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Rotation Shear Induced Fluctuation Decorrelation in a Toroidal Plasma

T.S. Hahm

Princeton University, Princeton Plasma Physics Laboratory,

P.O. Box 451, Princeton, NJ 08543

Abstract

The enhanced decorrelation of fluctuations by the combined effects of the $\mathbf{E}\times\mathbf{B}$ flow (V_E) shear, the parallel flow ($V_{\|}$) shear, and the magnetic shear is studied in toroidal geometry. A two–point nonlinear analysis previously utilized in a cylindrical model [Phys. Fluids B **2**, 1 (1990)] shows that the reduction of the radial correlation length below its ambient turbulence value (Δr_0) is characterized by the ratio between the shearing rate ω_s and the ambient turbulence scattering rate $\Delta\omega_T$. The derived shearing rate is given by

$$\omega_s^2 = (\Delta r_0)^2\left[\frac{1}{\Delta\phi^2}\left\{\frac{\partial}{\partial r}\left(\frac{qV_E}{r}\right)\right\}^2 + \frac{1}{\Delta\eta^2}\left\{\frac{\partial}{\partial r}\left(\frac{V_{\|}}{qR}\right)\right\}^2\right],$$

where $\Delta\phi$ and $\Delta\eta$ are the correlation angles of the ambient turbulence along the toroidal and parallel directions. This result deviates significantly from the cylindrical result for high magnetic shear or for ballooning-like fluctuations. For suppression of flute–like fluctuations, only the radial shear of qV_E/r contributes, and the radial shear of $V_{\|}/qR$ is irrelevant regardless of the plasma rotation direction.

[1]PACS numbers: 52.35.Ra, 52.35.Qz, 52.25.Gj, 52.55.Fa

I. Introduction

Since its discovery in Axisymmetric Divertor Experiment[1] (ASDEX), the high (H)-mode confinement regime has been reproduced in most tokamaks. Common signatures at the transition, which have received considerable theoretical interest, are a decrease of the plasma edge fluctuations, the development of the radial electric field (E_r) shear, and the formation of a transport barrier at the plasma edge. In particular, Doublet III–D (DIII–D) tokamak results[2] show that the region of the E_r shear correlates well with the region of the density fluctuation suppression and the transport improvement. The temporal change in E_r inferred by the rotation measurement of carbon VII (CVII) line is also accompanied by a sudden drop of the density fluctuations measured by the reflectometer.[3]

Naturally, the two–point theory of the eddy decorrelation by the sheared $\mathbf{E} \times \mathbf{B}$ velocity (V_E)[4] and consequent edge fluctuation suppression[5] and its related phenomenology[6] have received a lot of attention. Their predictions[5,6] have compared quite favorably with the experimental data up to now on DIII-D[7] and have led to significant progress in understanding the edge fluctuation suppression which occurs at the transition.

However, this aforementioned theory of the fluctuation suppression by the V_E shear is developed in cylindrical geometry where the poloidal direction remains as one of the symmetry directions. In a tokamak, poloidal symmetry no longer exists. Furthermore, given the fact that most extensive edge fluctuation measurements have been performed on strongly shaped tokamaks such as DIII–D[7] and Princeton Beta Experiment–Modified[8] (PBX-M), the poloidal asymmetry should be taken seriously for a more quantitative analysis.

In the very high (VH)-mode of DIII-D tokamak, the improved confinement regime broadens radially into the core region where the plasma rotates mainly in toroidal direction.[9] To test whether a paradigm of fluctuation suppression by flow shear[7] also works at the tokamak core, the relevant flow shear induced shearing rate which includes the contribution from the parallel flow, $V_{\parallel}(r)$, must be derived. In this regard, there exists a calculation in the context of a cylindrical model.[10] However, we find that the result of Ref. 10 is not applicable to the experimentally relevant case of multi-helicity turbulence when both parallel and perpendicular components of the flow shear exist.

