Rough Set Based Homogeneous Unsharp Masking for Bias Field Correction in MRI

Abhirup Banerjee and Pradipta Maji

Machine Intelligence Unit, Indian Statistical Institute, Kolkata, India
{abhirup_r,pmaji}@isical.ac.in

Abstract. A major issue in magnetic resonance (MR) image analysis is to remove the intensity inhomogeneity artifact present in MR images, which generally affects the performance of an automatic image analysis technique. In this context, the paper presents a novel approach for bias field correction in MR images by incorporating the merits of rough sets in estimating intensity inhomogeneity artifacts. Here, the concept of lower approximation and boundary region of rough sets deals with vagueness and incompleteness in filter structure definition and enables the algorithm to estimate optimum or near optimum bias field. A theoretical analysis is presented to justify the use of rough sets for bias field estimation. The performance of the proposed approach, along with a comparison with other bias field correction algorithms, is demonstrated on a set of MR images for different bias fields and noise levels.

Keywords: Magnetic resonance imaging, intensity inhomogeneity, bias field, rough sets.

1 Introduction

One of the key problems in any automatic image analysis technique on MR images is that it often provides incorrect results due to the presence of some degrading artifacts [1]. Among them, a specific artifact, known as intensity inhomogeneity or bias field, creates a shading effect in the images. This slow spatially varying artifact compels the intensity values of a specific tissue class to vary in different regions, thus increasing the overall variation of the tissue class. Although this artifact is hardly visible in human eyes, it is able to degrade the performance of any automatic image analysis tools such as segmentation or registration. Hence, before applying these tools, a preprocessing step is generally applied to remove such inhomogeneity artifacts from the MR images.

Several retrospective methods have been proposed in the past that try to remove bias field depending on the information of the acquired image. Some histogram based methods such as N3 [2] try to estimate this artifact by maximizing high frequency information of the tissue intensity distribution, while others try to remove it by simultaneously segmenting the image into meaningful tissue classes and estimating the bias field [3,4]. Pham and Prince [5] and Ahmed et al. [6] used fuzzy-c-means clustering algorithm to remove this artifact, while

A. Petrosino (Ed.): ICIAP 2013, Part II, LNCS 8157, pp. 542–551, 2013.

Ashburner and Friston [7] proposed a probabilistic framework for simultaneous image registration, tissue classification, and bias correction.

The simplest method to remove intensity inhomogeneity is filtering method, which is only dependent on the information of the acquired image. One of the popular filtering methods is homogeneous unsharp masking (HUM). The HUM was initially proposed by Axel et al. [8]. It is generally implemented either after masking out the background pixels from the image or by replacing the background pixels with average intensity values. However, Zhou et al. [9] also tried to remove the high-intensity structures and replaced them by the average intensity in their neighborhood. There exist some other methods in the literature that use median filter instead of mean filter to estimate the intensity inhomogeneity component [10,11]. However, Brinkmann et al. [12] showed experimentally that the mean filter outperforms median filter in estimating the inhomogeneity component from the MR images. In [12], they also tried to find the optimum window size or the optimum range of window size for the low-pass filter.

Generally, arithmetic mean (AM) filter is used as a low-pass filter in the HUM. But in AM filter, all the pixels in the neighborhood contribute equally in calculating the local average, which causes a problem in calculating the bias field component of the pixels in object-background edge area. In [12], Brinkmann et al. used a thresholding technique to differentiate background pixels from the object pixels. Recently, Banerjee and Maji [14] used contraharmonic mean filter instead of arithmetic mean filter to remove this problem. In HUM algorithm of Brinkmann et al., all the pixels are considered to contribute in estimating the bias field component. However, the pixels with similar intensity value are expected to contribute more than the other pixels in the neighborhood.

