RUSSELL'S USE OF TRUTH TABLES

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INTRODUCTION

Primarily, this article will present evidence to dispel a mistaken view by many logicians that Russell did not use the notion of truth tables. Russell used truth tables years before Wittgenstein and Post's work was published in the early 1920s. Whitehead and Russell seemed to have an idea of truth tables in their explanation of material implication in *Principia Mathematica*. Russell clearly used a modified truth table in Lecture III of "The Philosophy of Logical Atomism", given in London early in 1918.[1] Later, Russell continued independently to examine truth-table language in his *Introduction to Mathematical Philosophy* (1919).[2] Recent discovery of truth-table work by Wittgenstein (and perhaps Russell) in 1912, and an examination of T. S. Eliot's logic notes at Harvard in 1914, demonstrate an early knowledge, appreciation and use of this logical tool by Russell.

Anscombe claims that the truth table or "matrix" was "invented

[1] See *Russell's Logical Atomism*, edited and with an introduction by David Pears (La Salle, Ill.: Open Court, 1985; London: Fontana/Collins, 1972), pp. 71–3; *Papers* 8: 185–6. In the relevant portion of Lecture III, Russell discusses negation, implication and other operators in terms of their truth possibilities. These lectures were given before Russell re-established contact with Wittgenstein at the end of the First World War. While Russell begins the lectures explaining that many of the ideas he will discuss were learned from conversations with Wittgenstein, his most immediate attributions in this lecture were to Sheffer and Nicod, and how the use of the Sheffer Stroke could reduce the number of propositions necessary to begin *Principia Mathematica*. Relying on Frege's use of material equivalence, Sheffer needed at least the truth-table technique, as defined later in this paper, to make his logical discoveries.

[2] *IMP*, pp. 155–66.

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(independently)" by Wittgenstein and Post,[3] who each published truth tables in the early 1920s. Kneale and Kneale make a more modest claim that Wittgenstein and Post, following the work of Philo, Boole and Frege, provide an "account" of "a decision procedure for primary logic".[4] Quine attributes this "pattern of reasoning" to Frege, Peirce and Schröder, and truth tables themselves to Lukasiewicz, Post and Wittgenstein.[5] Post derives his own development of truth tables from Jevons, Venn, Schröder, Whitehead and Russell, and Lewis.[6] Wittgenstein introduced the "schemata" at 4.31 of the *Tractatus* without immediate attribution, but at 4.431 parenthetically mentions Frege's conception of a proposition as the expression of its truth conditions.[7] It is far from clear that any one person should be given the title of "inventor" of truth tables.

TERMINOLOGY

As we examine these claims, and the material that follows, it would perhaps be helpful to distinguish between two concepts: a truth-table

[3] G. E. M. Anscombe, *An Introduction to Wittgenstein's Tractatus*, 3rd ed. (Philadelphia: U. of Pennsylvania P., 1971), p. 23. I used this book in a class on the "Origins of Analytical Philosophy" at American University in the spring of 1995. Several students had earlier editions. My edition is the third. We discovered that in Chapter 10, which deals with the general form of propositions, Anscombe has changed her wording on Russell's understanding of Wittgenstein's use of truth-functions in each edition of her important book (note introductory comments to each of the three editions, found most accessibly in the third). The target appears to be Russell's explanation of truth-functionality and propositions in the introduction to the English version of the *Tractatus Logico-Philosophicus*. Far from missing the point, Russell had already worked through this problem on several occasions, as will be noted below.

[4] William Kneale and Martha Kneale, *The Development of Logic* (Oxford: Clarendon P., 1962), p. 532.

[5] Willard van Orman Quine, *Methods of Logic*, 4th ed. (Cambridge, Mass.: Harvard U.P., 1982), p. 39.

[6] Emil Leon Post, "Introduction to a General Theory of Elementary Propositions" (1921), in Jean van Heijenoort, ed., *From Frege to Gödel: a Sourcebook in Mathematical Logic, 1879–1931* (Cambridge, Mass.: Harvard U.P., 1967), pp. 267 n.6 and 269 n.7.

