S2P: A Stable 2-Pole RC Delay and Coupling Noise Metric[1]

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Abstract

The Elmore delay is the metric of choice for performance-driven design applications due to its simple, explicit form and ease with which sensitivity information can be calculated. However, for deep submicron technologies, the accuracy of the Elmore delay is insufficient. In this paper, we formulate a delay model using a provably stable two pole waveform response that provides a unique mapping between four moments and a specific delay value. Unlike traditional moment matching, this two-pole model permits us to precharacterize the delays, and store them in a table, as a mapped function of three parameters. The model also provides an explicit expression for the peak noise induced on a coupled line as a function of the same three moments. The results indicate runtimes comparable to an Elmore delay calculation but with the accuracy of an AWE approximation.

1. Introduction

With decreasing rise times and minimum feature sizes, Elmore delay [1][2] ceases to be the accurate metric for interconnect analysis and synthesis. It provides overly pessimistic delay measure for RC circuits with general finite-ramp inputs. Considering that 60% of the path delay in today's circuits are attributed to the interconnect [3], such an error is unacceptable. Also, since interconnect resistance is higher, its shielding effect is more important. Elmore delay, neglecting the resistance shielding, does not capture the correct sensitivities, which is very crucial.

Similar to the Elmore delay model for delay approxi-

mations, the first moment of the coupled node voltage multiplied with the input voltage slope, is shown to be a bound for coupling noise in RC networks [4]. However, this bound is a good approximation only for very slow rising inputs, and at most times yields very pessimistic results.

Timing driven physical design tools need accurate delay and noise metrics for deep submicron. These metrics should also be very efficient, thereby prohibiting the use of higher order Krylov space methods. For these reasons, we return back to AWE [7] with a Partial Padé capability that produces provably stable two-pole models using the moments at the driving point and the load end. The reasons are two-fold:

1. AWE with Partial Padé provides the most accurate stable approximation for low-orders. The entire frequency range for RC network responses can be captured using a few poles. The use of Krylov space methods [5][6] makes difference only for higher orders.

2. Any approximation with more than two poles does not permit the use of a direct formula for coupling noise and delay. It is known that the Newton-Raphson or regula-falsi iterations for the solution of this response is very costly. To avoid this cost, lookup tables can be used, however for a *q* pole formula, a *2q+1* dimension lookup table is needed. On the other hand, the two pole approximation lookup table can be made 3D with a proper implementation. In addition, there is an explicit formula for maximum coupling noise with the two pole model.

In the following sections, we will explain the proposed model and prove that our model is always stable.

2. S2P Approximation

The stable two pole (S2P) approximation consists of two stages: 1) Finding the poles from the driving point admittance moments. 2) Matching the moments at the load ends to find the residues.

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Let $Y(s)$ be an driving point admittance function of a general RC circuit and consider its representation in terms of poles and residues:

$$Y(s) = \sum_{n=1}^{q} \frac{k_n}{s - p_n} + k_0 \qquad (1)$$

where q is the exact order of the circuit. In terms of the poles and residues, the moments are given as

$$m_i = -\sum_{n=1}^{q} \frac{k_n}{p_n^{i+1}} \qquad i > 0 \qquad (2)$$

Let us now consider the second order Padé approximation of $Y(s)$ and denote it by $\hat{Y}(s)$:

$$\hat{Y}(s) = \frac{b_0 + b_1 s}{1 + a_1 s + a_2 s^2} \qquad (3)$$

In AWE technique, the coefficients b_0, b_1, a_1 and a_2 are obtained using a moment matching procedure which is equivalent to solving the following linear equation system:

$$\begin{aligned} 0 &= a_2 m_1 + a_1 m_2 + m_3 & b_0 &= m_0 \\ 0 &= a_2 m_1 + a_1 m_2 + m_3 & b_1 &= a_1 m_0 + m_1 \end{aligned} \qquad (4)$$