In this paper, we extend the previous two–point nonlinear theory of eddy decorrelation by the V_E shear in a cylindrical geometry[5] to include the effects of the radial shear of $V_{\parallel}$, and the magnetic shear in an axisymmetric high aspect ratio toroidal geometry. An important conclusion of Ref. 5 is that the edge fluctuation suppression becomes appreciable when the shearing rate

$$\omega_s = \left(\frac{\Delta r_0}{\Delta\theta}\right)\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right),$$

(here Δr_0 and $r\Delta\theta$ are the correlation lengths of the ambient turbulence in the radial and poloidal direction) due to the poloidal flow exceeds the random scattering rate of the ambient turbulence $\Delta\omega_T$. Our results can be characterized by a new formula for ω_s which includes the effects of the $V_{\parallel}$ shear, and the magnetic shear in toroidal geometry. Here, the magnetic shear dependence deserves considerable attention because higher quality H-modes are usually obtained with divertor operation. High magnetic shear may also be responsible for the H-mode transition induced by a current ramp–down in a limiter plasma[11] and the confinement improvement of the VH mode in DIII-D with higher triangularity.[9] There exists a class of dissipative fluid turbulence models which has a favorable dependence on the magnetic shear.[12]

The principal results of this paper are:

(i) The shearing rate ω_s in a tokamak is given by

$$\left(\frac{\omega_s}{\Delta r_0}\right)^2 = \frac{1}{\Delta\phi^2}\left\{\frac{\partial}{\partial r}\left(\frac{qV_E}{r}\right)\right\}^2 + \frac{1}{\Delta\eta^2}\left\{\frac{\partial}{\partial r}\left(\frac{V_\parallel}{qR}\right)\right\}^2,$$

where $\Delta\phi$ and $\Delta\eta$ are correlation angles of the ambient turbulence in the toroidal and parallel (to $\mathbf{B}$) directions. The radial shear of the angular frequency weighted by the inverse correlation angle add like components of a vector, because the nonlinear decorrelation occurs along $\mathbf{B}$ as well as across $\mathbf{B}$ due to coupling of the flow shearing and turbulent diffusion. The way in which the local magnetic field pitch ($q = rB_\phi/RB_\theta$) appears is a natural consequence of the poloidal symmetry breaking and the axisymmetry in toroidal direction. In general, the cylindrical formulas do not predict the magnetic shear dependence correctly.

(ii) Only the first term of the ω_s^2 formula is important if the fluctuations are flute-like ($\Delta\eta \gg 1$), and we obtain

$$\omega_s^2 \simeq \left(\frac{\Delta r_0}{\Delta\theta}\right)^2\left\{\frac{1}{q}\frac{\partial}{\partial r}\left(\frac{qV_E}{r}\right)\right\}^2,$$

regardless of the plasma rotation direction. This is true even when $V_\parallel$ is appreciable as expected for the core region of a DIII-D VH mode discharge. Here, $\Delta\theta$ is the correlation angle of the ambient turbulence in the poloidal direction which could be approximated by the inverse of the root mean square value of the poloidal mode number m, i.e., $\Delta\theta \simeq < m^2 >^{-1/2}$.

(iii) The second term shows that ballooning-like fluctuations (finite $1/\Delta\eta$) are more susceptible to the radial shear of $V_\parallel/qR$ than the flute-like fluctuations ($\Delta\eta \gg 1$).

(iv) If $V_\parallel \mathbf{b} + \mathbf{V}_E$ is in the toroidal direction ($\equiv V_\phi \phi$), the formula for ω_s^2 can be written as

$$\left(\frac{\omega_s}{\Delta r_0}\right)^2 \simeq \frac{1}{\Delta\phi^2}\left\{\frac{\partial}{\partial r}\left(\frac{V_\phi}{R}\right)\right\}^2 + \frac{1}{\Delta\eta^2}\left\{\frac{\partial}{\partial r}\left(\frac{V_\phi}{qR}\right)\right\}^2,$$

for high aspect ratio. Therefore, rigid toroidal rotation throughout the plasma ($V_\phi/R =$ constant, note that the diamagnetic flow is not included in the definition of V_ϕ) has no effect on the flute–like fluctuations.