[7] *Tractatus Logico-Philosophicus*, trans. C. K. Ogden, introduction by Russell (London: Routledge & Kegan Paul, 1922), pp. 93–5. The *Tractatus* had been published in German in 1921. (Russell had read a draft in 1919 where the truth tables weren't as extensive.—Ed.)

technique and a truth-table device. A truth-table technique is a logically exhaustive analysis of the truth-functions of a given proposition, examining each possible set of truth values, and deriving definitions or arguments based on the results of this logically exhaustive process. A truth-table device is the mechanical creation of vertical columns of possibilities, measured against horizontal rows of logically exhaustive options, with actual lines of valid argumentation tested against the horizontal and vertical matrix. In short, the truth-table technique is a reference to the logical process of examining all truth values for a proposition; the truth-table device is the creation of an actual truth table.

TRUTH-TABLE TECHNIQUES IN THE NINETEENTH CENTURY

Boole demonstrated a far-sighted understanding of the truth-table technique in 1854, in a well-known passage of *The Laws of Thought*. Boole argued for the expansive use of new notation in logic, notation which would lead to a more rapid and secure basis for logical reasoning. Adoption of "quantitative symbols" would assist in developing this new logic. Boole found that "we may in fact lay aside the logical interpretation of the symbols in a given equation; convert them into quantitative symbols, susceptible only of the values 0 and 1; perform upon them as such all requisite processes of solution; and finally restore to them their logical interpretation."[8] This explanation can be seen as a natural-language description of translating a proposition into symbols, gauging the truth value of the proposition in two-value logic, working out the permutations of logical possibilities, and re-translating the proposition back into a logically equivalent form. This is the truth-table technique.

In addition, Boole seemed to understand the importance of a truth-

[8] George Boole, *An Investigation of the Laws of Thought*, in *Collected Logical Works*, ed. P. E. B. Jourdain, Vol. 2 (La Salle, Ill.: Open Court, 1952; 1st ed., 1916), p. 76. The original edition was published in London: Walton and Maberly, 1854. This passage is found on page 70 of the original edition. Kneale and Kneale have some extremely revealing comments about Boole's use of truth-functionality on pages 531ff. in *The Development of Logic*. They claim that "but so far as we know, lists of possibilities were not constructed in tabular form until the last century, when Boole used them to explain the process of development by which any expression of his system could be transformed into a sum of products (i.e. a disjunction of conjunctions) and Frege used them explicitly for definition of the various truth-functions" (p. 531).

table device. In 1847, Boole published "A Mathematical Analysis of Logic".[9] This was a visionary essay concerning the psychological and logical components of human reasoning. In the essay, Boole examined hypothetical propositions of two or more categories linked by a logical copula. As he considered the range of potential hypothetical expressions, Boole posited the truth values of a given proposition as true or false. This means that when two propositions are logically linked together, as in the case of an assertion of a necessary connection, there are four potential cases: true/true, true/false, false/true, and false/false. However, Boole only analyzes these potential cases in terms of class claims about the entire universe of objects and expressions about those objects. It would be a far stretch to translate this view into a truth table recognizable to modern logicians, although it may be done with great ingenuity.[10] Boole did not himself combine the truth-table technique with his use of a embryonic truth-table device. However, it is surprising that he developed both techniques independently, and this realization demands far more attention to Boole's efforts than currently given by many modern logicians.

Even if he had combined his view of the truth-table technique and the truth-table device, the result would not have been the powerful weapon posited by Wittgenstein and Post. Boole needed two advances in logic that came after his own work: material implication and material equivalence. The former is necessary to draw theorems out of their axioms. The latter is a truth-functional restatement of what already has been proven. Both are used in most, if not all, logical systems developed in the twentieth century. In most logical systems used today, both now require truth-table techniques for definitional purposes.

The importance of material implication cannot be overlooked. An axiomatic logical system must have a rule of inference, one important example of which is material implication. The rule of modus ponens is

[9] *Studies in Logic and Probability*, ed. Rush Rhees, in Boole's *Collected Logical Works*, Vol. 1 (London: Watts; La Salle, Ill: Open Court, 1952), p. 89. The original manuscript was published by Cambridge University Press in 1847. The relevant analysis is found on page 50 of the Cambridge edition.

[10] Kneale and Kneale, *The Development of Logic*, p. 531. They demonstrate how Boole's use of this device, combined with his view of truth values, can implicitly lead to a truth table.

arguably the most important inference (the rule is, if you are given two premisses, [1] if *p*, then *q*, and [2] *p*, then you may infer *q* as a valid conclusion of those two premisses). This rule depends on a notion of material implication, or of some other form of implication. The vital concept is the valid inference of a conclusion that unpacks and extends information found in the premisses. Material implication is one rule used by Russell and others to explain how this process takes place. The rule itself requires some form of explanation—it is not merely a rule that can be posited and accepted on face value. The explanation is usually in the form of a definition. That definition usually explores the truth values for *p* and *q*, and stipulates truth values for the implication. This is a truth-table technique. Peano, Frege, Whitehead and Russell, and Lewis were each instrumental in underlining the importance of a rule of inference in building a logical system. Unfortunately, Boole did not have such a rule of inference.

