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SAMPLED-DATA MODELLING AND SIMULATION OF CYCLICALLY SWITCHED CONVERTERS

J.L. Duarte

Abstract : A procedure for the description of power electronics circuit dynamics is proposed with the intention of control system design and discrete-time system simulation. The approach is specially suited to be used along with computer-aided analysis and synthesis software packages such as MATLAB. The various modelling steps are illustrated by an application to a dc/dc converter under different control strategies.

Keywords : power convertors, discrete-time systems, sampled-data systems, sampled-data modelling, power circuit simulation.

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Sampled-Data Modelling and Simulation of Cyclically Switched Converters

1 Introduction

Models for the dynamics of power electronic circuits are of crucial importance in many applications, both for assessing stability and for designing compensators to enhance stability and performance.

A general sampled-data representation of power electronic circuit dynamics is presented in this report. It leads, via compact and powerful notation, to disciplined modelling for large-signal numerical simulations and to the derivation of small-signal linear models that describe perturbations about a nominal cyclic steady state.

Thus, starting from fundamental state-space techniques, the following approach has been chosen:

> Section 2 and Section 3 introduce a general formulation aiming at a straightforward discrete-time system description of cyclically switched converters. These two sections draw on the ideas in [1].
>
> Next, Section 4 places emphasis on an analytical method, through which the derivation of the system matrices can be carried out systematically, being especially suited to be used within the context of computer-aided analysis and design software tools such as MATLAB. The development in this section is close to the work in [3]-[4].
>
> Numerical techniques which complement the proposals in Section 4 are briefly sketched in Section 5. The back-ground for the formulations can be found in [2].
>
> Section 6 refers to application examples. By taking the properties of a simple dc/dc converter under different control strategies explicitly into account, the details related to the sampled-data modelling methodology are made clear.
>
> Supplementary techniques are summarized in appendices. MATLAB m-files are also provided to be considered along with the application examples.

Nomenclature:
Notational distinction is made between scalars, for instance x, and vectors, for instance $\underline{x}$. Matrices are represented by upper-case names, such as $\underline{A}$, or given between brackets [ ]. Appendix A introduces functions like $\underline{f}(x)$, $f(\underline{x})$, $\underline{f}(\underline{x})$ and their derivatives. If a lower-case name represents a variable that may be time dependent, such as (continuous-time) $x(t)$ or (discrete-time) $x(t_k)$, then the corresponding upper-case name, X in this case, is normally used to denote the nominal periodic steady-state value related to this variable.

2 Power Electronic Circuits as Cyclically Switched Systems

A general sampled-data representation of power electronic circuit dynamics is presented in this section. State-space techniques are employed to derive an equivalent nonlinear discrete-time model that describes the circuit exactly.

2.1 Continuous-time description of circuit operation

We consider a power electronic system model that is characterized as follows. The system operates cyclically. In the k-th cycle, extending from time $t = t_k$ to time $t = t_{k+1}$, the n-dimensional state vector $\underline{x}(t)$ of the system is governed by a succession of N linear time-invariant state-space equations of the form

$$\frac{d\underline{x}(t)}{dt} = \underline{A}_i\underline{x}(t) + \underline{B}_i\underline{u}(t); \quad t_k + T_{i-1,k} < t \leq t_k + T_{i,k}, \tag{1}$$

one for each of the N switch configurations in the k-th cycle. The state $\underline{x}(t)$ is continuous across each change in switch configuration, i.e., the final state in one configuration is the initial state in the next.

The index i in Eq.(1) runs from one to N, with

$$\begin{aligned} T_{0,k} &= 0 \\ t_{k+1} &= t_k + T_{N,k} \end{aligned}$$

as illustrated in Fig. 1. It goes without saying that $T_{N,k}$ is the *duration* of the k-th cycle.

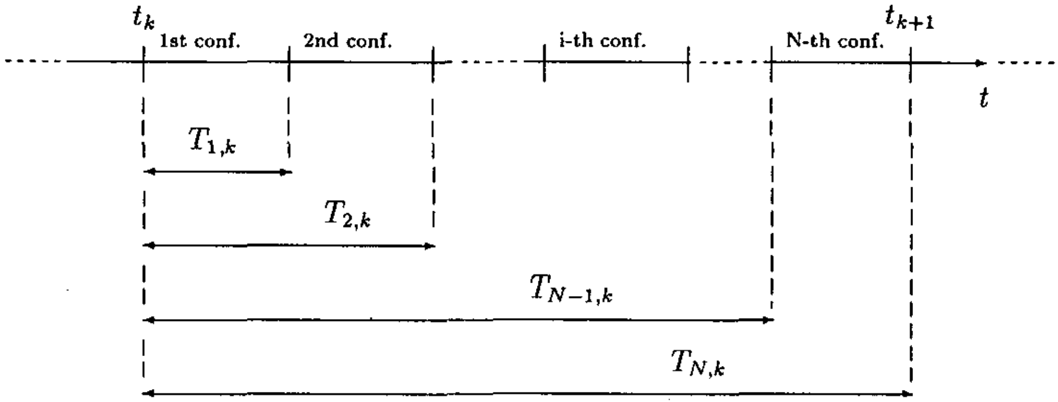

Figure 1: *Transition times at which the switch configuration changes, related to the beginning of the k-th cycle.*

The variables $T_{i,k}$ in Fig. 1 may be termed (relative) *transition times*, and they will be collected into the N-vector $\underline{T}_k$, with

$$\underline{T}_k = \begin{bmatrix} T_{1,k} \\ \vdots \\ T_{N,k} \end{bmatrix}.$$

The m-dimensional vector $\underline{u}(t)$ is a vector of time functions that typically represents sources acting on the circuit, and $\underline{A}_i$ and $\underline{B}_i$ in Eq.(1) are $n \times n$ and $n \times m$ matrices, respectively.

For a given $\underline{x}(t_k)$ the evolution of the system in Eq.(1) in the k-th cycle is completely determined by the source waveforms and the transition times at which the switch configurations change. In the cases of interest to us these source waveforms and transition times are in turn governed, directly or indirectly, by a set of independent *determining variables.* Some of the determining variables serve to impose directly all the source waveforms in the vector $\underline{u}(t)$ for the k-th cycle. Aiming at notational simplicity, it is advantageous to assemble all the independent determining variables into a vector labeled $\underline{p}_k$.

The transition times $T_{i,k}$ depend on external control action and on the system state. Therefore, they are essentially of two types. One type of transition may be *directly controlled* by external control action; this is usually the situation when, for example, thyristors are turned on or transistors are turned on or off (the exceptions correspond to those thyristors or transistors for which these particular operations have been made functions of the system state and are thus no longer direct functions of external control action). The corresponding transition times are then directly and explicitly determined by some of the determining variables.

The other type of transition only occurs when the system state reaches particular boundary's or *threshold conditions*; this is the case with, for example, thyristors turning off (threshold condition: zero thyristor current) or diodes turning on (threshold condition: zero diode voltage) or off (threshold condition: zero diode current). This type of transition is thus only *indirectly or implicitly controlled* by the determining parameters via the effect of external control action on the state trajectories of the system.

The expressions that give the $\underline{u}(t)$ in the the k-th cycle in terms of the entries of $\underline{p}_k$ are typically simple and explicit. Those that give the $T_{i,k}$ in terms of the entries of $\underline{p}_k$ can range from simple and explicit to complicated and implicit; simple explicit expressions are to be expected for the directly controlled transitions, while complicated implicit expressions are the norm for the indirectly controlled transitions.

Despite the distinction between the two types of transition times, the equations relating the N-vector $\underline{T}_k$ of transition times to the vector $\underline{p}_k$ of the determining variables, for both the directly and indirectly controlled cases and for any given $\underline{x}(t_k)$, can be seen to be summarized in a set of N equations that has the form

$$\underline{c}(\underline{x}(t_k), \underline{p}_k, \underline{T}_k) = \underline{0}. \tag{2}$$

From now on we shall refer to this set as the *constraint equation* for the system.

Note that the compact notation $\underline{c}(\cdot,\cdot,\cdot)$ is actually

$$\underline{c}(\cdot,\cdot,\cdot) = \begin{bmatrix} c_1(\cdot,\cdot,\cdot) \\ \vdots \\ c_N(\cdot,\cdot,\cdot) \end{bmatrix},$$

where each of the $c_i(\cdot,\cdot,\cdot)$ is a scalar function of three vector arguments, which determines the transition time $T_{k,i}$.

2.2 Large-signal sampled-data description

On integrating the governing description given by Eq.(1) over the interval from t_k to t_{k+1} and noting that $\underline{u}(t)$ in the k-th cycle is directly determined by $\underline{p}_k$, a *sampled-data description* of the form

$$\underline{x}(t_{k+1}) = \underline{f}(\underline{x}(t_k), \underline{p}_k, \underline{T}_k) \tag{3}$$

is obtained. Again note that the symbol $\underline{f}(\cdot,\cdot,\cdot)$ is being used to denote an n-vector, each of whose entries is a scalar function of three vector arguments.

For given $\underline{x}(t_k)$ and specified determining variables $\underline{p}_k$ the constraint Eq.(2) may be used to determine $\underline{T}_k$, typically by an iterative numerical computation (since some of the component functions of Eq.(2) will typically be implicit non-linear equations). Substitution of the resulting $\underline{T}_k$ in Eq.(3) then yields the the state $\underline{x}(t_{k+1})$ at the beginning of the next cycle. The process is then continued forward. We thus have in Eqs.(2-3) an exact large-signal sampled-data description of the dynamics of any power electronic circuit that can be modelled via Eq.(1).

It is often the case that one also wants to model the evolution of certain variables other than the state variables. For example, one may wish how the average value of some variable, taken over a cycle, varies as one goes from cycle to cycle, or one may be interested in the dynamic evolution of the peak value in each cycle of some variable. Any auxiliary variable in which the value for the k-th cycle is determined entirely by system behavior in the k-th cycle is completely determined by $\underline{x}(t_k)$, $\underline{p}_k$ and $\underline{T}_k$. Collecting all such auxiliary variables of interest into a vector $\underline{y}_k$, one can obtain an equation of the form

$$\underline{y}_k = \underline{h}(\underline{x}(t_k), \underline{p}_k, \underline{T}_k) \tag{4}$$

to be considered along with Eq.(2) and Eq.(3). Again, the constraints in Eq.(2) can be used to eliminate $\underline{T}_k$ from Eq.(4) as needed. Note as before that the notation $\underline{h}(\cdot,\cdot,\cdot)$ represents a vector function of three vector arguments.

3 Perturbations Around a Nominal Cyclic Steady State

Finding the *operating point* is usually the first step in the analysis of power electronic circuits. After this has been accomplished, it is possible to linearize the system about its equilibrium state to obtain a linear discrete-time model for small-signal performance evaluations, such as stability and transient response.

3.1 Periodic operating point

If a power electronic system model of the form given by Eq.(1) has a nominal cyclic operating point, then

$$\underline{x} = \underline{f}(\underline{x}, \underline{p}, \underline{T}), \tag{5}$$

and

$$\underline{c}(\underline{x}, \underline{p}, \underline{T}) = \underline{0} \tag{6}$$

for the functions $\underline{f}(\cdot,\cdot,\cdot)$ and $\underline{c}(\cdot,\cdot,\cdot)$ defined in Eqs.(2-3), where the vectors $\underline{p}$ and $\underline{T}$ denote the periodic steady-state values of the determining variables and transition times:

$$\begin{aligned} \underline{p}_{k-1} &= \underline{p}_k = \underline{p}_{k+1} = \cdots = \underline{p}, & (7) \\ \underline{T}_{k-1} &= \underline{T}_k = \underline{T}_{k+1} = \cdots = \underline{T}, & (8) \end{aligned}$$

(with corresponding constant cycle duration $T_{N,k} = T_N$) and

$$\underline{x}(t_{k-1}) = \underline{x}(t_k) = \underline{x}(kT_N) = \cdots = \underline{x}. \tag{9}$$

The conditions in Eqs.(5-6) follow from the fact that a cyclic steady state is characterized by values of the determining variables and initial state such that the system, after excursions and changes of switch configuration, returns at the end of the cycle to the same state.

3.2 Dynamics of perturbations from steady state

For the purpose of analyzing the dynamics of variations about a particular nominal cyclic steady state, we shall use the following notation to represent perturbations of the various system variables from their steady-state values in Eqs.(7-9):

$$\begin{aligned} \underline{x}_k &= \underline{x}(t_k) - \underline{x} \\ \underline{q}_k &= \underline{p}_k - \underline{p} \\ \underline{r}_k &= \underline{T}_k - \underline{T}. \end{aligned} \tag{10}$$

Note that the perturbation in the duration of the kth cycle, $T_{k,N} - T_N$, is actually the last component of $\underline{r}_k$.

