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# ROUGH SETS
# PRESENT STATE AND PERSPECTIVES*

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## Abstract

The paper presents basic concepts of rough set theory, outlines its applications and briefly discusses some further problems. Comparison to other similar approaches is also presented.

## 1 INTRODUCTION

The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. Recently it became also an important issue for computer scientists, particularly in artificial intelligence (AI). There is a variety of views in the AI community what is, and how to make use of imperfect knowledge. The most successful one approach to this question is, no doubt, fuzzy set theory.

In this paper we are going to outline still another attempt to this problem – rough set theory (Pawlak 1991). The theory has attracted attention of many researchers and practitioners all over the world, who contributed essentially to its development, and by now over a thousand papers have been published in this area and many successful application have been implemented.

Rough set theory overlaps with many other theories, especially with fuzzy set theory, evidence theory and Boolean reasoning methods – nevertheless it can be viewed in its own rights, as an independent, complementary, and not competing discipline.

## 2 BASIC PHILOSOPHY

Rough set philosophy is based on the assumption that, in contrast to the classical set theory, we have some additional information (knowledge, data) about elements of the universe we are interested in. Consider, for example, a group of patients suffering from a certain disease. In a hospital treating the patients there are data files containing information about patients – such as, e.g., body temperature, blood pressure, name, age, address and others. All patients revealing the same symptoms are indiscernible (similar) in view of the available information and form blocks, which can be understood as elementary granules of knowledge about patients. These granules are called *elementary sets or concepts*, and can be considered as elementary building blocks (types, units or classes) of our knowledge. Elementary concepts can be combined into compound concepts, i.e., concepts that are uniquely defined in terms of elementary concepts. Any union of elementary sets is called a *crisp* set, and any other sets are referred to *as rough* (*vague, imprecise*). With every set $X$ we can associate two crisp sets, called the *lower* and the *upper approximation* of $X$. The lower approximation of $X$ is the union of all elementary set which are included in $X$, whereas the upper approximation of $X$ is the union of all elementary set which have non-empty intersection with $X$. In other words the lower approximation of a set is the set of all elements that *surely* belongs to $X$, whereas the upper approximation of $X$ is the set of all elements that *possibly* belong to $X$ – in view of available knowledge. The difference of the upper and the lower approximation of $X$ is its *boundary region*. Obviously a set is rough if it has non empty boundary region; otherwise the set is crisp. Elements of the boundary region cannot be classified, employing the available knowledge, either to the set or its complement. Approximations of sets are basic operation in the rough set theory and are used as main tools to deal with vague and uncertain data.

Let us notice that sets are usually defined by employing a membership function, whereas rough sets are defined by approximations. Rough sets can be also defined using membership function, however the function is not a primitive notion in this approach, but is defined by employing knowledge about elements of the set.

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## 3 THE RELATIONSHIP TO OTHER THEORIES

The rough set concept overlaps – to some extent – with many other mathematical tools developed to deal with imperfect knowledge.

Frequently the rough set theory is contrasted with the fuzzy set theory. Basically the idea of fuzzy set and rough set are not competitive, but complementary since they refer to different aspects of imprecision, and consequently are meant to be used in different areas. In fuzzy set theory imprecision is expressed by a membership function, whereas the rough set approach is based on indiscernibility and approximations (Pawlak et al 1994).

Another relationship exists between the rough set theory and Dempster-Shafer theory of evidence (Skowron et al 1994). The main difference is that Dempster-Shafer theory uses belief functions as a main tool, while rough set theory makes use of sets – lower and upper approximations.

Furthermore, some relations exist between the rough set theory and discriminant analysis (Krusińska et al 1992) and the Boolean reasoning methods (Skowron et al 1992).

Despite of the relationships the rough set theory can be viewed in its own rights, as an independent discipline.

## 4 DATA ANALYSIS USING ROUHG SETS

Information about objects of interest is often available in a form of data tables, known also as attribute-values tables or information systems. An information system is a table column of which are labelled by *attributes*, rows – by *objects* and entries of the table are *attribute values*. Simple example of an information system is shown below.

Suppose we are given data about 6 patients, as shown in Table 1.

**Table 1:** Example of Information System

| Pat. | Headache | Muscle-pain | Temp. | Flu |
|---|---|---|---|---|
| p1 | no | yes | high | yes |
| p2 | yes | no | high | yes |
| p3 | yes | yes | very high | yes |
| p4 | no | yes | normal | no |
| p5 | yes | no | high | no |
| p6 | no | yes | very high | yes |

Columns of the table are labelled by attributes (symptoms) and rows by objects (patients), whereas entries of the table are attribute values. Thus each row of the table can be seen as information about specific patient. For example patient p2 is characterized in the table by the following attribute-value set

(Headache, yes), (Muscle-pain, no), (Temperature, high), (Flu, yes),

which form information about the patient.