In this regard, the paper presents a rough set theoretic bias field estimation technique. The concept of lower approximation and boundary region of rough sets deals with uncertainty, vagueness, and incompleteness in filter structure definition. The filter structure of each pixel consists of two parts, namely, lower approximation and boundary region. The bias field component for each pixel depends on the weighting average of these two parts. A theoretical analysis is presented to justify the use of rough sets for bias field estimation. The effectiveness of the proposed algorithm, along with a comparison with related algorithms, is demonstrated on a set of benchmark MR images both qualitatively and quantitatively for different bias fields and noise levels.

2 Basics of HUM and Rough Sets

This section presents the basic notions of homogeneous unsharp masking (HUM) filtering method and theory of rough sets. The proposed algorithm for estimating intensity inhomogeneity artifact is developed based on these concepts.

2.1 Homogeneous Unsharp Masking

The HUM assumes that intensity inhomogeneity is a low-frequency component in the high-frequency structure of the image. It is usually implemented with a noise threshold to prevent background pixels from distorting the bias field estimation.

The model of the HUM assumes that intensity inhomogeneity is multiplicative. If the ith pixel of the inhomogeneity-free image is u_i, and corresponding intensity inhomogeneity field and noise are b_i and n_i, respectively, then the ith pixel v_i of the acquired image is obtained as follows:

$$v_i = u_i b_i + n_i. \tag{1}$$

Generally one can estimate the bias field either from the noise-free image or from the noisy image. However, Guillemaud and Brady [4] showed that post-filtering is more preferable than pre-filtering. Also, intensity inhomogeneity is a low-frequency component. Hence, the model of the HUM can be rewritten as

$$u_i = \frac{v_i}{b_i} = \frac{v_i C_N}{LPF(v_i)}, \tag{2}$$

where $LPF(.)$ is the low-pass filter and C_N represents the normalizing constant, which depends on the low-pass filter. If the low-pass filter is an averaging filter, then the constant C_N is used to preserve the average intensity of the image.

2.2 Basics of Rough Sets

The theory of rough sets begins with the notion of an approximation space, which is a pair $< U, R >$, where $U = \{x_1, \cdots, x_i, \cdots, x_n\}$ be a non-empty set, the universe of discourse, and R is an equivalence relation on U. The relation R decomposes the set U into disjoint classes in such a way that two elements x_i and x_j are in the same class iff $(x_i, x_j) \in R$. Let denote by U/R the quotient set of U by the relation R, and

$$U/R = \{X_1, \cdots, X_i, \cdots, X_m\} \tag{3}$$

where X_i is an equivalence class of R. If two elements x_i and x_j in U belong to the same equivalence class $X_i \in U/R$, we say that x_i and x_j are indistinguishable. The equivalence classes of R and the empty set $\emptyset$ are the elementary sets in the approximation space $< U, R >$. Given an arbitrary set $X \in 2^U$, in general, it may not be possible to describe X precisely in $< U, R >$. One may characterize X by a pair of lower and upper approximations defined as follows [13]:

$$\underline{R}(X) = \bigcup_{X_i \subseteq X} X_i; \qquad \text{and} \qquad \overline{R}(X) = \bigcup_{X_i \cap X \neq \emptyset} X_i. \tag{4}$$

Hence, the lower approximation $\underline{R}(X)$ is the union of all the elementary sets, which are subsets of X, and the upper approximation $\overline{R}(X)$ is the union of all the elementary sets, which have a non-empty intersection with X. The interval $[\underline{R}(X), \overline{R}(X)]$ is the representation of an ordinary set X in the approximation space $< U, R >$ or simply called the rough set of X. The lower (respectively, upper) approximation $\underline{R}(X)$ (respectively, $\overline{R}(X)$) is interpreted as the collection of those elements of U that definitely (respectively, possibly) belong to X. $B(X) = \overline{R}(X) \setminus \underline{R}(X)$ is called the boundary region of X. Further, a set $X \in 2^U$ is said to be definable or exact in $< U, R >$ iff $\underline{R}(X) = \overline{R}(X)$.

