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Satellite attitude control system simulator

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Abstract. Future space missions will involve satellites with great autonomy and stringent pointing precision, requiring of the Attitude Control Systems (ACS) with better performance than before, which is function of the control algorithms implemented on board computers. The difficulties for developing experimental ACS test is to obtain zero gravity and torque free conditions similar to the SCA operate in space. However, prototypes for control algorithms experimental verification are fundamental for space mission success. This paper presents the parameters estimation such as inertia matrix and position of mass centre of a Satellite Attitude Control System Simulator (SACSS), using algorithms based on least square regression and least square recursive methods. Simulations have shown that both methods have estimated the system parameters with small error. However, the least square recursive methods have performance more adequate for the SACSS objectives. The SACSS platform model will be used to do experimental verification of fundamental aspects of the satellite attitude dynamics and design of different attitude control algorithm.

Keywords: Satellite simulator, control system, parameters estimation

1. Introduction

There are several methodologies to study the satellite ACS performance; depending on the investigation objectives computer simulation is not the most appropriate one. The use of experimental platforms have the important advantage of allowing the satellite dynamics representation by a prototype in laboratory and once validated this platform, it is possible to accomplish simulations and experiments that allow to evaluate satellites ACS with simple rigid dynamics as well as complex configurations involving flexible components. The preference for using experimental test is associated with the possibility of introducing more realism than the simulation, however, it has the difficulty of reproducing zero gravity and torque free space condition, extremely relevant for satellites with complex dynamics and ACS with great degree of precision. Experimental platforms with rotation around three axes are more complicated assemblies than rotation around one axis, but it is more representative. Examples of the use of experimental platforms for investigating different aspects of the satellite dynamics and control system can be found in [1,2]. A classic case of a phenomenon that was not investigated experimentally before launch was the dissipation energy effect that altered the satellite Explorer I rotation [3]. A pioneering experiment to study energy dissipation effect has been done by [4]. An important aspect that is possible to investigate through experimental platforms is the satellite inertia parameters variation and identification. An initial way to estimate inertia parameters is using CAD software. The results obtained by CAD can be compared to the results obtained through estimation techniques. The least square method with batch processing was used with success by the satellite simulator developed by [5–7]. For platforms where the dynamics has variation in time, like, mass center and inertia parameters variation, the application of parameters estimation methods in real time becomes more appropriate. In ref. [8] an algorithm was developed based on the least square method to identify a space vehicle mass parameters in rotation during attitude maneuvers. Methods with

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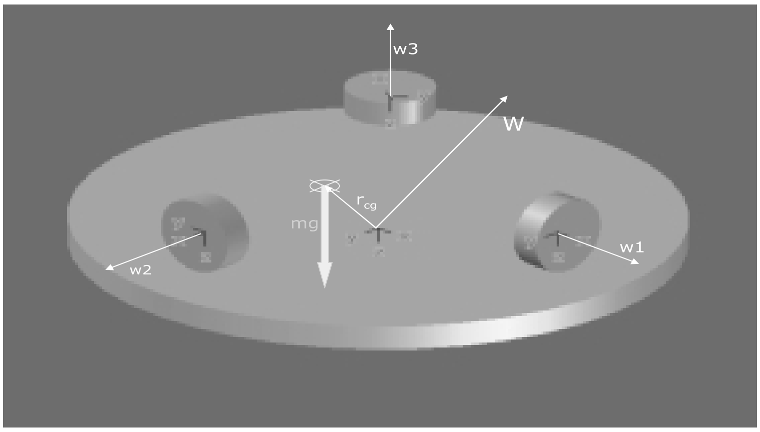

Fig. 1. Satellite Attitude Control System Simulator Platform.

the same objectives, but based on Kalman filter theory were used by [9]. Experimental platforms can also be used for familiarizing with modeling aspect and controllers design for complex space structures [10]. An experimental apparatus was used in [11] to investigate the dynamics and the control laws for a satellite composed of rigid and flexible parts. The results showed that the control of the flexible structure is extremely sensitive to the variation of the parameters of the system, indicating the need of more robust control strategies [12] in order to improve the controller performance.