In the table patients p2, p3 and p5 are indiscernible with respect to the attribute Headache, patients p3 and p6 are indiscernible with respect to attributes Muscle-pain and Flu, and patients p2 and p5 are indiscernible with respect to attributes Headache, Muscle-pain and Temperature. Hence, for example, the attribute Headache generates two elementary sets {p2,p3,p5} and {p1,p4,p6}, whereas the attributes Headache and Muscle-pain form the following elementary sets:
{p1,p4,p6}, {p2,p5} and {p3}. Similarly one can define elementary set generated by any subset of attributes.

Because patient p2 has flu, whereas patient p5 does not, and they are indiscernible with respect to the attributes Headache, Muscle-pain and Temperature, thus flu cannot be characterized in terms of attributes Headache, Muscle-pain and Temperature. Hence p2 and p5 are the boundary-line cases, which cannot be properly classified in view of the available knowledge. The remaining patients p1, p3 and p6 display symptoms which enable us to classify them with certainty as having flu, patients p2 and p5 cannot be excluded as having flu and patient p4 for sure has not flu, in view of the displayed symptoms. Thus the lower approximation of the set of patients having flu is the set {p1,p3,p6} and the upper approximation of this set is the set {p1,p2,p3,p5,p6}, where as the boundary-line cases are patients p2 and p5. Similarly p4 has not flu and p2, p5 can not be excludes as having flu, thus the lower approximation of this concept is the set {p4} whereas – the upper approximation is the set {p2,p4,p5} and the boundary region of the concept "not flu" is the set {p2,p5} the same as in the previous case.

We may also ask whether all attributes in this table are necessary to define the concept "flu". One can easily see, for example that, if a patient has very high temperature, he has for sure flu, but if he has normal temperature he has not flu whatsoever.

The problem of elimination of superfluous attributes boils down to finding so called reducts of the whole set of attributes.

One can compute that in the example shown in Tab.1 we have two reducts: {Headache, Temperature} and {Muscle-pain, Temperature}. That means that either the attribute Headache or Muscle-pain can eliminated from the table without changing its elementary sets. Hence instead of Tab 1. we can use either Tab.2

**Table 2:** Reduced Information System

| Pat. | Headache | Temp. | Flu |
|---|---|---|---|
| p1 | no | high | yes |
| p2 | yes | high | yes |
| p3 | yes | very high | yes |
| p4 | no | normal | no |
| p5 | yes | high | no |
| p6 | no | very high | yes |

or Tab.3

**Table 3:** Reduced Information System

| Pat. | Muscle-pain | Temp. | Flu |
|---|---|---|---|
| p1 | yes | high | yes |
| p2 | no | high | yes |
| p3 | yes | very high | yes |
| p4 | yes | normal | no |
| p5 | no | high | no |
| p6 | yes | very high | yes |

Either table can be used equivalently to analyze the decisions without loosing any information in comparison to Tab.1.

The tables considered above are also known as decision tables, where Headache, Muscle-pain and Temperature are refereed to as *condition attributes*, whereas Flu is called the *decision attribute*. Consequently each decision table can be seen as a set of decision rules of the form "IF ... THEN ...".

In fact the above considerations give rise to the question whether the attribute Flu depends on the attributes Headache, Muscle-pain and Temperature. As the above analysis shows this is not the case. In order to tackle this kind of situations partial dependency of attributes is needed.

Basis problems which can be solved by using rough set theory are the following:

- Characterisation of set of objects in terms of attribute values;
- Finding dependencies (total or partial) between attributes;
- Reduction of superfluous attributes (data);
- Finding the most significance attributes;
- Decision rule generation.

The theory offers simple algorithms and straightforward interpretation of obtained results.

Precise, mathematical formulation of the above presented ideas, can be found in many papers on rough set theory, in particular in (Pawlak 1991).

# 5 APPLICATIONS AND ADVANTAGES

The rough set theory has found many interesting applications. The rough set approach seems to be of fundamental importance to AI and cognitive sciences, especially in the areas of machine learning, knowledge acquisition, decision analysis, knowledge discovery from databases, expert systems, inductive reasoning and pattern recognition.