Material equivalence is a rule of substitution or logical synonymy. Material equivalence, as a rule of logical synonymy, functions as a rule of logical identity of truth values. The use of the concept demands that each side of the triple bar have the same truth variable, judging similar results as true and dissimilar results as false. The only realistic, rational, and workable method for determining this definition is with a truth-table technique. For two variables, given the four possible truth combinations, only the two possibilities with one side false and the other true can be judged false. The two possibilities of similar results (true/true and false/false) are adjudged true. The very act of defining the triple bar is the act of establishing a truth-table technique for the term, as Frege, Whitehead and Russell, Sheffer, and Wittgenstein well knew. Boole did not have a notion of material equivalence, which came later with Frege's concept of truth-functionality.

For example, consider the importance of material implication and material equivalence in modern logic. In a more recent attempt at building a logical system, Copi used nineteen rules of inference.[11] These rules are familiar to virtually every logic instructor and have become central to introductory instruction. The importance of material implication is ap-

[11] Irving Copi, *Symbolic Logic*, 5th ed. (New York: Macmillan, 1979), pp. 8–42. I reference this edition rather than the introductory collaborations between Copi and Carl Cohen.

parent when five of the nine rules of reasoning rely on the horseshoe symbol (modus ponens, modus tollens, hypothetical syllogism, constructive dilemma, and absorption). Of correlative importance, the remaining four rules (disjunctive syllogism, simplification, conjunction, and addition) rely on a truth-table technique to demonstrate their validity, although disjunctive syllogism could be primitively understood and used without a formal truth-table technique. Arguably, these four rules are not strong enough for building a logical system, although such an assertion is far from certain. The ten rules of replacement rely on material equivalence, which is undeniably tied to a truth-table technique.

In fact, there is a circular relationship between these rules and the use of truth tables. The rules are defined in terms of truth tables, and the truth-tables demonstrate the validity of the rules. Many modern logicians wonder how generations of logicians before 1921–22 worked logic without truth tables, given the relative ease of using this logical tool to develop systematic arguments. Once in conversation with Nick Griffin, he said, "It's incredible to imagine how logicians worked before truth tables." I share that fascination. Most logicians have probably felt the same wonder and awe at logical work before the ease of truth tables. But the history of logic may show that truth-table-type thinking was in use from Aristotle to Boole, and certainly from Boole onward to Frege, Whitehead and Russell, and others prior to 1921–22. A reasonable conjecture would be that the truth-table technique has been with modern logic since the mid-nineteenth century, if not earlier. The truth-table device merely made the application of the truth-table technique notion easier and quicker, analogous to the ease and speed of using a calculator for computational problems. In retrospect, it is shocking to see how close Boole came, underlining his genius in challenging Aristotelian logic and in setting the stage for later logical developments. If someone could demonstrate Boole's reliance on a truth-table device to develop his logical system, then Boole would have to be given credit for initiating the advance to the new logic with an even greater insight than his discovery of Boolean Algebra or the existential fallacy.

It is important to highlight Frege's contributions. He also used the truth-table technique. In his *Begriffsschrift* [*Concept Script*], Frege defines two key terms: conditionality and negation. In both, he uses the terms "affirms" and "denies" to, in effect, present a logically exhaustive definition. Van Heijenoort, in an introduction to the *Begriffsschrift*, argues that "Frege's use of the words 'affirmed' and 'denied', with his listing of all possible cases in the assignment of these terms to propositions, in fact amounts to the use of the truth-table method."[12] In other words, given the distinctions raised in this paper, Frege obviously understood the truth-table technique.

There is no evidence to suggest that Frege actually constructed the truth-table device.[13] He surely understood the importance of his version of implication "causal connection" (found in §5 of the *Begriffsschrift*). Frege invented logical equivalence, the triple-bar notation used to signify logical equivalence. He used both concepts to build an impressive conceptual system. However, while employing the truth-table technique, he did not utilize the truth-table device. Frege's work was a precondition to that of Russell and Wittgenstein, giving them the important technical advances found in the *Begriffsschrift*. Yet, the absence of any formal demonstration of a short-cut method to test the validity of propositions indicates that Frege did not use a truth-table device.