Actually, to find the coefficients of the denominator polynomial we can use any four successive moments. Using higher order moments give better approximations to the actual poles and is known as horizontal convergence in the Padé approximation literature. Therefore, in general we have

$$a_1 = \frac{m_{i+1}m_{i+2} - m_i m_{i+3}}{m_i m_{i+2} - m_{i+1}^2}, \qquad a_2 = \frac{m_{i+1}m_{i+3} - m_{i+2}^2}{m_i m_{i+2} - m_{i+1}^2} \qquad (5)$$

2.1. Stability and Realness of the Poles

It is proved in [8] that second order Padé approximations for the driving point admittance function of a RC circuit is always stable, and the two poles of the approximation are real. The proofs are based on the monotonicity property of the admittance moment ratios in RC circuits [8]:

$$\frac{m_{i-1}}{m_i} < \frac{m_i}{m_{i+1}} \qquad i > 1 \qquad (6)$$

2.2. The Algorithm

1. Compute m_1, m_2, m_3 and m_4 for $Y(s)$
2. Find the two poles at the driving point admittance

$$a_1 = \frac{m_2 m_3 - m_1 m_4}{m_1 m_3 - m_2^2}, \qquad a_2 = \frac{m_2 m_4 - m_3^2}{m_1 m_3 - m_2^2} \qquad (7)$$

$$p_1 = \frac{(-a_1 + \sqrt{a_1^2 - 4a_2})}{2a_2} \qquad p_2 = \frac{(-a_1 - \sqrt{a_1^2 - 4a_2})}{2a_2} \qquad (8)$$

3. To match the voltage moments at the response nodes, solve the Vandermonde equations:

$$k_2 = \frac{(m_0^*/p_1 - m_1^*)}{1/p_2 - 1/p_1} p_2 \qquad k_1 = (-m_0^* - k_2/p_2)p_1 \qquad (9)$$

where m_0^* and m_1^* are the moments at the response nodes. (Note that m_0^* is known as the Elmore delay at the response node.)

4. The S2P approximation is then expressed as:

$$\hat{h}(s) = \frac{k_1}{s - p_1} + \frac{k_2}{s - p_2} \qquad (10)$$

The computation of the additional moments beyond the first moment comes with very little incremental cost. This process can be implemented using a vectorized path tracing algorithm like that in RICE [9]. The poles in (10) are guaranteed to be stable and real, thereby eliminating the need for using heuristics to get real, stable poles in traditional moment matching.

3. Delay and Coupling Noise Metrics

3.1. Delay Metric

If the input is a finite-ramp signal with a transition time of T_{rise}, the waveform at the response node can be written as:

$$\begin{aligned} g(t) &= \sum_{n=1}^{2} \left(\frac{k_n}{p_n^2} e^{p_n t} - \frac{k_n}{p_n^2} - \frac{k_n}{p_n} t \right) \frac{1}{T_{rise}} \\ &- \sum_{n=1}^{2} \left(\frac{k_n}{p_n^2} e^{p_n(t - T_{rise})t} - \frac{k_n}{p_n^2} - \frac{k_n}{p_n}(t - T_{rise}) \right) \frac{1}{T_{rise}} u(t - T_{rise}) \end{aligned} \qquad (11)$$

For the 50% delay evaluation, the equation $g(t_d) = 0.5$ needs to be solved for t_d. We obtain the S2P delay estimate using $T_{S2P} = t_d - T_{Rise}/2$. The solution of (11) requires Newton-Raphson or Regula-Falsi iterations. To avoid these costly iterations, we build a lookup table with pre-computed delays for various parameter values.

The table for a two pole model has five parameters $(p_1, k_1, p_2, k_2, T_{rise})$, but it is possible to reduce it to 3-D by normalizing the system with respect to p_1 and using the fact that $m_0^* = 1$ for RC trees. Then, the parameters

for our table is T_{rise}/p_1, m_1*/p_1 ,and p_2/p_1. The delay value obtained from the table is then divided by p_1 for back-normalization. With this selection of parameters, it is easier to predict the ranges. For example, it is empirically known that after $T_{rise} > 7p_1$, the 50% delay is almost equal to m_1* (Elmore delay), so the range for T_{rise}/p_1 in the table should be between 0 and 7.