The main advantage of rough set theory is that it does not need any preliminary or additional information about data – like probability in statistics, or basic probability assignment in Dempster-Shafer theory, grade of membership or the value of possibility in fuzzy set theory

The rough set theory has been successfully applied in many real-life problems in medicine, pharmacology, engineering, banking, financial and market analysis and others.

There are many applications of rough set theory in medicine, (Grzymala-Busse et al 1995, Peterson 1993, Słowiński et al 1988, Słowiński 1992, Słowiński et al 1995, Tanaka et al 1992). In pharmacology the analysis of relationships between the chemical structure and the antimicrobial activity of drugs (Krysiński 1992, 1995) has been successfully investigated. Banking applications include evaluation of a bankruptcy risk (Słowiński et al 1993, 1994) and market research (Ziarko et al 1993, Golan et al 1993). Very interesting results have been also obtained in speaker independent speech recognition (Czyżewski 1993, 1995, Czyżewski et al 1995, Brindle 1994) and acoustics (Kostek 1995). The rough set approach seems also important for various engineering applications, like diagnosis of machines using vibroacoustics, symptoms (noise, vibrations) (Nowicki et al 1992, Słowiński et al 1995), material sciences (Jackson et al 1995) and process control (Lin 1995, Mrózek 1992, Munakata 1995, Płonka et al 1995, Szladow et al 1992, (Ziarko et al 1989). Application in linguistics (Moradi et al 1995, Haines 1994, 1995, Kobayashi 1995) and environment (Gunn et al 1994) databases (Beaubouef 1993, 1994, 1995) are other important domains.

More about applications of the rough set theory can be found in (Słowinski 1992, Ziarko 1993, Lin 1994, Lin et al 1995) and (Wang 1995). Besides, many other fields of application, e.g. time series analysis, image processing and character recognition, are being extensively explored.

Application of rough sets requires a suitable software. Many software systems for workstations and personal computers based on rough set theory have been developed. The most known include LERS (Grzymała-Busse 1992), Rough DAS and Rough Class (Słowiński et al 1992) and DATALOGIC (Szladow 1993). Some of them are available commercially.

One of the most important and difficult problem in software implementation of the presented approach is optimal decision rule generation from data. Many various approaches to solve this task can be found in (Bazan et al 1994, Grzymała-Busse et al 1993, Lenarcik et al 1993, Piasta 1995, Stefanowski et al 1993), Tsumoto et al 1993, 1995, Kryszkiewicz et al 1993). The relation to other methods of rule generation is analysed in (Grzymała-Busse et al 1995).

Complexity of computing all reducts in an information system is rather high. However, in many applications we do not need to compute all reducts, but only some of them, satisfying specific requirements, which is much simpler. There are many approaches to compute reducts, including statistical methods (Bazan et al 1994), genetic algorithms (Wróblewski 1995), Boolean reasoning (Skowron et al 1992) and others.

The proposed method has many important advantages. Some of them are listed below.

- Provides efficient algorithms for finding hidden patterns in data
- Finds minimal sets of data (data reduction)
- Evaluates significance of data
- Generates minimal sets of decision rules from data
- It is easy to understand and offers straightforward interpretation of results

The method is particularly suited for parallel processing, but in order to exploit this feature fully a new hardware solution are necessary.

# 6 FURTHER RESEARCH

Despite many important theoretical contributions and extensions of the original model some essential research problems still remain open. Some of them are listed below.

- Many efficient, optimal decision rule generation methods from data, have developed in recent years based on rough set theory – however more research in this are is needed, particularly, when quantitative attributes are involved.
- Discretization methods for quantitative attribute values are badly needed.
- The relationship between neural network and rough set approach for feature extraction from data seems of particular interest.
- Rough logic, based on the concept rough truth seems to be a very important issue.
- Theory of rough relation and rough function is necessary in many applications.

Rough mereology seems to be a very perspective area of research and applications (Polkowski, Skowron 1995).

Besides, some practical problems related with application of rough sets in many domains are of great importance.

- Efficient and widely assessable software is necessary to further development of various applications.
- Development of rough set computer seems to be a must in order to pursue many new applications.

Last but not least "rough control" seems to be a very promising area of application of the rough set concept.

# 7 CONCLUSIONS

Rough set theory has reached a certain degree of maturity, both from theoretical and practical points of view. It is based on sound mathematical foundations and many of its successful real life applications have shown that it can be treated as a new useful tool for data analysis. The theory is not competing with other existing similar approaches, but it is rather complementary.

Despite of its achievements it requires further research and development. Particularly important seems to be the design of rough set computer. This kind of computer could speed up the computations and extend areas of possible applications.

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