From Eqs.(2-3) we then get

$$\underline{x}_{k+1} = \underline{f}(\underline{x} + \underline{x}_k, \underline{p} + \underline{q}_k, \underline{T} + \underline{r}_k) - \underline{x} \tag{11}$$

with

$$\underline{c}(\underline{x} + \underline{x}_k, \underline{p} + \underline{q}_k, \underline{T} + \underline{r}_k) = \underline{0}. \tag{12}$$

Carrying out (multivariable) Taylor series expansions in Eqs.(11-12), retaining only linear terms, and using Eq.(10) yields what is essentially the small-signal model (though still in implicit form):

$$\underline{x}_{k+1} = \left[\partial \underline{f}/\partial \underline{x}\right] \underline{x}_k + \left[\partial \underline{f}/\partial \underline{p}\right] \underline{q}_k + \left[\partial \underline{f}/\partial \underline{T}\right] \underline{r}_k \tag{13}$$

and

$$\left[\partial \underline{c}/\partial \underline{x}\right] \underline{x}_k + \left[\partial \underline{c}/\partial \underline{p}\right] \underline{q}_k + \left[\partial \underline{c}/\partial \underline{T}\right] \underline{r}_k = \underline{0}. \tag{14}$$

The compact notation makes things look simple, but power over the symbols requires understanding them completely. The (Jacobian) symbol $\left[\partial \underline{f}/\partial \underline{x}\right]$ is been used to denote the partial derivative of the vector $\underline{f}(\cdot,\cdot,\cdot)$ with respect to its first vector argument (see App. A and App. B) evaluated at the cyclic steady state specified by $\underline{x}$, $\underline{p}$, and $\underline{T}$. Similar definitions hold for the other matrices of derivatives in Eq.(13) and Eq.(14). Note that $\left[\partial \underline{c}/\partial \underline{T}\right]$ is a $N \times N$ square matrix.

The results in Eq.(13) and Eq.(14) can be combined by solving for $\underline{r}_k$ from Eq.(14) and substituting the resulting expression in Eq.(13) to get

$$\underline{x}_{k+1} = \underline{F}_0 \underline{x}_k + \underline{G}_0 \underline{q}_k, \tag{15}$$

where

$$\underline{F}_0 = \left[\partial \underline{f}/\partial \underline{x}\right] - \left[\partial \underline{f}/\partial \underline{T}\right] \left[\partial \underline{c}/\partial \underline{T}\right]^{-1} \left[\partial \underline{c}/\partial \underline{x}\right] \tag{16}$$

and

$$\underline{G}_0 = \left[\partial \underline{f}/\partial \underline{p}\right] - \left[\partial \underline{f}/\partial \underline{T}\right] \left[\partial \underline{c}/\partial \underline{T}\right]^{-1} \left[\partial \underline{c}/\partial \underline{p}\right]. \tag{17}$$

This constitutes the final form of the linear time-invariant sampled-data model for perturbations of the system away from a cyclic steady state. In particular, the periodic steady state is locally asymptotically stable (without further control action) iff all eigenvalues of $\underline{F}_0$ have magnitude less than one.

Perturbations of the auxiliary variables in Eq.(4) around their steady-state values $\underline{y} = \underline{h}(\underline{x}, \underline{p}, \underline{T})$ can also be readily modeled. By defining in analogy to Eq.(16)

$$\underline{v}_k = \underline{y}_k - \underline{y}, \quad \underline{y} = \underline{h}(\underline{x}, \underline{p}, \underline{T}), \tag{18}$$

it follows that (by the same procedure of Taylor expansion in Eq.(4), truncation at linear terms, and by using Eq.(14))

$$\underline{v}_k = \underline{H}_0 \underline{x}_k + \underline{K}_0 \underline{q}_k \tag{19}$$

where

$$\underline{H}_0 = \left[\partial \underline{h} / \partial \underline{x}\right] - \left[\partial \underline{h} / \partial \underline{T}\right] \left[\partial \underline{c} / \partial \underline{T}\right]^{-1} \left[\partial \underline{c} / \partial \underline{x}\right], \tag{20}$$

and

$$\underline{K}_0 = \left[\partial \underline{h} / \partial \underline{p}\right] - \left[\partial \underline{h} / \partial \underline{T}\right] \left[\partial \underline{c} / \partial \underline{T}\right]^{-1} \left[\partial \underline{c} / \partial \underline{p}\right]. \tag{21}$$

4 Looking for the Cyclic Steady State

A cyclic steady state of the power electronic circuit model is characterized by the conditions established at Eqs.(5-6). Since the elements of $\underline{p}$ (the determining variables) are supposed to be well known, the problem of finding a periodic operating point can be stated as one of finding stationary solutions $\underline{x}$ and $\underline{T}$, for which both equations

$$\underline{f}(\underline{x},\underline{p},\underline{T}) - \underline{x} = \underline{0} \tag{22}$$

$$\underline{c}(\underline{x},\underline{p},\underline{T}) = \underline{0} \tag{23}$$

are fulfilled simultaneously.

The solution of Eq.(22) and Eq.(23) can be obtained by iterative computation using the Newton formula (see App. B). Hence, starting with initial values $\underline{x}^0$ and $\underline{T}^0$, the $m+1$ iteration step is given by

$$\begin{bmatrix} \underline{x}^{m+1} \\ \underline{T}^{m+1} \end{bmatrix} = \begin{bmatrix} \underline{x}^{m} \\ \underline{T}^{m} \end{bmatrix} - \left[\underline{J}(\underline{x}^m,\underline{p},\underline{T}^m)\right]^{-1} \cdot \underline{R}(\underline{x}^m,\underline{p},\underline{T}^m), \tag{24}$$

where $\underline{J}$ is the Jacobian and $\underline{R}$ the residue of the set of Eqs.(22-23) :

$$\underline{J} = \begin{bmatrix} \partial \underline{f}/\partial \underline{x} - \underline{I} & \partial \underline{f}/\partial \underline{T} \\ \partial \underline{c}/\partial \underline{x} & \partial \underline{c}/\partial \underline{T} \end{bmatrix} \tag{25}$$

and

$$\underline{R} = \begin{bmatrix} \underline{f} - \underline{x} \\ \underline{c} \end{bmatrix}. \tag{26}$$

The solution of Eq.(22) and Eq.(23) is found when

$$||\underline{R}|| < \epsilon, \quad \text{a small number.}$$

Notice that the submatrices of the Jacobian also provides the basis for the description of perturbations about the steady state, Eqs.(15-21). Therefore, solving the steady state problem by the Newton method implies that the small-signal model will be partially constructed.

Numeric algorithms for determining the cyclic steady state (and the partial derivatives in Eq.(25)) are well established [2]. However, a better insight of cyclic power electronic converters will be acquainted if the expressions of the Jacobian matrix are determined through an analytical formulation. For this reason, an analytical approach aiming at determining the derivatives in Eq.(25) will be discussed in the following.

4.1 Sensitivity matrices

As it will be clear in the next section, for the purpose of evaluating analytically the Jacobian matrix in Eq.(25), we have to deal with the dependency of the state vector at the transient time T_i from variations in the state vector at the transition time T_j, and from variations in the transition time T_j self. Otherwise stated, it is necessary to look for expressions for the derivatives

$$\frac{\partial \underline{x}(T_i)}{\partial \underline{x}(T_j)} \quad \text{and} \quad \frac{\partial \underline{x}(T_i)}{\partial T_j}.$$

(*Note*: since we focus on the steady-state environment, from now on the absolute time t_k will be chosen equal to zero, and the inessential subscripts k will be dropped, without further loss of generality).

In order to compute such derivatives, it is advantageous to introduce a *sensitivity* matrix $\underline{\Phi}_i(t)$, which is obtained by differentiating the state vector within a switch configuration with respect to the initial state value:

$$\underline{\Phi}_i(t) = \frac{\partial \underline{x}(t)}{\partial \underline{x}(T_{i-1})}; \quad T_{i-1} < t \leq T_i. \tag{27}$$

By rewriting Eq.(1) in its integral form,

$$\underline{x}(t) = \int_{T_{i-1}}^{t} \underline{g}_i(\underline{x}, \underline{u}, \tau) d\tau \; + \underline{x}(T_{i-1}), \quad T_{i-1} < t \leq T_i, \; 1 \leq i \leq N; \tag{28}$$

where

$$\underline{g}_i(\underline{x}, \underline{u}, t) = \underline{A}_i \underline{x}(t) + \underline{B}_i \underline{u}(t), \tag{29}$$

it follows that[1]

$$\frac{\partial \underline{x}(t)}{\partial \underline{x}(T_{i-1})} = \int_{T_{i-1}}^{t} \left. \frac{\partial \underline{g}_i(\underline{x}, \underline{u}, \tau)}{\partial \underline{x}} \right|_{\underline{x}(T_{i-1})} \cdot \frac{\partial \underline{x}(\tau)}{\partial \underline{x}(T_{i-1})} \cdot d\tau \; + \underline{I}. \tag{30}$$

Comparing Eq.(27) to Eq.(30) leads to

$$\frac{d\underline{\Phi}_i(t)}{dt} = \left. \frac{\partial \underline{g}_i(\underline{x}, \underline{u}, t)}{\partial \underline{x}} \right|_{\underline{x}(T_{i-1})} \cdot \underline{\Phi}_i(t); \quad \underline{\Phi}_i(T_{i-1}) = \underline{I}. \tag{31}$$

[1] *Hint:* Leibniz's theorem for differentiation of an integral:

$$\frac{d}{dc} \int_{a(c)}^{b(c)} f(x, c) dx = \int_{a(c)}^{b(c)} \frac{\partial f(x, c)}{\partial c} dx + f(b, c) \frac{db}{dc} - f(a, c) \frac{da}{dc}.$$

In view of Eq.(29)

$$\left.\frac{\partial \underline{g}_i(\underline{x},\underline{u},t)}{\partial \underline{x}}\right|_{\underline{x}(T_{i-1})} = \underline{A}_i, \tag{32}$$

and the solution of Eq.(31) is found to become (see App. C) :

$$\underline{\Phi}_i(t) = \exp\{\underline{A}_i \cdot (t - T_{i-1})\}; \quad T_{i-1} < t \leq T_i, 1 \leq i \leq N. \tag{33}$$

Consider now the expression for the derivative

$$\frac{\partial \underline{x}(T_i)}{\partial \underline{x}(T_j)}, \text{ with } i > j.$$

Since the final state in one switch configuration is the initial state in the next, by using the chain rule of differentiation, and in view of Eq.(27), the above derivative can be expanded to

$$\begin{aligned}\frac{\partial \underline{x}(T_i)}{\partial \underline{x}(T_j)} &= \frac{\partial \underline{x}(T_i)}{\partial \underline{x}(T_{i-1})} \cdot \frac{\partial \underline{x}(T_{i-1})}{\partial \underline{x}(T_{i-2})} \cdot \cdots \cdot \frac{\partial \underline{x}(T_{j+1})}{\partial \underline{x}(T_j)};\\ &= \underline{\Phi}_i(T_i) \cdot \underline{\Phi}_{i-1}(T_{i-1}) \cdot \cdots \cdot \underline{\Phi}_{j+1}(T_{j+1});\\ &\qquad 0 \leq j < N, \;\; j < i \leq N.\end{aligned} \tag{34}$$

Another relation of importance to us is the one which describes the dependency of the state vector from variations in the transition times, that is,

$$\frac{\partial \underline{x}(T_i)}{\partial T_j}.$$

Now the distinction between $i = j$ and $i > j$ is conceptually relevant. If $i = j$, then T_i has to be seen as an upper limit time, and the system is supposed to present *no* perturbations until (and including) the transition time previous to T_i. Hence, with regard to Eq.(28),

$$\frac{\partial \underline{x}(T_i)}{\partial T_i} = \frac{\partial}{\partial T_i}\left[\int_{T_{i-1}}^{T_i} \underline{g}_i(\underline{x},\underline{u},\tau)d\tau + \underline{x}(T_{i-1})\right]$$

which implies[2]

$$\frac{\partial \underline{x}(T_i)}{\partial T_i} = \underline{g}_i(\underline{x},\underline{u},T_i), \quad 1 \leq i \leq N. \tag{35}$$

[2]See footnote 1.

If $i > j$, then T_j has to be considered as a transition time inside the history of the system, and the effects of variations in T_j propagate until the upper limit time T_i. Once again the chain rule of derivation can be used :

$$\frac{\partial \underline{x}(T_i)}{\partial T_j} = \frac{\partial \underline{x}(T_i)}{\partial \underline{x}(T_{i-1})} \cdot \frac{\partial \underline{x}(T_{i-1})}{\partial \underline{x}(T_{i-2})} \cdot \cdots \cdot \frac{\partial \underline{x}(T_{j+1})}{\partial \underline{x}(T_j)} \cdot \frac{\partial \underline{x}(T_j)}{\partial T_j}. \tag{36}$$

In view of Eq.(28), and from the fact that a variation in T_j has influence in *two* circuit topologies, the latest derivative in Eq.(36) is found to become[3]:

$$\begin{aligned} \frac{\partial \underline{x}(T_j)}{\partial T_j} &= \lim_{t \to T_j} \frac{\partial}{\partial T_j} \left[\int_{T_j}^{t} \underline{g}_{j+1}(\underline{x}, \underline{u}, \tau) d\tau + \int_{T_{j-1}}^{T_j} \underline{g}_j(\underline{x}, \underline{u}, \tau) d\tau + \underline{x}(T_{j-1}) \right], \\ &= -\underline{g}_{j+1}(\underline{x}, \underline{u}, T_j) + \underline{g}_j(\underline{x}, \underline{u}, T_j), \quad 1 \le j < N. \end{aligned} \tag{37}$$

Finally, the results from Eqs.(35-37) yield

$$\frac{\partial \underline{x}(T_i)}{\partial T_j} = \begin{cases} \underline{\Phi}_i(T_i) \cdot \underline{\Phi}_{i-1}(T_{i-1}) \cdot \cdots \cdot \underline{\Phi}_{j+1}(T_{j+1}) \cdot \left[-\underline{g}_{j+1}(\underline{x}, \underline{u}, T_j) + \underline{g}_j(\underline{x}, \underline{u}, T_j) \right], \\ \qquad 1 \le j < N, \ \ j < i \le N \ ; \\ \underline{g}_j(\underline{x}, \underline{u}, T_j), \quad 1 \le j \le N, \ \ i = j. \end{cases} \tag{38}$$

Obviously, for $j > i$, both Eq.(34) and Eq.(38) are identical $\underline{0}$.