The remainder of this paper is organized as follows. In Sec. II, we present our basic theoretical model and the two–point correlation function evolution equation in a toroidal geometry. In Sec. III, the eddy decorrelation dynamics due to the flow shear is investigated and the shearing rate which characterizes the radial correlation length reduction is derived. We discuss various limiting cases and their possible experimental relevance in Sec. IV. Finally, the Appendix provides a simple illustration of the overall structure of our shearing rate formula.

II. Theoretical Model

We consider a standard fluid model in which fluctuating quantities obey the following nonlinear equation,

$$\left(\frac{\partial}{\partial t} + \boldsymbol{V}_0 \cdot \boldsymbol{\nabla} + \nu\right)\delta H = -\tilde{\boldsymbol{V}}_E \cdot \boldsymbol{\nabla}\delta H + S, \tag{1}$$

where δH is the fluctuating field in configuration space, $\mathbf{V}_0 = \mathbf{V}_E + V_\| \mathbf{b}$, and $\mathbf{V}_E = \mathbf{E}_\mathbf{r}^{(\mathbf{o})} \times \mathbf{B}/B^2$. The right hand side of Eq. (1) consists of the usual $\mathbf{E} \times \mathbf{B}$ nonlinearity ($\tilde{\boldsymbol{V}}_E = \mathbf{B} \times \nabla\delta\Phi/B^2$) due to the fluctuating electric field and the driving source of the turbulence, S. Other subdominant nonlinearities which contain $\mathbf{b} \cdot \nabla$ are ignored for simplicity. In Eq. (1), ν is the linear dissipative operator which ensures the finiteness of the two point correlation function at infinitesimal separation.[13] Since this does not affect the flow shear induced decorrelation dynamics significantly at finite separation, we ignore it in the analysis. Finally, the contribution from $\mathbf{V}_d \cdot \nabla$ (the particle drift across $\mathbf{B}$) is not kept, assuming $\omega_d \ll \Delta\omega_T$ (the ambient turbulent scattering rate). Furthermore, in some fluid turbulence models, this term is absent.[14]

In tokamaks, the poloidal symmetry in a cylinder is broken due to either the toroidicity or strong shaping (e.g., DIII-D and PBX-M). A natural representation of the high-m (poloidal mode number) fluctuating field is provided by the ballooning mode formalism.

$$\delta H(r,\theta,\phi) = \sum_n e^{in\phi} \sum_m e^{-im\theta} \int d\eta e^{i(m-nq)\eta} \delta H_n(\eta, r), \tag{2}$$

where ϕ is the toroidal angle, and η is the ballooning coordinate in the direction of the magnetic field that can be regarded as a Fourier-conjugate of the radial distance from the rational surface. Fast variation of the fluctuation across $\mathbf{B}$ is extracted by the $e^{i(m-nq)\eta}$ factor. Therefore, the dominant radial variation is characterized by $k_r = -n(dq/dr)\eta$. The residual radial variation in δH_n is very slow and r is considered as a parameter in the lowest order ballooning mode formalism. In this paper, we use the (η, ϕ, r) coordinate, where the variation perpendicular to both $\mathbf{B}$ and $\mathbf{r}$ is captured by the ϕ coordinate. The advection part due to the equilibrium flow in Eq. (2) can be decomposed into

$$V_0 \cdot \nabla = \frac{V_E q}{r} \frac{\partial}{\partial \phi} + \frac{V_\parallel}{qR} \frac{\partial}{\partial \eta}.$$

We note that $\mathbf{V}_0$ does not include the diamagnetic flow and this fact must be remembered even when we call $\mathbf{V}_0$ "the equilibrium flow" in this paper.

