

Routes to left-handed materials by magnetoelectric couplings

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(Received 21 January 2007; revised manuscript received 28 February 2007; published 28 June 2007)

Magnetoelectric materials, which couple electric fields with magnetic fields, are intensively studied for physical realization and potential synthesis of left-handed materials (LHM). Both isotropic and anisotropic magnetoelectric materials are the realization of backward waves and/or negative refraction within certain frequency bands. Since the magnetoelectric coupling (scalar or tensorial) parameters may significantly reduce the refractive indices and even cause the anisotropic nihility, left-handed materials can be realized without requiring the resonances of the permittivity and permeability. Wave-field theorem is thus applied to obtain the effective parameters in their equivalent isotropic counterparts, and dispersion models associated with mixing rules are considered as an alternative important factor to achieve LHM.

DOI: [10.1103/PhysRevB.75.245214](https://doi.org/10.1103/PhysRevB.75.245214)

PACS number(s): 78.20.Ci, 78.66.Sq, 42.25.Bs, 77.84.Lf

I. INTRODUCTION

Recently, composite materials have attracted considerable attention in various physics and engineering areas.¹⁻³ Among these materials, the double negative (DNG⁴) materials exhibit a left-handedness ruling the polarizations of electromagnetic fields, which is also referred to as backward-wave media (BWM) or left-handed media.⁵⁻¹⁰ Those materials possess left-handedness and negative refraction, and thus are considered to be new avenues for achieving unprecedented physical properties and functionality unattainable with natural materials.¹¹⁻¹⁵ The LHMs in the microwave region and their related applications have been extensively explored, including metamaterial waveguide,¹⁶ split ring resonators (SRRs) and spiral resonators,^{17,18} layered metamaterial cylinders,¹⁹ and subwavelength cavity resonators.^{20,21} Since the negative refraction by the artificial LHM was experimentally verified by Shelby *et al.*,²² more studies on metamaterials have been recently carried out, such as tensor-parameter retrieval using quasistatic Lorentz theory,^{23,24} S-parameter retrieval using the plane wave incidence,²⁵ and constitutive relation retrieval using the transmission line method.^{26,27} However, the artificial metamaterials can be synthesized based on the creation of metal inclusions of strong magnetic response, especially in the optical region. It is a challenge if we want to practically realize negative refraction and superlens for actual optical applications. Alternative approaches for creating backward-wave materials thus have to be considered.

Recently, some works^{28,29} have shown that chiral media, which represent a subset of magnetoelectric materials, can also exhibit backward-wave phenomenon and negative refraction in the condition of chiral nihility (i.e., $\epsilon, \mu=0$ and $\kappa \neq 0$). The nihility in chiral media requires that at least one of the permittivity and permeability is at its resonance, and both imaginary and real parts are zero,³⁰ which is not physically realizable in electromagnetic materials. Due to those assumptions, the potential applications of chiral nihility become limited. Instead, a special chiral material with

gyrotropy^{31,33,35} has been proposed for optical applications, such as subwavelength resonators, phase compensators, and super lens. Moving further another step, we will, in this paper, focus on the magnetoelectric materials³⁶⁻³⁸ (in which the electric field could generate a magnetization) with a particular interest in backward waves and left-handed phenomena. In particular, such effects can be exhibited by magnetoelectric materials; and a backward-wave regime can be, in principle, realized even if the medium has very weak magnetic properties or no magnetic property. Thus it appears that using magnetoelectric materials, one could realize LHM in the optical region without creating artificial magnetic materials or requiring permittivity-permeability resonance(s) operational in that frequency range. Various types of magnetoelectric materials are considered and their special wave properties are addressed and studied, while our emphasis is made on the discussion of realization of backward waves through isotropic-gyrotropic magnetoelectric couplings. It is also shown that the dispersion and gyrotropy in the magnetoelectric coupling play important roles in reducing values of certain eigenmodes, thus achieving negative refraction at the backward-wave regime off the resonances of permittivity and permeability. Throughout the paper, a time dependence $e^{-i\omega t}$ is assumed but always suppressed.