3 Rough Sets for Bias Field Correction

This section presents a new approach, using the merits of rough sets, for estimating bias field present in the MR images.

3.1 Bias Field Correction Using Rough Sets

Using the merits of rough sets, next a new bias field correction technique is described. The proposed algorithm adds the concept of lower and upper approximations of rough sets into the HUM algorithm, because the rough sets have the ability to deal with uncertainty, vagueness, and incompleteness in filter structure definition.

Generally, all pixels within the filtered area do not contribute equally in estimating the bias field component of the center pixel. The pixels with similar intensity value with respect to the center pixel are expected to contribute more in estimating the bias field as they lie in same or similar cluster. Hence, all pixels in the filtered area should not be given equal priority in estimating the bias field component of the center pixel.

Let the coordinate i of a pixel in the image I be (i_x, i_y). Given an arbitrary filter N_i of size $\varDelta_x \times \varDelta_y$ corresponding to the ith pixel, one may characterize N_i by a pair of upper and lower approximations defined as follows:

$$\overline{R}(N_i) = \left\{ j = (j_x, j_y) : \ |j_x - i_x| < \frac{\varDelta_x}{2}, |j_y - i_y| < \frac{\varDelta_y}{2} \right\};$$

$$\text{and} \ \ \underline{R}(N_i) = \left\{ j : \ |v_j - v_i| < \delta_i, j \in \overline{R}(N_i) \right\}; \tag{5}$$

therefore, the boundary region of N_i is given by

$$B(N_i) = \left\{ j : \ |v_j - v_i| \geq \delta_i, j \in \overline{R}(N_i) \right\}; \tag{6}$$

where δ_i is a predefined threshold corresponding to the ith pixel. Hence, each filter N_i, corresponding to the ith pixel, consists of two parts, namely, lower approximation $\underline{R}(N_i)$ and boundary region $B(N_i)$. The pixels in the lower approximation are given higher priority than the pixels in the boundary region as they definitely contribute in estimating the bias field component. Hence, the model of the HUM, using the merits of rough sets, can be rewritten as

$$u_i'' = \frac{v_i}{b_i''} \tag{7}$$

where the estimated bias field at coordinate i is given by

$$b_i'' = \{\omega_i \mathcal{A}_i + (1 - \omega_i)\mathcal{B}_i\} \left\{ \frac{|I|}{\displaystyle\sum_{j \in I} v_j} \right\} \tag{8}$$

$$\text{where } \mathcal{A}_i = \frac{\sum_{j \in \underline{R}(N_i)} v_j}{|\underline{R}(N_i)|}; \text{ and } \mathcal{B}_i = \frac{\sum_{j \in B(N_i)} v_j}{|B(N_i)|} \tag{9}$$

ω_i and $(1 - \omega_i)$ represent the relative importance of lower approximation and boundary region of filter N_i. The threshold δ_i controls the size of ith equivalence class or information granule. In practice, the following definition works well:

$$\delta_i = 3 \times \sqrt{\frac{1}{|\bar{R}(N_i)|} \sum_{j \in \bar{R}(N_i)} (v_j - v_i)^2}. \tag{10}$$

Hence, the pixels in the lower approximation are selected in such a way that their intensity values lie near the intensity value of the center pixel. This selection is achieved by introducing the threshold δ_i that enables the algorithm to select only those pixels that lie in the same or similar cluster with respect to the center pixel and contain information about its bias field.

