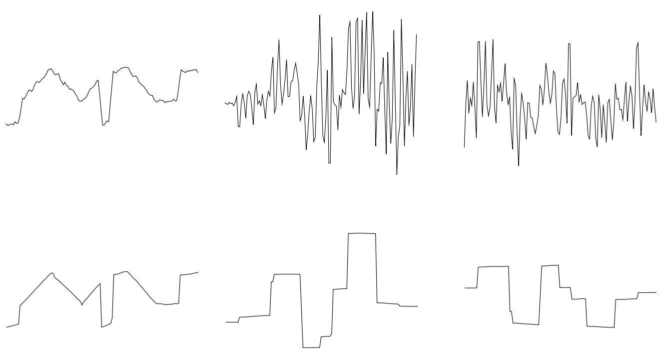

Fig. 3. Smoothing 1D surface. Top (from left to right): surface data $d(x)$, initial estimates for $a(x)$ and $b(x)$ obtained by using finite difference. Bottom (from left to right): the smoothed surface $z^*(x) = a^*(x) + b^*(x) \cdot x$, and the MAP estimates $a^*(x)$ and $b^*(x)$.

Before presenting results for image surfaces on the 2D domain, let us look at a result for 1D surface. For the clean ideal roof in Fig.2, the proposed model produces results which are very close to the analytically calculated shapes shown in the top row of Fig.2, with sharp steps in a^* and b^* and hence a sharp roof in z^*. For a noisy input (Fig.3), the discontinuities in a^* and b^* are well preserved and there are no round-up's over the discontinuities. This demonstrates significant improvement over the line process model result shown at the bottom of Fig.2.

Now, let us look at two results for 2D surfaces. The first is obtained from a synthetic pyramid image composed of piecewise planar surfaces (Fig.4) containing ideal roof edges. The smoothed image has clean surfaces and clear roof boundaries as can be seen in the 3D plots. The the a, b, c and the edge maps before and after smoothing demonstrate the removal of noise and the preservation of discontinuities in the a, b and c parameter functions. The detection of step edges (the out-most

