

Journal of Composite Materials

<http://jcm.sagepub.com/>

Saint-Venant End Effects In Composites

Cornelius O. Horgan

Journal of Composite Materials 1982 16: 411

DOI: 10.1177/002199838201600506

The online version of this article can be found at:

<http://jcm.sagepub.com/content/16/5/411>

Published by:



<http://www.sagepublications.com>

On behalf of:



[American Society for Composites](http://www.americansocietyforcomposites.com)

Additional services and information for *Journal of Composite Materials* can be found at:

Email Alerts: <http://jcm.sagepub.com/cgi/alerts>

Subscriptions: <http://jcm.sagepub.com/subscriptions>

Reprints: <http://www.sagepub.com/journalsReprints.nav>

Permissions: <http://www.sagepub.com/journalsPermissions.nav>

Citations: <http://jcm.sagepub.com/content/16/5/411.refs.html>

Saint-Venant End Effects In Composites

CORNELIUS O. HORGAN

*College of Engineering
Michigan State University
East Lansing, Michigan 48824*

ABSTRACT

In this paper, we demonstrate that the neglect of elastic end effects, usually justified by appealing to Saint-Venant's principle, cannot be applied routinely in problems involving composite materials. In particular, for fiber reinforced composites, the characteristic decay length over which end effects are significant is, in general, several times longer than the corresponding length for isotropic materials. For plane strain or generalized plane stress of a highly anisotropic transversely isotropic (or orthotropic) material, modeling a fiber-reinforced composite, the characteristic decay length is of order $b(E/G)^{1/2}$, where b is the maximum dimension perpendicular to the fibers and E , G are the longitudinal Young's modulus and shear modulus respectively. Thus when E/G is large, as for fiber-reinforced composites, end effects are transmitted over a distance which is of the order of several specimen widths. This is *in marked contrast* with the situation for isotropic materials where decay lengths of *one* specimen width are typical. Similar results hold for axisymmetric problems and for sandwich laminates. The results have widespread implications for the mechanics of composite materials.

1. INTRODUCTION

In the application of elasticity theory to problems of practical interest, an essential simplification is made by ignoring "end effects" through consideration of load resultants. We cite, for example, the theories for strength of materials, involving beams, plates and shells having such relaxed boundary conditions as cornerstones of their development. The justification for such approximations is usually based on some form of Saint-Venant's principle characterizing the boundary layer behavior involved. Thus Saint-Venant's principle is appealed to in neglecting local or end effects and experience with homogeneous isotropic materials in linear elasticity has served to establish this standard procedure.

J. COMPOSITE MATERIALS, Vol. 16 (September 1982), p. 411

0021-9983/82/05 0411—12 \$04.50/0

©1982 Technomic Publishing Co., Inc.

It is natural to ask to what degree this practice may be applied in design with composite materials. We address this question here in the course of a review of current knowledge regarding Saint-Venant end effects for highly anisotropic and composite materials. It will be seen that, for such materials, local stress effects persist over distances far greater than is typical for isotropic materials. The practical implications of this for composites are many: we cite, for example, end constraint problems in mechanical testing (see e.g., [1,2]), influence of fasteners, joints, cut-outs etc. in composite structures, and limitations of strength of materials formulas when applied to composites. Of chief interest for design purposes is the current availability of order of magnitude estimates (and explicit formulas) for the *characteristic decay length* over which end effects are significant for a variety of material and load configurations. In particular, for plane strain or generalized plane stress of a highly anisotropic transversely isotropic (or orthotropic) material, modeling a fiber-reinforced composite, the characteristic decay length is of order $b(E/G)^{1/2}$, where b is the maximum dimension perpendicular to the fibers and E , G are the longitudinal Young's modulus and shear modulus respectively. Thus when E/G is large, as for fiber-reinforced composites, end effects are transmitted over a distance which is of the order of several specimen widths. This is in marked contrast with the situation for isotropic materials where decay lengths of *one* specimen width are typical. Similar results hold for axisymmetric problems and for sandwich laminates.