II. ISOTROPIC MAGNETOELECTRIC MATERIALS

A chiral medium, in which the magnetoelectric coupling is present in terms of the chirality and Tellegen parameters, is of our particular interest in this work. There are two definitions widely used to describe chiral media:

(i) Post's relations

$$\begin{aligned} \mathbf{D} &= \epsilon_p \mathbf{E} + i\xi \mathbf{B}, \\ \mathbf{H} &= i\xi \mathbf{E} + (1/\mu_p) \mathbf{B}, \end{aligned} \quad (1)$$

and (ii) Tellegen's relations

$$\mathbf{D} = \epsilon_T \mathbf{E} + i\kappa \sqrt{\mu_0 \epsilon_0} \mathbf{H},$$

$$\mathbf{B} = -i\kappa\sqrt{\mu_0\epsilon_0}\mathbf{E} + \mu_T\mathbf{H}, \quad (2)$$

where ϵ_0 and μ_0 stand for the permittivity and permeability in free space, and κ (or ξ) denotes the chirality used in the Tellegen (or Post) constitutive relations.

It can be found that the following relations exist:

$$\begin{aligned} \epsilon_T &= \epsilon_P + \xi^2\mu_P, \\ \kappa &= \xi\mu_P/\sqrt{\mu_0\epsilon_0}, \\ \mu_T &= \mu_P, \end{aligned} \quad (3)$$

where P and T denote permittivity and permeability under Post and Tellegen constitutive relations, respectively. These two constitutive relations were found to be applicable to chiral media composed of short wire helices as well as reciprocal chiral objects of arbitrary shape.⁴⁰ One can note when the chirality (i.e., κ or ξ) represents the manifestation of handedness in the chiral media:

(1) when chirality is positive, the polarization is right-handed and the medium is right-handed;

(2) when chirality is negative, the right-handed system is reversed to the left-handed system; and

(3) when no chirality is present, no magnetoelectric couplings or optical activity exists. Note that the source-free Maxwell equations have the following form:

$$\begin{aligned} \nabla \times \mathbf{E} &= i\omega\mathbf{B}, \\ \nabla \times \mathbf{H} &= -i\omega\mathbf{D}. \end{aligned} \quad (4)$$

Hence one can see from Eqs. (1), (2), and (4) that \mathbf{D}/\mathbf{B} at a given point also depends on the value of derivatives of \mathbf{E}/\mathbf{H} at that particular point, which can be characterized by the spatial dispersion.

In view of the chiral route to negative refraction, the chiral nihility²⁸ was proposed, which means at a certain frequency, ϵ and μ become zero while chirality is nonzero. Apparently, this restriction will lead to negative refractive indices since the product of $\epsilon\mu$ gives a zero. However, one may raise the question: is it necessary or physically realizable? The answer is obviously not. As is shown in Eq. (3), the chiral nihility defined in Ref. 28 was under the Tellegen's relations. Thus the restriction of ϵ_T and μ_T being zero leads us to zero κ .⁴¹ In such a case, the Maxwell equations cannot be established and the volume occupied by such a material becomes a *null* space.

In order to force chiral materials to fall in the backward wave regime, one only needs to make either permittivity or permeability resonant, which will still produce a very small value of the product of $\epsilon\mu$. Besides, the dispersion effects of the chiral media should be also taken into account, and especially the dispersion in chirality can play an important role in realization of backward waves and negative refractive indices. Moreover, the exact chiral nihility lacks physical meaning because at resonant frequency, it is generally impossible for normal materials to have zero imaginary parts of permittivity-permeability. One has to, however, bear in mind that chiral media still provide alternative possibilities to realize backward wave and negative refraction phenomena due

to the magnetoelectric couplings, which deserves further investigations.

