# Routing with Opportunistically Coded Exchanges in Wireless Mesh Networks

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*Abstract*— **Network coding is known to improve network throughput by mixing information from different flows and conveying more information in each transmission. Recently there have been some proposals for applying network coding to wireless mesh networks leveraging the broadcast nature of wireless transmissions. These approaches exploit coding opportunities passively while forwarding packets but they do not proactively change routing of flows to create more coding opportunities. In this paper, we attempt to investigate the extent of performance gain possible when routing decisions are made with the awareness of coding. We first define the expected number of coded transmissions for a successful exchange of packets between two nodes through an intermediate node. We then formulate optimal routing with coding, given the topology and traffic, as a linear programming problem. We conduct a preliminary evaluation of coding-aware routing and show that it offers significant gain particularly when there are many long distance flows.**

## I. INTRODUCTION AND MOTIVATION

The proliferation of wireless networks despite their capacity limitations is spurring networking researchers to find various ways to improve their throughput. Network coding is one of the few options available for stretching the capacity of existing wireless technologies. The essence of network coding is to convey more information in each transmission by mixing information from different sources resulting in increased network throughput. Though the idea of network coding is not new, in the past, it has been applied mainly in the context of multicasting in traditional wired networks [1]. Only recently, it is found that wireless mesh networks offer fertile ground for network coding given that wireless transmissions are inherently broadcast at physical layer and thus with coding yield better throughput even for unicast applications.

The benefit of coding in wireless networks can be illustrated using a simple example of two nodes exchanging information through a common intermediate node [5]. Consider the scenario shown in Fig. 1, where node 2 has a packet 'a' that needs to be delivered to node 3 via node 1 and similarly node 3 has a packet 'b' intended for node 2 via node 1. Let us assume that the channel is perfect, i.e., no transmissions are lost. Without coding, it takes a total of 4 wireless transmissions ('a' from 2 to 1 and then 1 to 3 and 'b' from 3 to 1 and 1 to 2) to complete the exchange. On the other hand, if coding is employed, once node 2 receives both the packets 'a' and 'b', it can transmit a single coded packet 'a' xor 'b' which is received by both nodes 2 and 3. They can then extract the desired packet as they have the other packet. Such a coded exchange effectively reduces the total number of transmissions from 4 to 3.

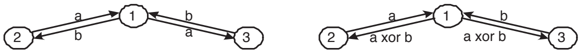

Fig. 1. An example of information exchange with network coding

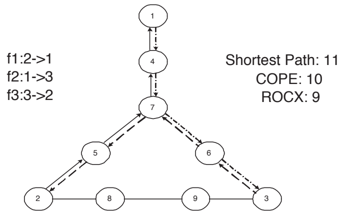

Fig. 2. An example of routing with opportunistically coded exchange

A practical scheme, referred to as COPE, based on network coding for wireless networks is proposed in [3]. COPE extends the gain of coding beyond the above information exchange scenario through opportunistic coding of two or more packets in a single transmission. COPE does this by having nodes overhear others transmissions in their neighborhood and issue reception reports to let neighbors learn about the packets they currently have. It is reported that COPE with its opportunistic listening and coding achieves several fold increase in throughput. However, we argue that further gains are possible if routing decisions are made with the awareness of coding instead of routing and coding independently.

Consider the scenario in Fig. 2 with three flows $f_1 : 2 \rightarrow 1$, $f_2 : 1 \rightarrow 3$, $f_3 : 3 \rightarrow 2$. If all the links are perfect with no loss, the corresponding shortest paths are 2→5→7→4→1, 1→4→7→6→3, and 3→9→8→2 respectively. Without coding, the total number of transmissions needed to deliver one packet of each flow is 11. Under COPE, since nodes 4, 5 and 6 can not overhear each other, there is only one coding opportunity at node 4 as node 1 and node 7 exchange packets through node 4, reducing the total from 11 to 10. On the other hand, if the route of $f_3$ is changed to 3→6→7→5→2, we need no more than 9 transmissions. Though this path has one more hop (one more transmission) than the shortest path, it creates coding opportunities at nodes 5 and 6[1]. This leads us to investigate the extent of gain possible when routing is performed with the awareness of coding opportunities.