The order of magnitude results just described were first obtained in [3,4] for plane problems (a similar "stress channelling" phenomenon was observed by Everstine and Pipkin [5] in the context of idealized theories for fiber-reinforced composites) and in [6] for the torsionless axisymmetric problem of a circular cylinder, in the course of general investigations concerning the effect of anisotropy on Saint-Venant's principle. (See [7] for an extensive survey of recent results on Saint-Venant's principle.) A more detailed analysis of the *exact* stress decay in highly anisotropic rectangular strips was carried out in [8], leading to explicit formulas (see equations (3.6), (3.7) here) for the stress decay rate and characteristic decay length for plane deformations. (Similar estimates for "load diffusion lengths" in fiber reinforced composites were found in [9]). These results are summarized and elaborated in Sections 2 and 3 of the present paper, while Section 4 is concerned with the torsionless axisymmetric problem for a circular cylinder. In Section 5, we describe analogous results, established in [10], for symmetric sandwich laminates with isotropic phases. For the case of a relatively soft inner core, a slow decay rate is obtained, in qualitative agreement with photoelastic experimental studies of Alwar [11].

Slow stress decay for highly anisotropic materials has also been observed experimentally. In the course of conducting torsional pendulum tests designed to measure the longitudinal shear modulus of a polymeric composite, Folkes and Arridge [12] encountered difficulties (because of end effects) in obtaining

values of this modulus which are independent of specimen aspect ratios (length/width ratio). Meaningful results were obtained only for samples whose aspect ratios exceeded 100. The data in [12] for polystyrene fibers in a matrix of polybutadiene indicate that E/G is about 280 and so characteristic decay lengths of the order of several specimen widths are predicted by the theoretical results. Further tests are described in [13,14], the latter reference also containing some finite element calculations. Other experimental studies concerned with end effects in highly oriented polymers may be found in [15-18].

Additional finite element investigations confirming the order estimates for the characteristic decay length obtained in [3,4] are described in [19,20]. A recent paper [21] in this journal uses finite difference and relaxation methods to study extended end effects in a T300/5208 graphite/epoxy laminate (modeled as a homogeneous orthotropic material) in a short beam shear test (similar to the ASTM D 2344 test). The author, however, appears to be unaware of the previous work outlined above.

Finally here we note that the present paper is concerned solely with Saint-Venant end effects in *linear* elasticity. End effects in composites (and in particular, the extended edge phenomena for polymers observed experimentally in [12-18]) may well require consideration of inelastic and nonlinear effects. Some discussion of Saint-Venant's principle in the latter contexts is carried out in [7,22] but explicit results comparable to those available for linear elasticity are at present unknown.

2. Plane Deformation for Transversely Isotropic Materials

The stress-strain relations governing the in-plane stresses and strains in plane strain of an homogeneous anisotropic elastic solid, transversely isotropic about the x-axis, are given by ([3,4,23])

$$\begin{aligned} e_{xx} &= \beta_{11} \sigma_x + \beta_{12} \sigma_y \\ e_{yy} &= \beta_{21} \sigma_x + \beta_{22} \sigma_y \\ 2e_{xy} &= \beta_{66} \tau_{xy}, \end{aligned} \tag{2.1}$$

where the elastic constants $\beta_{pq} = \beta_{qp}$ ($p, q = 1, 2, 6$) may be written in terms of the usual engineering constants (see e.g. [24]) as

$$\begin{aligned} \beta_{11} &= \frac{1}{E_L} (1 - \nu_{LT}^2 E_T/E_L), \quad \beta_{12} = -\nu_{LT}(1 + \nu_{TT})/E_L, \\ \beta_{22} &= (1 - \nu_{TT}^2)/E_T, \quad \beta_{66} = 1/G_{LT}. \end{aligned} \tag{2.2}$$

Here L denotes the direction parallel to the x -axis, T the transverse direction and ν , E , G denote Poisson's ratio, Young's modulus and shear modulus respectively. The strain-energy density is assumed to be positive definite and so the elastic constants are such that

$$\beta_{11} > 0, \beta_{11} \beta_{22} - \beta_{12}^2 > 0, \beta_{66} > 0. \tag{2.3}$$

We are concerned with the traction boundary-value problem of linear elasticity for plane domains in the x - y Cartesian coordinate plane. Introducing the Airy stress function $\phi(x, y)$ in the usual way by

$$\sigma_x = \phi_{yy}, \sigma_y = \phi_{xx}, \tau_{xy} = -\phi_{xy}, \tag{2.4}$$

where the subscript notation on ϕ denotes partial differentiation, we obtain the governing elliptic partial differential equation

$$\beta_{22} \phi_{xxxx} + (2\beta_{12} + \beta_{66}) \phi_{xxyy} + \beta_{11} \phi_{yyyy} = 0. \tag{2.5}$$

For the special case of an isotropic material $\beta_{11} = \beta_{22} = (1-\nu)/2G$, $\beta_{66} = 1/G$, $\beta_{12} = -\nu/2G$ and (2.5) reduces to the familiar biharmonic equation.