A. Molecular model for chiral media

A chiral medium can be considered as, in a macroscopic view, a continuous medium composed of chiral composites which are uniformly distributed and randomly placed. The optical activity and circular dichroism of chiral media have been studied, and the chirality of the media's molecules can be seen as the cause of optical activity. Born⁴² put forward the interpretation of optical activity for a particular molecular model, in which a coupled-oscillator model was used. In what followed, Condon gave a single-oscillator model^{43,44} in dissymmetric field for an optically active material, based on the molecular theories of Drude, Lorentz, and Livens. The constitutive relations were suggested as

$$\mathbf{D} = \epsilon_c\mathbf{E} + \frac{i\omega\alpha}{c_0}\mathbf{H}, \quad (5)$$

$$\mathbf{B} = -\frac{i\omega\alpha}{c_0}\mathbf{E} + \mu_c\mathbf{H}, \quad (6)$$

where c_0 is the speed of light in free space, the subscript c denotes the parameters under the Condon model, and α stands for the rotatory parameter. The parameter of α for rotatory power is frequency dependent,

$$\alpha(\omega) \sim \sum_b \frac{R_{ba}}{\omega_{ba}^2 - \omega^2 + i\omega\Gamma_{ba}}, \quad (7)$$

where a and b stand for quantum states, ω_{ba} is the frequency of the light absorbed in the jump from $a \rightarrow b$, R_{ba} means the rotational strength of the absorbed line,⁴⁵ and the damping term of Γ_{ba} has been included for the consideration of absorption. Finally, by comparing Tellegen's relations and Condon's model, the dispersion of the dimensionless chirality κ can be expressed by substituting Eq. (7) into Eqs. (5) and (6) and then comparing Eqs. (5) and (6) with Eq. (2) as follows:

$$\kappa(\omega) = \frac{\omega\omega_c}{\omega_c^2 - \omega^2 + id_c\omega\omega_c}, \quad (8)$$

where ω_c represents the characteristic frequency, and d_c means the damping factor. Note that Eq. (8) is valid for the one-phase transition, in which only one rotatory term in Eq. (7) is counted due to the assumption that each transition between quantum states lies far off the others. Using the wave-field theory,⁴⁶ a chiral medium can be characterized as two sets of equivalent permittivity and permeability: ϵ_{\pm} and μ_{\pm} given by

$$\epsilon_{\pm}(\omega) = \epsilon_c \left(1 \pm \frac{\kappa(\omega)\sqrt{\mu_0\epsilon_0}}{\sqrt{\mu_c\epsilon_c}} \right), \quad (9)$$

$$\mu_{\pm}(\omega) = \mu_c \left(1 \pm \frac{\kappa(\omega)\sqrt{\mu_0\epsilon_0}}{\sqrt{\mu_c\epsilon_c}} \right). \quad (10)$$

The derivation of the above relations is straightforward and will not be given here. Details of deriving them can be re-

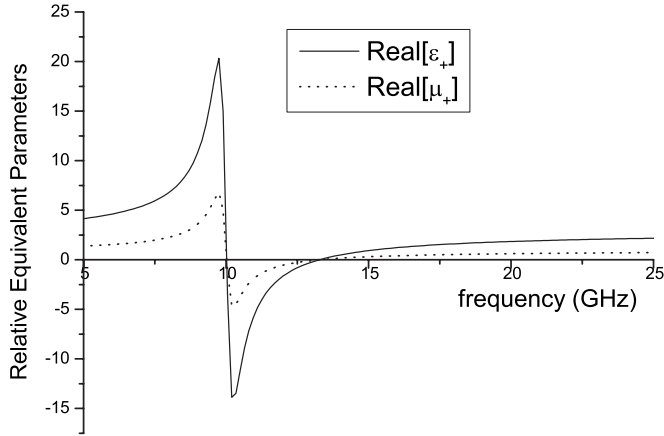


FIG. 1. The frequency dependence of relative (ϵ_+, μ_+) in the range of 5–25 GHz, where the chirality's characteristic frequency $\omega_c = 2\pi 10^9$ (rad/s), $d_c = 0.05$, $\epsilon = 3\epsilon_0$, and $\mu = \mu_0$.