3.2. Noise Metric

To understand the behavior of coupling noise in victim nodes of RC circuits, we should investigate the exact noise response for a saturated ramp input. Noticing that there is no dc coupling between the aggressor and victim line, therefore $m_0* = 0$. For the victim node, the exact noise response can be written as:

$$n(t) = \begin{cases} \dfrac{1}{T_{rise}}\left(\displaystyle\sum_{n=1}^{q} \dfrac{k_n}{p_n^2} e^{p_n t} + m_1* \right) & t < T_{rise} \\[4mm] \dfrac{1}{T_{rise}}\left(\displaystyle\sum_{n=1}^{q} \dfrac{k_n}{p_n^2} e^{p_n(t - T_{rise})}(e^{p_n T_{rise}} - 1) \right) & t \ge T_{rise} \end{cases} \quad (12)$$

Under the "wrong" assumption that $p_n T_{rise}$ is a very big negative number for all n, the transient part of $n(t)$ dies out before $t < T_{rise}$. Therefore, with this assumption, it can be shown that m_1*/T_{rise} appears as an upper found for $n(t)$. The same argument has been found using a different way in [4]. However for today's technologies with fast rise times, this measure is overly pessimistic, and this motivates the need for a two-pole approximation.

Under the S2P model, there exists an explicit formula for the maximum coupling noise. We compute k_1, k_2, p_1 and p_2 for the victim node(s). Writing out formula with two poles and solving for the time where derivative is equal to zero, we obtain $t*$ when voltage gets maximum.

$$t* = T_{rise} + \ln\left(\frac{e^{p_2 T_{rise}} - 1}{e^{p_1 T_{rise}} - 1} \right) / (p_1 - p_2) \quad (13)$$

From (13), $t*$ is always later than T_{rise}, and evaluating the noise at that time point, we obtain:

$$N_{max} = \frac{1}{T_{rise}}\left(\sum_{n=1}^{2} \frac{k_n}{p_n^2} e^{p_n(t* - T_{rise})}(e^{p_n T_{rise}} - 1) \right) \quad (14)$$

4. Results

In this section, the new metrics that are derived from S2P models are compared against conventional measures. We compare the delay estimation performances of S2P approximation result T_{S2P}, T_{Elmore} (Elmore delay), $0.7 \times T_{Elmore}$ and the exact delay values found by SPICE. The $0.7 \times T_{Elmore}$ measure is widely used as a remedy to reduce the pessimistic behavior of simple Elmore delay. In fact, it is the 50% delay point for the step response of a one pole approximation. For the evaluation of our proposed metric for peak noise, S2P models are compared with m_1*/T_{Rise} bound and exact peak noise value found by SPICE

The S2P delay metric is tested on an industrial IC with over 700 nets and 1200 load points. The relative percent errors of S2P, T_{Elmore} and $0.7 \times T_{Elmore}$ metrics with respect to SPICE results are shown in Figure 1. As seen clearly, Elmore delay has up to 140% errors, whereas the new S2P delay metric has less than 5% error in all of the cases.

To test the noise measure, a coupled circuit extracted from an IC with 56 outputs is used. The percent errors of S2P noise and m_1*/T_{rise} metrics with respect to SPICE results are compared in Figure 2. The m_1*/T_{rise} measure is overly pessimistic, sometimes reaching up to 240% errors, while S2P model predicts the maximum coupling noise perfectly within 1% percent error.

As another example to show that the exact coupling waveform also matches perfectly with the S2P waveform, the two-bit coupled bus problem has been chosen (Figure 3). As demonstrated in this example, m_1*/T_{rise} metric gives a very pessimistic bound for the peak noise, whereas S2P waveform matches the exact response in the victim line perfectly.