4.2 Analytical evaluation of Jacobian matrices

Aiming notational clarity, we shall rewrite Eq.(25) as follows:

$$\underline{J} = \begin{bmatrix} \partial \underline{x}(T_N)/\partial \underline{x}(0) - \underline{I} & \partial \underline{x}(T_N)/\partial \underline{T} \\ \partial \underline{c}/\partial \underline{x}(0) & \partial \underline{c}/\partial \underline{T} \end{bmatrix}, \tag{39}$$

because

$$\underline{x}(0) \equiv \underline{x} \quad \text{and} \quad \underline{x}(T_N) \equiv \underline{f}(\underline{x}, \underline{p}, \underline{T})$$

emphasizes more sharply that we are focusing on the state vector at the beginning and at the end of the cyclic steady-state period.

In view of Eq.(34)

$$\frac{\partial \underline{x}(T_N)}{\partial \underline{x}(0)} = \underline{\Phi}_N(T_N) \cdot \underline{\Phi}_{N-1}(T_{N-1}) \cdot \cdots \cdot \underline{\Phi}_1(T_1), \tag{40}$$

and with this, the computation of the upper left submatrix in Eq.(39) is determined.

[3]See footnote 1.

The right upper submatrix in Eq.(39) can also be evaluated by using Eq.(38). Its i-th *column* is found to become

$$\frac{\partial \underline{x}(T_N)}{\partial T_i} = \begin{cases} \underline{\Phi}_N(T_N) \cdot \underline{\Phi}_{N-1}(T_{N-1}) \cdot \cdots \cdot \underline{\Phi}_{i+1}(T_{i+1}) \cdot \left[-\underline{g}_{i+1}(\underline{x}, \underline{u}, T_i) + \underline{g}_i(\underline{x}, \underline{u}, T_i) \right], \\ \qquad 1 \leq j < N \; ; \\ \\ \underline{g}_i(\underline{x}, \underline{u}, T_i), \quad i = N. \end{cases} \tag{41}$$

For the submatrix $\left[\partial \underline{c} / \partial \underline{x}(0)\right]$ the chain rule also applies. The i-th *row* of this submatrix can be expressed as being:

$$\frac{\partial c_i}{\partial \underline{x}(0)} = \frac{\partial c_i}{\partial \underline{x}(T_i)} \cdot \underline{\Phi}_i(T_i) \cdot \underline{\Phi}_{i-1}(T_{i-1}) \cdot \cdots \cdot \underline{\Phi}_1(T_1); \quad 1 \leq i \leq N. \tag{42}$$

The derivation of the remaining submatrix, which also depends on the function $\underline{c}$, is a little more subtle. With regard to the results in Eq.(38), this last submatrix is a lower-triangular one, whose ij-elements are of the form:

$$\left[\partial \underline{c} / \partial \underline{T}\right]_{ij} = \begin{cases} \partial c_i / \partial T|_{T=T_i} \; + \\ \quad + \left[\partial c_i / \partial \underline{x}(T_i)\right] \cdot \underline{g}_i(\underline{x}, \underline{u}, T_i); \quad 1 \leq i \leq N, \; j = i \\ \\ \partial c_i / \partial T|_{T=T_j} \; + \\ \quad + \left[\partial c_i / \partial \underline{x}(T_i)\right] \cdot \underline{\Phi}_i(T_i) \cdot \underline{\Phi}_{i-1}(T_{i-1}) \cdot \cdots \cdot \underline{\Phi}_{j+1}(T_{j+1}) \cdot \\ \quad \cdot \left[-\underline{g}_{j+1}(\underline{x}, \underline{u}, T_j) + \underline{g}_j(\underline{x}, \underline{u}, T_j) \right]; \quad 2 \leq i \leq N, \; 1 \leq j < i \\ \\ 0\,; \quad 1 \leq i < N, \; j > i. \end{cases} \tag{43}$$

It is worthwhile to notice that in all submatrices only the sensitivity matrix $\underline{\Phi}_i$, the functions $\underline{g}_i$ and the derivatives of the function $\underline{c}$ appears. Therefore the Jacobian can be evaluated very easily, and moreover by steps.

5 Numerical Evaluation of Derivatives

Once the cyclic steady-state operation point has been found, the sensitivity of the solution to changes in the determining variables (circuit element values, for instance) should be determined for the purpose of constructing the small-signal model (Eqs.(15-21)).

By differentiating Eqs.(22-23) with respect to one determining variable p_j at a time, it follows that

$$\frac{\partial \underline{f}}{\partial p_j} = -\left[\frac{\partial \underline{f}}{\partial \underline{x}} - \mathrm{I}\right] \cdot \frac{\partial \underline{x}}{\partial p_j} - \left[\frac{\partial \underline{f}}{\partial \underline{T}}\right] \cdot \frac{\partial \underline{T}}{\partial p_j}, \tag{44}$$

$$\frac{\partial \underline{c}}{\partial p_j} = -\left[\frac{\partial \underline{c}}{\partial \underline{x}}\right] \cdot \frac{\partial \underline{x}}{\partial p_j} - \left[\frac{\partial \underline{c}}{\partial \underline{T}}\right] \cdot \frac{\partial \underline{T}}{\partial p_j}, \tag{45}$$

where the matrices between [ ] are supposed to be evaluated at the operating point. In fact, in view of the results of the previous sections, these matrices are already available. Therefore, the problem reduces on finding

$$\frac{\partial \underline{x}}{\partial p_j} \equiv \frac{\partial \underline{x}(0)}{\partial p_j} \quad \text{and} \quad \frac{\partial \underline{T}}{\partial p_j}, \tag{46}$$

i.e, the dependency of the state vector and transition times at the steady state from variations in the parameter p_j.

An analytical solution for Eq.(46) is not always straightforward in account of the switching sequence. For this reason we will propose a numerical resolution.

A classical approach for computing derivatives consists on perturbating the parameter p_j by Δp_j and finding the resultant variations in $\underline{x}$ or $\underline{T}$ (you are right, dear reader: for *each* determining variable at a time, a new periodic steady state has to be computed after *each* perturbation Δp_j, according to the procedure in Section 4, indeed!). Obviously, it is a brute-force evaluation of the derivatives, where it is important to select the appropriate increments Δp_j. Some experimentation with the increment size is advisable, since the accuracy of the partial derivatives depends on it. If the function varies rapidly, a very small increment is clearly required. On the other hand, if the increment is chosen needless small, then the accuracy decreases because of numerical computation errors; i.e., a difference quotient assumes numerically the value close to 0/0. Study on the increment size and its effect on the results also has physically significant implications. If the linearized system shows high sensitivity to incremental size, then this points out that the non-linear system changes its behavior rather rapidly as it moves alway from its equilibrium, and the result obtained for the linearized system is only valid for very small perturbations about the equilibrium.

Of course, a similar numerical approach could be followed, aiming at the determination of the cyclic steady state (and the partial derivatives related to it) [2]. However such a formulation results in high computational costs if compared to the analytical method of Section 4.

6 Application Examples

In this section the methodology presented in the preceding sections for modelling cyclically switched systems will be highlighted through step-by-step examples concerning a simple dc/dc converter. First, the method will be applied to the particular case of open-loop duty-ratio control of an up/down converter. Next, the sampled-data modelling approach will be demonstrated by operating the same converter under feed-forward control and current-mode control. In all cases, general equations for steady-state and dynamic performance are obtained. The resulting equations yield discrete recurrence relationships which can be readily used for circuit simulation with the MATLAB software program. Numerical results from simulations are provided, together with the corresponding m-files.

Parameter values for the converter under consideration have been taken from [5], which is a good reference book on modelling techniques in Power Electronics. The reader is encouraged to compare the results presented here to the examples in [5], where the same converter is used several times to illustrate various aspects of dynamic modelling and control design.

6.1 Up/down converter

For the purpose of illustrating the proposed modelling procedure, an up/down (or buck/boost) converter will be considered. The power circuit is build according to the circuit schematic in Fig. 2, with the following component values

$$L = 250\mu H, \quad C = 220\mu F, \quad \text{and} \quad R = 2\Omega, \tag{47}$$

being operated at a fixed switching frequency of $50kHz$, which implies a period of $T_s = 20\mu s$. The nominal input voltage and output power are

$$U_s = 12V \quad \text{and} \quad P_o = 40W. \tag{48}$$

As a first approximation, it is assumed that the transistor and the diode function as ideal switches, and that there is no parasitics or other non-linearities in the lumped components.

We will only take into consideration the so-called *continuous-conduction mode* in which the instantaneous inductor current does not fall to zero at any instant. Therefore, on the

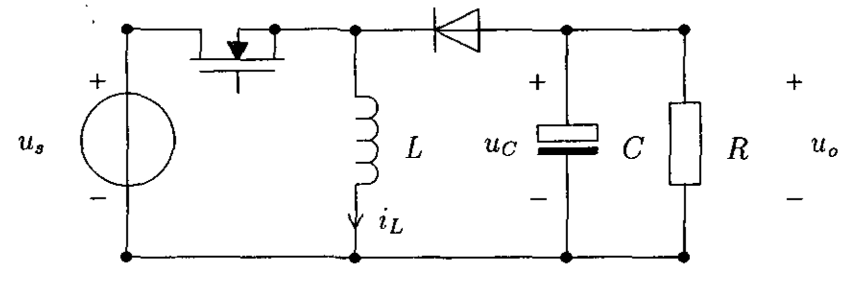

Figure 2: *Up/down converter.*

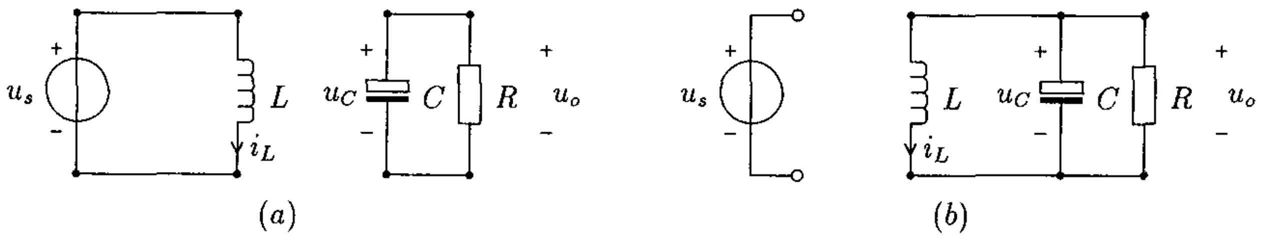

Figure 3: *Operating modes of the circuit in Fig. 2, under the assumption of ideal switches; (a) transistor on, (b) transistor off.*

assumption of ideal switches, there will be only two different circuit topologies, as shown in Fig. 3, which also implies two transition times within the switching period. Fig. 4 shows typical time wave-forms over a generic period. The definition of the *duty ratio* per cycle is shown, as well.

6.2 Duty-ratio control

If the transistor in Fig. 2 is turned on periodically and operated with constant duty-ratio D, then it follows from the Vs-balance for the inductor L that, in the case of constant input voltage $u_s(t) \equiv U_s$, the average output voltage U_o is given by

$$U_o = \langle u_C(t) \rangle = -\frac{D}{D'} U_s, \tag{49}$$

where $D' = 1 - D$ (note the polarity reversal in Eq.(49)). Therefore, the duty ratio must be set at a nominal value of

$$D = -U_r/(-U_r + U_s) \tag{50}$$

in order to obtain a desired average output voltage $U_o = U_r < 0$. For instance, in order to establish the nominal operation conditions in Eq.(48), which implies $U_r = -9V$, the duty ratio for the transistor in Fig. 2 should be set to $D = 0.43$.

Certainly, if the duty ratio is changed, or perturbations occur at the input voltage, the voltage at the output will reach another average value after a transient.