3.2 Importance of Rough Sets

Let the intensity values of the pixel restored by the HUM with and without rough sets be denoted by u_i'' and u_i', respectively. Now better restoration can be achieved by the rough set based bias field estimation method if the error in estimating the intensity value of the restored pixel is minimum, that is,

$$(u_i - u_i'')^2 < (u_i - u_i')^2. \tag{11}$$

Hence, the better restoration will be achieved if

$$u_i' < u_i'' \qquad \text{and} \qquad u_i'' < 2u_i - u_i'. \tag{12}$$

$$\text{Now, } u_i' < u_i'' \qquad \Leftrightarrow \frac{v_i}{b_i'} < \frac{v_i}{b_i''} \qquad \Leftrightarrow b_i'' < b_i'$$

$$\Leftrightarrow \omega_i \frac{\sum_{j \in \underline{R}(N_i)} v_j}{|\underline{R}(N_i)|} + (1 - \omega_i) \frac{\sum_{j \in B(N_i)} v_j}{|B(N_i)|} < \frac{\sum_{j \in N_i} v_j}{|N_i|}; \tag{13}$$

the right-hand term of (13) can be rewritten as

$$\frac{\sum_{j \in N_i} v_j}{|N_i|} = \frac{\sum_{j \in \underline{R}(N_i)} v_j + \sum_{j \in B(N_i)} v_j}{|\underline{R}(N_i)| + |B(N_i)|}$$

$$\Leftrightarrow \omega_i \frac{a_1}{b_1} + (1-\omega_i)\frac{a_2}{b_2} < \frac{a_1+a_2}{b_1+b_2}$$

$$\text{where } a_1 = \sum_{j \in \underline{R}(N_i)} v_j; \qquad b_1 = |\underline{R}(N_i)|;$$
$$a_2 = \sum_{j \in B(N_i)} v_j; \qquad b_2 = |B(N_i)|.$$

$$\Leftrightarrow \omega_i \left(\frac{a_1}{b_1} - \frac{a_1+a_2}{b_1+b_2}\right) + (1-\omega_i)\left(\frac{a_2}{b_2} - \frac{a_1+a_2}{b_1+b_2}\right) < 0$$

$$\Leftrightarrow (a_1 b_2 - a_2 b_1)\left[\frac{\omega_i}{b_1(b_1+b_2)} - \frac{(1-\omega_i)}{b_2(b_1+b_2)}\right] < 0$$

$$\Leftrightarrow \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)\left[\omega_i \frac{b_2}{b_1+b_2} - (1-\omega_i)\frac{b_1}{b_1+b_2}\right] < 0$$

$$\text{Hence, either } \frac{a_1}{b_1} < \frac{a_2}{b_2} \qquad \text{and} \qquad \frac{\omega_i}{1-\omega_i} > \frac{b_1}{b_2} \tag{14}$$

$$\Leftrightarrow \frac{\sum_{j \in \underline{R}(N_i)} v_j}{|\underline{R}(N_i)|} < \frac{\sum_{j \in B(N_i)} v_j}{|B(N_i)|} \text{ and } \frac{\omega_i}{1-\omega_i} > \frac{|\underline{R}(N_i)|}{|B(N_i)|};$$

$$\text{or } \frac{a_1}{b_1} > \frac{a_2}{b_2} \qquad \text{and} \qquad \frac{\omega_i}{1-\omega_i} < \frac{b_1}{b_2} \tag{15}$$

$$\Leftrightarrow \frac{\sum_{j \in \underline{R}(N_i)} v_j}{|\underline{R}(N_i)|} > \frac{\sum_{j \in B(N_i)} v_j}{|B(N_i)|} \text{ and } \frac{\omega_i}{1-\omega_i} < \frac{|\underline{R}(N_i)|}{|B(N_i)|}.$$

The terms $\frac{a_1}{b_1}$ and $\frac{a_2}{b_2}$ denote the local average within the lower approximation and boundary region, respectively, while the terms $\frac{b_1}{b_1+b_2}$ and $\frac{b_2}{b_1+b_2}$ denote the original weight assigned to the local average of lower approximation and boundary region, respectively, while calculating the combined average considering all the pixels in the filtered area. Let us assume that $\omega_{i_0} = \frac{b_1}{b_1+b_2}$. Combining (9), (14), and (15), we get

$$\omega_i = \begin{cases} \omega_{i_0} + \epsilon_i \text{ if } \mathcal{A}_i < \mathcal{B}_i \\ \omega_{i_0} - \epsilon_i \text{ if } \mathcal{A}_i > \mathcal{B}_i \\ \omega_{i_0} \quad \text{ if } \mathcal{A}_i = \mathcal{B}_i \end{cases} \tag{16}$$