3. Saint-Venant's Principle for Plane Deformation of Highly Anisotropic Materials

(i) Stress Estimates

Our interest here is to discuss the effect of anisotropy on the exponential decay of stresses inherent in Saint-Venant's principle. In particular, we are chiefly concerned with the case of transversely isotropic materials with a high degree of anisotropy in the axial direction. Such transversely isotropic materials can be viewed as models for fiber-reinforced composites. For highly anisotropic materials, we assume that

$$E_T/E_L \ll 1, G_{LT}/E_L \ll 1, E_T/G_{LT} \approx 1. \tag{3.1}$$

Methods involving energy-decay inequalities can be used (see [3,4]) to investigate Saint-Venant's principle in plane elasticity for a wide class of anisotropic materials in a general bounded domain. *Lower bounds* (in terms of the elastic constants) for the rate of exponential decay of stresses (with distance from self-equilibrated boundaries) are thus obtained, giving rise to *upper bounds* for "characteristic decay lengths." The typical results obtained in [3,4] for a representative stress component τ are of the form

$$|\tau| \leq K e^{-kx}, \quad x \geq 0, \tag{3.2}$$

where K is a constant and the decay rate k is characterized explicitly in terms of the elastic constants and cross-sectional dimensions of the body. For highly anisotropic transversely isotropic materials satisfying (3.1), it is shown in [4] that

$$k = O \left[\frac{(G_{LT}/E_L)^{1/2}}{b} \right], \tag{3.3}$$

where b is the maximum cross-sectional dimension. Thus the *characteristic decay length* $\lambda = 1/k$ is of the form

$$\lambda = O [(E_L/G_{LT})^{1/2} b]. \tag{3.4}$$

This characteristic decay length is the axial distance over which the stress (at fixed height y) decays to a fraction $1/e$ (e : exponential constant) of its value at the end $x = 0$ (at height y). (Alternatively, one could take the distance over which this stress decays to 1% of its value at $x = 0$, in which case $\lambda = (\ln 100)/k = 4.6/k$). When E_L/G_{LT} is large, therefore, very slow stress decay is anticipated with end effects being transmitted a considerable distance from the loaded ends. For example, for a typical graphite/epoxy composite, this ratio has the value 33.3 (see Equation (3.8) here) compared with a range of values of between 2 and 3 for the isotropic case. Thus for the problem of a rectangular strip loaded only at the short ends, the usual engineering approximation of neglecting Saint-Venant end effects at distances of about one width from the ends is justified in the isotropic case (see e.g. Fig. 1 here). However, for a

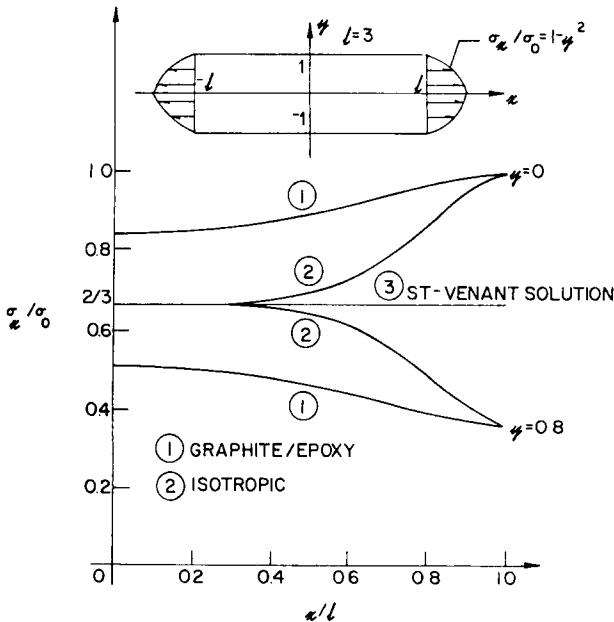


Figure 1. Axial Stress Decay for Extension Problem (from Choi and Horgan [8]).

graphite/epoxy composite, such approximations are justified only at distances about four times larger. Of course, for even more highly anisotropic materials the effects are more pronounced.