6.2.1 Basic modelling equations

It is customary and convenient in electrical networks to adopt inductor currents and capacitor voltages as state variables. In the case of the circuit in Fig. 2, the natural choice for the state-space vector is then

$$\underline{x}(t) = \begin{bmatrix} i_L(t) \\ u_C(t) \end{bmatrix}. \tag{51}$$

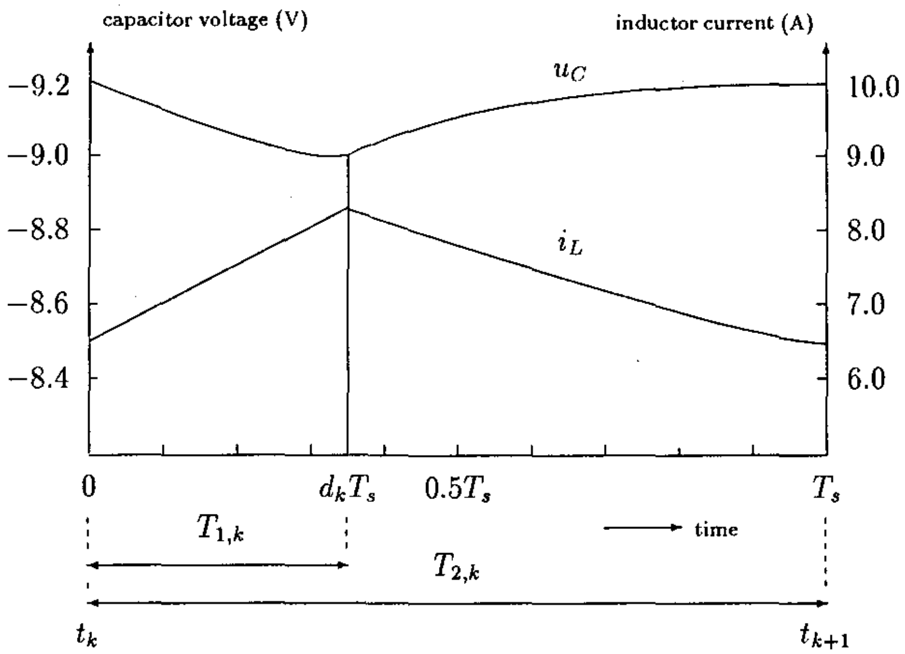

Figure 4: *Typical wave-forms over a switching period for an up/down converter in continuous-conduction mode. Notice that the capacitor voltage is negative.*

The matrices related to the set of state-space differential equations for the two possible circuit configurations in Fig. 3 are found to be, in view of Eq.(1),

$$\underline{A}_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1/RC \end{bmatrix}, \quad \underline{B}_1 \underline{u}(t) = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u_s(t) = \underline{b}_1 u_s(t); \tag{52}$$

$$\underline{A}_2 = \begin{bmatrix} 0 & 1/L \\ -1/C & -1/RC \end{bmatrix}, \quad \underline{B}_2 \underline{u}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} u_s(t) = \underline{0}. \tag{53}$$

The switching frequency is supposed to be held constant, but the duty ratio d is normally a directly controlled parameter. Also the input voltage $u_s(t)$ in Fig. 2 can be seen as an independent parameter. Therefore, to keep the notation proposed in Sec. 2.1, the set of transition times and determining variables under consideration will be

$$\underline{T}_k = \begin{bmatrix} T_{1,k} \\ T_{2,k} \end{bmatrix}, \text{ and } \underline{p}_k = \begin{bmatrix} u_s(t_k) \\ d_k \end{bmatrix}, \tag{54}$$

where d_k denotes the duty ratio of the transistor at each switching period and $u_s(t_k)$ is the value of the input voltage at the beginning of a switching cycle.

According to Eq.(2) and with regard to Fig. 4, the constraint equation for the system can be expressed as being

$$\underline{c}_k = \begin{bmatrix} T_{1,k} - d_k T_s \\ \\ T_{2,k} - T_s \end{bmatrix} = \underline{0}. \tag{55}$$

Notice that both transition times are directly controlled by external control action.

6.2.2 Large-signal model

Under the supposition that eventual perturbations in the input voltage can be modelled by step changes that take place only at the beginning of a switching period, the exact discrete-time solution for the set of differential equations given by Eqs.(51-55) is found to be (cf. App. C) :

$$\underline{x}(T_{1,k}) = \underline{\Phi}_1(T_{1,k})\underline{x}(t_k) + \underline{b}_1\, u_s(t_k)\, d_k T_s, \tag{56}$$

$$\underline{x}(T_{2,k}) = \underline{\Phi}_2(T_{2,k})\underline{x}(T_{1,k}), \quad \underline{x}(t_{k+1}) = \underline{x}(T_{2,k}), \tag{57}$$

with

$$\underline{\Phi}_1(T_{1,k}) = \exp\left\{\underline{A}_1 T_{1,k}\right\}, \ T_{1,k} = d_k T_s, \tag{58}$$

$$\underline{\Phi}_2(T_{2,k}) = \exp\left\{\underline{A}_2 \cdot (T_{2,k} - T_{1,k})\right\}, \ T_{2,k} = T_s. \tag{59}$$

Eqs.(56-59) provide then a complete and exact large-signal recurrence description for the up/down converter under open-loop duty-ratio control. A MATLAB file is given in App. D, based on the discrete equations above, for the purpose of simulating the dynamics of the system subject to step variations in the input voltage $u_s(t_k)$ or in the duty ratio d_k.

6.2.3 Nominal operating condition

Let us now concentrate on the steady-state solution for Eqs.(56-59), that is, $\underline{x}(t_{k+1}) = \underline{x}(t_k) = \underline{x}(0)$, which is found to be (with all inessential subscripts k dropped):

$$\underline{x}(0) = \begin{bmatrix} i_L(0) \\ \\ u_C(0) \end{bmatrix} = \left[\underline{I} - \underline{\Phi}_2(T_2)\underline{\Phi}_1(T_1)\right]^{-1} \left[\underline{\Phi}_2(T_2)\underline{b}_1\right] U_s D T_s. \tag{60}$$

Although an exact analytical description for Eq.(60) is possible (cf. App. C), we will keep things simple if we notice that the output time constant of an up/down converter is normally large enough (product $RC = 440\mu s >> 20\mu s$ in Fig. 2) that the steady-state output voltage at the beginning of a switching period $u_C(0)$ can be approximate by its average value within the switching period. Hence, in view of Eq.(49),

$$u_C(0) \approx -\frac{D}{D'} U_s. \tag{61}$$

Under steady-state operation and taking also the approximation in Eq.(61) into account, the voltage across the inductor L is then a square wave symmetrical about zero. Therefore, the steady-state value of the inductor current at the beginning of the switching period is found to be, after some calculations,

$$i_L(0) \approx \left(1 - \frac{T_s/2}{L/R}\, D'^2\right) \frac{D}{D'^2} \frac{1}{R} U_s. \tag{62}$$

6.2.4 Small-signal model

The small-signal model describing perturbations around a nominal cyclic steady state follows from Eqs.(15-21) as being

$$\underline{x}_{k+1} = \underline{F}_0 \underline{x}_k + \underline{G}_0 \underline{q}_k, \tag{63}$$
$$\underline{v}_k = \underline{H}_0 \underline{x}_k + \underline{K}_0 \underline{q}_k, \tag{64}$$

where

$$\underline{F}_0 = \left[\partial \underline{f}/\partial \underline{x}\right] - \left[\partial \underline{f}/\partial \underline{T}\right] \left[\partial \underline{c}/\partial \underline{T}\right]^{-1} \left[\partial \underline{c}/\partial \underline{x}\right], \tag{65}$$
$$\underline{G}_0 = \left[\partial \underline{f}/\partial \underline{p}\right] - \left[\partial \underline{f}/\partial \underline{T}\right] \left[\partial \underline{c}/\partial \underline{T}\right]^{-1} \left[\partial \underline{c}/\partial \underline{p}\right], \tag{66}$$
$$\underline{H}_0 = \left[\partial \underline{h}/\partial \underline{x}\right] - \left[\partial \underline{h}/\partial \underline{T}\right] \left[\partial \underline{c}/\partial \underline{T}\right]^{-1} \left[\partial \underline{c}/\partial \underline{x}\right], \tag{67}$$
$$\underline{K}_0 = \left[\partial \underline{h}/\partial \underline{p}\right] - \left[\partial \underline{h}/\partial \underline{T}\right] \left[\partial \underline{c}/\partial \underline{T}\right]^{-1} \left[\partial \underline{c}/\partial \underline{p}\right]. \tag{68}$$

After evaluating ancillary matrices (again without inessential subscripts) for a given nominal operation point,

$$\underline{\Phi}_1(T_1) = \exp\left\{\underline{A}_1 T_1\right\} = \exp\left\{\underline{A}_1\, DT_s\right\}, \tag{69}$$
$$\underline{\Phi}_2(T_2) = \exp\left\{\underline{A}_2\,(T_2 - T_1)\right\} = \exp\left\{\underline{A}_2\, D'T_s\right\}, \tag{70}$$
$$\underline{x}_1(T_1) = \underline{\Phi}_1 \underline{x}(0) + \underline{b}_1\, U_s DT_s, \tag{71}$$
$$\underline{g}_1(T_1) = \underline{A}_1 \underline{x}(T_1) + \underline{b}_1 U_s, \tag{72}$$
$$\underline{g}_2(T_1) = \underline{A}_2 \underline{x}(T_1), \quad \underline{g}_2(T_2) = \underline{A}_2 \underline{x}(0), \tag{73}$$

with $\underline{x}'(0) = [i_L(0) \quad u_C(0)]$ calculated from Eqs.(61-62), the Jacobians related to the small-signal model follow:
a) from Eq.(40):

$$\left[\partial \underline{f}/\partial \underline{x}\right] = \frac{\partial \underline{x}(T_2)}{\partial \underline{x}(0)} = \underline{\Phi}_2(T_2)\underline{\Phi}_1(T_1); \tag{74}$$

b) from Eq.(41):

$$\left[\partial \underline{f}/\partial \underline{T}\right] = \left[\ \partial \underline{f}/\partial T_1 \quad \partial \underline{f}/\partial T_2\ \right], \tag{75}$$
$$\partial \underline{f}/\partial T_1 = \underline{\Phi}_2(T_2)\left[-\underline{g}_2(T_1) + \underline{g}_1(T_1)\right], \tag{76}$$
$$\partial \underline{f}/\partial T_2 = \underline{g}_2(T_2); \tag{77}$$

c) from Eq.(42):

$$\left[\partial \underline{c} / \partial \underline{x}\right] = \begin{bmatrix} \partial c_1/\partial \underline{x} \\ \partial c_2/\partial \underline{x} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \tag{78}$$

d) from Eq.(43):

$$\left[\partial \underline{c} / \partial \underline{T}\right] = \begin{bmatrix} \partial c_1/\partial T_1 & 0 \\ \partial c_2/\partial T_1 & \partial c_2/\partial T_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \tag{79}$$

e) from Eqs.(44-45):

$$\left[\partial \underline{f} / \partial \underline{p}\right] = -\left[\frac{\partial \underline{f}}{\partial \underline{x}} - \underline{I}\right]\left[\frac{\partial \underline{x}}{\partial \underline{p}}\right] - \left[\frac{\partial \underline{f}}{\partial \underline{T}}\right]\left[\frac{\partial \underline{T}}{\partial \underline{p}}\right], \tag{80}$$

$$\left[\partial \underline{c} / \partial \underline{p}\right] = -\left[\frac{\partial \underline{c}}{\partial \underline{x}}\right]\left[\frac{\partial \underline{x}}{\partial \underline{p}}\right] - \left[\frac{\partial \underline{c}}{\partial \underline{T}}\right]\left[\frac{\partial \underline{T}}{\partial \underline{p}}\right] = -\left[\frac{\partial \underline{T}}{\partial \underline{p}}\right], \tag{81}$$

where

$$\left[\partial \underline{T} / \partial \underline{p}\right] = \begin{bmatrix} \partial T_1/\partial U_s & \partial T_1/\partial D \\ \partial T_2/\partial U_s & \partial T_2/\partial D \end{bmatrix} = \begin{bmatrix} 0 & T_s \\ 0 & 0 \end{bmatrix}, \tag{82}$$

which can be derived from the steady-state version of Eq.(55), and

$$\left[\partial \underline{x} / \partial \underline{p}\right] = \begin{bmatrix} \partial i_L(0)/\partial U_s & \partial i_L(0)/\partial D \\ \partial u_C(0)/\partial U_s & \partial u_C(0)/\partial D \end{bmatrix}, \tag{83}$$

with, after differentiation of Eqs.(61-62),

$$\partial i_L(0)/\partial U_s = \left(1 - \frac{T_s/2}{L/R} D'^2\right) \frac{D}{D'^2} \frac{1}{R}, \tag{84}$$

$$\partial u_C(0)/\partial U_s = -\frac{D}{D'}, \tag{85}$$

$$\partial i_L(0)/\partial D = \left(-\frac{T_s/2}{L/R} + \frac{1+D}{D'^3}\right) \frac{1}{R} U_s, \tag{86}$$

$$\partial u_C(0)/\partial D = -\frac{1}{D'^2} U_s. \tag{87}$$

For the purpose of observing only the output voltage, we write then

$$\underline{H}_0 = \left[\partial \underline{h} / \partial \underline{x}\right] = \begin{bmatrix} 0 & 1 \end{bmatrix}, \tag{88}$$

$$\underline{K}_0 = \underline{0}. \tag{89}$$

It is worthwhile to emphasize that only the computation of the matrices exponential in Eq.(52) asks some amount of labour. However, MATLAB has sound algorithms for the purpose of solving these matrices numerically.

6.2.5 Simulation results

By inputing the nominal determining variables given by Eqs.(47-48), which are $U_s = 12V$ and $D = 0.43$, into Eqs.(69-87), and after substitution of the resulting matrices in Eqs.(65-68), yields

$$\underline{F}_0 = \begin{bmatrix} 0.9988 & 0.0442 \\ -0.0513 & 0.9544 \end{bmatrix} \quad \underline{G}_0 = \begin{bmatrix} 0.0339 & 1.6792 \\ -0.0014 & 0.6550 \end{bmatrix}$$
$$\underline{H}_0 = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \underline{K}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{90}$$

In App. E a m-file is given based on the recurrence equations (63-64) with the numerical values from Eq.(90). Fig. 5 shows simulation results from a step change in the input voltage from 12V to 8V, while Fig. 6 presents results concerning a step change in the duty ratio from 0.43 to 0.5. In both cases, data from the exact large-signal model are also given for the purpose of comparison.

On the basis of MATLAB tools, it is then straightforward to get the input-to-output transfer functions in the z-domain. By assuming $u_o(t) \approx u_C(t_k)$, it follows from Eq.(90) that

$$\frac{u_o(z)}{u_s(z)} = -0.0014\frac{(z+0.2208)}{(z-0.9766-j0.0421)(z-0.9766+j0.0421)}, \tag{91}$$
$$\frac{u_o(z)}{d(z)} = +0.6550\frac{(z-1.1302)}{(z-0.9766-j0.0421)(z-0.9766+j0.0421)}. \tag{92}$$

As expected, the system poles in Eqs.(91-92) are complex and stable. Notice also a system zero outside the unit circle in Eq.(92), which explains the non-minimal phase behavior in Fig. 6.

