Sampling Density Compensation in MRI: Rationale and an Iterative Numerical Solution

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Introduction

MRI data which is sampled nonuniformly in k-space is often interpolated onto a Cartesian grid for fast reconstruction using the method outlined in Jackson(1), after the data is weighted to correct for the sampling density. Unlike previous methods for calculating the density compensation weights (1-4), the proposed method only requires the coordinates of the sampled data; it does not require knowledge of the sampling trajectories, easily handles overlapping sampling trajectories, and provides correct estimates in regions where the Nyquist criterion is not met.

Theory

Gridding has been well characterized by Jackson (1) , and is in part given (in k-space) by the formula

$$M_C(u,v) = [\,(M * S * W) \otimes C\,] * R, \qquad [1]$$

where $\otimes$ is the convolution function, M_C is the gridded data , (u,v) are k-space coordinates, M is the MR signal, S is the sampling function, W is the density compensating weighting function, C is the convolution function, and R defines a Cartesian grid. Defining W to be zero everywhere except for at the sampling coordinates means W = S*W, and

$$M_C(u,v) = [\,(M * W) \otimes C\,] * R, \qquad [2]$$

which is

$$m_C(x,y) = [\,(m \otimes w) * c\,] \otimes r \qquad [3]$$

in the image domain. Accurate reconstruction requires

$$(m \otimes w) * c = m * c, \qquad [4]$$

which requires that

$$\begin{aligned} w(x,y) &= 1, \quad x = y = 0 \\ &= 0, \quad 0 < \sqrt{x^2 + y^2} < \zeta \end{aligned}, \qquad [5]$$

where ζ is the FOV of m in the image domain. This is shown to give

$$W * (W \otimes C) = W, \qquad [6]$$

where the region of support of C in the image domain has a diameter ζ. Dividing both sides of Eq. [6] by W gives a stable iteration for W as

$$W_{i+1} = \frac{W_i}{W_i \otimes C} \qquad [7]$$

with the first iteration obtained by

$$W_1 = \frac{S}{S \otimes C}. \qquad [8]$$

The weighting function in Eq. [8] is that given by Jackson. As Eq. [7] converges,

$$(W \otimes C) \rightarrow S, \qquad [9]$$

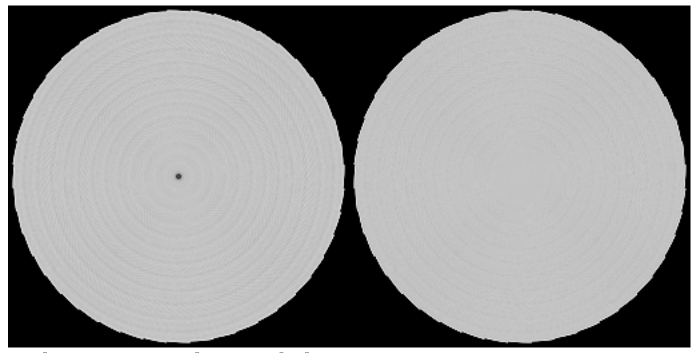
that is, **the sampling density is forced to be unity at the sampling coordinates.**

Application to Spiral MRI

The density of sampling coordinates for a spiral acquisition (44 interleafs, 6π rotation per interleaf) was calculated using the method of Jackson (Eq. [8]) and the proposed method. The function

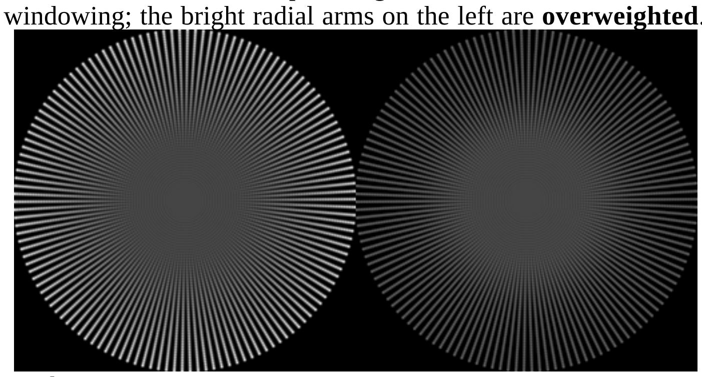
$$E = (W \otimes C) \qquad [10]$$

was evaluated at the sampling coordinates. A perfect weighting function would cause the value of E to be unity at every sampling location. For the method of Jackson, the range was 0.649 to 1.176; for the proposed method (100 iterations), the range was 0.989 to 1.007. The function E is shown below, illustrating that the error in the weighting function of Jackson (on the left) occurs near the k-space origin, where there are rapid changes in the sampling density; this is removed with the proposed method (on the right).



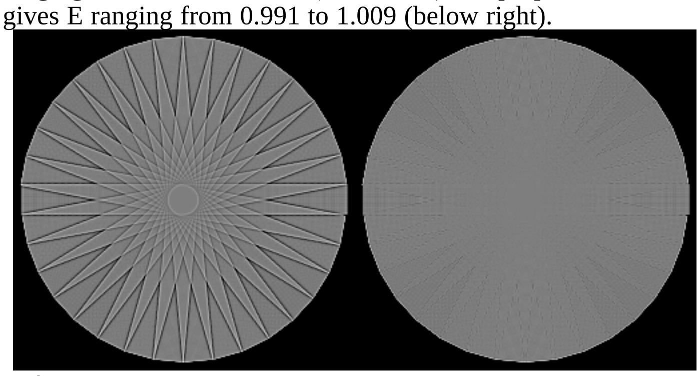
Application to Undersampled Projection Reconstruction

Sampling coordinates for a center-out projection-reconstruction scan were calculated for an effective 128 diameter matrix, using only 134 projections (3x azimuthal undersampling at the periphery of k-space). For a Jacobian-based density function, the value of E ranged from 0.998 to 2.273 (below, left); for the proposed method, E ranged from 1.000 to 1.001 (below, right). The dark center at each k-space origin show E = 1.0 due to windowing; the bright radial arms on the left are **overweighted**.



Application to PROPELLER MRI

PROPELLER MRI collects data in strips of parallel lines which rotate about the center of k-space (see other abstracts at this meeting by Pipe). Jacobian-based methods will not work for these overlapping trajectories. The method of Jackson gave E ranging from 0.860 to 1.416 (below, left); the proposed method gives E ranging from 0.991 to 1.009 (below right).



References

1. J. I. Jackson, C. H. Meyer, D. G. Nishimura, A. Macovski, Selection of a Convolution Function for Fourier Inversion Using Gridding. *IEEE Trans Med Im*, **10**, 473-478 (1991).
2. C. H. Meyer, B. S. Hu, D. G. Nishimura, A. Macovski, Fast Spiral Coronary Artery Imaging. *Magn Reson Med*, **28**, 202-213 (1992).
3. R. D. Hoge, R. K. S. Kwan, B. G. Pike, Density Compensation Functions for Spiral MRI. *Magn Reson Med*, **38**, 117-128 (1997).
4. N. G. Papadakis, T. A. Carpenter, L. D. Hall, An Algorithm for Numerical Calculation of the K-space Data-Weighting for Polarly Sampled Trajectories: Application to Spiral Imaging. *Mag Res Im*, **15**, 785-794 (1997).