The slow exponential decay rate (3.3) predicted by the analyses of [3,4] might also be anticipated from another viewpoint. Introducing the notation $\epsilon_t^2 = G_{LT}/E_L$, and using the conditions (3.1) characterizing a highly anisotropic material, the differential equation (2.5) can be written in this case as

$$A \phi_{xxxx} + B \phi_{xyyy} + \epsilon_t^2 \phi_{yyyy} = 0, \tag{3.5}$$

where A and B are constants. In the limit as $\epsilon_t \rightarrow 0$, the equation (3.5) ceases to be elliptic—in fact, the equation becomes parabolic with the straight lines $y = \text{const.}$ being characteristic curves. Thus, in this limit (3.3) implies the complete breakdown of a Saint-Venant principle—the “end effects” are transmitted without attenuation along the fibers (characteristics). A similar stress “channelling” phenomenon was observed in [5] in the context of idealized theories for fiber-reinforced composites (see also [25]).

(ii) Exact Solutions

The order estimates (3.3), (3.4) may be made explicit by considering exact solutions for rectangular strips with traction free sides. Such an analysis has been carried out by Choi and Horgan [8] using analogues of the well-known Papkovitch-Fadle eigenfunctions. For the *exact* stress decay rate, they obtained the asymptotic estimate

$$k \sim \frac{2\pi}{b} (G_{LT}/E_L)^{1/2} \text{ as } G_{LT}/E_L \rightarrow 0, \text{ (} b = \text{strip width),} \tag{3.6}$$

thus suggesting a value of 2π for the constant inherent in the order estimate (3.3). We observe that this constant has not been taken into consideration by many authors (see [12-19]) in their interpretation of the results in [3,4]. The characteristic decay length $\lambda = 1/k$ corresponding to (3.6) is thus given by

$$\lambda \sim \frac{b}{2\pi} (E_L/G_{LT})^{1/2} \text{ as } G_{LT}/E_L \rightarrow 0. \tag{3.7}$$

We advocate the use of the formulas (3.6), (3.7) for design purposes (in both plane strain and generalized plane stress for highly anisotropic transversely isotropic, or orthotropic, materials) for a general simply-connected plane domain which is close to rectangular, where b is now the maximum lateral dimension. For a typical graphite/epoxy composite with elastic constants given by (we use the values supplied' in [26])

$$E_T = 6.833, E_L = 137.65, G_{LT} = 4.13 \text{ GPa}, \quad (3.8)$$

$$\nu_{TT} = .25, \nu_{LT} = .25,$$

we obtain from (3.6) the result

$$k \approx 1.0884/b, \quad (3.9)$$

while the *exact* value for a rectangular strip of width b , computed numerically in [8], is given by

$$k = 1.1280/b. \quad (3.10)$$

The corresponding value for an isotropic material★ is given by (see e.g. [27] pp. 61-62 or [8])

$$k \equiv k_i = 4.2124/b. \quad (3.11)$$

Thus the exact stress decay rate k for the graphite/epoxy composite is almost four times smaller than that for an isotropic material.

The comparison between isotropic and highly anisotropic materials may be conveniently described, on using (3.6), (3.11), by the ratio of decay rates

$$\frac{k}{k_i} \cong \frac{2\pi}{4.2124} \left(\frac{G_{LT}}{E_L} \right)^{1/2} \cong 1.49 \left(\frac{G_{LT}}{E_L} \right)^{1/2}, \quad (3.12)$$

or equivalently, in terms of the characteristic decay length ratio

$$d \equiv \frac{\lambda}{\lambda_i} \cong .67 \left(\frac{E_L}{G_{LT}} \right)^{1/2}. \quad (3.13)$$

The (dimensionless) measure (3.13) is similar to the “load diffusion length” concept for fiber reinforced composites introduced in [9]. For a graphite/epoxy composite, with elastic constants given by (3.8), $d = 3.87$.

Specific examples illustrating the contrast between the decay of end effects in isotropic and highly anisotropic materials are treated in [8]. Consider the problem of extension of a finite strip. For uniform tension of $(2/3)\sigma_0$ at both ends, we have the Saint-Venant solution

$$\sigma_x = (2/3)\sigma_0, \sigma_y = 0, \tau_{xy} = 0. \quad (3.14)$$

★ Note that the stresses in the traction boundary value problem are independent of the elastic constants in the isotropic case.