ferred to the wave-field theory⁴⁶ and the example procedure.³⁵ The imaginary parts of (ϵ_\pm, μ_\pm) are not shown here, which are almost zero over the whole region except in the vicinity of ω_c . From Fig. 1, one can find that (ϵ_+, μ_+) becomes a double negative (DNG) material in the frequency band of 10–13.3 GHz. When the frequency drops below ω_c or exceeds, it turns to a double positive (DPS) medium. In Fig. 2, such a DNG-DPS reversion also happens. In a frequency band of 7.52–13.3 GHz, the negative refraction occurs to positive and negative effective materials, alternatively.

Hence the electromagnetic fields inside the chiral media can be obtained by the superposition of components as follows:

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_-, \quad (11)$$

$$\mathbf{H} = \mathbf{H}_+ + \mathbf{H}_-, \quad (12)$$

where \pm fields correspond to the results calculated from two separate sets of effective materials (ϵ_+, μ_+) and (ϵ_-, μ_-) , respectively.

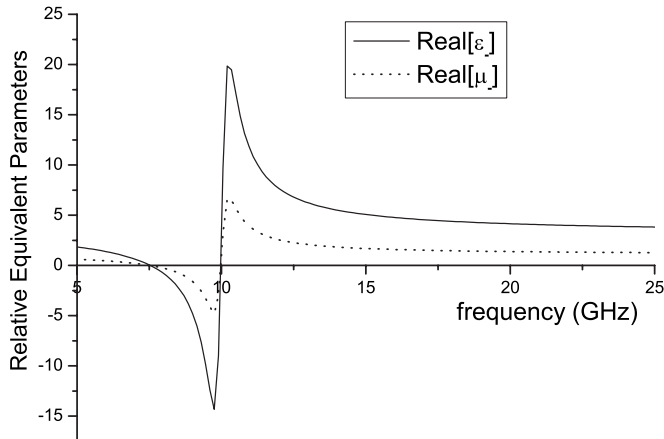


FIG. 2. The same as Fig. 1, for the frequency dependence of relative (ϵ_-, μ_-) .

Interestingly, if we consider the case of a plane wave impinging upon an air-chiral interface, there exist two particular frequencies at which no chirality is actually present:

(i) if $f_l = 7.52$ GHz, the chiral medium is only characterized by the + equivalent medium composed of (ϵ_+, μ_+) , which results in that only half of the power can be transmitted from the air to the chiral medium; and

(ii) if $f_h = 13.3$ GHz, only the pair of (ϵ_-, μ_-) remains.

It can be observed that their geometrical significance is the characteristic frequency of chirality (i.e., $f_c = \omega_c / 2\pi$), demonstrating the logarithmic symmetry of the dispersion, i.e.,

$$f_l f_h = f_c^2. \quad (13)$$

To summarize, the chirality dispersion in the Condon's model, derived based on the molecular theory for quantum mechanics, can lead to negative-index materials (i.e., $n_\pm = \text{Re}[\sqrt{\epsilon_\pm \mu_\pm}]$) at certain frequency bands. One has to, however, note that n_\pm cannot be simultaneously negative within the region of (f_l, f_h) . The plus and minus signs of refractive indices will be exchanged when the working frequency oversteps resonant frequency f_c .