A useful form for the $\mathbf{E} \times \mathbf{B}$ nonlinearity in this coordinate system has been derived in Ref. 15, and has been utilized for the two-point theory.[13] Then the two-point correlation evolution equation is derived by multiplying Eq. (1) for $\delta H(\eta_1, \phi_1, r_1)$ by $\delta H(\eta_2, \phi_2, r_2)$, ensemble averaging, and symmetrizing the result with respect to $\delta H(\eta_1, \phi_1, r_1)$ and $\delta H(\eta_2, \phi_2, r_2)$. Defining the average and relative coordinates via $(\eta_\pm, \phi_\pm, r_\pm) = (\eta_1, \phi_1, r_1) \pm (\eta_2, \phi_2, r_2)$, and averaging over ϕ_+, we obtain the two-point correlation evolution equation[16] in terms of the relative coordinate,

$$\left\{\frac{\partial}{\partial t} + r_-(\omega'_\perp \frac{\partial}{\partial \phi_-} + \omega'_\| \frac{\partial}{\partial \eta_-}) - (\frac{q}{r})^2 D_- \frac{\partial^2}{\partial \phi_-^2}\right\} < \delta H(1)\delta H(2) >= S_2. \tag{3}$$

Here, S_2 is the source term for the two-point correlation function, and the $\mathbf{E} \times \mathbf{B}$ nonlinearity is approximated as a diffusion process along the perpendicular direction via the standard renormalization.[13]

For small separations, the relative diffusion D_- has a simple form,

$$D_- = 2D_\phi \left\{(\frac{\eta_-}{\Delta\eta})^2 + (\frac{\phi_-}{\Delta\phi})^2 + (\frac{r_-}{\Delta r_0})^2\right\}. \tag{4}$$

Here, D_ϕ is the diffusion coefficient at large separation, and $Rq\Delta\eta$, $R\Delta\phi$, and Δr_0 are the correlation lengths of the ambient turbulence in the parallel, toroidal, and radial directions, respectively.

The radial shear of the angular frequency in perpendicular and parallel directions is given by

$$\omega'_\perp = \frac{\partial}{\partial r}(\frac{V_E q}{r}), \tag{5}$$

and

$$\omega'_\| = \frac{\partial}{\partial r}(\frac{V_\|}{qR}). \tag{6}$$

Here we note that Ref. 4 is the first work which has considered the effects of the radial electric field shear on turbulence using the ballooning mode formalism. However, the effects associated with the $V_{\parallel}$ shear, and the mode structure in toroidal geometry which are the focal points of this paper, have not been investigated. Also, the results of Ref. 4 are only valid when $|dlnE_r/dlnr| \gg |\hat{s}| \equiv |dlnq/dlnr|$.

III. Flow Shear Induced Decorrelation Dynamics

In this section, we derive the appropriate shearing rate ω_s, which should be compared to the ambient turbulent scattering rate, $\Delta\omega_T = 4(q/r\Delta\phi)^2 D_\phi$ (this $\Delta\omega_T$ is identical to the one defined in Ref. 5, our D_ϕ differs from the D of Ref. 5 in normalization). The correlation dynamics due to flow shear can be analyzed by taking various moments of the left hand side of Eq. (3).[16]

$$\partial_t < r_-^2 >= 0, \tag{7}$$

$$\partial_t < \phi_-^2 >= 4D_\phi(\frac{q}{r})^2 \left\{ \frac{< \eta_-^2 >}{\Delta\eta^2} + \frac{< \phi_-^2 >}{\Delta\phi^2} + \frac{< r_-^2 >}{\Delta r_o^2} \right\} + 2\omega_\perp' < r_-\phi_- >, \tag{8}$$

$$\partial_t < r_-\phi_- >= \omega_\perp' < r_-^2 >, \tag{9}$$

$$\partial_t < \eta_-^2 >= 2\omega_\parallel' < r_-\eta_- >, \tag{10}$$

$$\partial_t < r_-\eta_- >= \omega_\parallel' < r_-^2 >, \tag{11}$$

and

$$\partial_t < \eta_-\phi_- >= \omega_\perp' < r_-\eta_- > + \omega_\parallel' < r_-\phi_- >, \tag{12}$$

where

$$< A(\eta_-, \phi_-, r_-) >\equiv \int d\eta_-' d\phi_-' dr_-' G(\eta_-, \phi_-, r_- | \eta_-', \phi_-', r_-') A(\eta_-', \phi_-', r_-'),$$

and G is the two point Green's function for the left hand side of Eq. (3).