Consider the statically equivalent load conditions shown in Fig. 1. By subtraction, we obtain a boundary-value problem involving self-equilibrating load conditions. Using an eigenfunction expansion technique, the resulting problem was solved numerically [8] for both the graphite/epoxy (with elastic constants given by (3.8)) and isotropic materials (with Poisson's ratio equal to zero). In Fig. 1, the decay of σ_x with distance from the end $x = l$ is illustrated for both materials. In the latter case, the normal stress distribution reaches that of the Saint-Venant solution at approximately one width from the end. For the highly anisotropic material, the contrasting slow decay is evident—the characteristic decay length is about four times larger.

4. Torsionless Axisymmetric Problem for Circular Cylinders

Similar results to those described above for plane problems have also been obtained for axisymmetric problems. In [6] the torsionless axisymmetric problem for a circular cylinder, transversely isotropic about the axial direction, under the action of self-equilibrated end loads is considered. Using energy-decay inequalities, a result of the form (3.2) is obtained for the exponential decay of stresses and compared with previous results obtained in the isotropic case [28]. The exact decay rate is given by the root of a transcendental equation involving Bessel functions (see equation (9.5) of [6]). For highly anisotropic materials, with $\epsilon \equiv E_T/E_L \ll 1$, the decay rate k predicted in [6] is of the form

$$k = O\left(\frac{\epsilon^{1/2}}{c}\right) \text{ as } \epsilon \rightarrow 0, \tag{4.1}$$

where c is the radius of the cylinder. An asymptotic analysis of the transcendental equation for the exact decay rate also yields the result (4.1). In fact, it can be shown that the exact decay rate has the asymptotic form

$$k \sim \frac{3.83}{c} (G_{LT}/E_L)^{1/2} \text{ as } E_T/E_L \rightarrow 0, \tag{4.2}$$

with corresponding characteristic decay length

$$\lambda \sim \frac{c}{3.83} (E_L/G_{LT})^{1/2} \text{ as } E_T/E_L \rightarrow 0. \tag{4.3}$$

(cf. eqns. (3.6), (3.7) here for plane elasticity). For a graphite/epoxy composite (with elastic constants given by (3.8) and for which $\epsilon = 1/20$) we obtain from (4.2) the result

$$k \cong .66/c \tag{4.4}$$

which is in excellent agreement with the *exact* value (computed numerically in [29]) given by

$$k = .69/c. \tag{4.5}$$

The corresponding exact value for an isotropic material (with Poisson's ratio $\nu = .25$) is (see e.g. [28])

$$k = 2.70/c. \tag{4.6}^*$$

Thus the exact stress decay rate k for the graphite/epoxy composite is almost four times smaller than that for an isotropic material, when $\nu = .25$.

5. Symmetric Sandwich Laminates

Finally, we discuss briefly the case of plane deformation of a sandwich strip, composed of identical homogeneous isotropic face materials occupying two layers of equal thickness enclosing a dissimilar homogeneous isotropic core. For the case when the Young's modulus of the core is small compared with that for the face layers, it might be expected that the decay of Saint-Venant end effects should be much slower than that for a single strip. Such a result was demonstrated experimentally by Alwar [11] using photoelasticity techniques.

Recently a theoretical analysis of this issue was carried out in [10], through investigation of the *exact* decay rates characterized as eigenvalues. An exponential decay result of the form (3.2) was established. Using the notation $f = (4C_f)/(4C_f + 2C_c)$ for the volume fraction of face material (where $2C_f, 2C_c$ denote the width of face layer and core layer respectively), and E_f, E_c, ν_f, ν_c for the respective Young's moduli and Poisson ratios, it is shown in [10] that when $E_f/E_c \gg 1$,

$$kx \text{ (total strip width)} \sim 2 \left[\frac{2(f^2 - 3f + 3)(1 - \nu_f^2)E_c}{f^3(1-f)(1 + \nu_c)E_f} \right]^{1/2}, \tag{5.1}$$

yielding slow exponential decay for the case of a relatively soft inner core. If we assume that $E_f/E_c = 3600$, $f = 0.8$ as in [11], and let $\nu_f = \nu_c = 0.3$, then from (5.1) we obtain the estimate

$$kx \text{ (width)} \sim 0.14, \tag{5.2}$$

which is about thirty times smaller than the value for a homogeneous strip. Thus the neglect of Saint-Venant end effects is justified in this case only at a distance of about thirty widths from the ends. Related finite element calculations have been reported in [30,31].