B. Nonreciprocity route

In view of Eqs. (1) and (2), both constitutive relations are applicable to reciprocal media only. When the nonreciprocity is present in the chiral magnetoelectric materials, the constitutive relations are expressed as follows:

For the Post's relations

$$\mathbf{D} = \epsilon_p \mathbf{E} + (i\xi - \nu) \mathbf{B}, \quad (14a)$$

$$\mathbf{H} = (i\xi + \nu) \mathbf{E} + (1/\mu_p) \mathbf{B}, \quad (14b)$$

and for Tellegen's relations

$$\mathbf{D} = \epsilon_T \mathbf{E} + (\chi + i\kappa) \sqrt{\mu_0 \epsilon_0} \mathbf{H},$$

$$\mathbf{B} = (\chi - i\kappa) \sqrt{\mu_0 \epsilon_0} \mathbf{E} + \mu_T \mathbf{H}, \quad (15)$$

where χ and ν denote the nonreciprocity parameters used in these two commonly used constitutive relations. The conversion rule between these two sets of relations is given:

$$\epsilon_T = \epsilon_p + \mu_p (\xi^2 + \nu^2),$$

$$\chi = \mu_p \nu c_0,$$

$$\kappa = \mu_p \xi c_0,$$

$$\mu_T = \mu_p. \quad (16)$$

In particular, we only consider the Tellegen's relations as an example for the nonreciprocal nihility discussion, since such a condition can be transformed to the Post's relations. The chiral nihility in Ref. 28 is generalized from the two-phase mixture of positive and negative materials with null effective parameters for the macroscopic mixture.⁴⁷ However, this generalization has some restrictions because the requirement

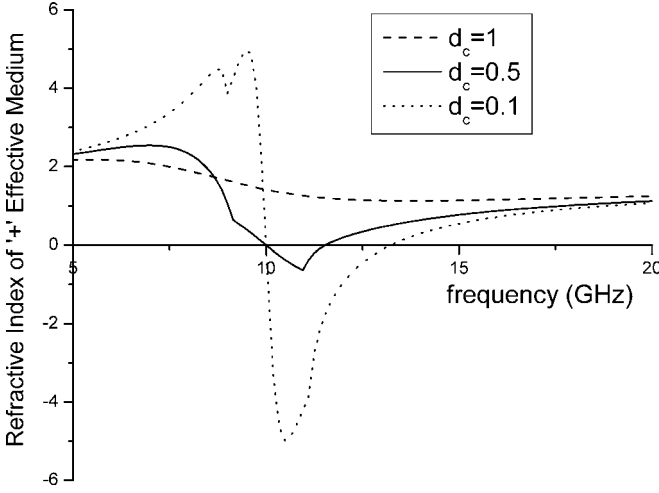


FIG. 3. The frequency dependence of refractive indices for the “+” effective medium in the range of 5–20 GHz with the same parameters as in Fig. 1 except d_c .

of $\epsilon = \mu = 0$ is physically impossible and too strict to achieve the resonances for both parameters. As a result, the practical applications for the chiral nihility is quite limited. Therefore we propose another condition, that is, a nonreciprocity route, to achieve backward waves and negative refraction:

$$\sqrt{\epsilon\mu/\epsilon_0\mu_0 - \chi^2} \pm \kappa < 0. \quad (17)$$

Apparently, the requirement of the value of $\epsilon\mu$ becomes less strict since both the magnetoelectric parameters (i.e., χ and κ) reduce the refractive indices, leading to negative-index materials. In what follows, the problematic chiral nihility can be modified as the condition $\sqrt{\epsilon\mu/\epsilon_0\mu_0 - \chi^2} = 0$. The dispersion of nonreciprocity has not been clearly worked out independently so far, but it can be envisioned from general considerations that the dispersion relations of χ and κ in Eq. (17) could be a similar alternative of the Condon model:

$$\chi(\omega) = \frac{d_c \omega^2 \omega_c^2}{\omega^4 + \omega_c^2 - (2 - d_c^2) \omega^2 \omega_c^2}, \quad (18)$$

$$\kappa(\omega) = \frac{(\omega_c^2 - \omega^2) \omega \omega_c}{\omega^4 + \omega_c^2 - (2 - d_c^2) \omega^2 \omega_c^2}. \quad (19)$$

