Sasakian manifolds with vanishing C-Bochner curvature tensor

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1 Introduction

As a complex analogue to the Weyl conformal curvature tensor, Bochner and Yano [1], [15] (See also, Tachibana [13]) introduced a Bochner curvature tensor in a Kählerian manifold. Many subjects for vanishing Bochner curvature tensors with constant scalar curvature have been studied by Ki and Kim [6], Kubo [8], Matsumoto [9], Matsumoto and Tanno [11], Yano and Ishihara [16] and so on. One of those, done by Ki and Kim, asserts that the following theorem:

Theorem A ([6]) *Let M be a Kählerian manifold with vanishing Bochner curvature tensor. Then the scalar curvature is constant if and only if* $\operatorname{Tr}\operatorname{Ric}^{(m)}$ *is constant for a positive integer* $m\,(\geq 2)$.

In a Sasakian manifold, a C-Bochner curvature tensor is constructed from the Bochner curvature tensor in a Kählerian manifold by the fibering of Boothby-Wang. Recently, the Sasakian manifold with vanishing C-Bochner curvature tensor and the constant scalar curvature is studied, and in [12], the following theorem was proved

Theorem B *Let $M^n\,(n \geq 5)$ be a Sasakian manifold with constant scalar curvature whose C-Bochner curvature tensor vanishes. If the Ricci tensor is positive semi-definite, then M is a space of constant ϕ-holomorphic sectional curvature.*

Also, when M is compact, the following theorems were proved:

Theorem C ([4]) *Let $M^n\,(n \geq 5)$ be a compact Sasakian manifold with vanishing C-Bochner curvature tensor. If the length of the Ricci tensor is constant and the length of the η-Einstein tensor is less than $\frac{\sqrt{2}(R-n+1)}{\sqrt{(n-1)(n-3)}}$, then M is a space of constant ϕ-holomorphic sectional curvature.*

Theorem D ([10]) *Let $M^n\,(n \geq 5)$ be a compact Sasakian manifold with vanishing C-Bochner curvature tensor and the constant scalar curvature. If the smallest Ricci curvature greater than -2, then M is a space of constant ϕ-holomorphic sectional curvature.*

We shall prove Theorem A as a Sasakian analogue in §3. Moreover in §4 we shall discuss when the smallest Ricci curvature is greater than or equal to -2 in a Sasakian manifold with vanishing C-Bochner curvature tensor and $\operatorname{Tr}\operatorname{Ric}^{(m)}$ is constant for a positive integer m.

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2 Preliminaries

Let M be an n-dimensional Riemannian manifold. Throughout this paper, we assume that manifolds are connected and of class C^∞. Denoting respectively by g_{ji}, $R_{kji}{}^h$, $R_{ji} = R_{rji}{}^r$ and R the metric tensor, the curvature tensor, the Ricci tensor and the scalar curvature of M in terms of local coordinates $\{x^h\}$, where Latin indices run over the range $\{1, 2, \ldots, n\}$.

An $n\,(= 2l+1)$-dimensional Riemannian manifold is called a Sasakian manifold if there exists a unit Killing vector field ξ^h satisfying

$$(2.1)\quad \begin{cases} \eta_i = g_{ir}\xi^r, \quad \phi_{ji} = \nabla_j \eta_i, \quad \phi_{ji} + \phi_{ij} = 0, \quad \phi_r{}^h \xi^r = 0, \quad \phi_j{}^r \eta_r = 0, \\ \phi_i{}^r \phi_r{}^h = -\delta_i{}^h + \eta_i \xi^h, \qquad \nabla_k \phi_{ji} = -g_{kj}\eta_i + g_{ki}\eta_j, \end{cases}$$

where ∇ denotes the operator of the Riemannian covariant derivative.