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[1] It is possible to accomplish the three-way exchange of 5→4, 4→6, and 6→5 via node 7 with 2 coded transmissions, reducing the total number of transmissions to 8. But in this paper, we only consider the pair-wise coded exchanges such as the ones through nodes 4, 5 and 6.

## II. CODING-AWARE ROUTING

In this section, we first define a new metric ECX that captures the expected number of coded transmissions needed for a successful exchange of packets between two nodes via an intermediate node. Based on ECX, we formulate the optimal coding-aware routing as a linear programming problem given the delivery probability and traffic intensity between each node pair. We then evaluate the performance of optimal coding-aware routing against shortest path routing with coding.

### A. Expected Number of Coded Transmissions for an Exchange

The illustrations in the previous section assumed that channel conditions are perfect with no loss. While this makes it convenient to determine the number of coded transmissions needed for an exchange, it is not a realistic assumption considering that wireless transmissions are prone to various forms of interference. This begs the question, how to determine the number of coded transmissions needed for a successful exchange when retransmissions are necessary to recover from lost packets, which is answered in the following.

Let us again consider the scenario in Fig. 1 but now assume that the one-way delivery probability from node $i$ to node $j$ is given by $r_{i\to j}$. We use $r_{i,j}$ to denote the probability of successful two-way delivery including the acknowledgment, i.e., $r_{i,j} = r_{i\to j}r_{j\to i}$. According to the ETX metric [2], the expected number of transmissions for a successful exchange without coding is $\frac{2}{r_{1,2}} + \frac{2}{r_{1,3}}$. Even with coding, since only node 1 performs coding, we still need $\frac{1}{r_{1,2}} + \frac{1}{r_{1,3}}$ transmissions for nodes 2 and 3 to deliver both packets to node 1. To complete the exchange, node 1 needs to successfully deliver the coded packet to both nodes 2 and 3, which requires

$$\sum_{k=1}^{\infty}(s_{1,2}^{k-1}r_{1,2}\sum_{i=1}^{k}s_{1,3}^{i-1}r_{1,3}+s_{1,3}^{k-1}r_{1,3}\sum_{i=1}^{k}s_{1,2}^{i-1}r_{1,2}-s_{1,2}^{k-1}s_{1,3}^{k-1}r_{1,2}r_{1,3})k \tag{1}$$

coded transmissions, where $s_{i,j} = 1-r_{i,j}$, which boils down to $\frac{1}{r_{1,2}} + \frac{1}{r_{1,3}} - \frac{1}{r_{1,2}+r_{1,3}-r_{1,2}r_{1,3}}$. Therefore, the expected number of transmissions for a successful exchange with coding, which we refer to as ECX, is $\frac{2}{r_{1,2}} + \frac{2}{r_{1,3}} - \frac{1}{r_{1,2}+r_{1,3}-r_{1,2}r_{1,3}}$. Effectively, the gain with coding is $\frac{1}{r_{1,2}+r_{1,3}-r_{1,2}r_{1,3}}$.

### B. Optimal Coding-aware Routing

| Notation used in our formulation | |
|---:|---|
| $N$ | the number of nodes in the network |
| $r_{i,j}$ | the two-way successful delivery probability between $i$ and $j$ |
| $T$ | the number of traffic flows |
| $M_t$ | amount of traffic of flow $t$ |
| $(s_t, d_t)$ | the source and destination of flow $t$ |
| $m_{k,i,j}^t$ | units of flow $t$ traffic on link $(k, i)$ with next hop $j$ |
| $x_{i,j}$ | the amount of traffic on link $(i, j)$ |
| $u_{i,j}^t$ | total amount of traffic of flow $t$ on link $(i, j)$ |
| $w_{k,i,j}$ | total amount of traffic on link $(k, i)$ with next hop $j$ |
| $c_{k,i,j}$ | the amount of traffic between $k$ and $j$ that can be coded at $i$ |