The refractive indices for corresponding eigenmodes are shown in Fig. 3, noting that the indices for the minus “−” effective medium carry a similar fashion by mirroring the curves of the “+” medium along the vertical line at $f = 10$ GHz. When the damping factor $d_c = 1$, the refractive index varies in a limited frame against the frequency even in the characteristic frequency of ω_c , and it can be proved that a high damping due to the chiral material will hold back the power rotatory and the curve appears more flat (approaching to $\sqrt{3}$ over all frequencies), which means that the chirality does not resonate for chiral media of high damping. When the damping factor becomes smaller, more power is rotated and the resonant phenomena becomes fairly clear. The resonance will further induce the negative refraction of eigenmodes within certain frequency bands. Those negative-index

bands are inversely proportional to the damping factor.

III. GYROTROPIC MAGNETOELECTRIC MATERIALS

Generally, bianisotropic media can be considered as the most generalized magnetoelectric materials in form. However, in a practical case, parameters in those four dyadics characterizing bianisotropy effects can only be retrieved for particular structures.³² In this section, those gyrotropic magnetoelectric materials^{33,34} (which can be practically manufactured) are of great interest, instead of conceptual bianisotropic materials whose parameters are manually set. In particular, we consider gyroelectric chiral materials,⁴⁸ of which the constitutive relations are shown:

$$\mathbf{D} = \epsilon_0 \epsilon_r \begin{bmatrix} \epsilon & -ig & 0 \\ ig & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \cdot \mathbf{E} + i\xi \mathbf{B},$$

$$\mathbf{H} = i\xi \mathbf{E} + \frac{1}{\mu_0 \mu_r} \mathbf{B}, \quad (20)$$

where

$$\epsilon = \left(1 - \frac{\omega_p^2 (\omega + i\omega_{eff})}{\omega [(\omega + i\omega_{eff})^2 - \omega_g^2]} \right), \quad (21)$$

$$g = \frac{\omega_p^2 \omega_g}{\omega [(\omega + i\omega_{eff})^2 - \omega_g^2]}, \quad (22)$$

$$\epsilon_z = 1 - \frac{\omega_p^2}{\omega^2}, \quad (23)$$

with ω_{eff} , ω_g , and ω_p representing the collision frequency, electron gyrofrequency, and plasma frequency,⁴⁹ respectively. Such gyroelectric chiral materials can be managed by distributing chiral objects into a controllable biasing magnetic field, which is applied externally. If the plane wave $\mathbf{E}e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ is propagating inside gyroelectric chiral media, the wave equations can be expressed as

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + 2i\omega\mu_r\mu_0\xi\mathbf{k} \times \mathbf{E} + k_0^2 \mu_r \epsilon_r \begin{bmatrix} \epsilon & -ig & 0 \\ ig & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \cdot \mathbf{E} = 0, \quad (24)$$

where k_0 represents the wave number in free space. Algebraically, wave numbers corresponding to parallel and antiparallel eigenmodes for two mutually perpendicular polarizations can be obtained from nontrivial solutions in terms of a quartic polynomial, which would be cumbersome to solve. Thereafter, to yield some physical insight, longitudinal waves with respect to the external biasing field are considered with the interest in backward waves and negative phase velocity. For the longitudinally propagating eigenwaves along the biasing plasma, one can yield four wave numbers corresponding to eigenmodes:

$$k_2^1 = \omega \left[\mp \mu_0 \mu_r \xi \pm \sqrt{\mu_0^2 \mu_r^2 \xi^2 + \mu_0 \mu_r \epsilon_0 \epsilon_r (\epsilon \mp g)} \right], \quad (25)$$

$$k_4^3 = \omega[\pm\mu_0\mu_r\xi \pm \sqrt{\mu_0^2\mu_r^2\xi^2 + \mu_0\mu_r\epsilon_0\epsilon_r(\epsilon \pm g)}]. \quad (26)$$