It is well known that in a Sasakian manifold the following equations hold:

(2.2) $R_{jr}\xi^r = (n-1)\eta_j$,

(2.3) $H_{ji} + H_{ij} = 0$,

(2.4) $R_{ji} = R_{rs}\phi_j{}^r \phi_i{}^s + (n-1)\eta_j\eta_i$,

(2.5) $\nabla_k R_{ji} - \nabla_j R_{ki} = (\nabla_t R_{kr})\phi_j{}^r \phi_i{}^t - \eta_j\{H_{ki} - (n-1)\phi_{ki}\} - 2\eta_i\{H_{kj} - (n-1)\phi_{kj}\}$,

(2.6) $\nabla_k R_{ji} - (\nabla_k R_{rs})\phi_j{}^r \phi_i{}^s = -\eta_i\{H_{kj} - (n-1)\phi_{kj}\} - \eta_j\{H_{ki} - (n-1)\phi_{ki}\}$,

(2.7) $\xi^r \nabla_r R_{kji}{}^h = 0$,

where we put $H_{ji} = \phi_j{}^r R_{ri}$.

We denote a tensor field $\mathrm{Ric}^{(m)}$ with components $R_{ji}^{(m)}$ and a function $R_{(m)}$ as follows:

$$R_{ji}^{(m)} = R_{ji_1} R_{i_2}^{i_1} \cdots R_i^{i_{m-1}}, \qquad R_{(m)} = \mathrm{Tr}\,\mathrm{Ric}^{(m)} = g^{ji} R_{ji}^{(m)}.$$

Then, from (2.2) and (2.3), we get

$$(2.8)\qquad R_{jr}^{(m)}\xi^r = (n-1)^m \eta_j,$$

$$(2.9)\qquad R_{jr}^{(m)}\phi_i{}^r + R_{ir}^{(m)}\phi_j{}^r = 0.$$

Also, we define the η-Eintein tensor T_{ji} by

$$(2.10)\qquad T_{ji} = R_{ji} - \left(\frac{R}{n-1} - 1\right) g_{ji} + \left(\frac{R}{n-1} - n\right)\eta_j\eta_i.$$

If the η-Einstein tensor vanishes, then M is called an η-Einstein manifold. From (2.2) and (2.3), we have

$$(2.11)\qquad \mathrm{Tr}\, T = 0,$$

$$(2.12)\qquad T_{jr}\xi^r = 0,$$

$$(2.13)\qquad T_{jr}\phi_i{}^r + T_{ir}\phi_j{}^r = 0.$$

A Sasakian manifold M is called a space of constant ϕ-holomorphic sectional curvature c if the curvature tensor of M has the form:

$$\begin{aligned} R_{kji}{}^h &= \frac{c+3}{4}(g_{ji}\delta_k{}^h - g_{ki}\delta_j{}^h) \\ &+\frac{c-1}{4}(g_{ki}\eta_j\xi^h - g_{ji}\eta_k\xi^h + \eta_k\eta_i\delta_j{}^h - \eta_j\eta_i\delta_k{}^h - \phi_{ki}\phi_j{}^h + \phi_{ji}\phi_k{}^h - 2\phi_{kj}\phi_i{}^h). \end{aligned}$$

Matsumoto and Chūman ([10]) introduced the C-Bochner curvature tensor $B_{kji}{}^h$ defined by

$$\begin{aligned} (2.14)\quad B_{kji}{}^h &= R_{kji}{}^h + \frac{1}{n+3}(R_{ki}\delta_j{}^h - R_{ji}\delta_k{}^h + g_{ki}R_j{}^h - g_{ji}R_k{}^h + H_{ki}\phi_j{}^h \\ &\quad -H_{ji}\phi_k{}^h + \phi_{ki}H_j{}^h - \phi_{ji}H_k{}^h + 2H_{kj}\phi_i{}^h + 2\phi_{kj}H_i{}^h \\ &\quad -R_{ki}\eta_j\xi^h + R_{ji}\eta_k\xi^h - \eta_k\eta_iR_j{}^h + \eta_j\eta_iR_k{}^h) \\ &-\frac{k+n-1}{n+3}(\phi_{ki}\phi_j{}^h - \phi_{ji}\phi_k{}^h + 2\phi_{kj}\phi_i{}^h) \\ &-\frac{k-4}{n+3}(g_{ki}\delta_j{}^h - g_{ji}\delta_k{}^h) \\ &+\frac{k}{n+3}(g_{ki}\eta_j\xi^h - g_{ji}\eta_k\xi^h + \eta_k\eta_i\delta_j{}^h - \eta_j\eta_i\delta_k{}^h), \end{aligned}$$

where $k = \frac{R+n-1}{n+1}$. It is well-known that if a Sasakian manifold with vanishing C-Bochner curvature tensor is an η-Einstein manifold, then it is a space of constant ϕ-holomorphic sectional curvature.