It could be claimed that Frege did not need a truth-table device because the truth-table technique made such a device redundant. However, the more revealing fact may be that Frege's use of causal connection was not completely dependent on a truth table for its definition, as is the case with material implication, the version of implication used by Russell in *The Principles of Mathematics* and by Whitehead and Russell in *Principia Mathematica*.

[12] Jean van Heijenoort, "Introduction to the *Begriffsschrift*, a formula language, modeled upon that of arithmetic, for pure thought", *From Frege to Gödel*, p. 2.

[13] Kneale and Kneale claim that Philo, Boole, and Frege each used what this paper defines as the truth-table technique. In an effort to prove their claim, they provide actual truth tables of each philosopher's use of this technique. Such a visual aid may be misleading, because it may suggest that one, two, or all three actually used truth tables, an assertion for which there is no evidence of which I am aware at this time. Such a mistaken claim is not a straw man argument on my part, because the table constructed for Boole looks suspiciously like the type of device used by Wittgenstein or Post, and the device for Frege is very similar to the truth tables used by Russell at Harvard in 1914 (see below). Kneale and Kneale are not claiming that truth tables were used by Philo, Boole, or Frege. They are only making the point that the use of the truth-table technique by these three logicians easily translates into the modern use of the truth-table device (*Development of Logic*, pp. 531ff.).

"PRINCIPIA MATHEMATICA" AND MATERIAL IMPLICATION

Russell conducted significant prior exploratory work in his *Principles of Mathematics*, especially in Chapter 2 ("Symbolic Logic") and Chapter 3 ("Implication and Formal Implication").[14] This work was foundational for *Principia Mathematica*.[15] By 1910, Whitehead and Russell utilized the truth-table technique in crafting and building their logical system in *Principia Mathematica*. They were logical historians who carefully borrowed from previous logicians after critical examination of previous use of symbols and definitions. Of relevant interest, they borrowed the notion of material implication from Peano, and the use of material equivalence from Frege. Both material implication and material equivalence require a truth-table technique for a definition. Also, the importance of truth-functionality was a key lesson learned from Frege. As a result, Whitehead and Russell began *Principia Mathematica* with primitive propositions explained in terms of truth values. Each inferential rule is posited and developed with these primitive propositions.

Whitehead and Russell were evidently familiar with the truth-table technique as early as 1910, if not before. In *Principia Mathematica*, they offer a definition of material implication that is a verbal explanation of a truth-table technique, unlike Frege's weaker version of causal connection. Whitehead and Russell stipulate "the essential property that we require of implication is this: 'what is implied by a true proposition is true.' It is in virtue of this property that implication yields proofs." The relationship of possible truth values is the definition: "the most convenient interpretation of implication is to say, conversely, that if either p is false or q is true, then 'p *implies* q' is to be true. Hence, 'p implies q' is to be defined to mean: 'either p is false or q is true'" (*PM* I: 94).

[14] In Chapter 3 Russell claims that material and formal implication are "found to be essential to every kind of deduction" (*PoM*, p. 33). Interestingly, the discussion of implication of either sort is mostly theoretical, with little use of truth values as definition or explanation. That is not the case seven years later in *Principia Mathematica*, when material implication is defined explicitly in terms of truth values.

[15] Russell himself never published an explicit account of the truth-table technique. However, by the second decade of this century, Russell had accumulated a logical arsenal more powerful than either that possessed by Boole or Frege: he had both material implication (via Peano), which was much stronger than Frege's logical connection, and he had material equivalence (via Frege), which was a quantum leap past Boole's notions of logical identity. Russell put the two concepts (material implication and material equivalence) together, which made *Principia Mathematica* a more profound and useful logical exploration than many of its immediate predecessors. And again, each of these two notions required an understanding of the truth-table technique.

The notion of material implication demands a truth-table technique. Parry finds that "the meaning of the conditional" can be traced back to the Megarian philosophers, particularly Diodorus Cronus and Philo, and the scholastics, such as Jean Buridan and Albert of Saxony.[16] Some form of logical connection is necessary to offer a rule of inference. Whitehead and Russell's proposition *1.1 is typical of the function performed by logical connection: "anything implied by a true elementary proposition is true."