With reference to the energy transportation direction, eigenwaves corresponding to eigenwave numbers k_1 and k_2 may become backward waves because the handedness of these two eigenwaves will change within certain frequency bands. Note that the k_1 -related eigenwave is parallel to the energy transportation while the k_2 -related eigenwave is opposite; and in backward-wave frequency bands both eigenwaves are right-circularly polarized.³⁹ In particular, the phase velocity versus the frequency is studied in order to observe characteristics of LHM.

In Fig. 4(a), it can be observed that when no magnetoelectric coupling is present the phase velocity of the k_2 eigenmode is always negative and that of the k_1 eigenmode is positive. Substituting those two eigenmodes into Eq. (24), one can note that negative phase velocity in Fig. 4(a) does not mean a backward-wave phenomenon. Instead, when $\xi = 0$, negative phase velocity represents that the k_2 eigenmode is left-handed with the reference to the opposite direction of external magnetic field, and positive velocity shows that the k_1 eigenmode is left-handed along with the direction of external field. When slight magnetoelectric coupling exists [e.g., $\xi = 10^{-3}$ in Fig. 4(b)], backward-wave phenomena arise for both k_1 and k_2 eigenmodes, in which resonances can be observed. In what follows, we consider the gyroelectric chiral medium with bigger magnetoelectric coupling effect as shown in Fig. 4(c). Compared with the case shown in Fig. 4(b), one can note that the shift of resonant frequencies is neglectable, while resonant amplitudes in Fig. 4(c) are drastically enhanced. In both weak-coupling and strong-coupling cases, it can be found that backward-wave regions arise before respective resonances. After passing the resonant frequency, the handedness and polarization status of those eigenmodes reduce to those of nonmagnetoelectric materials. Such analogy only occurs when the working frequency is higher than the intrinsic resonant frequency.

As another type of special gyrotropic magnetoelectric materials, the gyrotropic Ω medium is of particular interest, of which the constitutive relations are given for the Post's description:

$$\mathbf{D} = \bar{\epsilon} \cdot \mathbf{E} + i \begin{bmatrix} 0 & B & 0 \\ A & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \mathbf{B},$$

$$\mathbf{H} = i \begin{bmatrix} 0 & A & 0 \\ B & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \mathbf{E} + \bar{\mu}^{-1} \cdot \mathbf{B}, \quad (27)$$

where $\bar{\epsilon}$ and $\bar{\mu}$ represent permittivity and permeability tensors, respectively. In order to create those gyrotropic magnetoelectric materials, two crossed external fields (i.e., electric and magnetic) are required to apply upon the materials. Here, we still consider the plane waves propagating along the optical z -axis in such materials. To emphasize the role of magnetoelectric couplings, the permittivity/permeability param-