Here, we make the following observations: Eq. (7) is simple because the radial coordinate is only a parameter to lowest order in the ballooning representation. The $\mathbf{E}\times\mathbf{B}$ nonlinearity, acting as a turbulent diffusion in the ϕ_- direction due to random $\tilde{E}_r$, couples various moments in Eq. (8). The radial shear in the angular rotation frequencies, $\omega'_\perp$ and $\omega'_\parallel$ cause the cross correlation of the radial and the angular motion.

Integration of Eqs. (7) through (12) yields a solution which has the following asymptotic form for $\Delta\omega_T t > 1$:

$$\frac{<\phi_-^2>(t)}{\Delta\phi^2} = \left[\frac{r_-^2}{(\Delta r_0)^2}\left\{1+\left(\frac{\omega'_\perp\Delta r_0}{\Delta\phi\Delta\omega_T}\right)^2+\left(\frac{\omega'_\parallel\Delta r_0}{\Delta\eta\Delta\omega_T}\right)^2\right\}\right.$$
$$\left. + \frac{1}{\Delta\phi^2}\left(\phi_- + \frac{\omega'_\perp}{\Delta\omega_T}r_-\right)^2 + \frac{1}{\Delta\eta^2}\left(\eta_- + \frac{\omega'_\parallel}{\Delta\omega_T}r_-\right)^2\right]e^{\Delta\omega_T t}. \qquad (13)$$

Defining the fluid eddy lifetime as the time required for the relative separation between two nearby points to reach eddy size, we find

$$\tau_{fe} \simeq \Delta\omega_T^{-1}\ln([\cdots]^{-1}), \qquad (14)$$

where $[\cdots]$ is the expression multiplying $e^{\Delta\omega_T t}$ on the right hand side of Eq. (13), and Eq. (14) is valid when $[\cdots] < 1$. We can note that the radial correlation length has been reduced by the radial shear of the angular rotation frequencies relative to its value Δr_o, determined by ambient turbulence alone:

$$\left(\frac{\Delta r_o}{\Delta r_t}\right)^2 = 1 + \frac{\omega_s^2}{\Delta\omega_T^2}, \qquad (15)$$

where Δr_t is the radial correlation length reduced by the flow shear, and ω_s is the shearing rate which quantifies the effectiveness of the flow shear in reducing the radial correlation length

$$\omega_s^2 = \Delta r_0^2\left\{\left(\frac{\omega'_\perp}{\Delta\phi}\right)^2 + \left(\frac{\omega'_\parallel}{\Delta\eta}\right)^2\right\}. \qquad (16)$$

We note that the way in which each component of the angular frequency shear contributes to ω_s^2 in Eq. (15) is a general consequence of the nonlinear decorrelation due to coupling between the turbulent diffusion and each component of the flow shear. Each component of the angular frequency shear weighted by the inverse correlation angle contributes in a similar fashion to the shearing of the fluctuations, and they add like components of a vector. Therefore, the efficiency of the shear suppression of turbulence depends not only on the equilibrium flow shear, but also on the spatial structure of the fluctuations. This point regarding the summation rule for ω_s^2 in Eq. (16) is illustrated in the context of a simpler cylindrical model in the Appendix. A previous work[10] based only upon modification of the linear propagator has produced a different result which is not applicable to the multi-helicity fluctuations considered here. Finally, we note that other modifications in Eq. (13) due to the flow shear describe the minor distortion of the eddy shape.