It is not beside the mark to speculate that the entire foundation of *Principia Mathematica* rests on knowledge and subtle employment of the truth-table technique. In addition, material equivalence, *prima facie*, demanded definition by a truth-table technique.

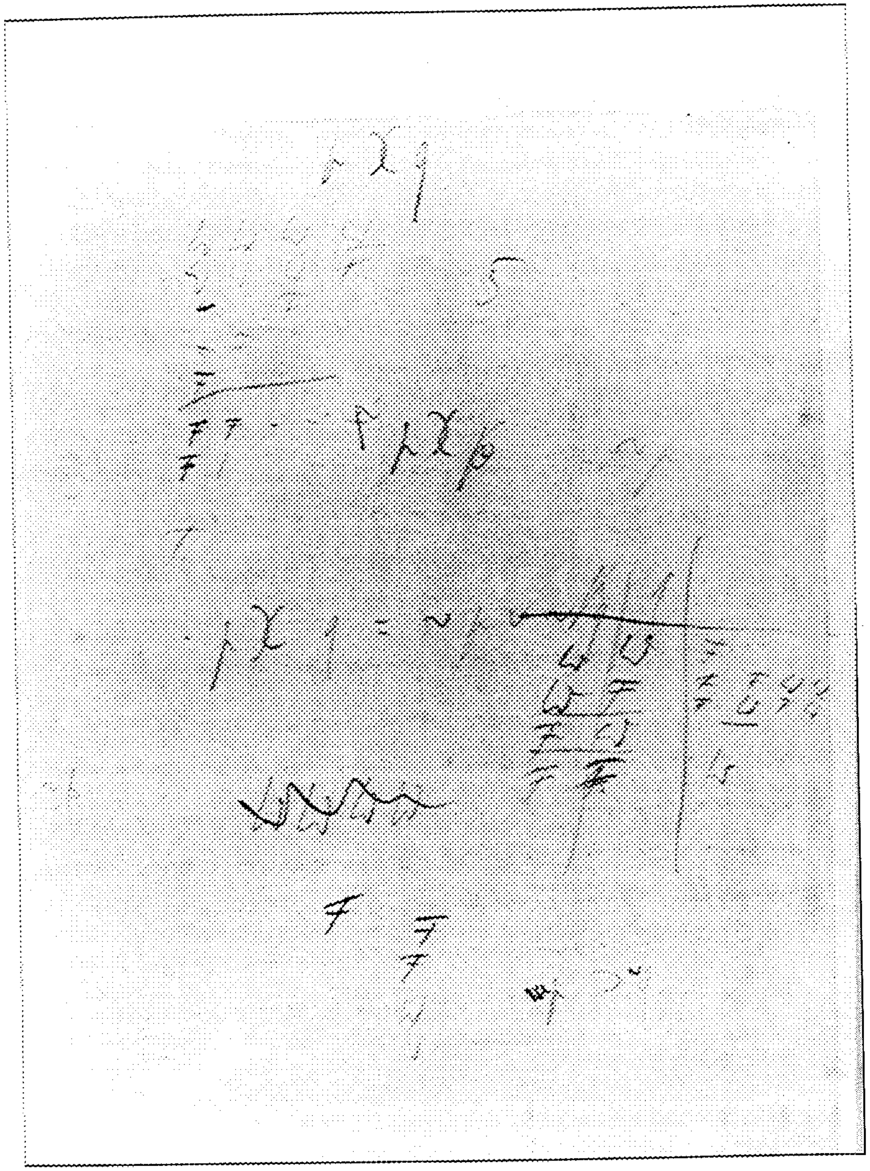
WITTGENSTEIN

Having shown that Russell did understand and use a truth-table technique, I now turn to a discussion of the truth-table device. Wittgenstein and Russell evidently discussed the truth-table device before Wittgenstein left Cambridge in 1913, probably no later than October 1912. On 25 October, Russell read a paper entitled "On Matter" before the Cambridge Moral Sciences Club, and Wittgenstein was there to hear the paper.[17] They had discussed the first draft of the paper in May 1912. On the back of one leaf of a cognate manuscript from this period, "Matter. The Problem Stated",[18] there are pencilled symbolic comments or jottings (actually more like doodling) identified to be in Wittgenstein's

[16] W. T. Parry, "The Logic of C. I. Lewis", *The Philosophy of C. I. Lewis*, ed. P. A. Schilpp, The Library of Living Philosophers (La Salle, Ill.: Open Court, 1968), pp. 116–18. This brilliant essay provides a history of conditionality and demonstrates the many differences between Russell's material implication and Lewis' strict implication.