6.3 Feed-forward control

A feed-forward approach to achieve immunity with respect to perturbations at the input consists of forcing the duty ratio to change dynamically according to Eq.(50), that is,

$$d_k = -U_r/(-U_r + u_s(t_k)). \tag{93}$$

By doing so, the steady-state average voltage at the output will be independent of the voltage value at the input.

The large-signal behavior of the system under feed-forward control can be easily simulated in MATLAB by using Eq.(93) in combination with Eqs.(56-59). Fig. 7 shows numerical results when a step change occurs at the input voltage.

Although the steady-state value of the output voltage remains constant in Fig. 7, the system transient is not better than under open-loop control, which is to be expected since the feed-forward action has no influence upon the system poles. This can be concluded from the small-signal model, Eqs.(63-64), as follows.

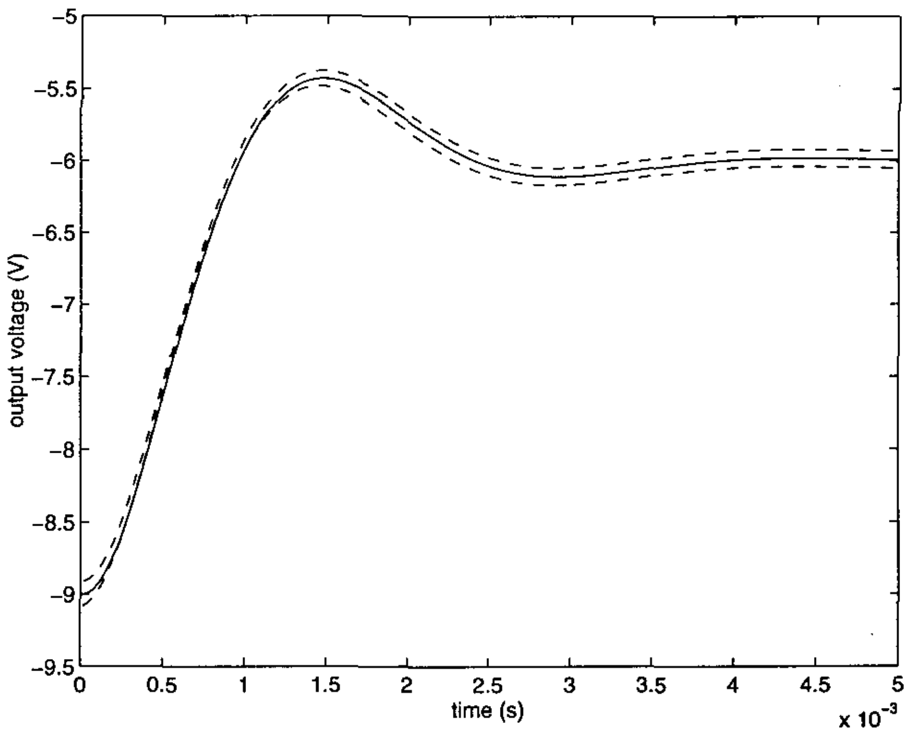

Figure 5: *Up/down converter under duty-ratio control: output voltage transient as a consequence of a step change in the input voltage (from 12V to 8V); large-signal (- -) and small-signal (–) models. Only the boundaries of the switching ripple is plotted for the large-signal data.*

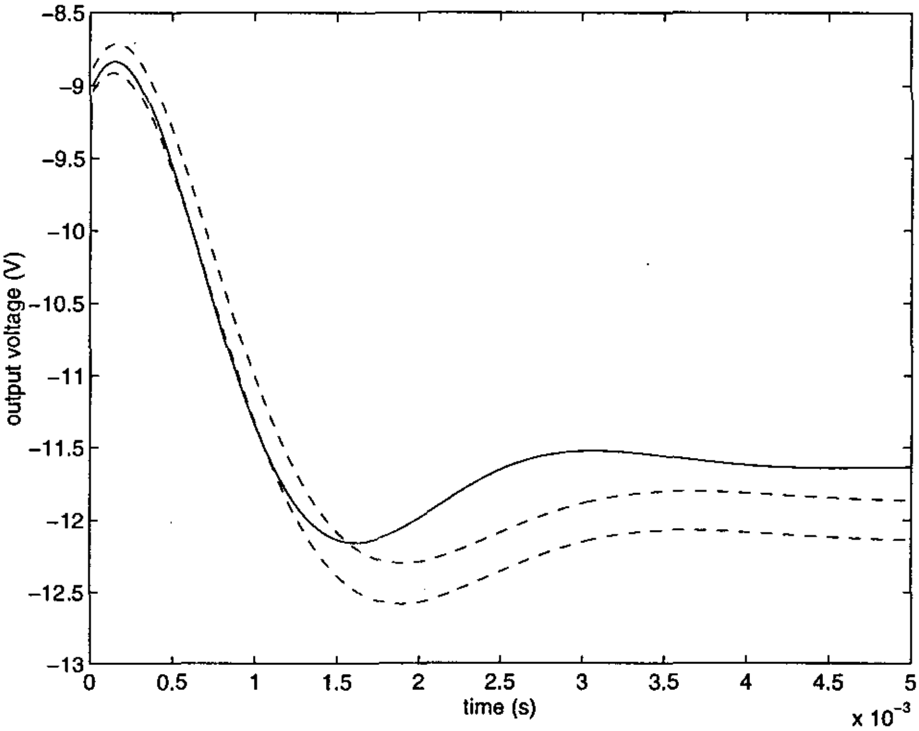

Figure 6: *Up/down converter under duty-ratio control: output voltage transient as a consequence of a step change in the duty ratio (from 0.43 to 0.5); large-signal (- -) and small-signal (–) models. Only the boundaries of the switching ripple is plotted for the large-signal data.*

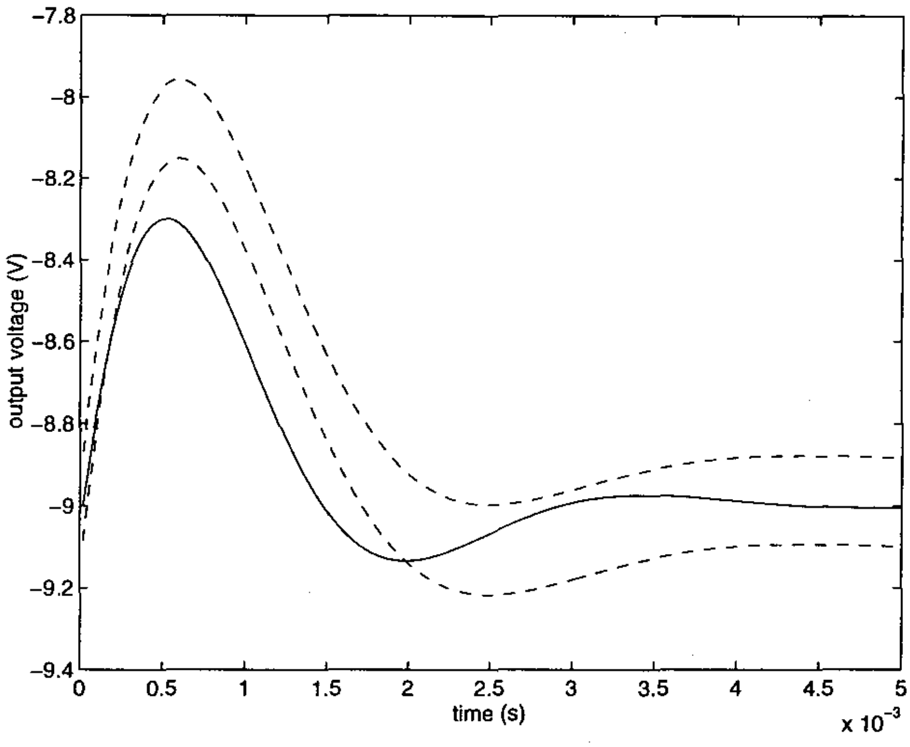

Figure 7: *Up/down converter under feed-forward control: output voltage transient as a consequence of a step change in the input voltage (from 12V to 8V); large-signal (- -) and small-signal (-) models. Only the boundaries of the switching ripple is plotted for the large-signal data.*

By defining $\nu_k = u_s(t_k) - U_s$, a linear approximation for small variations in the duty ratio is found from Eq.(93) to be

$$d_k - D \approx \frac{U_r}{(-U_r + U_s)^2}\nu_k. \tag{94}$$

Substitution of Eq.(94) into Eq.(63) yields

$$\underline{x}_{k+1} = \underline{F}_0\underline{x}_k + \underline{G}_0\underline{m}_0\nu_k, \quad \text{with} \quad \underline{m}_0 = \begin{bmatrix} 1 \\ U_r/(-U_r + U_s)^2 \end{bmatrix}, \tag{95}$$

which implies a linear system with the same poles as in Eq.(91).

Fig. 7 also shows numerical results from Eq.(95), that are in good agreement with the large-signal data.

6.4 Current-mode control

In Sec. 6.2.3 we obtained an approximate sampled-data model for an up/down converter, with the duty ratio as the control variable. In *current-mode* control, however, the controller specifies a peak switching current $i_p(t)$ in each cycle rather than the duty ratio. The switch

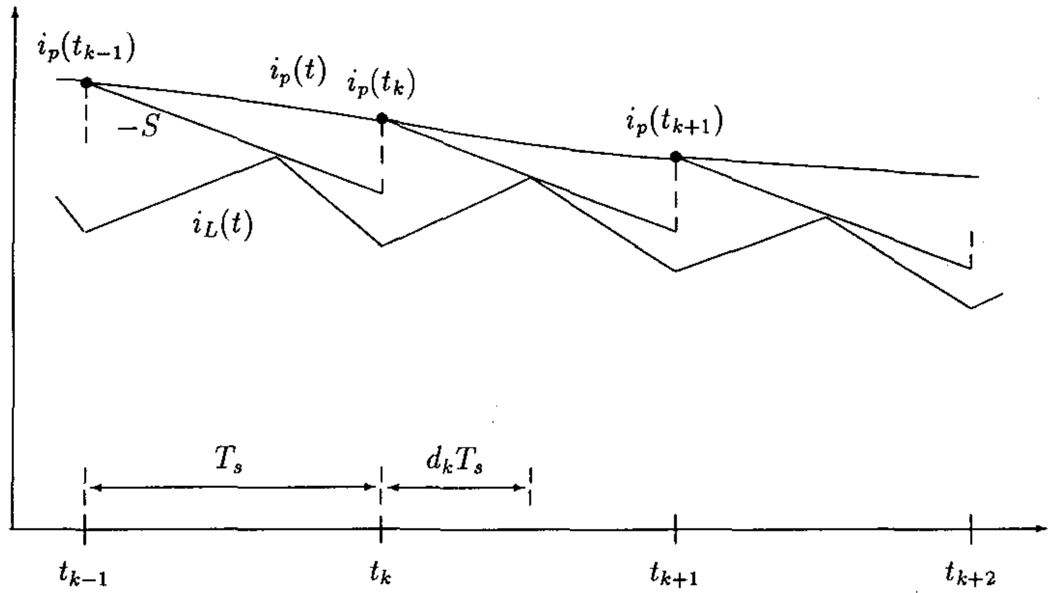

Figure 8: *Wave-forms over a switching period for an up/down converter under current-mode control; with stabilizing ramp.*

may be turned on regularly as earlier, but it is turned off when the inductor current reaches a threshold value, as illustrated in Fig. 8. The duty ratio becomes now an indirectly determined auxiliary variable, the peak inductor current $i_p(t)$ being the primary control variable.

As suggested in Fig. 8, the threshold current signal is build up as the sum of two signals: a slowly varying signal $i_p(t)$ determined by the controller on the basis of the discrepancy between the actual and nominal average output voltages; and a regular sawtooth ramp of slope $-S$ at the switching frequency, termed a stabilizing ramp for reasons that will be clear when analyzing the stability aspects of this control approach.

6.4.1 Basic modelling equations

The state-space vector $\underline{x}(t)$ and the system matrices $\underline{A}_1, \underline{A}_2$ and $\underline{b}_1$ remain the same, being given by Eqs.(51-52).

Under current-mode control, the determining variables vector becomes

$$\underline{p}_k = \begin{bmatrix} u_s(t_k) \\ i_p(t_k) \end{bmatrix}, \tag{96}$$

where $i_p(t_k)$ is the value of $i_p(t)$ in beginning of the k-th cycle; along with the constraint equations vector

$$\underline{c}_k = \begin{bmatrix} \underline{\ell}'\underline{x}(T_{1,k}) - i_p(t_k) + ST_{1,k} \\ T_{2,k} - T_s \end{bmatrix} = \underline{0}, \quad \text{with} \quad \underline{\ell}' = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{97}$$

6.4.2 Large-signal model

In order to simulate the large-signal behavior of the system, it is convenient to rewrite Eq.(97) as

$$i_L(t_k) + \frac{d_k T_s}{L} u_s(t_k) = i_p(t_k) - S d_k T_s, \tag{98}$$

resulting then the duty ratio at each k-th cycle

$$d_k = \frac{L}{T_s}\left\{\frac{i_p(t_k) - i_L(t_k)}{u_s(t_k) + LS}\right\}. \tag{99}$$

Eq.(99) in combination with Eqs.(56-59) provide a complete and exact large-signal recurrence description for simulating the up/down converter under current-mode control. A MATLAB m-file is given in App. F for the purpose of simulating the dynamics of the system subject to step variations in the input voltage $u_s(t_k)$ or in the peak inductor current reference $i_p(t_k)$.