IV. Applications and Discussion

Within the context of the one field model described by Eq. (1), Eq. (14) shows that the reduction of the radial correlation length is appreciable when $\omega_s \gtrsim \Delta\omega_T$, where $\Delta\omega_T = 4D_\phi(q/r\Delta\phi)^2$, and the general fomula for ω_s is given by,

$$\left(\frac{\omega_s}{\Delta r_0}\right)^2 = \frac{1}{\Delta\phi^2}\left[\frac{\partial}{\partial r}\left(\frac{qV_E}{r}\right)\right]^2 + \frac{1}{\Delta\eta^2}\left[\frac{\partial}{\partial r}\left(\frac{V_\parallel}{qR}\right)\right]^2 . \tag{17}$$

Now we discuss the relevance of each term in Eq. (17) and the relationship to the simpler cases studied previously. First, we recall that the way in which the local magnetic field pitch q appears in Eq. (17) is a consequence of the fundamental symmetry of an axisymmetric toroidal system. The breakdown of poloidal symmetry leads to the nonconservation of poloidal mode number m. For high-m fluctuations, this is replaced by another approximate symmetry, the quasi-translational invariance utilized via the ballooning mode representation. This is shown in the decomposition of $\boldsymbol{V}_0 \cdot \nabla$ on page 6.

In several relevant limiting cases, the second term on the right hand side of Eq. (17) is a lot smaller than the first term.

i) Flute–like fluctuations:

A flute–like fluctuation aligns itself along $\boldsymbol{B}$ for a long parallel scale length, and thus has $\Delta\eta \gg 1$. Since the fluctuation pitch is almost identical to the pitch of $\boldsymbol{B}$, we have $\Delta\phi^2 \simeq q^2\Delta\theta^2$. Then, Eq. (17) reduces to

$$\left(\frac{\omega_s}{\Delta r_0}\right)^2 \simeq \frac{1}{\Delta\phi^2}\left[\frac{\partial}{\partial r}\left(\frac{qV_E}{r}\right)\right]^2 \simeq \frac{1}{\Delta\theta^2}\frac{1}{q^2}\left[\frac{\partial}{\partial r}\left(\frac{qV_E}{r}\right)\right]^2. \tag{18}$$

Therefore, we recover the previous cylindrical results when

$$\left|r\frac{\partial}{\partial r}\ln\left(\frac{V_E}{r}\right)\right| \gg |\hat{s}|.$$

Equation (18), however, can deviate significantly from the cylindrical result, at the vicinity of separatrix for example, where the magnetic shear $\hat{s}$ becomes large. We emphasize that Eq. (18) is valid regardless of the direction of $\boldsymbol{V}_E + V_{\|}\boldsymbol{b}$ (even when it is along the toroidal direction), and therefore, is applicable to the plasma core as well as the edge.

ii) Ballooning-like fluctuations:

The effectiveness of the parallel angular frequency shear in suppressing turbulence depends on the spatial structure of the fluctuations. The ballooning-like fluctuations with $\Delta\eta \sim \pi$ are more susceptible to the suppression due to the radial shear of $V_{\|}/qR$. As the mode structure becomes more flute–like with increasing $\Delta\eta$, the $\partial(V_{\|}/qR)/\partial r$ term becomes irrelevant.

iii) Purely toroidal $\boldsymbol{V}_0$: $(V_\parallel \boldsymbol{b} + \boldsymbol{V}_E \equiv V_\phi \boldsymbol{\phi})$

Neoclassical theory predicts that, in the tokamak core, $V_\parallel$ and $V_\perp$ couple through magnetic pumping and result in $V_\theta = 0$. If the poloidal component of $\boldsymbol{V}_0$ (which excludes the diamagnetic flow) is negligible,

$$\frac{V_E}{r} \simeq \frac{V_\parallel}{qR} \simeq \frac{V_\phi}{qR} \quad \text{for} \quad \frac{r}{qR} \ll 1.$$

Then, Eq. (17) can be written in an alternative form,

$$\left(\frac{\omega_s}{\Delta r_0}\right)^2 = \frac{1}{\Delta\phi^2}\left[\frac{\partial}{\partial r}\left(\frac{V_\phi}{R}\right)\right]^2 + \frac{1}{\Delta\eta^2}\left[\frac{\partial}{\partial r}\left(\frac{V_E}{r}\right)\right]^2. \tag{19}$$