3 A Sasakian manifold with vanishing C-Bochner curvature tensor.

Let M^n $(n \geq 5)$ be a Sasakain manifold with vanishing C-Bochner curvature tensor. By a straitforward computation, we can prove

$$\begin{aligned} (3.1)\qquad \frac{n+3}{n-1}\nabla_rB_{kji}{}^r &= \nabla_kR_{ji} - \nabla_jR_{ki} - \eta_k\{H_{ji} - (n-1)\phi_{ji}\} \\ &\quad +\eta_j\{H_{ki} - (n-1)\phi_{ki}\} + 2\eta_i\{H_{kj} - (n-1)\phi_{kj}\} \\ &\quad +\frac{1}{2(n+1)}\{(g_{ki} - \eta_k\eta_i)\delta_j{}^r - (g_{ji} - \eta_j\eta_i)\delta_k{}^r \\ &\qquad +\phi_{ki}\phi_j{}^r - \phi_{ji}\phi_k{}^r + 2\phi_{kj}\phi_i{}^r\}R_r, \end{aligned}$$

where we put $R_j = \nabla_jR$.

By virtue of (2.1), (2.2), (2.5) - (2.7) and (3.1), we obtain

$$\begin{aligned} (3.2)\qquad \nabla_kR_{ji} &= \{R_{kr} - (n-1)g_{kr}\}(\phi_j{}^r\eta_i + \phi_i{}^r\eta_j) \\ &\quad +\frac{1}{2(n+1)}\{2R_k(g_{ji} - \eta_j\eta_i) + R_j(g_{ki} - \eta_k\eta_i) \\ &\qquad +R_i(g_{kj} - \eta_k\eta_j) - \phi_{kj}\phi_i{}^rR_r - \phi_{ki}\phi_j{}^rR_r\} \end{aligned}$$

and consequently from (2.7), we find

$$(3.3)\qquad (n+1)(\nabla_kR_{ji})R^jR^i = 2\lambda^2R_k,$$

where we put $\lambda^2 = R_r R^r$.

The following lemma is needed for the later use.

LEMMA 3.1 *Let M^n $(n \geq 5)$ be a Sasakian manifold with vanishing C-Bochner curvature tensor. Then $\overset{(m)}{R_{jr}} R^r = 0$ holds for a positive integer m if and only if the scalar curvature R is constant.*

Proof. If $\overset{(m)}{R_{jr}} R^r = 0$ holds, then we get $\overset{(2m-2)}{R_{jr}} R^r = 0$ which implies that $| \overset{(m-1)}{R_{jr}} R^r |^2 = 0$. Accordingly, we obtain $\overset{(m-1)}{R_{jr}} R^r = 0$. By the inductive method, we get $R_{jr} R^r = 0$. Operating ∇_k to this, we find $(\nabla_k R_{jr}) R^j R^r = 0$. By means of (3.3), we see that the scalar curvature R is constant. The converse is trivial.

For the sake of brevity, we shall define a function $\alpha(m)$ as follows:

$$\alpha(m) = \overset{(m)}{R_{ji}} R^j R^i.$$

Then, it is clear from (3.2) that

$$(3.4) \quad 2(n+1)(\nabla_k R_{ji}) R^j (R^{ir(m)} R_r) = \lambda^2 \overset{(m)}{R_{kr}} R^r + 3\alpha(m) R_k,$$

$$(3.5) \quad 2(n+1)(\nabla_k R_{ji})(R^{jr(\ell)} R_r)(R^{is(m)} R_s) = \alpha(\ell) \overset{(m)}{R_{kr}} R^r + \alpha(m) \overset{(\ell)}{R_{kr}} R^r + 2\alpha(\ell + m) R_k,$$

where we have used (2.7), (2.8) and (2.9).