[17] "My paper on Matter last night was not a success. No one except Wittgenstein understood it at all." Letter from Bertrand Russell to Lady Ottoline Morrell, 26 October 1912, RA REC. ACQ. 70, #608.

[18] Located in the Russell Archives at RAI 220.011450.



Verso, turned upside down, of folio 1, "Matter. The Problem Stated", with Wittgenstein's truth-table sketches. The "∼p" on the far left is in Russell's hand.

hand. They record truth possibilities in both a horizontal and vertical presentation.[19] Surprisingly, at least one logical operator ("∼p" on the far left of the illustration) has been identified to be in Russell's hand.[20]

McGuinness has already commented on these doodlings. He notes that "not all of the jottings are easily intelligible".[21] But he does find four points of interest in them. They represent the truth possibilities of a pair of propositions, include a third column to indicate which of the possibilities are allowed, express all of the possible truth-functions, and record a successful attempt to combine the use of negation and disjunction into a single operator.[22] We do not know the extent of Russell's input in the construction of these truth tables, if any. But we now know that Russell was aware of this new use of the truth-table device.[23] And, at the very least, the set of doodles indicates that truth tables were known to both Wittgenstein and Russell almost ten years before the publication of the *Tractatus* or of Post's dissertation.[24]

[19] Interestingly, both manuscripts have been published in *Papers* 6: 80–95, 98–9, but the editors missed the significance of the jottings, failing even to mention them. They probably didn't see them or, in the unlikely event that they did, may have thought that the doodles were an unfortunate defacing of a valuable document, instead of an early recording of a central thought revealed a decade later with the publication of the *Tractatus.*

[20] See Brian McGuinness, *Wittgenstein: a Life: Young Ludwig, 1889–1921* (Berkeley: U. of California P., 1988), pp. 160–2. As far as I know, McGuinness is the only person to comment on this truth table. He surely is owed a great debt of thanks by myself and others who may find this discovery of interest.

[21] *Ibid.*, p. 160.

[22] This was a year before Sheffer performed that operation with the "stroke". See Henry Maurice Sheffer, "A Set of Five Independent Postulates for Boolean Algebras, with an Application to Logical Constants", *Transactions of the American Mathematical Society*, 14 (1913): 481–8.

[23] As McGuinness notes, this was before Wittgenstein dictated his *Notes on Logic* to Russell in 1913. He also makes a very important observation for Wittgenstein scholars: "These jottings are a valuable reminder of how little we know about the genesis of the *Tractatus* and how misleading the fragmentary preliminary work we have can be" (*ibid.*, p. 162).

[24] It is tempting to comment on Russell's potential input into the development of this truth table. Wittgenstein may have been showing him a new discovery, or he may have taken notes from Russell's own work. I, for one, believe that Russell had to have some idea of the truth-table device in order to conduct the work of *PM.* I confess that this is merely a hunch, backed by no solid evidence at present. I also realize that some Wittgenstein scholars will be horrified by this hunch, perhaps rightly so. But I find it

LECTURES ON ADVANCED LOGIC AT HARVARD

Russell later employed the truth-table device in his teaching. During the first months of 1914, Russell gave a course entitled "Advanced Logic" at Harvard College.[25] One of the graduate students in the class was T. S. Eliot. The lecture course began in Russell's absence, conducted by a graduate student, Harry Costello, perhaps with input from other faculty members.[26] There were no required texts. In each lecture, extensive citations were given so students could read outside materials to further understand the logical concepts under discussion. The difficulty of the material and the frequent citations forced Eliot to take careful, extensive notes. These notes are now part of the Eliot Papers housed in Harvard's Houghton Library.