This form could be more useful for comparison to experimental data from the core due to the absence of an explicit q dependence. Furthermore, $\Delta\phi^2 = q^2\Delta\theta^2$ would be a useful approximation (even when $1/\Delta\eta$ is non-negligible) for comparison to the experimental data, because many diagnostics tend to measure the poloidal correlation length, $r\Delta\theta$, rather than the toroidal correlation length, $R\Delta\phi$. Equation (19) may look similar to the cylindrical model results presented in the Appendix. However, the second term is different. For the following discussion regarding rigid rotation, we can write Eq. (19) using V_ϕ in favor of V_E,

$$\left(\frac{\omega_s}{\Delta r_0}\right)^2 \simeq \frac{1}{\Delta\phi^2}\left[\frac{\partial}{\partial r}\left(\frac{V_\phi}{R}\right)\right]^2 + \frac{1}{\Delta\eta^2}\left[\frac{\partial}{\partial r}\left(\frac{V_\phi}{qR}\right)\right]^2. \tag{20}$$

Therefore, there is no fluctuation suppression if the toroidal rotation is rigid throughout the plasma ($V_\phi/R =$ constant) and the fluctuation is flute–like ($\Delta\eta \gg 1$) .

Since $\Delta\phi < \Delta\eta$ for realistic unstable fluctuations and $q \gtrsim 1$, the first term on the right hand side of Eq. (20) is nominally a larger term. However, for rigid toroidal rotation throughout the plasma, only the second term survives

$$\left(\frac{\omega_s}{\Delta r_0}\right)^2 \simeq \frac{1}{\Delta\eta^2}\left(\frac{V_\phi}{R}\right)^2\left[\frac{\partial}{\partial r}\left(\frac{1}{q}\right)\right]^2. \tag{21}$$

We note that there is a small residual fluctuation suppression due to magnetic shear which could be non-negligible near the separatrix. This residual fluctuation suppression due to magnetic shear may be related to the facts that the divertor H–mode discharges are typically higher quality than the limiter H-mode discharges and the transition to H-mode can be induced by a current ramp–down.[11] Also, the high magnetic shear may contribute to the confinement improvement in VH-modes in DIII-D tokamak through the triangularity.[9] Of course, a quantitative comparison requires an analysis in flux coordinates which account for the shaping effects such as elongation and triangularity.

Acknowledgments

The author would like to thank K.H. Burrell, L. Chen, P.H. Diamond, Y.B. Kim, and W.M. Tang for useful discussions. This work was supported by U.S. Department of Energy Contract No. DE–AC02–76–CHO–3073.

Appendix A. Cylindrical Model

To illustrate the fact that the summation rule for ω_s shown in Eq. (16) is a natural consequence of the multi–helicity fluctuation decorrelation when both components of the radial shear of the angular frequency exist, we consider a simple cylindrical model. Here, the equilibrium flow is decomposed into the poloidal and the toroidal directions,

$$\boldsymbol{V}_0 = \boldsymbol{V}_E + V_{\|}\boldsymbol{b} \equiv V_\theta \boldsymbol{\theta} + V_\phi \boldsymbol{\phi}.$$

After the standard procedures of the two–point renormalization analysis,[4,5,13,16] the evolution equation for the two–point correlation function in the relative coordinate (ϕ_-, θ_-, r_-) can be obtained,

$$\left\{\frac{\partial}{\partial t} + r_-\left(\omega'_\theta \frac{\partial}{\partial \theta_-} + \omega'_\phi \frac{\partial}{\partial \phi_-}\right) - D_- \frac{\partial^2}{\partial r_-^2}\right\} < \delta H \delta H >= S_2, \tag{A1}$$

where $\omega'_\theta = \partial(V_\theta/r)/\partial r$ and $\omega'_\phi = \partial(V_\phi/R)/\partial r$. Here, D_- is the relative diffusion coefficient which has the usual asymptotic form at small separation,

$$D_- = 2D\left\{\left(\frac{x_-}{\Delta x}\right)^2 + \left(\frac{y_-}{\Delta y}\right)^2 + \left(\frac{z_-}{\Delta z}\right)^2\right\}, \tag{A2}$$