The accuracy of the approximate formula (5.1) may be assessed by comparing with *exact* values for the decay rate computed from the transcendental equation (3.5) of [10] (see also Fig. 3 of [10]). If we let $\nu_f = \nu_c = 0.3$ as in [10], then (5.1) may be written as

* As ν varies from 0 to 1/2, k ranges from $2.56/c$ to $2.81/c$.

$$kx \text{ (width)} \sim 2.3664 \left(\frac{E_c}{E_f} \right)^{1/2} \left[\frac{f^2 - 3f + 3}{f^3 (1 - f)} \right]^{1/2} \quad (5.3)$$

When $E_f/E_c = 256$, the estimated values (5.3) and corresponding exact values are tabulated below for various values of the volume fraction f . The agreement is seen to be excellent. For *larger* values of E_f/E_c , the formula (5.3) furnishes even more accurate estimates, which are valid for values of the volume fraction f arbitrarily close to zero.

f	0.3	0.4	0.5	0.6	0.7	0.8	0.9
(5.3)	1.592	1.057	0.783	0.628	0.544	0.515	0.577
exact results (from [10])	1.616	1.060	0.783	0.628	0.543	0.514	0.576

6. Conclusions

The neglect of elastic end effects, usually justified by appealing to Saint-Venant's principle, cannot be applied routinely in problems involving composite materials. In particular, for fiber reinforced composites, the characteristic decay length over which end effects are significant is, in general, several times longer than the corresponding length for isotropic materials. For plane strain or generalized plane stress of an equivalent homogeneous transversely isotropic (or orthotropic) material the stress decay rate which may be used for design purposes is given by the formula (3.6) for highly anisotropic materials while for torsionless axisymmetric problems the corresponding formula is given by (4.2). For plane deformation of symmetric sandwich laminates, with isotropic phases, the stress decay rate for a relatively soft inner core is given by (5.1).

ACKNOWLEDGEMENT

This work was supported by the U.S. National Science Foundation under Grant MEA - 7826071.

Symbols

- E Young's modulus
- G Shear modulus
- ν Poisson's ratio
- σ normal stress

τ	shear stress
e	strain
β	elastic constant
k	stress decay rate
λ	characteristic decay length
x	coordinate in axial direction
y	coordinate in transverse direction
b	maximum width of plane domain
f	volume fraction of face material in sandwich laminate

REFERENCES

1. Pagano, N. J., and J. C. Halpin, "Influence of End Constraint in the Testing of Anisotropic Bodies," *J. of Composite Materials*, Vol. 2, 1968, p. 18-31.
2. Guess, T. R., and C. B. Haizlip, Jr., "End-grip Configurations for Axial Loading of Composite Tubes," *Experimental Mechanics*, Vol. 20, 1980, p. 31-36.
3. Horgan, C. O., "On Saint-Venant's Principle in Plane Anisotropic Elasticity," *J. of Elasticity*, Vol. 2, 1972, p. 169-180.
4. Horgan, C. O., "Some Remarks on Saint-Venant's Principle for Transversely Isotropic Composites," *J. of Elasticity*, Vol. 2, 1972, p. 335-339.
5. Everstine, G. C., and A. C. Pipkin, "Stress Channelling in Transversely Isotropic Elastic Composites," *J. Appl. Math. Phys.*, (ZAMP), Vol. 22, 1971, p. 825-834.
6. Horgan, C. O., "The Axisymmetric End Problem for Transversely Isotropic Circular Cylinders," *Int. J. Solids Structures*, Vol. 10, 1974, p. 837-852.
7. Horgan, C. O., and J. K. Knowles, "Recent Developments Concerning Saint-Venant's Principle" in *Advances in Applied Mechanics* (J. W. Hutchinson, editor). Academic Press, Vol. 23, 1983 (in press).
8. Choi, I., and C. O. Horgan, "Saint-Venant's Principle and End Effects in Anisotropic Elasticity," *J. Applied Mechanics* (Trans. ASME), Vol. 44, 1977, p. 424-430.
9. Mansfield, E. H., and D. R. Best, "The Concept of Load Diffusion Length in Fibre Reinforced Composites." *Aeronautical Research Council Current Paper* No. 1338. Her Majesty's Stationery Office, London, 1976.
10. Choi, I., and C. O. Horgan, "Saint-Venant End Effects for Plane Deformation of Sandwich Strips." *Int. J. Solids Structures*, Vol. 14, 1978, p. 187-195.
11. Alwar, R. S., "Experimental Verification of Saint-Venant's Principle in a Sandwich Beam." *AIAA J.*, Vol. 8, 1970, p. 160-162.
12. Folkes, M. J., and R. G. C. Arridge, "The Measurement of Shear Modulus in Highly Anisotropic Materials: The Validity of St. Venant's Principle." *J. Physics D: Appl. Phys.*, Vol. 8, 1975, p. 1053-1064.
13. Arridge, R. G. C., P. J. Barham, C. J. Farrell and A. Keller, "The Importance of End Effects in the Measurement of Moduli of Highly Anisotropic Materials," *J. of Materials Science*, Vol. 11, 1976, p. 788-790.
14. Arridge, R. G. C., and M. J. Folkes, "Effect of Sample Geometry on the Measurement of Mechanical Properties of Anisotropic Materials," *Polymer*, Vol. 17, 1976, p. 495-500.
15. Odell, J. A., and A. Keller, "Deformation Behavior of an S-B-S Copolymer." *Polymer Engineering and Science*, Vol. 17, 1977, p. 544-559.
16. Keller, A., and J. A. Odell, "A Method for Obtaining High Modulus Polyethylene from the Melt: The Influence of Crystal Nucleation." *Journal of Polymer Science: Polymer Symposium*, Vol. 63, 1978, p. 155-164.