Russell arrived in March 1914, mid-way through the semester, and took over the lecturing duties. According to Eliot's record, on 4 April 1914, Russell presented three truth tables to illustrate the notions of negation, disjunction, and material implication. Eliot copied these truth tables into his notes. Aside from the Wittgenstein's jottings in 1912, these are the first recorded, verifiable, cogent, and attributable truth tables in modern logic.[27]

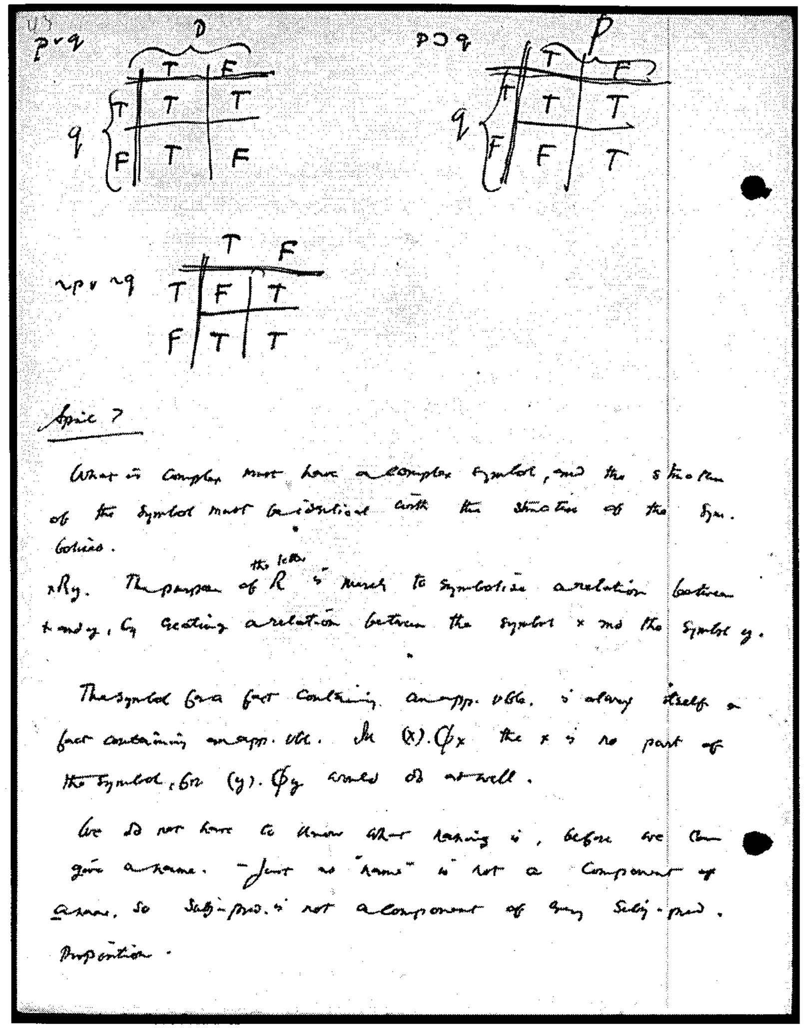
very hard to believe that Russell could use material implication, material equivalence, and modus ponens in 1910 without relying on a truth-table device.

[25] According to Russell's handwritten proposal sent to Harvard in the spring of 1913, the course would offer an

> Outline of recent developments: Peano, Frege, Cantor and the influence of mathematics. Definition of logic: generalization and logical form. Logic not concerned with thought or with any special subject-matter. Classification of complexes; particulars, predicates, and relations of various orders. Premisses and inference; distinction between premisses and hypotheses. The meaning of "existence"; descriptions, classes. Identity of logic and mathematics. Relation of logic to the natural sciences, to metaphysics, and to theory of knowledge. Unsolved problems of logic. (*Papers* 7: 183)

[26] See his "Logic in 1914 and Now", *The Journal of Philosophy*, 54 (1957): 245–64.

[27] T. S. Eliot, personal papers, unpublished materials, Houghton Library, Harvard University, 4 April 1914, bMS AM 1619.14 (13), p. 24. The illustration is reproduced with the permission of his widow, Valerie Eliot.



Truth tables in T. S. Eliot's notes on Russell's logic course at Harvard, April 1914 (by permission of the Houghton Library, Harvard; bMS Am 1619.14 (13)).

Throughout the semester at Harvard, Costello and Russell had explicated the work in *The Principles of Mathematics* and *Principia Mathematica*, as well as the work of Boole, Schröder, Frege, and Peano, among others. In the lecture on 4 April, Russell was explaining the notion of truth-functionality, and how the notions of definability and indefinability are "properties of symbols, not of things".[28] The truth tables were used to illustrate that point.

The truth tables are fully understandable to the modern logician, and are strikingly similar to the Wittgenstein and Post variety. The main difference is that they do not explicitly develop arguments, but are used only to define terms and what those terms do in a logical system. It is interesting to notice that Russell clearly sketches the use of material implication found in *Principia Mathematica*, which previously was only a verbal definition. He is also interested in $\sim p \vee \sim q$, which is central to the Wittgenstein (and Russell?) tables from 1912.