Hence, the analysis reported above establishes the fact that if the local average intensity of the boundary region is higher than that of the lower approximation, higher weightage has to be given to the lower approximation than boundary region to achieve better restoration by the proposed rough set based bias field estimation technique. On the other hand, higher weightage has to be applied to the boundary region if the local average intensity of the boundary region is lower than that of lower approximation. In effect, the optimum value ω_{optimum} of the weight parameter ω_i can be estimated by gradually increasing or decreasing the value of ω_i, satisfying the condition $u''_i < 2u_i - u'_i$.

4 Experimental Results and Discussion

The performance of the proposed bias field estimation method is extensively studied and compared with the HUM [8,12] and N3 algorithm [2]. In [12], Brinkmann et al. showed that the optimal window size of the low-pass filter lies in the range 65 to 127. Hence, in the present research work, the optimal window size is fixed at 121 for all the experiments.

To analyze the performance of different algorithms, the experimentation is done on some benchmark images obtained from "BrainWeb: Simulated Brain Database" (http://www.bic.mni.mcgill.ca/brainweb/). The results are reported for different noise levels and intensity inhomogeneity. The performance of different methods is evaluated using the RMSE value. A good bias field correction procedure should make the value of RMSE as low as possible.

4.1 Performance of Different Algorithms

To find out the effectiveness of the proposed algorithm over the HUM algorithm of Brinkmann et al. [12] and N3 algorithm of Sled et al. [2] for bias field estimation, extensive experimentation is carried out on several image volumes. Fig. 1 presents the performance of the proposed bias field correction method, HUM algorithm and N3 algorithm, in terms of RMSE value. From the results reported in Fig. 1, it is observed that the proposed algorithm provides better restoration in 6 cases out of the total 12 cases, in terms of RMSE value and comparable performances in all other cases. The second, third and fourth columns of Fig. 2 and 3 compare the reconstructed images produced by the proposed algorithm, HUM algorithm and N3 algorithm for different bias fields and noise levels. All the results reported in Fig. 2 and 3 establish the fact that the proposed method estimates the bias field more accurately than the existing HUM and N3 algorithms irrespective of the bias fields and noise levels.

4.2 Unbiased Estimation

One of the caveats about the HUM algorithm is that it can alter an image even when no inhomogeneity is present, while a perfect correction algorithm should be expected to leave the image unchanged.

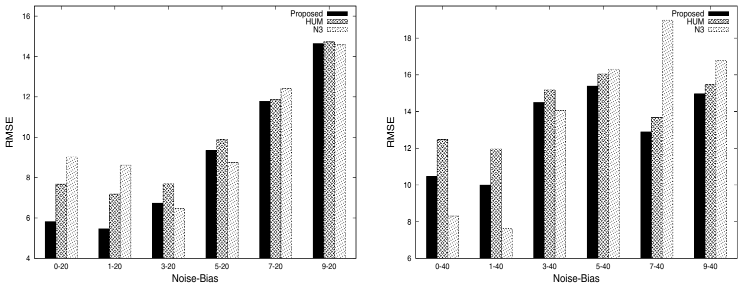

Fig. 1. Performance of the proposed method, HUM algorithm and N3 algorithm for bias affected images