Operating $R^{ji(m)}$ to (3.2) and owing to (2.1), (2.7), (2.8) and (2.9), we find

$$(3.6) \qquad (n+1)\nabla_k R_{(m+1)} = (m+1)[2\overset{(m)}{R_{kr}} R^r + \{R_{(m)} - (n-1)^m\} R_k].$$

Therefore, if the scalar curvature R is constant, then $R_{(m)}$ is constant for any integer $m\,(\geq 2)$.

Now, we shall prove that the scalar curvature R is constant if $R_{(m)}$ is constant for any fixed integer $m\,(\geq 2)$.

At first, suppose that $R_{(2\ell+3)}$ $(\ell = 0, 1, 2, \dots)$ is constant. Then, from (3.6), we can get

$$2\overset{(2\ell+2)}{R_{kr}} R^r + \{R_{(2\ell+2)} - (n-1)^{2\ell+2}\} R_k = 0,$$

which yields that $2\alpha(2\ell + 2) + \lambda^2 \{R_{(2\ell+2)} - (n-1)^{2\ell+2}\} = 0$, that is,

$$2 \,| \overset{(\ell+1)}{R_{jr}} R^r |^2 + \lambda^2 \,| \overset{(\ell+1)}{R_{ji}} - (n-1)^{\ell+1} \eta_j \eta_i |^2 = 0.$$

Thus, from Lemma 3.1, the scalar curvature R is constant.

In the next place, we shall consider when $R_{(2\ell+2)}$ $(\ell = 0, 1, 2, \dots)$ is constant. From (3.6), we have

$$(3.7) \qquad 2\overset{(2\ell+1)}{R_{jr}} R^r + \{R_{(2\ell+1)} - (n-1)^{2\ell+1}\} R_j = 0.$$

Operating ∇_k to this and owing to (3.7), we get

$$(3.8) \qquad 2(\nabla_k \overset{(2\ell+1)}{R_{jr}}) R^j R^r + \lambda^2 \nabla_k R_{(2\ell+1)} = 0.$$

From (3.3) and (3.8), we find the scalar curvature R is constant if $\ell = 0$. Because of (3.4), (3.5) and (3.6), equation (3.8) is rewritten as follows:

$$(3.9) \qquad 4(\ell+1)\lambda^2 \overset{(2\ell)}{R_{kr}} R^r + 2\sum_{i=1}^{2\ell-1} \alpha(i) \overset{(2\ell-i)}{R_{kr}} R^r$$

$$+4(\ell+1)\alpha(2\ell) R_k + (2\ell+1)\lambda^2 \,| \overset{(\ell)}{R_{ji}} - (n-1)^\ell \eta_j \eta_i |^2 \, R_k = 0.$$

By virtue of (3.9) and Lemma 3.1, it is clear that the scalar curvature R is constant if $\ell = 1$.

On the other hand, we have

$$\begin{aligned} (3.10) \qquad & \lambda^6\alpha(2\ell) + 2\lambda^4\alpha(s)\alpha(2\ell - s) + \lambda^4\alpha(2s)\alpha(2\ell - 2s) \\ & = \lambda^2 \,|\, \lambda^2 R_{jr}^{(\ell)} R^r + \alpha(s) R_{jr}^{(\ell-s)} R^r \,|^2 + \alpha(2\ell - 2s) \,|\, \lambda^2 R_{jr}^{(s)} R^r - \alpha(s) R_j \,|^2 . \end{aligned}$$

Because of (3.9) and (3.10), it is to see that the following equations hold:
if $\ell = 2, 6, 10, \ldots$,

$$\begin{aligned} & (7\ell + 8)\lambda^6\alpha(2\ell) + (2\ell + 1)\lambda^8 \,|\, R_{ji}^{(\ell)} - (n-1)^\ell \eta_j \eta_i \,|^2 \\ & + 4\lambda^4 \sum_{i=1}^{(\ell-2)/4} \alpha(4i)\alpha(2\ell - 4i) \\ & + 2\lambda^2 \sum_{i=1}^{\ell/2} |\, \lambda^2 R_{js}^{(\ell)} R^s + \alpha(2i-1) R_{js}^{(\ell-2i+1)} R^s \,|^2 \\ & + 2 \sum_{i=1}^{\ell/2} \alpha(2\ell - 4i + 2) \,|\, \lambda^2 R_{js}^{(2i-1)} R^s - \alpha(2i-1) R_j \,|^2 = 0, \end{aligned}$$