The aim of optimal coding-aware routing is to find paths for flows such that the total expected number of coded transmissions needed for their successful delivery is minimized. We formulate this as a linear programming problem. The notation used in the formulation is listed below. Suppose there are $N$ nodes in the network and $r_{i,j}$ is the probability of successful two-way delivery including the acknowledgment between nodes $i$ and $j$. Further assume that the number of flows are $T$ where $(s_t, d_t)$ denotes the source and destination pair of a flow $t$ and $M_t$ is its traffic intensity. Let $m_{k,i,j}^t$ denote the units of flow $t$ on link $(i, j)$ whose next hop is node $k$, which effectively determines the routing of each flow. Then, our objective is to find $m_{k,i,j}^t$ that minimizes

$$\sum_{i,j=1}^{N}\frac{x_{i,j}}{r_{i,j}} - \sum_{i,j,k=1}^{N}\frac{c_{k,i,j}}{r_{i,k}+r_{i,j}-r_{i,k}r_{i,j}} \tag{2}$$

under the following constraints. First, we have

$$\sum_{t=1}^{T}u_{i,j}^t - x_{i,j} = 0 \tag{3}$$

$$\sum_{t=1}^{T}m_{i,j,k}^t - w_{i,j,k} = 0 \tag{4}$$

$$\sum_{i=1}^{N}u_{s_i,i}^t - M_t = 0 \tag{5}$$

$$\sum_{i=1}^{N}u_{i,d_i}^t - M_t = 0 \tag{6}$$

Then, there are flow conservation constraints that require that the amount of traffic in and out of a node is the same except for the source and destination of each flow, i.e.,

$$\sum_{k=1}^{N}m_{i,j,k}^t - u_{i,j}^t = 0 \qquad j \neq d_t \tag{7}$$

$$\sum_{k=1}^{N}m_{k,i,j}^t - u_{i,j}^t = 0 \qquad i \neq s_t \tag{8}$$

$$\sum_{i=1}^{N}u_{i,s_i}^t = 0 \tag{9}$$

$$\sum_{i=1}^{N}u_{d_i,i}^t = 0 \tag{10}$$

Finally, there are constraints on the number of packets that can be coded at each node, i.e.,

$$c_{k,i,j} - w_{k,i,j} \leq 0 \tag{11}$$

$$c_{k,i,j} - w_{j,i,k} \leq 0 \tag{12}$$

$$c_{k,i,j} - c_{j,i,k} = 0 \tag{13}$$

We refer to our approach of achieving optimal trade-off between coding opportunities and shortest paths as *routing with opportunistically coded exchanges* (ROCX).

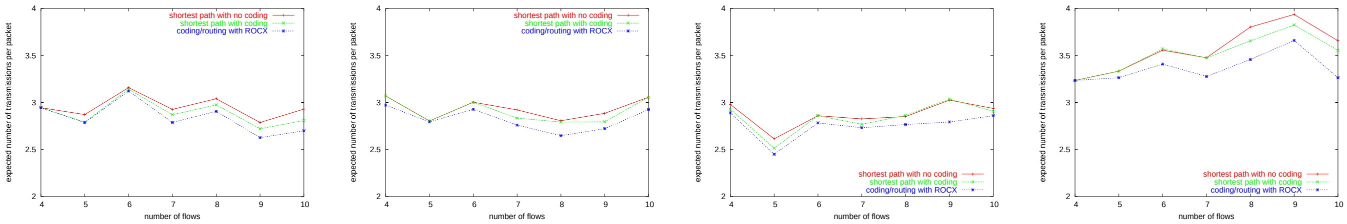

Fig. 3. Performance under varying transmission rates when minimum hop count of each flow is 2: (a) 1 Mbps; (b) 2 Mbps; (c) 5.5 Mbps (d) 11 Mbps