17. Lewis, E. L. V., "An Experimental Study of End Effects in the Extensional Deformation of Polymers." *J. of Materials Science*, Vol. 14, 1979, p. 2343-2352.
18. Lewis, E. L. V. and I. M. Ward, "The Measurement of Shear Compliances for Oriented Polyethylene Terephthalate Sheet." *J. of Materials Science*, Vol. 15, 1980, p. 2354-2364.
19. Gibson, A. G., Holt, J. S., and P. S. Hope, "Factors Affecting Stress Transmission in Crystalline Polymers." *J. of Polymer Science: Polymer Physics Edition*, Vol. 17, 1979, p. 1375-1394.
20. Holt, J. S., and P. S. Hope, "Displacement Oscillation in Plane Quadratic Isoparametric Elements in Orthotropic Situations." *Int. J. for Numerical Methods in Engineering*, Vol. 14, 1979, p. 913-920.
21. Sandorff, P. E., "Saint-Venant Effects in an Orthotropic Beam." *J. of Composite Materials*, Vol. 14, 1980, p. 199-212.
22. Horgan, C. O., and J. K. Knowles, "The Effect of Nonlinearity on a Principle of Saint-Venant Type." *J. of Elasticity*, Vol. 11, 1981, p. 271-291.
23. Lekhnitskii, S. G., *Theory of Elasticity of an Anisotropic Elastic Body* (translated by P. Fern), Holden-Day, San Francisco, 1963.
24. Tsai, S. W., and H. T. Hahn, *Introduction to Composite Materials*, Technomic Publishing Co., Westport, Connecticut, 1980.
25. Pipkin, A. C., "Stress Analysis For Fiber-Reinforced Materials" in *Advances in Applied Mechanics* (C.-S. Yih, ed.), Vol. 19, 1979, p. 1-51.
26. Pagano, N. J., and J. M. Whitney, "Geometric Design of Composite Cylindrical Characterization Specimens," *J. of Composite Materials*, Vol. 4, 1970, p. 360-378.
27. Timoshenko, S. P., and J. N. Goodier, *Theory of Elasticity*, 3rd edition, McGraw-Hill, New York, 1970.
28. Knowles, J. K., and C. O. Horgan, "On the Exponential Decay of Stresses in Circular Elastic Cylinders Subject to Axisymmetric Self-Equilibrating End Loads." *Int. J. Solids Structures*, Vol. 5, 1969, p. 33-50.
29. Horgan, C. O., "Saint-Venant's Principle in Anisotropic Elasticity Theory," in *Mechanical Behavior of Anisotropic Solids* (J. P. Boehler, ed), Proc. of Euromech Colloquium 115, Villard de Lans, France, 1979, Centre National de la Recherche Scientifique, France 1982, p. 855-870.
30. Rao, N. R., and K. V. Valsarajan, "Saint-Venant's Principle in Sandwich Strip." *Computers and Structures*, Vol. 12, 1980, p. 185-188.
31. Dong, S. B., and D. B. Goetschel, "Edge Effects in Laminated Composite Plates." *J. Appl. Mech.* (Trans. ASME), Vol. 49, 1982, p. 129-135.