$$y_- \equiv r_+\theta_-, \qquad z_- \equiv R_+\phi_-,$$

in which Δx, Δy, and Δz are correlation lengths of the ambient turbulence in the radial, poloidal, and toroidal directions, respectively. We note that a more conventional form of the turbulent diffusion is adopted in Eq. (A1). A slight difference from the one used in Ref. 5 does not change our conclusions regarding the summation rule for ω_s^2 and the scalings. Taking various moments of Eq. (A1), we obtain,

$$\partial_t < x_-^2 >= 4D\left\{\frac{< x_-^2 >}{\Delta x^2} + \frac{< y_-^2 >}{\Delta y^2} + \frac{< z_-^2 >}{\Delta z^2}\right\}, \tag{A3}$$

$$\partial_t < y_-^2 >= 2r\omega_\theta' < x_- y_- >, \tag{A4}$$

$$\partial_t < z_-^2 >= 2R\omega_\phi' < x_- z_- >, \tag{A5}$$

$$\partial_t < x_- y_- >= r\omega_\theta' < x_-^2 >, \tag{A6}$$

$$\partial_t < x_- z_- >= R\omega_\phi' < x_-^2 > . \tag{A7}$$

Equations (A5), (A7), and the last term in Eq. (A3) are the additional features due to the toroidal component of the angular frequency shear. We note that z_- and ω_ϕ' couple to the radial motion in exactly the same way that y_- and ω_θ' do.

Utilizing this fact, a straight forward manipulation of Eqs. (A3)-(A7) yields,

$$\partial_t < x_-^2 >= \Delta\omega_T(< x_-^2 > + < w_-^2 >), \tag{A8}$$

$$\partial_t < w_-^2 >= 2(\Delta r_0)^2\left(\frac{r\omega_\theta'}{\Delta y^2} + \frac{R\omega_\phi'}{\Delta z^2}\right) < P_- x_- >, \tag{A9}$$

$$\partial_t < P_- x_- >= \left\{\frac{(r\omega_\theta')^2/\Delta y^2 + (R\omega_\phi')^2/\Delta z^2}{(r\omega_\theta')/\Delta y^2 + (R\omega_\phi')/\Delta z^2}\right\} < x_-^2 >, \tag{A10}$$

where

$$< w_-^2 > \equiv \Delta r_0^2\left(\frac{< x_-^2 >}{\Delta x^2} + \frac{< y_-^2 >}{\Delta y^2}\right),$$

$$P_- \equiv \frac{(r\omega_\theta'/\Delta y^2)y_- + (R\omega_\phi'/\Delta z^2)z_-}{(r\omega_\theta'/\Delta y^2) + (R\omega_\phi'/\Delta z^2)},$$

$$\Delta\omega_T \equiv 4D/\Delta r_0^2 \qquad \Delta r_0 \equiv \Delta x.$$

Now, the mathematical structure of Eqs. (A8)–(A10) is identical to that of the equations in Ref. 5, in the strong flow shear limit, $\omega_s > \Delta\omega_T$:

$$\frac{\partial^3}{\partial t^3} < x_-^2 >= 2\omega_s^2\Delta\omega_T < x_-^2 >, \tag{A11}$$

where

$$\omega_s^2 = (\Delta r_0)^2 \left\{ \frac{(r\omega'_\theta)^2}{\Delta y^2} + \frac{(R\omega'_\phi)^2}{\Delta z^2} \right\}$$
$$= (\Delta r_0)^2 \left\{ \left(\frac{\omega'_\theta}{\Delta\theta}\right)^2 + \left(\frac{\omega'_\phi}{\Delta\phi}\right)^2 \right\}. \tag{A12}$$

Now, we can observe that the summation rule shown in Eq. (16) emerges in a simpler cylindrical model. Again, we emphasize that the Fourier decomposition used in a cylindrical model ($\delta H = \sum_{n,m} \delta H_{n,m}(r) e^{i(n\phi - m\theta)}$) is **not** a natural decomposition of fluctuations in a toroidal system because m is not a good quantum number. Equation (A12) is derived only for the limited purpose of illustrating the summation rule and Eq. (16) provides the relevant ω_s in a toroidal geometry.

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