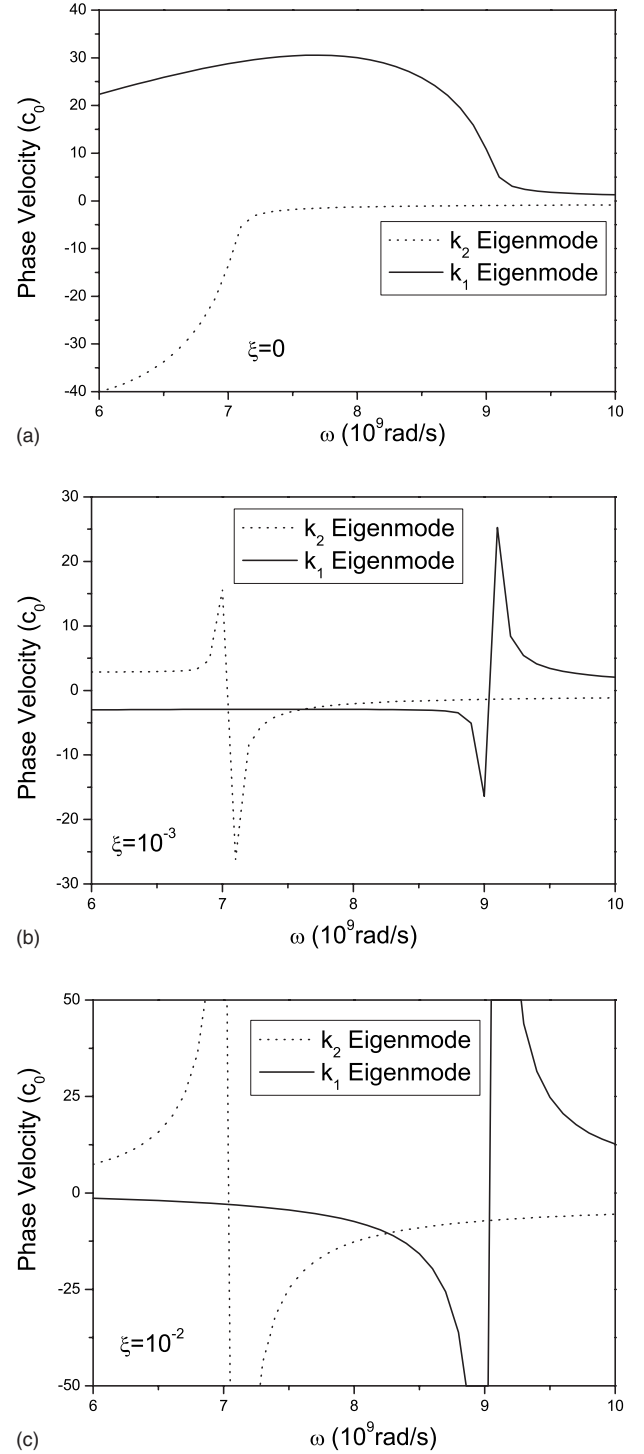


FIG. 4. Phase velocities for backward-wave eigenmodes as a function of frequency near plasma frequency with parameters $\omega_p = 8 \times 10^9$ rad/s, $\omega_{eff} = 0.1 \times 10^9$ rad/s, and $\omega_g = 2 \times 10^9$ rad/s under different degrees of magnetoelectric couplings.

eters are chosen to be scalar quantities, and thus one can yield the wave numbers of eigenmodes

$$k_{\pm} = \omega \sqrt{\mu \epsilon \pm \mu^2 (A^2 - B^2)}. \quad (28)$$

It is also noted that backward-wave and left-handed phenomena can be realized even for the case of $\mu < 0$ and $\epsilon > 0$,

since the magnetoelectric parameters would make the effective permittivity negative by properly applying perpendicular electromagnetic external fields.

IV. CONCLUSION

In this paper, we studied the possibility of realizing left-handed materials through magnetoelectric couplings. Both isotropic and gyrotropic magnetoelectric materials, which are manufacturable in practice, have been revisited with the interest in studying left-handed phenomena. Such phenomena (e.g., backward waves, negative refractive indices, negative effective parameters, and negative phase velocities) have been fully investigated, in which dispersion was also included. To summarize, the magnetoelectric coupling pro-

vides us various routes to realize left-handed materials for optical applications.

ACKNOWLEDGMENTS

The authors are grateful for the support from SUMMA Foundation, USA, to the Joint Program offered by the National University of Singapore (in Singapore) and Supélec (in Paris, France) in terms of the financial support by both parties; and to the joint project supported by the France-Singapore “Merlion Project.” The financial supports by the US AOARD/AFOSR via Grants No. FA4869-06-1-0054 and No. FA4869-07-1-4024 are greatly appreciated. The authors are also grateful to the reviewers for their time and great effort, and especially for their useful suggestions for revising this paper.

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