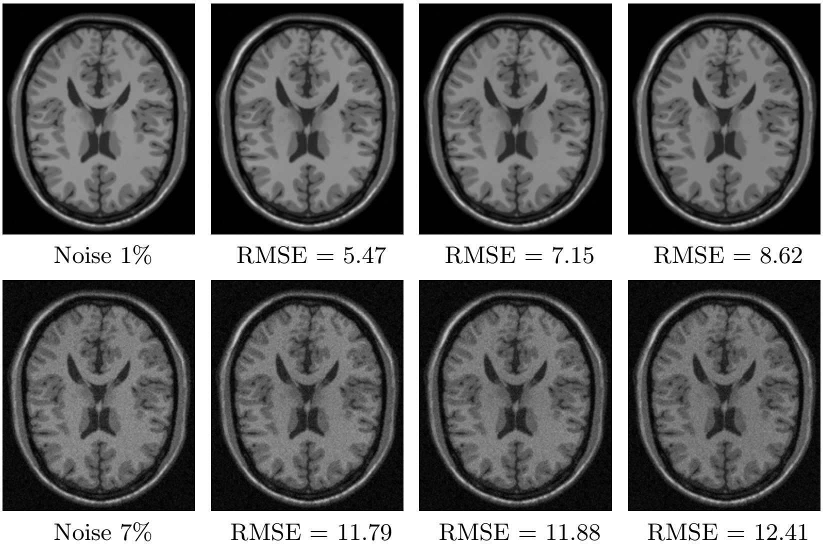

Fig. 2. Input image with 20% bias field and images restored by the proposed algorithm, HUM algorithm of Brinkmann et al. and N3 algorithm

To find the effectiveness of the proposed algorithm over HUM and N3 for unbiased estimation, extensive experimentation is carried out on several image volumes. Fig. 4 presents the performance of the proposed method, HUM algorithm and N3 algorithm in terms of RMSE value.

From the results reported in Fig. 4, it is observed that the proposed algorithm provides better restoration in all of the 6 cases, in terms of RMSE value. The HUM algorithm of Brinkmann et al. and N3 algorithm of Sled et al. severely changes the input image in spite of absence of intensity inhomogeneity artifacts, whereas the proposed algorithm leaves the input image more or less unchanged.

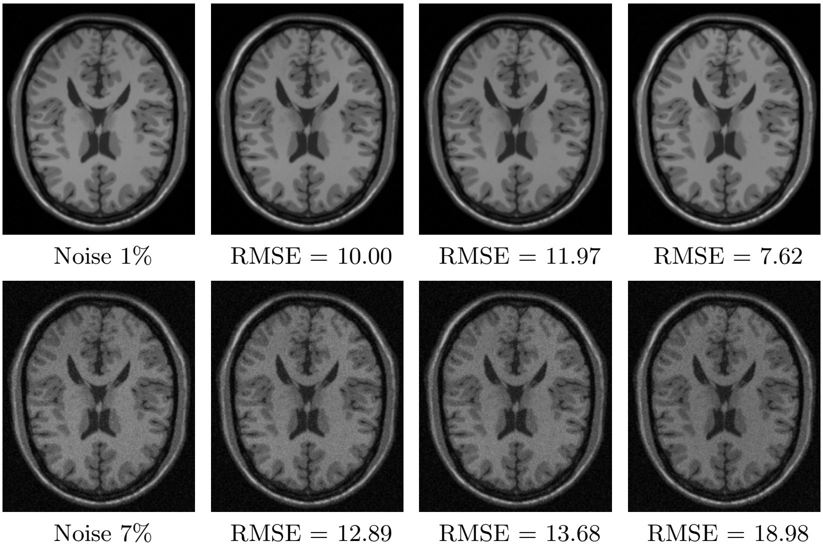

Fig. 3. Input image with 40% bias field and images restored by the proposed algorithm, HUM algorithm of Brinkmann et al. and N3 algorithm

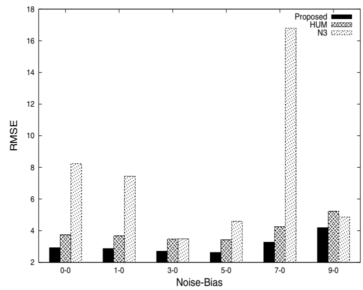

Fig. 4. Performance of the proposed method, HUM algorithm and N3 algorithm for bias-free images

5 Conclusion

The problem of removing intensity inhomogeneity artifact from MR images is of high importance as it degrades the performance of any automatic image analysis technique. In this regard, the contribution of the paper is two fold, namely, the development of a bias field correction algorithm using the merits of rough sets and demonstrating the effectiveness of the proposed algorithm, along with a comparison with other algorithms, on a set of MR images for different bias fields and noise levels.