LECTURES ON LOGICAL ATOMISM

In addition, Russell used the truth-table device in his lectures on "The Philosophy of Logical Atomism", a series of eight public presentations during the first months of 1918. He defines "$p \vee q$" via a "schema": *TT* for *p and q both true* and *TF* for *p true and q false*, etc.[29] He then lays out the grid of

TT	TF	FT	FF
T	T	T	F

This is the modern notion of a truth table for disjunction. Interestingly, Russell uses the truth tables to explain truth-functionality, the construction of an atomistic system, and how all of this leads to the use of the Sheffer Stroke. Again, this is well before 1921–22, and before Wittgenstein re-enters Russell's life after World War I.

[28] *Ibid.*, p. 23. These are Eliot's recorded notes, and perhaps not Russell's exact words.
[29] *PLA*, in *Russell's Logical Atomism*, p. 72; *Papers* 8: 186.

CONCLUSION

Of course, in 1914, Russell may simply be utilizing a device learned earlier from Wittgenstein. The evidence from 1912 would support this thesis. One could argue the real lesson here is that Wittgenstein's discovery of the truth-table device is almost a decade prior to the publication of the *Tractatus Logico-Philosophicus*, and years before Wittgenstein's early drafts. Of course, many of the notebooks have been destroyed, but it is of interest that a truth-table device is absent from the published notebooks from 1914–16.[30] I would not be adverse to this conclusion, which may be easily argued by Wittgenstein scholars.

But even if that conclusion is challenged, it is now clear that Russell understood and used the truth-table technique and the truth-table device. By 1910, Russell had already demonstrated a well-documented understanding of the truth-table technique in his work on *Principia Mathematica*. Now, it would seem that by 1912, and surely by 1914, Russell understood, and used, the truth-table device. Of course, the combination of logical conception and logical engineering by Russell in his use of truth tables is the culmination of work by Boole and Frege, who were closely studied by Russell. Wittgenstein and Post still deserve recognition for realizing the value and power of the truth-table device. But Russell also deserves some recognition on this topic, as part of this pantheon of logicians.

In this paper I have shown that neither the truth-table technique nor the truth-table device was "invented" by Wittgenstein or Post in 1921–22. The truth-table technique may originally be a product of Philo's mind, but it was clearly in use by Boole, Frege, and Whitehead and Russell. The truth-table device is found in use by Wittgenstein in 1912, perhaps with some collaboration from Russell. Russell used the truth-table technique at Harvard in 1914 and in London in 1918. So the truth-table technique and the truth-table device both predate the early 1920s.

Another lesson here may be that truth tables are an indispensable device in modern deductive logic precisely because of their use in definition, valid inference, proof theory, and system construction. The genesis

[30] *Notebooks, 1914–16*, ed. G. H. von Wright and G. E. M. Anscombe, 2nd ed. (Oxford: Blackwell, 1979).

of Russell's use of truth tables tells us much about the need to study the history of mathematical logic, and to recognize the need for truth-table-type reasoning in the rise of modern deductive logic. This exercise in excavating truth-table use, therefore, is much more than polite philosophical history; it is a blatant demonstration of the inherent mutuality between truth-table-type thinking, and material implication and material equivalence, as understood by Boole, Frege, Russell, Whitehead, and Wittgenstein.[31]

[31] I would like to thank Nicholas Griffin, Kenneth Blackwell, Jeffrey Cothren, Paul Summers, Dennis Burke, Catherine Kendig, Silvia Roloff, Mitchell Haney, Jason Adsit, and Bob Barnard for their helpful insights during the preparation of this manuscript. I also thank Valerie Eliot for permission to examine the T. S. Eliot papers at Houghton Library, Harvard University. All passages taken from Eliot's notes are used with the kind permission of Valerie Eliot and Houghton Library. My appreciation is also extended to the Bertrand Russell Archives at McMaster University for allowing me to use Wittgenstein and Russell's truth-table doodles, overlaid on the original manuscript of the "Matter. The Problem Stated". Most of all, I would thank my good friend, David Rodier, chairman of the Department of Philosophy and Religion at the American University, for encouraging me in the strongest possible terms to visit the Houghton Library "to see what Eliot heard".