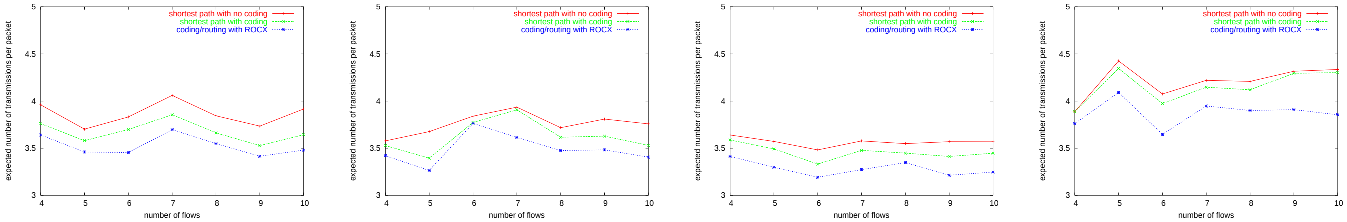

Fig. 4. Performance under varying transmission rates when minimum hop count of each flow is 3: (a) 1 Mbps; (b) 2 Mbps; (c) 5.5 Mbps (d) 11 Mbps

### C. Preliminary Evaluation

We now evaluate the performance of ROCX and compare it with coding under shortest path routing based on ETX metric which represents the approach of COPE. We use the MIT Roofnet [4] trace data consisting of 38 nodes to set up the experiments after excluding the unidirectional links and isolated nodes. The delivery ratio between two nodes under this topology are given for four different transmission rates: 1, 2, 5.5, and 11 Mbps. Based on these delivery ratios, we compute the path with the shortest ETX for each node pair under each transmission rate. We vary the number of flows from 4 to 10 and assume that each flow offers one unit of traffic. In each experiment, the source and destination of a flow are randomly chosen and we repeat the experiment with five different random seeds and record the average number of transmissions per flow. Under shortest path routing, this is just the average of the shortest path ETX of each flow. In case of shortest path routing with coding, we use ECX to compute the average number of transmissions per flow given the ETX based shortest paths. Under ROCX, the LP formulation yields the average number of transmissions per flow.

The results of the performance comparison are shown in Fig. 3 where the shortest ETX path of each flow is at least 2 hops long and in Fig. 4 where it is at least 3 hops long. They also compare the performance under varying number of flows and four different transmission rates. It can be seen that in all settings, ROCX yields the minimum number of transmissions. It is also apparent that as the number of flows increases, ROCX has better relative performance. It also performs better than others when each flow traverses more hops. This is not unexpected since when there are more long distance flows, it is likely that more alternate routes with better coding opportunities exist in the network. It is interesting to note that the gain of ROCX is more when nodes are transmitting at high data rates which needs further investigation.

## III. CONCLUSIONS AND FUTURE WORK

In this paper, we introduced a new approach for routing called ROCX and investigated the potential gain when routing is performed with the awareness of coding in wireless mesh networks. We proposed a new metric ECX to capture the number of transmissions needed with coding for a successful exchange of packets in case of lossy wireless links. We formulated the optimal coding-aware routing problem and compared its performance with coding under shortest path routing which is similar to COPE. Our preliminary evaluation shows that ROCX approach can further reduce the number of transmissions compared to COPE. The gain seems particularly significant when there are many long distance flows and when nodes are transmitting at high data rates. Further evaluation is needed to investigate the effects of data rate, traffic and topology characteristics on the performance of ROCX.

We can also devise an on-demand routing scheme based on ECX metric with route establishment and maintenance framework similar to an ETX-based on-demand routing protocol. The key difference however is that, under ECX-based scheme, whenever a node receives a route request, it assigns a different cost to each next hop depending on the previous hop of the request, based on the amount of traffic flowing between those two nodes through this node. We plan to develop these ideas into a practical coding-aware routing scheme in the near future.

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