6.4.3 Nominal operating condition

If the steady-state solution for the duty ratio is found on the basis of Eq.(99), the cyclic steady-state values for the state variables can be readily obtained from Eqs.(61-62).

In order to find an analytical expression for the duty ratio under steady-state operation, one should consider again the Vs-balance for the inductor L, which implies the following relationships between averaged state variables

$$\langle u_C(t)\rangle = -\frac{D}{D'}U_s, \quad D'\langle i_L(t)\rangle = -\frac{\langle u_C(t)\rangle}{R}, \tag{100}$$

and, from Eq.(98),

$$\langle i_L(t)\rangle + \frac{1}{2}DT_s\frac{U_s}{L} = I_p - SDT_s. \tag{101}$$

The combination of Eqs.(100-101) yields

$$\frac{D}{D'^2} + \gamma D = \Gamma, \tag{102}$$

where

$$\gamma = \left(1 + 2\frac{LS}{U_s}\right)\frac{T/2}{L/R} \quad \text{and} \quad \Gamma = \frac{RI_p}{U_s}. \tag{103}$$

An (approximate) explicit solution for D as described by Eq.(102) is found to be, after some calculations,

$$D \approx (3/4)\left[1 - \frac{1}{1 + \Gamma - (3/4)\gamma}\right], \tag{104}$$

which provides the the duty ratio as function of the circuit parameters and the steady-state determining variables U_s and I_p.

For building the small-signal model in the next section, it is necessary to compute the partial derivatives of the nominal cyclic operation point with respect to the steady-state determining variables. By using D as an auxiliary variable, it follows

$$\frac{\partial i_L(0)}{\partial U_s} = \frac{\partial i_L(0)}{\partial D}\frac{\partial D}{\partial U_s} + \frac{\partial i_L(0)}{\partial U}|_{U=U_s}, \tag{105}$$

$$\frac{\partial u_C(0)}{\partial U_s} = \frac{\partial u_C(0)}{\partial D}\frac{\partial D}{\partial U_s} + \frac{\partial u_C(0)}{\partial U}|_{U=U_s}, \tag{106}$$

$$\frac{\partial i_L(0)}{\partial I_p} = \frac{\partial i_L(0)}{\partial D}\frac{\partial D}{\partial I_p}, \tag{107}$$

$$\frac{\partial u_C(0)}{\partial I_p} = \frac{\partial u_C(0)}{\partial D}\frac{\partial D}{\partial I_p}. \tag{108}$$

Analytical expressions for $\partial i_L(0)/\partial D$ and $\partial u_C(0)/\partial D$ have been already calculated in Eqs.(86-87); while, with regard to Eqs.(84-85),

$$\frac{\partial i_L(0)}{\partial U}|_{U=U_s} = \left(1 - \frac{T_s/2}{L/R}D'^2\right)\frac{D}{D'^2}\frac{1}{R}, \tag{109}$$

$$\frac{\partial u_C(0)}{\partial U}|_{U=U_s} = -\frac{D}{D'}. \tag{110}$$

Further, taking also Γ and γ as intermediate variables, it is possible to write

$$\frac{\partial D}{\partial U_s} = \frac{\partial D}{\partial \Gamma}\frac{\partial \Gamma}{\partial U_s} + \frac{\partial D}{\partial \gamma}\frac{\partial \gamma}{\partial U_s}, \tag{111}$$

$$\frac{\partial D}{\partial I_p} = \frac{\partial D}{\partial \Gamma}\frac{\partial \Gamma}{\partial I_p}, \tag{112}$$

with, as a consequence of Eqs.(103-104),

$$\frac{\partial D}{\partial \Gamma} = (3/4)\frac{1}{(1+\Gamma-(3/4)\gamma)^2}, \tag{113}$$

$$\frac{\partial D}{\partial \gamma} = -(3/4)^2\frac{1}{(1+\Gamma-(3/4)\gamma)^2}, \tag{114}$$

$$\frac{\partial \Gamma}{\partial U_s} = -RI_p/U_s^2, \quad \frac{\partial \Gamma}{\partial I_p} = R/U_s, \tag{115}$$

$$\frac{\partial \gamma}{\partial U_s} = -2\frac{LS}{U_s^2}\frac{T_s/2}{L/R}. \tag{116}$$

(117)

6.4.4 Small-signal model

For the sake of clarity, the small-signal model describing perturbations about a nominal cyclic steady state will be rewritten again as in Sec. 6.2.4:

$$\underline{x}_{k+1} = \underline{F}_0\underline{x}_k + \underline{G}_0\underline{q}_k, \tag{118}$$

$$\underline{v}_k = \underline{H}_0\underline{x}_k + \underline{K}_0\underline{q}_k, \tag{119}$$

where

$$\underline{F}_0 = \left[\partial\underline{f}/\partial\underline{x}\right] - \left[\partial\underline{f}/\partial\underline{T}\right]\left[\partial\underline{c}/\partial\underline{T}\right]^{-1}\left[\partial\underline{c}/\partial\underline{x}\right], \tag{120}$$

$$\underline{G}_0 = \left[\partial\underline{f}/\partial\underline{p}\right] - \left[\partial\underline{f}/\partial\underline{T}\right]\left[\partial\underline{c}/\partial\underline{T}\right]^{-1}\left[\partial\underline{c}/\partial\underline{p}\right], \tag{121}$$

$$\underline{H}_0 = \left[\partial\underline{h}/\partial\underline{x}\right] - \left[\partial\underline{h}/\partial\underline{T}\right]\left[\partial\underline{c}/\partial\underline{T}\right]^{-1}\left[\partial\underline{c}/\partial\underline{x}\right], \tag{122}$$

$$\underline{K}_0 = \left[\partial\underline{h}/\partial\underline{p}\right] - \left[\partial\underline{h}/\partial\underline{T}\right]\left[\partial\underline{c}/\partial\underline{T}\right]^{-1}\left[\partial\underline{c}/\partial\underline{p}\right]. \tag{123}$$

After evaluating ancillary matrices using the value of D given by Eq.(104) and the cyclic operation point value $\underline{x}'(0) = [i_L(0) \;\; u_C(0)]$ given by Eqs.(61-62),

$$\underline{\Phi}_1(T_1) = \exp\left\{\underline{A}_1 T_1\right\} = \exp\left\{\underline{A}_1\, DT_s\right\}, \tag{124}$$

$$\underline{\Phi}_2(T_2) = \exp\left\{\underline{A}_2 (T_2 - T_1)\right\} = \exp\left\{\underline{A}_2\, D'T_s\right\}, \tag{125}$$

$$\underline{x}_1(T_1) = \underline{\Phi}_1\underline{x}(0) + \underline{b}_1 U_s DT_s, \tag{126}$$

$$\underline{g}_1(T_1) = \underline{A}_1\underline{x}(T_1) + \underline{b}_1 U_s, \tag{127}$$

$$\underline{g}_2(T_1) = \underline{A}_2\underline{x}(T_1), \quad \underline{g}_2(T_2) = \underline{A}_2\underline{x}(0), \tag{128}$$

the Jacobians related to the small-signal model follow:
a) from Eq.(40):

$$\left[\partial\underline{f}/\partial\underline{x}\right] = \frac{\partial\underline{x}(T_2)}{\partial\underline{x}(0)} = \underline{\Phi}_2(T_2)\underline{\Phi}_1(T_1); \tag{129}$$

b) from Eq.(41):

$$\left[\partial\underline{f}/\partial\underline{T}\right] = \left[\begin{array}{cc} \partial\underline{f}/\partial T_1 & \partial\underline{f}/\partial T_2 \end{array}\right], \tag{130}$$

$$\partial\underline{f}/\partial T_1 = \underline{\Phi}_2(T_2)\left[-\underline{g}_2(T_1) + \underline{g}_1(T_1)\right], \tag{131}$$

$$\partial\underline{f}/\partial T_2 = \underline{g}_2(T_2); \tag{132}$$

c) from Eq.(42):

$$\left[\partial\underline{c}/\partial\underline{x}\right] = \left[\begin{array}{c} \partial c_1/\partial\underline{x}(0) \\ \partial c_2/\partial\underline{x}(0) \end{array}\right] = \left[\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array}\right], \tag{133}$$

because now, on the basis of Eq.(97),

$$\frac{\partial c_1}{\partial \underline{x}(0)} = \frac{\partial c_1}{\partial \underline{x}(T_1)}\underline{\Phi}_1(T_1) = \underline{\ell}'\underline{\Phi}_1(T_1) = \begin{bmatrix} 1 & 0 \end{bmatrix}, \tag{134}$$

$$\frac{\partial c_2}{\partial \underline{x}(0)} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \tag{135}$$

d) from Eq.(43):

$$\left[\partial \underline{c}/\partial \underline{T}\right] = \begin{bmatrix} \partial c_1/\partial T_1 & 0 \\ \partial c_2/\partial T_1 & \partial c_2/\partial T_2 \end{bmatrix} = \begin{bmatrix} S + U_s/L & 0 \\ 0 & 1 \end{bmatrix}, \tag{136}$$

since, by taking again Eq.(97) into consideration,

$$\frac{\partial c_1}{\partial T_1} = \frac{\partial c_1}{\partial T}|_{T=T_1} + \frac{\partial c_1}{\partial \underline{x}(T_1)}\underline{g}_1(T_1) = S + \underline{\ell}'\underline{g}_1(T_1) = S + U_s/L, \tag{137}$$

$$\frac{\partial c_2}{\partial T_1} = 0, \quad \frac{\partial c_2}{\partial T_2} = 1; \tag{138}$$

e) from Eqs.(44-45):

$$\left[\partial \underline{f}/\partial \underline{p}\right] = -\left[\frac{\partial \underline{f}}{\partial \underline{x}} - \underline{I}\right]\left[\frac{\partial \underline{x}}{\partial \underline{p}}\right] - \left[\frac{\partial \underline{f}}{\partial \underline{T}}\right]\left[\frac{\partial \underline{T}}{\partial \underline{p}}\right], \tag{139}$$

$$\left[\partial \underline{c}/\partial \underline{p}\right] = -\left[\frac{\partial \underline{c}}{\partial \underline{x}}\right]\left[\frac{\partial \underline{x}}{\partial \underline{p}}\right] - \left[\frac{\partial \underline{c}}{\partial \underline{T}}\right]\left[\frac{\partial \underline{T}}{\partial \underline{p}}\right], \tag{140}$$

where

$$\left[\partial \underline{T}/\partial \underline{p}\right] = \begin{bmatrix} \partial T_1/\partial U_s & \partial T_1/\partial I_p \\ \partial T_2/\partial U_s & \partial T_2/\partial I_p \end{bmatrix}, \tag{141}$$

with, from the fact that $T_1 = DT_s$ and $T_2 = T_s$,

$$\frac{\partial T_1}{\partial U_s} = \frac{\partial T_1}{\partial D}\frac{\partial D}{\partial U_s} = T_s\frac{\partial D}{\partial U_s}, \quad \frac{\partial T_2}{\partial U_s} = 0, \tag{142}$$

$$\frac{\partial T_1}{\partial I_p} = \frac{\partial T_1}{\partial D}\frac{\partial D}{\partial I_p} = T_s\frac{\partial D}{\partial I_p}, \quad \frac{\partial T_2}{\partial I_p} = 0, \tag{143}$$

the partial derivatives $\partial D/\partial U_s$ and $\partial D/\partial I_p$ being given by Eqs.(111-112); and finally

$$\left[\partial \underline{x}/\partial \underline{p}\right] = \begin{bmatrix} \partial i_L(0)/\partial U_s & \partial i_L(0)/\partial I_p \\ \partial u_C(0)/\partial U_s & \partial u_C(0)/\partial I_p \end{bmatrix}, \tag{144}$$

the entries above being already calculated in Eqs.(105-108).

Related to the output voltage, we repeat again

$$\underline{H}_0 = \left[\partial \underline{h}/\partial \underline{x}\right] = \begin{bmatrix} 1 & 0 \end{bmatrix}, \tag{145}$$

$$\underline{K}_0 = \underline{0}. \tag{146}$$

6.4.5 Simulation results

By choosing the nominal determining variables as $U_s = 12V$ and $I_p = 9A$ with $S = 0.3\,U_s/L$, it leads approximately to the same output power as given by Eq.(48).

After inputing the nominal values into Eqs.(124-144) and then into Eqs.(120-123), the resulting matrices are found to be

$$\underline{F}_0 = \begin{bmatrix} -0.3841 & 0.0435 \\ -0.5871 & 0.9545 \end{bmatrix} \quad \underline{G}_0 = \begin{bmatrix} -0.1115 & 1.4767 \\ -0.0587 & 0.5738 \end{bmatrix}$$
$$\underline{H}_0 = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \underline{K}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix} \qquad (147)$$

In App. G a m-file is given based on the recurrence equations (118-119) with the numerical values from Eq.(147). Fig. 9 shows simulation results from a step change in the input voltage from $12V$ to $8V$, while Fig. 10 presents results concerning a step change in the peak current from $9A$ to $10.5A$. As earlier, data from the exact large-signal model are also given for the purpose of comparison.

On the basis of MATLAB tools, it follows from Eq.(147) that

$$\frac{u_o(z)}{u_s(z)} = -0.0587\frac{(z+0.7315)}{(z+0.3638)(z-0.9351)}, \qquad (148)$$
$$\frac{u_o(z)}{i_p(z)} = +0.5738\frac{(z-1.1269)}{(z+0.3638)(z-0.9351)}, \qquad (149)$$

Both poles in Eqs.(148-149) are real, which explains the over-damped transient in Figs.(9-10). This transient is, however, much faster than the situation in Figs. 5-7. In fact, one pole is dominant, making the circuit now to behavior much like a first-order system. Notice also that the system became much less sensitive to perturbations in the input voltage.

