

Sampling Theory in Fourier and Signal Analysis

Foundations

J. R. HIGGINS

*Division of Mathematics and Statistics
Anglia Polytechnic University, Cambridge*

CLARENDON PRESS • OXFORD

1996

CONTENTS

1	An introduction to sampling theory	1
1.1	General introduction	2
1.2	The seventeenth to the mid-twentieth century	7
1.3	Some further introductory remarks	11
2	Background in Fourier analysis	15
2.1	The Fourier series	16
2.2	The Fourier transform	17
2.3	Poisson's summation formula	19
2.4	Tempered distributions — some basic facts	20
3	Hilbert spaces, bases and frames	23
3.1	Bases for Banach and Hilbert spaces	23
3.2	Riesz bases and unconditional bases	25
3.3	Frames	27
3.4	Reproducing kernel Hilbert spaces	29
3.5	Direct sums of Hilbert spaces	30
3.6	Sampling and reproducing kernels	31
4	Finite sampling	33
4.1	A general setting for finite sampling	33
4.2	Sampling on the sphere	37
5	From finite to infinite sampling series	41
5.1	The change to infinite sampling series	42
5.2	The theorem of Hinsin and Klösters	45
6	Sampling for Bernstein and Paley–Wiener spaces	48
6.1	Bernstein spaces	48
6.2	Convolution and the cardinal series	50
6.3	Paley–Wiener classes	52
6.4	The cardinal series for Paley–Wiener classes	53
6.5	The space $\mathfrak{R}_\epsilon H^1$	54
6.6	A convergence principle for Paley–Wiener spaces	55
6.7	Ordinary Paley–Wiener space and its reproducing kernel	57
6.8	Sampling and entire functions of polynomial growth	60
7	More about Paley–Wiener spaces	67
7.1	Paley–Wiener theorems — a review	67
7.2	Bases for Paley–Wiener spaces	69
7.3	Transformations on the Paley–Wiener space	70

7.4	Oscillatory properties of Paley–Wiener functions	72
8	Kramer’s Lemma	78
8.1	Kramer’s Lemma	78
8.2	The Walsh sampling theorem	85
9	Contour integral methods	88
9.1	The Paley–Wiener theorem	88
9.2	Some formulae of analysis and their equivalence	90
9.3	A general sampling theorem	96
10	Irregular sampling	102
10.1	Stable sampling, interpolation and uniqueness	102
10.2	Irregular sampling at minimal rate	105
10.3	Frames and over-sampling	108
11	Errors and aliasing	113
11.1	Errors	113
11.2	The time jitter error	114
11.3	The aliasing error	118
12	Single channel and multi-channel sampling	124
12.1	Single channel sampling	125
12.2	Two channels	130
13	Multi-band sampling	137
13.1	Regular sampling	138
13.2	Optimal regular sampling	139
13.3	An algorithm for the optimal regular sampling rate	141
13.4	Selectively tiled band regions	142
13.5	Harmonic signals	146
13.6	Band-pass sampling	147
14	Multi-dimensional sampling	153
14.1	Remarks on multi-dimensional Fourier analysis	154
14.2	The rectangular case	155
14.3	Regular multi-dimensional sampling	155
15	Sampling and eigenvalue problems	164
15.1	Preliminary facts	164
15.2	Direct and inverse Sturm–Liouville problems	166
15.3	Further types of eigenvalue problem — some examples	172
16	Campbell’s generalized sampling theorem	177
16.1	L.L. Campbell’s generalization of the sampling theorem	177
16.2	Band-limited functions	179
16.3	Non-band-limited functions — an example	181

17	Modelling, uncertainty and stable sampling	184
17.1	Remarks on signal modelling	184
17.2	Energy concentration	188
17.3	The uncertainty principle of signal theory	193
17.4	Prolate spheroidal wave functions	196
17.5	The Nyquist–Landau minimal sampling rate	196
Appendix A	Fourier transforms	200
Appendix B	Hilbert transforms	206
References		207
Index		217