if $\ell = 4, 8, 12, \ldots$,

$$\begin{aligned} & (7\ell + 8)\lambda^6\alpha(2\ell) + (2\ell + 1)\lambda^8 \,|\, R_{ji}^{(\ell)} - (n-1)^\ell \eta_j \eta_i \,|^2 \\ & + 4\lambda^4 \sum_{i=1}^{(\ell-4)/4} \alpha(4i)\alpha(2\ell - 4i) + 2\lambda^4\alpha(\ell)^2 \\ & + 2\lambda^2 \sum_{i=1}^{\ell/2} |\, \lambda^2 R_{js}^{(\ell)} R^s + \alpha(2i-1) R_{js}^{(\ell-2i+1)} R^s \,|^2 \\ & + 2 \sum_{i=1}^{\ell/2} \alpha(2\ell - 4i + 2) \,|\, \lambda^2 R_{js}^{(2i-1)} R^s - \alpha(2i-1) R_j \,|^2 = 0 \end{aligned}$$

and if $\ell = 3, 5, 7, \ldots$,

$$\begin{aligned} & (7\ell + 9)\lambda^6\alpha(2\ell) + (2\ell + 1)\lambda^8 \,|\, R_{ji}^{(\ell)} - (n-1)^\ell \eta_j \eta_i \,|^2 \\ & + 2\lambda^4 \sum_{i=1}^{(\ell-1)/2} \alpha(2i)\alpha(2\ell - 2i) + 2\lambda^4\alpha(\ell)^2 \\ & + 2\lambda^2 \sum_{i=1}^{(\ell-1)/2} |\, \lambda^2 R_{js}^{(\ell)} R^s + \alpha(2i-1) R_{js}^{(\ell-2i+1)} R^s \,|^2 \\ & + 2 \sum_{i=1}^{(\ell-1)/2} \alpha(2\ell - 4i + 2) \,|\, \lambda^2 R_{js}^{(2i-1)} R^s - \alpha(2i-1) R_j \,|^2 = 0. \end{aligned}$$

Thus we find from Lemma 3.1 that the scalar curvature R is constant if $R_{(2\ell+2)}$ ($\ell = 2, 3, 4, \ldots$) is constant. Hence, we have

THEOREM 3.2 *Let M^n $(n \geq 5)$ be a Sasakian manifold with vanishing C-Bochner curvature tensor. Then the scalar curvature R is constant if and only if* $\mathrm{Tr}\,\mathrm{Ric}^{(m)}$ *is constant for an integer m (≥ 2).*

REMARK. In the proof of Theorem 3.2, we use only equation (3.1). Thus Theorem 3.2 is valid for the parallel C-Bochner curvature tensor.

Also, we have from Theorems B and 3.2

THEOREM 3.3 *Let M^n $(n \geq 5)$ be a Sasakian manifold whose C-Bochner curvature tensor vanishes. If the Ricci tensor is positive semi-definite and* $\mathrm{Tr}\,\mathrm{Ric}^{(m)}$ *is constant for a positive integer m, then M is a space of constant ϕ-holomorphic sectional curvature.*

Furthermore, it is easy to see from the proof of Theorem C and Theorem 3.2 that the following theorem hold:

THEOREM 3.4 *Let M^n $(n \geq 5)$ be a Sasakian manifold with vanishing C-Bochner curvature tensor. If* $\mathrm{Tr}\,\mathrm{Ric}^{(m)}$ *is constant for a positive integer m and the length of the η-Einstein tensor is less than $\frac{\sqrt{2}(R-n+1)}{\sqrt{(n-1)(n-3)}}$, then M is a space of constant ϕ-holomorphic sectional curvature.*

