Rotational Reshaping and Yield Stress of Rubble-Pile Asteroids

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Using a Soft-Sphere DEM code we simulate the reshaping and disruption of a self-gravitating 3D granular aggregate by increasing its spin rate. During the process, we monitor the evolution of the internal stresses in the aggregate to find its yield stress using the Von Mises and Maximum Stress yield criteria. In the simulations we either increase the number of particles or their density to increase the total mass of the aggregate and find an increment in the yield stress. In addition, once a reshaping spin rate (density dependent) has been reached, its further increase causes further reshape (ultimately fission) and a decrease in the spin rate itself.

Introduction

Given the current understanding that asteroids can be in fact rubble-piles held together by gravitational and cohesive forces, it is necessary to understand their internal mechanics if we are to understand how they have evolved to show the variety of sizes, shapes and configurations that we can attest today. One of the possible reasons for these varied features could be the rotation and subsequent fission of the primary asteroid. The research presented here will deal, not with the reasons for increased rotation, but with the results, i.e., the re-shaping process and the internal stress that the primary undergoes as a result and before fission occurs.

Simulation Details

In our studies we simulate the particles forming the asteroid by means of a Soft-Sphere Discrete Element Method code [1, 2]. Contact forces are calculated through a linear spring-dashpot potential [3]. The overlap of the particles provide the measure of the restoring force of the modeled spring; the dashpot provides a velocity-dependent dissipation. Frictional forces are calculated as the minimum between a tangential spring, located at the contact point of two particles, and dynamic friction.

To calculate gravitational forces [4], we divide the available space using a regular-static 3D grid and keep track of the absolute position of the particles in the grid. The mass of the particles in each cell is assigned to a virtual particle in its centre. The distance between the centres of these cells are calculated and kept in memory. This is then used to calculate the force between two mass points plus a first order correction term that accounts for the granularity and inhomogeneity of the system.

The density of the particles is 3200 kg/m^3 and their sizes are randomly distributed between 8 and 9m. They are contained in a cubic box, 600 m per side, with periodic boundary conditions. The particles are positioned in as many horizontal layers as needed, forming a hexagonal closed packed lattice at the centre of the box and covering its entire cross-sectional area. To start the simulation, friction is turned off and the particles are given a random speed between -2.5 and 2.5 mm/s in each coordinate direction. When the kinetic energy of the system has reached a near steady value, the simulation is started and friction is turned back on. The angular velocity is increased by 9×10^{-5} rad/s every 30000s up until when the total simulation time is 300000s. The stress field in the aggregate is monitored every 500s, and in much more detail during the first 500s after each increase.

Average Stress Tensor due to Contact Forces

In the simulations, the formed asteroid is divided in regular cubes of 25m in size and the average stress tensor of the particles inside the cube is calculated as:

$$\bar{\sigma} = \frac{1}{V} \sum_{c \in K_V} \vec{f} \otimes \vec{l} \tag{1}$$

where V is the volume in which the calculation is taking place, K_V is the set of contacts between the particles in the volume V, $\vec{f}$ is the reaction force due to a contact and $\vec{l}$ is the relative position of one particle with respect to the other (branch vector). The contribution of contacts between particles in different volumes has a weight of 0.5 for the sum in either volume [5]. The principal stresses of the stress tensor of the particles in each volume are then calculated. These values define the stress state of the particles inside each individual volume and will later be used to calculate the value of the yield stress. By convention they will be noted: $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

The exact value of the static part of the stress tensor, calculated as a force per unit area in the faces of the volumes, showed numerical agreement with this averaging process. As the system is not in static equilibrium, in general, the stress tensor is asymmetric and its eigenvalues could be complex. If this happened they were not calculated and left out of the analysis.

Yield Criteria

In order to find the yield stress (σ_y) of an aggregate we have used two definitions. 1. The *Maximum Principal Stress Theory*: Yield takes place when the largest principal stress exceeds the uniaxial tensile yield strength.

$$\sigma_1 \leq \sigma_y \tag{2}$$

2. *Distortion Energy Theory*: The total strain energy can be separated into two components: the volumetric (hydrostatic) strain energy and the shape (distortion or shear) strain energy. Yield takes place when the distortion component exceeds that at the yield point for a simple tensile test. This is generally known as the Von Mises yield criterion and is expressed as:

$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]} \leq \sigma_y \quad (3)$$

Results and Observations

In order to observe whether or not the yield is dependent on the mass/volumen of the aggregate, we carried out our simulations with 2000, 3000, 4000 and 8000 particles with a density of 3200 $\mathrm{kg/m^3}$. The calculated σ_y for these systems are tabulated in table 1.

Table 1: Average σ_y for aggregates with different number of particles and constant density (values given in Pa).

Criterium	2000ptc	3000ptc	4000ptc	8000ptc
MS(Pa)	3.4	4	4.7	7.7
VM(Pa)	1.8	2.4	2.8	4.7

These values of σ_y show a monotonic increase with the mass of the aggregate as expected. In addition, reshaping was found to occur at a spin rate near or smaller than 5.4×10^{-4} rad/s. This value is in good agreement with the predicted value of 4×10^{-4} rad/s ($\omega_{crit} = 0.53\sqrt{4\pi G\rho/3}$) [6].

Next, we keep a constant number of particles (3000) and change their density to 3900 and 4600 $\mathrm{kg/m^3}$. The calculated σ_y for these systems are tabulated in table 2. Here also, the increased mass of the aggregate showed an increase in the value of the σ_y. The spin rate at which reshaping started also changed with the change in mass, from near to 5.4×10^{-4} rad/s to 6.3×10^{-4}. In all cases here studied, after a reshaping spin rate was reached, its continued increase would only cause further reshaping (finally fission) with the consequent decrease in spin rate due to the increased inertia. Figure 1 shows the evolution of the spin-rate for equal-size aggregates with different densities.

Table 2: Average σ_y for aggregates with different density and constant number of particles (values given in Pa).

Criterium	$3200\mathrm{kg/m^3}$	$3900\mathrm{kg/m^3}$	$4600\mathrm{kg/m^3}$
MS(Pa)	4	6.4	8.4
VM(Pa)	2.4	3.6	5

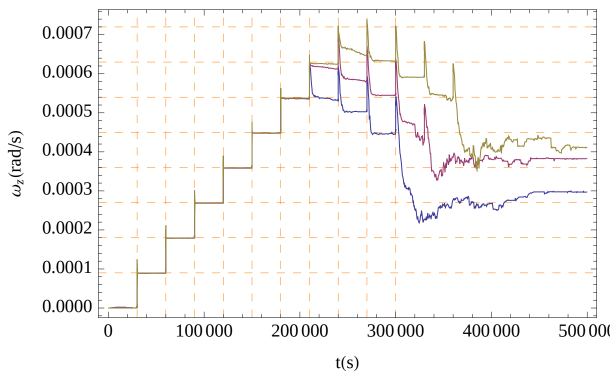

Figure 1: Spin-rate for aggregates formed by 3000 particles and densities of 3200(blue), 3900(red) and 4600 $\mathrm{kg/m^3}$ (green). The dashed lines represent the nominal spin rate (horizontal) and the instants when it was increased (vertical).

Conclusions

In this paper we present the preliminary results of the simulation of self-gravitating granular aggregates that undergo a reshaping and fission process due to the increase in angular velocity. It is observable that σ_y always increases with the mass of the aggregate. However, the angular velocity for reshaping depends on its bulk density. The maximum stress is mostly found near centre of the aggregate. In addition, after a reshaping spin rate has been reached, its further increase only causes more reshaping and decrease in the spin rate itself.

References

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