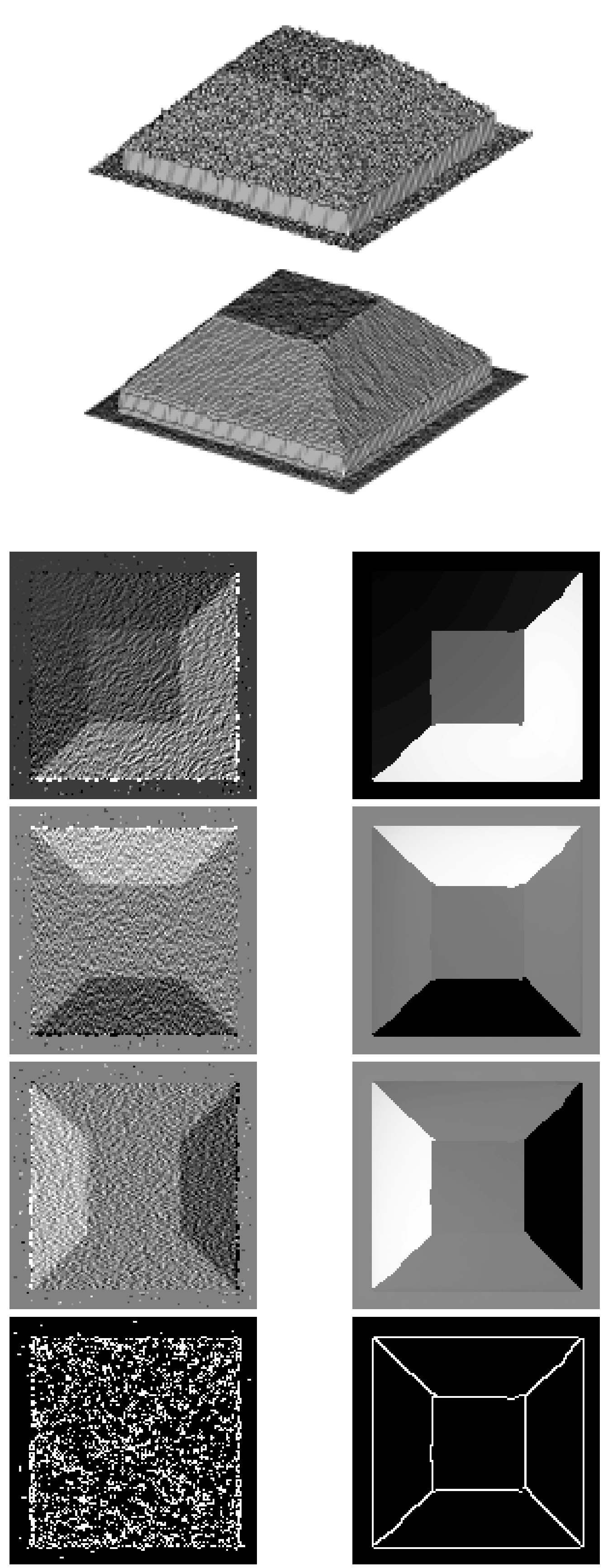

Fig. 4. Upper part: 3D plots of a pyramid image before and after smoothing. Lower part: The parameter and edge maps of the pyramid before (left) and after (right) smoothing; from top to bottom are: parameter maps a, b and c and the detected edges.

part of the edge map) and roof edges (the inner part) is nearly perfect even with the simple thresholding edge detector.

The second result is obtained on a Renault part image composed of free-form surfaces with roof edges (Fig.5), noting that the orginal image is severely quantized in depth. In this case, a, b and c are piecewise continuous nonlinear functions (of x and y) rather than piecewise constant. True roof edges emerge as a result of the edge-preserving smoothing. This result demonstrates that the proposed model also works well for free-form surfaces containing general roof edges.

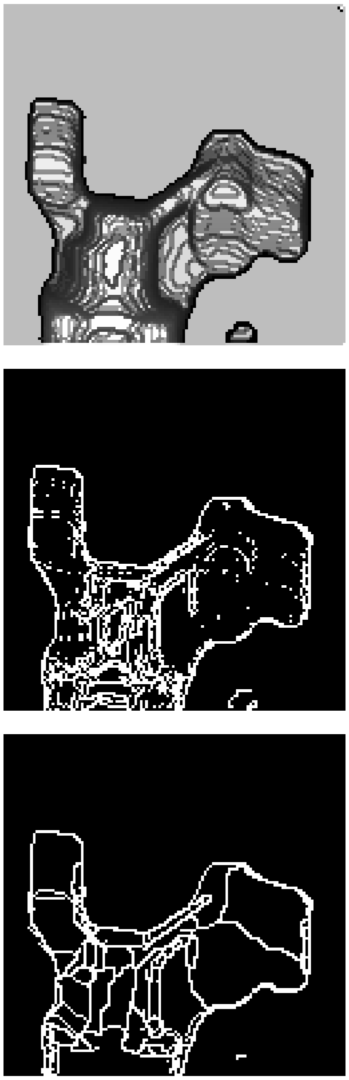

Fig. 5. The Renault part image (top, shaded for visualization of the shape) and edges detected before (middle) and after (bottom) roof edge preserving smoothing.

IV. Conclusion

A new MRF model has been proposed for roof-edge (as well as step-edge) preserving smoothing. The approach is the following: The image surface function is modeled using a parametric polynomial representation. Based on this, the problem of preserving C^1 discontinuities in the image surface function is converted to preserving C^0 discontinuities in the governing parameter functions; in other words, the piecewise smoothness is imposed on the parameter functions rather directly on the image surface height function itself. In this way, only the first derivatives are required to preserve roof edges. This avoids the instability of using higher order derivatives and is the advantage of the proposed MAP-MRF model. As such, the model extends the ability of the existing first order piecewise smoothness models in edge preserving smoothing.

Acknowledgment This work was supported by NTU-AcRF RG 43/95 and RG 51/97.

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