4 The smallest Ricci curvature.

Let M be an $n\,(\geq 5)$-dimensional Sasakian manifold with vanishing C-Bochner curvature tensor. Suppose that $R_{(m)}$ is constant for any positive integer m. By Theorem 3.2, equation (3.2) is reduced to

$$\nabla_k R_{ji} = \{R_{kr} - (n-1)g_{kr}\}(\phi_j{}^r \eta_i + \phi_i{}^r \eta_j), \tag{4.1}$$

which implies $\nabla_k R_{ji} + \nabla_j R_{ik} + \nabla_i R_{kj} = 0$, namely, the Ricci tensor is cyclic parallel. Therefore, using the Ricci formula, we find

$$\nabla^k \nabla_k R_{ji} = 2(R_{rjis}R^{rs} - R_{ji}^{(2)}).$$

Applying ∇^k to (4.1) and owing to (2.1) and (2.2), we get

$$\nabla^k \nabla_k R_{ji} = -2[R_{ji} - (n-1)g_{ji} - \{R - n(n-1)\}\eta_j \eta_i].$$

On the other hand, by virtue of (2.1) - (2.4) and (2.14), it is clear that the following equation holds:

$$\begin{aligned}(n+3)R_{rjis}R^{rs} &= 4R_{ji}^{(2)} - (4n - R + 2k)R_{ji} + \{R_{(2)} - (k-4)R + (n-1)k\}g_{ji} \\ &\quad - \{R_{(2)} + (n-1)^2 - (n-1)k - kR\}\eta_j \eta_i.\end{aligned}$$

From the last three equations, we have

$$R_{ji}^{(2)} = \beta R_{ji} + \gamma g_{ji} + \{(n-1)^2 - (n-1)\beta - \gamma\}\eta_j \eta_i, \tag{4.2}$$

where constants β and γ are given by

$$(n+1)\beta = R - 3n - 5, \tag{4.3}$$

$$(n-1)\gamma = R_{(2)} - \frac{1}{n+1}R^2 + 4R - \frac{n-1}{n+1}(n^2 + 3n + 4). \tag{4.4}$$

Thus, equation (4.2) tells us that M has at most three constant Ricci curvatures $n-1$, x_1 and x_2, where we have put

$$x_1 = \frac{1}{2}(\beta - \sqrt{D}), \qquad x_2 = \frac{1}{2}(\beta + \sqrt{D}), \qquad D = \beta^2 + 4\gamma\,(\geq 0), \tag{4.5}$$

moreover, the multiplicities of x_1 and x_2 denote by s and $n-1-s$, respectively. Therefore we have (cf. [7])

Lemma 4.1 *Let M^n $(n \geq 5)$ be a Sasakian manifold with vanishing C-Bochner curvature tensor such that* $\operatorname{Tr}\operatorname{Ric}^{(m)}$ *is constant for a positive integer m. Then M has at most three constant Ricci curvatures.*

Now, we shall prove the following theorem.

Theorem 4.2 *Let M^n $(n \geq 5)$ be a Sasakian manifold with vanishing C-Bochner curvature tensor such that* $\operatorname{Tr}\operatorname{Ric}^{(m)}$ *is constant for a positive integer m. If the smallest Ricci curvature is greater than or equal to -2, then M is a space of constant ϕ-holomorphic sectional curvature -3.*

Proof. By means of (4.3), (4.5) and Lemma 4.1, we find

$$R+n-1=\frac{n+1}{n+3}(n-1-2s)\sqrt{D}. \tag{4.6}$$

Because of (4.3), (4.4) and (4.6), we have

$$\frac{n-1}{4}\left\{1-\left(\frac{n-1-2s}{n+3}\right)^2\right\}D=R_{(2)}-\frac{1}{n+1}\{R^2-2(n+3)R+(n-1)^2(n+2)\},$$

which yields that

$$(n+1)R_{(2)}\geq R^2-2(n+3)R+(n-1)^2(n+2). \tag{4.7}$$

Let x_1 be the smallest Ricci curvature. Then, by virtue of (4.5), we obtain $\gamma \leq 2\beta+4$ which means from (4.4) that

$$(n+1)R_{(2)}\leq R^2-2(n+3)R+(n-1)^2(n+2).$$

Combining this with (4.7), we get D vanishes identically, which implies that equation (4.6) gives $R=-n+1$. We find $\mid R_{ji}+2g_{ji}-(n+1)\eta_j\eta_i\mid^2=0$ which yields that M is an η-Einstein manifold. Thus, it is easy to see from (2.14) that M is of constant ϕ-holomorphic sectional curvature -3.

Remark. In [10], this theorem was proved under the condition that M is compact.

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