A theoretical analysis is presented to justify the use of rough sets for bias field estimation. The proposed algorithm has been proven to be more effective than the existing HUM algorithm both theoretically and experimentally. Using the concept of rough sets, the algorithm provides better restoration of MR images than using the HUM algorithm and N3 algorithm. The efficiency of the algorithm has been tested on "BrainWeb: Simulated Brain Database".

Acknowledgment. This work is partially supported by the Indian National Science Academy, New Delhi (grant no. SP/YSP/68/2012).

References

1. Suetens, P.: Fundamentals of Medical Imaging. Cambridge University Press (2002)
2. Sled, J.G., Zijdenbos, A.P., Evans, A.C.: A Nonparametric Method for Automatic Correction of Intensity Nonuniformity in MRI Data. IEEE Transactions on Medical Imaging 17(1), 87–97 (1998)
3. Wells III, W.M., Grimson, W.E.L., Kikins, R., Jolezs, F.A.: Adaptive Segmentation of MRI Data. IEEE Transactions on Medical Imaging 15(8), 429–442 (1996)
4. Guillemaud, R., Brady, M.: Estimating the Bias Field of MR Images. IEEE Transactions on Medical Imaging 16(3), 238–251 (1997)
5. Pham, D.L., Prince, J.L.: Adaptive Fuzzy Segmentation of Magnetic Resonance Images. IEEE Transactions on Medical Imaging 18(9), 737–752 (1999)
6. Ahmed, M.N., Yamany, S.M., Mohamed, N., Farag, A.A., Moriarty, T.: A Modified Fuzzy *C*-Means Algorithm for Bias Field Estimation and Segmentation of MRI Data. IEEE Transactions on Medical Imaging 21(3), 193–199 (2002)
7. Ashburner, J., Friston, K.J.: Unified Segmentation. NeuroImage 26(3), 839–851 (2005)
8. Axel, L., Costantini, J., Listerud, J.: Intensity Correction in Surface-Coil MR Imaging. American Journal of Roentgenology 148, 418–420 (1987)
9. Zhou, L.Q., Zhu, Y.M., Bergot, C., Laval-Jeantet, A.M., Bousson, V., Laredo, J.D., Laval-Jeantet, M.: A Method of Radio-Frequency Inhomogeneity Correction for Brain Tissue Segmentation in MRI. Computerized Medical Imaging and Graphics 25(5), 379–389 (2001)
10. Narayana, P.A., Borthakur, A.: Effect of Radio Frequency Inhomogeneity Correction on the Reproducibility of Intra-Cranial Volumes Using MR Image Data. Magnetic Resonance in Medicine 33, 396–400 (1994)
11. Bedell, B.J., Narayana, P.A., Wolinsky, J.S.: A Dual Approach for Minimizing False Lesion Classifications on Magnetic-Resonance Images. Magnetic Resonance in Medicine 37(1), 94–102 (1997)
12. Brinkmann, B.H., Manduca, A., Robb, R.A.: Optimized Homomorphic Unsharp Masking for MR Grayscale Inhomogeneity Correction. IEEE Transactions on Medical Imaging 17(2), 161–171 (1998)
13. Pawlak, Z.: Rough Sets: Theoretical Aspects of Reasoning about Data. Kluwer Academic, Dordrecht (1991)
14. Banerjee, A., Maji, P.: Contraharmonic Mean Based Bias Field Correction in MR Images. In: Wilson, R., Hancock, E., Bors, A., Smith, W. (eds.) CAIP 2013, Part I. LNCS, vol. 8047, pp. 523–530. Springer, Heidelberg (2013)