In these examples, RICE [9] is used to obtain the first four moments of the circuit. The run time costs are dominated by the circuit compilation time, where the circuit graph is extracted and compiled. The circuit compilation in RICE corresponds to LU decomposition on matrix based moment solvers. Since by the use of lookup tables in delay evaluation and a simple explicit formula for the noise measure, the complexities of S2P metrics are very close to single moment based criteria.

5. Conclusion

In this paper, we have proved that the second order

AWE approximation of the driving point admittance functions of RC circuits are always stable. Then, transfer functions are approximated using the Partial Padé matching. This no-heuristic, three-moment based S2P approximation is observed to have superior accuracy in RC circuits, for both delay and coupling evaluation. By the use of lookup tables for delay and explicit formulas for maximum coupling noise, our metrics avoid the costly iterations and practically achieve the speed of single moment metrics.

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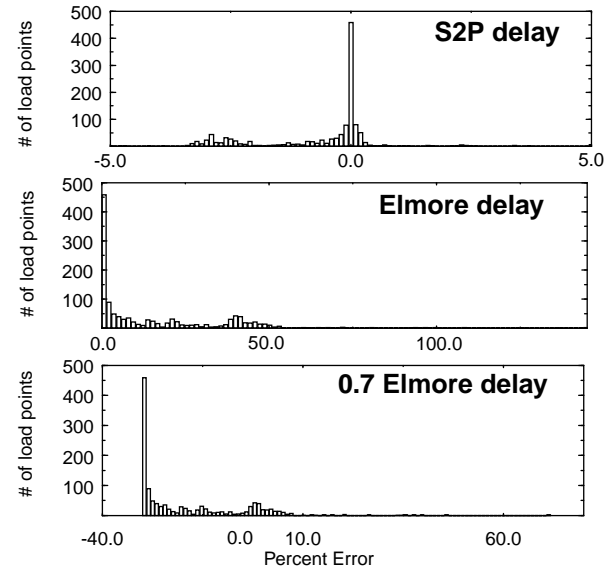
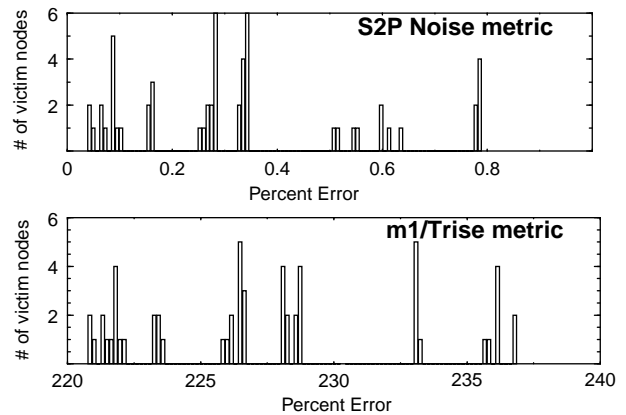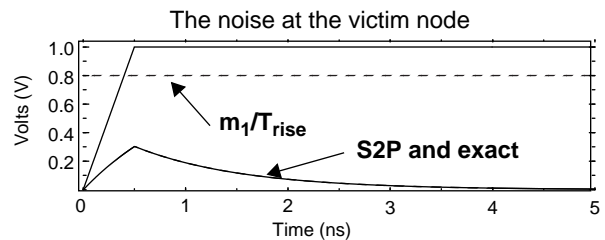**Fig. 1   Relative percent error comparisons of the 50% delay metrics.**

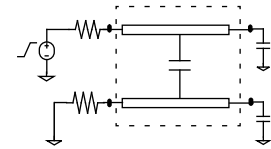**Fig. 2   Comparison of coupling noise metrics with respect to SPICE.**

Fig. 3   Waveform comparisons of S2P and SPICE for coupling noise