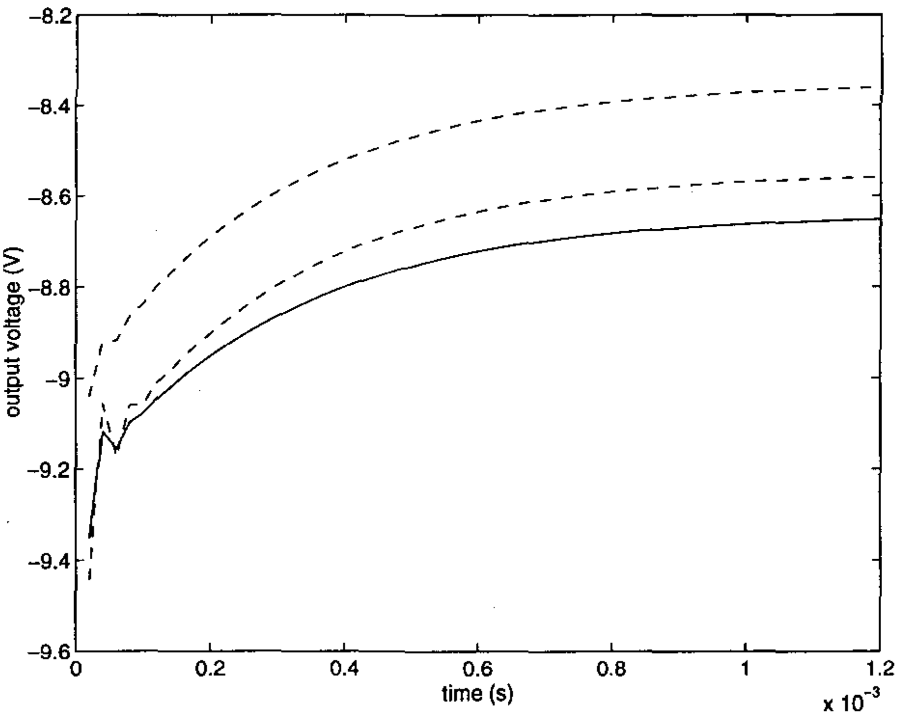

Figure 9: *Up/down converter under current-mode control: output voltage transient as a consequence of a step change in the input voltage (from 12V to 8V); large-signal (- -) and small-signal (–) models. Only the boundaries of the switching ripple is plotted for the large-signal data.*

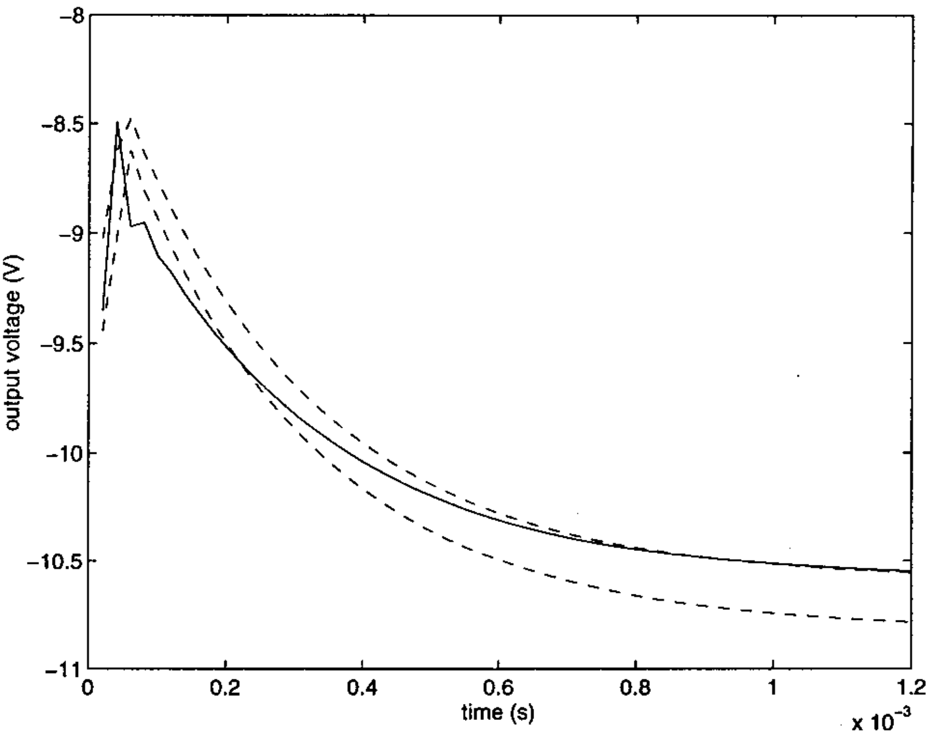

Figure 10: *Up/down converter under current-mode control: output voltage transient as a consequence of a step change in the controlling peak current (from 9A to 10.5A); large-signal (- -) and small-signal (–) models. Only the boundaries of the switching ripple is plotted for the large-signal data.*

APPENDICES

Ancyllary techniques are summarized in the following sections, together with MATLAB m-files concerning the application examples.

A Vector gradient functions

If $\underline{f}(x)$ is a p-vector which is a function of the scalar x, then

$$d\underline{f}(x)/dx = \begin{bmatrix} df_1(x)/dx \\ \vdots \\ df_p(x)/dx \end{bmatrix}$$

If $f(\underline{x})$ is a scalar function of the q-dimensional vector $\underline{x}$, then the gradient vector is

$$\partial f(\underline{x})/\partial \underline{x} = \left[\partial f(\underline{x})/\partial x_1 \cdots \partial f(\underline{x})/\partial x_q\right]$$

where the gradient is a row vector by definition.

If $\underline{f}(\underline{x})$ is a p-vector function of a q-vector $\underline{x}$, then the Jacobian matrix is the p by q matrix

$$\partial \underline{f}(\underline{x})/\partial \underline{x} = \begin{bmatrix} \partial f_1(\underline{x})/\partial x_1 & \cdots & \partial f_1(\underline{x})/\partial x_q \\ \vdots & & \vdots \\ \partial f_p(\underline{x})/\partial x_1 & \cdots & \partial f_p(\underline{x})/\partial x_q \end{bmatrix}.$$

The matrix *identity* (whose elements in the main diagonal are equal to one, all other elements equal to zero) is represented by $\underline{I}$. A vector (or a matrix) which has all elements equal to zero shall be denoted by $\underline{0}$.

B Newton algorithm

Consider the system of n non-linear equations f_i in n variables x_i:

$$\begin{aligned} f_1(x_1, x_2, \cdots, x_n) &= 0 \\ f_2(x_1, x_2, \cdots, x_n) &= 0 \\ &\vdots \\ f_n(x_1, x_2, \cdots, x_n) &= 0. \end{aligned} \tag{150}$$

Denote the vector of variables by $\underline{x}$ and the vector of functions by $\underline{f}$. Then Eq.(150) has a compact form

$$\underline{f}(\underline{x}) = \underline{0} \tag{151}$$

Assume that the system has a solution; denote it by $\underline{x}^*$ and expand each function in a Taylor series about $\underline{x}$:

$$\begin{aligned}
f_1(\underline{x}^*) &= f_1(\underline{x}) + \frac{\partial f_1}{\partial x_1}\cdot(x_1^* - x_1) + \frac{\partial f_1}{\partial x_2}\cdot(x_2^* - x_2) + \cdots + \frac{\partial f_1}{\partial x_n}\cdot(x_n^* - x_n) + \cdots \\
f_2(\underline{x}^*) &= f_2(\underline{x}) + \frac{\partial f_2}{\partial x_1}\cdot(x_1^* - x_1) + \frac{\partial f_2}{\partial x_2}\cdot(x_2^* - x_2) + \cdots + \frac{\partial f_2}{\partial x_n}\cdot(x_n^* - x_n) + \cdots \\
&\vdots \\
f_n(\underline{x}^*) &= f_n(\underline{x}) + \frac{\partial f_n}{\partial x_1}\cdot(x_1^* - x_1) + \frac{\partial f_n}{\partial x_2}\cdot(x_2^* - x_2) + \cdots + \frac{\partial f_n}{\partial x_n}\cdot(x_n^* - x_n) + \cdots
\end{aligned}$$

Assuming that $\underline{x}$ is close to $\underline{x}^*$, higher order terms may be negleted and the system may be writen in the linearized form:

$$\underline{f}(\underline{x}^*) \approx \underline{f}(\underline{x}) + \underline{J}\cdot(\underline{x}^* - \underline{x}) \tag{152}$$

where

$$\underline{J} = \begin{bmatrix} \partial f_1/\partial x_1 & \partial f_1/\partial x_2 & \cdots & \partial f_1/\partial x_n \\ \partial f_2/\partial x_1 & \partial f_2/\partial x_2 & \cdots & \partial f_2/\partial x_n \\ \vdots & \vdots & & \\ \partial f_n/\partial x_1 & \partial f_n/\partial x_2 & \cdots & \partial f_n/\partial x_n \end{bmatrix}_{|\underline{x}} \tag{153}$$

is the Jacobian matrix of the function $\underline{f}$, evaluated at $\underline{x}$. If we set Eq.(152) equal to zero and solve, the result will not be the vector $\underline{x}^*$ (because the high-order terms have been negleted) but some new value for $\underline{x}$. Using superscripts to indicate iteration sequence we have

$$\underline{f}(\underline{x}^m) + \underline{J}\cdot(\underline{x}^{m+1} - \underline{x}^m) = \underline{0}. \tag{154}$$

Formally, the solution of Eq.(154) is obtained by writing

$$\underline{x}^{m+1} = \underline{x}^m - \underline{J}^{-1}\underline{f}(\underline{x}^m). \tag{155}$$

In practice, the Jacobian matrix is not inverted. Instead, define

$$\Delta\underline{x}^m = \underline{x}^{m+1} - \underline{x}^m.$$

Then

$$\underline{J}\,\Delta\underline{x}^m = -\underline{f}(\underline{x}^m)$$

is solved by **LU** factorization [7] and the new $\underline{x}^{m+1}$ is obtained from

$$\underline{x}^{m+1} = \underline{x}^m + \Delta\underline{x}^m.$$

The algorithm has fast convergence (quadratic close to the solution). The reader is referred to any good mathematical book for more detail information, since this is a well-known procedure.

C Recurrence equation between states

Generally, a power electronic converter has several modes of operation. The state-space description in Eq.(1) has been derived for each mode (usually a physical structure consisting of two energy storage elements) in the classical form

$$\frac{d\underline{x}}{dt} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}. \tag{156}$$

The solution of Eq.(156) gives the recurrence equation between states at different times, which is found to be [6]

$$\underline{x}(t_1) = \exp\{\underline{A}\cdot(t_1-t_0)\}\underline{x}(t_0) + \int_{t_0}^{t_1}\exp\{\underline{A}\cdot(t_1-\tau)\}\underline{B}\,\underline{u}\,d\tau \tag{157}$$

with

$$\exp\{\underline{A}\cdot(t_1-t_0)\} = \underline{I} + \underline{A}\cdot(t_1-t_0) + \frac{1}{2!}\underline{A}^2\cdot(t_1-t_0)^2 + \frac{1}{3!}\underline{A}^3\cdot(t_1-t_0)^3 + \cdots$$

If $\underline{u}$ remains invariant during the time interval $[t_0, t_1]$, then Eq.(157) becomes

$$\underline{x}(t_1) = \exp\{\underline{A}\cdot(t_1-t_0)\}\underline{x}(t_0) + \underline{A}^{-1}\left[\exp\{\underline{A}\cdot(t_1-t_0)\} - \underline{I}\right]\underline{B}\,\underline{u} \tag{158}$$

under the condition that $\underline{A}^{-1}$ exists.

When $\underline{A}$ is a second-order matrix, a very simple procedure for evaluating $\exp\{\underline{A}(t_1-t_0)\}$ is obtained by making use of the Caley-Hamilton theorem [6]. In this case, consider

$$\underline{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$

whose characteristic equation

$$\lambda^2 - \lambda(a_{11}+a_{22}) + (a_{11}a_{22} - a_{12}a_{21}) = 0$$

has the roots

$$\lambda_1 = \alpha + \beta \quad \text{and} \quad \lambda_2 = \alpha - \beta.$$

After some manipulations, the solution of

$$\exp\{\underline{A}(t_1-t_0)\} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}$$

is found to be

$$
\begin{aligned}
\Omega_{11} &= \left\{\frac{a_{11}-\alpha}{\beta}\sinh[\beta(t_1-t_0)]+\cosh[\beta(t_1-t_0)]\right\}\mathrm{e}^{\alpha(t_1-t_0)},\\
\Omega_{12} &= \left\{\frac{a_{12}}{\beta}\sinh[\beta(t_1-t_0)]\right\}\mathrm{e}^{\alpha(t_1-t_0)},\\
\Omega_{22} &= \left\{\frac{a_{22}-\alpha}{\beta}\sinh[\beta(t_1-t_0)]+\cosh[\beta(t_1-t_0)]\right\}\mathrm{e}^{\alpha(t_1-t_0)},\\
\Omega_{21} &= \left\{\frac{a_{21}}{\beta}\sinh[\beta(t_1-t_0)]\right\}\mathrm{e}^{\alpha(t_1-t_0)}.
\end{aligned}
$$

A considerable amount of labour is involved when dealing with matrices of order higher than two. In such cases, sound algorithms exist for the purpose of numerically computing matrices exponential [7].

D M-file large-signal model DRC

```
%******************************************
% Large-signal discrete model of a up/down converter
% under duty-ratio control
%
%*****************************************
%
%            Ts
%       .---/---.---<---.---.    +
%       .       .       .   .
%       Us      L       C   R    Uo
%       .       .       .   .
%       .-------.-------.---.    -
%
%******************************************

%******************************************
% Basic parameters
%--------------------------------
% Passive components
L=250e-6; C=220e-6; R=2.0;
%
% Nominal determining variables
Us=12; Uo=-9; Ts=20e-6;
D=-Uo/(-Uo+Us);
%
Dp=1.0-D; qsi=(Ts/2)/(L/R);
%*****************************************

%******************************************
% System matrices
%-------------------------------
A1 = [ 0  0 ;   0  -1/(R*C)]; B1 = [ 1/L; 0];
A2 = [ 0 1/L; -1/C -1/(R*C)];
%******************************************

%******************************************
% Periodic operating point
%-------------------------------
T1 = D*Ts; T2 = Ts;
Psi1 = expm(A1*T1);
Psi2 = expm(A2*(T2-T1));
%
x0 = inv(eye(2)-Psi2*Psi1)*Psi2*B1*Us*D*Ts;
%******************************************

%******************************************
% Simulating the converter
%-------------------------------
```

```
% simulation range:       (1, 2, ..., SmRng)*Ts
SmRng = 250;              % simulating 250*20us=5ms
%
% step change from operating point
Usk=8; Dk=D;
%
% new system matrices
T1k = Dk*Ts; T2k = Ts;
Psi1k = expm(A1*T1k);
Psi2k = expm(A2*(T2k-T1k));
%
%--------------------------------
% Simulation loop
%
clear td yd1 yd2;
I=0; xk=x0;                     % initial conditions
%
for I=1:SmRng                   % simulation loop
%
xk1 = Psi1k*xk + B1*T1k*Usk;    % recurrence eqs.
xk2 = Psi2k*xk1;                %
%
yd1(I)=xk1(1); td(I)=I*Ts;      % store results
yd2(I)=xk2(1);                  %
xk=xk2;                         % next state
end
%******************************************

%******************************************
% Presentation
%-------------------------------
plot(td,yd1,'y--')
hold on
plot(td,yd2,'y--')
hold off
clear td yd1 yd2;
%******************************************
```

E M-file small-signal model DRC

```
%*******************************************
% Small-signal discrete model of an up/down converter
% under duty-ratio control
%
%*******************************************
%
%           Ts
%       .---/---.---<---.---.   +
%       .       .       .   .
%       Us      L       C   R   Uo
%       .       .       .   .
%       .-------.-------.---.   -
%
%*******************************************

%*******************************************
% Basic parameters
%---------------------------------
% Passive components
L=250e-6; C=220e-6; R=2.0;
%
% Nominal determining variables
Us=12; Uo=-9; Ts=20e-6;
D=-Uo/(-Uo+Us);
Dp=1.0-D; qsi=(Ts/2)/(L/R);
%
% Simulation range:     (1, 2, ..., SmRng)*Ts
SmRng = 250;            % simulating 250*20us=5ms
%
% Step change from determining variables
Usk=Us;         usk=Usk-Us;
Dk=0.5;         dk=Dk-D;
%*******************************************

%*******************************************
% Periodic operating point & derivatives
%--------------------------------
iL0 =  (1-qsi*Dp^2)*(D/Dp^2)*(Us/R);
vC0 = -(D/Dp)*Us;
%
diL0_dUs = (1-qsi*Dp^2)*(D/Dp^2)/R;
dvC0_dUs = -D/Dp;
%
diL0_dD = (-qsi+(1+D)/Dp^3)*Us/R;
dvC0_dD = -(1/(Dp^2))*Us;
%*******************************************

%*******************************************
```

```
% System matrices & co
%--------------------------------
A1 = [ 0  0 ;    0  -1/(R*C)]; B1 = [ 1/L; 0];
A2 = [ 0 1/L; -1/C -1/(R*C)];
%
T1 = D*Ts; T2 = Ts;
Psi1 = expm(A1*T1);
Psi2 = expm(A2*(T2-T1));
%
x0 = [ iL0  ; vC0 ];
x1 = Psi1*x0 + B1*T1*Us;
%
g1_T1 = A1*x1 + B1*Us;
g2_T2 = A2*x0;  g2_T1 = A2*x1;
%********************************************

%********************************************
% Jacobians
%--------------------------------
df_dx = Psi2*Psi1;
%
df_dT1 = Psi2*(-g2_T1 + g1_T1);
df_dT2 = g2_T2;
df_dT  = [ df_dT1 df_dT2 ];
%
dc_dT  = [1 0; 0 1];
%
dT_dp  = [0 Ts; 0 0];
%
dx_dp  = [ diL0_dUs diL0_dD
           dvC0_dUs dvC0_dD ];
%
df_dp = -(df_dx -eye(2))*dx_dp -df_dT*dT_dp;
%
dc_dp = - dc_dT*dT_dp;
%********************************************

%********************************************
% Small-signal model
%--------------------------------
F0 = df_dx;                        % dc_dx = 0
G0 = df_dp - df_dT*dc_dp;          % dc_dT = 1
H0 = [0 1];
K0 = [0 0];
%********************************************

%********************************************
% Simulating the converter
%--------------------------------
clear td yd1 yd2;
```

```
I=0; xk1=[0;0]; xk=[0;0];         % from steady state
%
for I=1:SmRng                     % simulation loop
%
qk= [usk; dk];                    % input to the system
xk1= F0*xk + G0*qk;               % discrete recurrence
%
yd1(I)=xk(1)+iL0; td(I)=I*Ts;     % store results
yd2(I)=xk(2)+vC0;                 %
xk=xk1;                           % next state
end
%*********************************************

%*********************************************
% Presentation
%--------------------------------
plot(td,yd2,'b')
%*********************************************
```

F M-file large-signal model CMC

```
%****************************************
% Large-signal discrete model of an up/down converter
% under current-mode control
%
%****************************************
%
%          Ts
%     .---/---.---<---.---.   +
%     .       .       .   .
%     Us      L       C   R   Uo
%     .       .       .   .
%     .-------.-------.---.   -
%
%
%     - IP     -        -
%     :  -     :  -     :  -  S  :
%     :     *  :     *  :     -  :
%     :   *  * -   *  * -   *    -
%     : *     *: *     *: *      :
%     *        *        *        :
%
%*******************************************

%*******************************************
% Passive components
L=250e-6; C=220e-6; R=2.0;
%----------------------------------
% Nominal determining variables
Us=12; Ip=9; Ts=20e-6;
S=0.3*Us/L;
% --------
% in order to show instability,
% just make: S=0.0; R=4;
%-----------------------------------
% Step change from operating point
Usk=Us;          usk=Usk-Us;
Ipk=10.5;        ipk=Ipk-Ip;
%
SmRng=60; % Simulation range: 60*20us=1.2ms
%****************************************

%****************************************
% System matrices
%
A1 = [ 0  0 ;    0  -1/(R*C)]; B1 = [ 1/L; 0];
A2 = [ 0 1/L; -1/C -1/(R*C)];
%
%------------------------------
```

```
% Auxiliary variables
%
qsi=(Ts/2)/(L/R);
Gamma=R*Ip/Us;
gam=(1 +2*L*S/Us)*qsi;
Ogam=1/(1+Gamma-gam*3/4);
%*****************************************

%*******************************************
% Periodic operating point
%
D=(3/4)*(1-Ogam); Dp=1.0-D;
%
T1 = D*Ts; T2 = Ts;
Psi1 = expm(A1*T1);
Psi2 = expm(A2*(T2-T1));
%
x0 = inv(eye(2)-Psi2*Psi1)*Psi2*B1*Us*D*Ts;
%********************************************

%********************************************
% Simulation loop
%
clear td yd1 yd2;               % reset vectors
I=0; xk=x0;                     % initial cond.
%
for I=1:SmRng                   % simulation loop
%
%-- approx duty ratio           %
T1k = (Ipk -xk(1))/(S +Usk/L);  %
if T1k<0  T1k=0; end            % boundaries
if T1k>Ts T1k=Ts; end           %
T2k = Ts;                       %
%
Psi1k = expm(A1*T1k);           % sensitivy matrices
Psi2k = expm(A2*(T2k-T1k));     %
%
xk1 = Psi1k*xk + B1*T1k*Usk;    % recurrence eqs.
xk2 = Psi2k*xk1;                %
%
yd1(I)=[0 1]*xk; td(I)=I*Ts;    % store results
yd2(I)=[0 1]*xk1;               %
xk=xk2;                         % next state
end
%***************************************

%***************************************
% Presentation
%
plot(td,yd1,'y--')
```

```
hold on
plot(td,yd2,'y--')
hold off
%****************************************
```

G M-file small-signal model CMC

```
%****************************************
% Small-signal discrete model of an up/down converter
% under current-mode control
%
%****************************************
%
%          Ts
%       .---/---.---<---.---.   +
%       .       .       .   .
%       Us      L       C   R   Uo
%       .       .       .   .
%       .-------.-------.---.   -
%
%
%       - Ip    -       -
%       :  -    :  -    :  -  S  :
%       :    *  :    *  :     -  :
%       :  *  * -  *  * -  *     -
%       : *    *: *    *: *      :
%       *       *       *        :
%
%*******************************************

%*******************************************
% Passive components
L=250e-6; C=220e-6; R=2.0;
%-----------------------------------
% Nominal determining variables
Us=12; Ip=9; Ts=20e-6;
S=0.3*Us/L;
% --------
% in order to show instability,
% just make: S=0.0; R=4;
%------------------------------------
% Step change from operating point
Usk=Us;         usk=Usk-Us;
Ipk=10.5;       ipk=Ipk-Ip;
%
SmRng=60; % Simulation range= 60*20us=1.2ms
%****************************************

%***************************************
% Auxiliary variables
%
qsi=(Ts/2)/(L/R);
Gamma=R*Ip/Us;
gam=(1 +2*L*S/Us)*qsi;
Ogam=1/(1+Gamma-gam*3/4);
```

```
%*****************************************

%*****************************************
% Cyclic operation point & derivatives
%
D=(3/4)*(1-Ogam); Dp=1.0-D;
%
iL0 =  (1-qsi*Dp^2)*(D/Dp^2)*Us/R;
vC0 = -(D/Dp)*Us;
%
% ------
dGamma_dUs = -R*Ip/Us^2;        dGamma_dIp = R/Us;
dgam_dUs   = -2*qsi*L*S/Us^2;   dgam_dIp   = 0;
%
dD_dGamma =  (3/4)*Ogam^2;
dD_dgam   = -(3/4)^2*Ogam^2;
%
dD_dUs = dD_dGamma*dGamma_dUs +dD_dgam*dgam_dUs;
dD_dIp = dD_dGamma*dGamma_dIp;
%
% ------
diL0_dD =  (-qsi +(1+D)/Dp^3)*Us/R;
dvC0_dD = -(1/Dp^2)*Us;
%
% ------
diL0_dUs = (1-qsi*Dp^2)*(D/Dp^2)/R +diL0_dD*dD_dUs;
dvC0_dUs = -D/Dp +dvC0_dD*dD_dUs;
%
diL0_dIp = diL0_dD*dD_dIp;
dvC0_dIp = dvC0_dD*dD_dIp;
%*******************************************

%*******************************************
% System matrices & co
%
A1 = [ 0  0 ;   0  -1/(R*C)]; B1 = [ 1/L; 0];
A2 = [ 0 1/L; -1/C -1/(R*C)];
%
T1 = D*Ts; T2 = Ts;
Psi1 = expm(A1*T1);
Psi2 = expm(A2*(T2-T1));
%
x0 = [ iL0  ; vC0 ];
x1 = Psi1*x0 + B1*T1*Us;
%
g1_T1 = A1*x1 + B1*Us;
g2_T2 = A2*x0;  g2_T1 = A2*x1;
%********************************************

%********************************************
```

```
% Jacobians
%-------------
df_dx = Psi2*Psi1;
%
df_dT1 = Psi2*(-g2_T1 + g1_T1);
df_dT2 = g2_T2;
df_dT  = [ df_dT1 df_dT2 ];
%
%-------------
dc_dx  = [ 1 0 ; 0 0 ];
dc_dT  = [(S+Us/L) 0 ; 0 1];
%
%------------
dT_dp  = [ Ts*dD_dUs Ts*dD_dIp ; 0 0 ];
%
dx_dp  = [ diL0_dUs diL0_dIp
           dvC0_dUs dvC0_dIp ];
%
%-------------
df_dp = -(df_dx -eye(2))*dx_dp - df_dT*dT_dp;
%
dc_dp = -dc_dx*dx_dp -dc_dT*dT_dp;
%
%*********************************************

%*********************************************
% Small-signal model
%
F0 = df_dx - df_dT*inv(dc_dT)*dc_dx;
G0 = df_dp - df_dT*inv(dc_dT)*dc_dp;
H0 = [0 1];
K0 = [0 0];
%*********************************************

%*********************************************
% simulating the convertor
%
clear td yd1 yd2;                % reset vectors
I=0; xk1=[0;0]; xk=[0;0];        % initial cond.
%
for I=1:SmRng                    % begin loop
qk= [usk; ipk];                  % input to the system
xk1= F0*xk + G0*qk;
%
yd1(I)=xk(1)+iL0; td(I)=I*Ts;    % store results
yd2(I)=xk(2)+vC0;                %
%
xk=xk1;                          % next state
end
%---------------
```

```
plot(td,yd2,'b')
%
%*******************************************
```

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