2. Platform equations of motion

The SACSS consists of a platform in disk form that is supported by a film of air. Over this platform is possible to accommodate several satellite attitude control system components with their respective interfaces and connections. Usually the main equipments are sensors, actuators, computers, interfaces, batteries and etc.

Figure 1 shows the coordinates system (x, y, z) that is fixed to the platform with origin in its rotation center. The coordinate systems $(x, y, z)_{1,2,3}$ are fixed to the reaction wheels 1, 2 and 3 with origin in their respective mass centers and aligned with their rotation axes. The vectors $\vec{R}_{1,2,3}$ indicate the reaction wheels position and the vector $\vec{r}$ locates the platform elements of mass dm with respect to (xyz). The vectors $\vec{\rho}_{1,2,3}$ locates the reaction wheels elements of mass $\mathrm{dm}_{1,2,3}$ with respect to the coordinates $(x, y, z)_{1,2,3}$.

The platform angular velocity is given by

$$\vec{W} = p\vec{i} + q\vec{j} + r\vec{k} \tag{1}$$

and the reaction wheels angular velocity are given by $\vec{w}_1$, $\vec{w}_2$ and $\vec{w}_3$.

The total angular moment for the platform is the sum of the base and reaction wheels angular moment given by

$$\vec{H} = \int_B (\vec{r} \times \vec{v})dm + \sum_{i=1}^{3} \int_{RW} (\vec{r} \times \vec{v})dm \tag{2}$$

where the angular velocity of the base is $\vec{v} = \vec{W} \times \vec{r}$ and the reaction wheels is $\vec{v} = \vec{W} \times \vec{R}_i + \vec{w}_i \times \vec{\rho}_i$.

After manipulating Eq. (2) is given by

$$\vec{H} = \int_{B+RW} \vec{r} \times (\vec{W} \times \vec{r})dm + \sum_{i=1}^{3} R \int_{RW} \vec{\rho}_i \times (\vec{w}_i \times \vec{\rho}_i)dm \tag{3}$$

which in compact form can be represented by

$$\vec{H} = \vec{h} + \sum_{i=1}^{3} \vec{h}_i \tag{4}$$

The equations of motion of the platform is obtained deriving the total angular moment and equalizing to the external torques acting on the platform given by

$$\vec{r}_{cg} \times (m\vec{g}) = \frac{d\vec{H}}{dt} = (\dot{\vec{h}})_r + \vec{W} \times \vec{h} + \sum_{i=1}^{3} (\dot{\vec{h}}_i)_r + \vec{W} \times \left(\sum_{i=1}^{3} \vec{h}_i \right) \tag{5}$$

where $\vec{r}_{cg}$ is the center of gravity location, where the gravitational force (mg) acts.

Applying the same principle, the reaction wheels equations of motion are given by

$$\begin{aligned} T_1 &= I_1 \left[\dot{w}_1 + \dot{p} \right] \\ T_2 &= I_2 \left[\dot{w}_2 + \dot{q} \right] \\ T_3 &= I_3 \left[\dot{w}_3 + \dot{r} \right] \end{aligned} \tag{6}$$

where $T_{1,2,3}$ and $\mathrm{I}_{1,2,3}$ are the control input and inertia moments of the reaction wheels, respectively.

The kinematic equations describing the temporal variations of the Euler angles (ϕ, θ, ψ) in the sequence 3-2-1 are given by

$$\begin{aligned} \dot{\phi} &= p + \tan(\theta) \left[q \sin(\phi) + r \cos(\phi) \right] \\ \dot{\theta} &= q \cos(\phi) - r \sin(\phi) \\ \dot{\psi} &= \sec(\theta) \left[q \sin(\phi) + r \cos(\phi) \right] \end{aligned} \tag{7}$$

Putting together Eqs (6) and (7) with respect to the same coordination system yields

$$\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} & 0 & 0 & 0 & I_1 & 0 & 0 \\ I_{xy} & I_{yy} & I_{yz} & 0 & 0 & 0 & 0 & I_2 & 0 \\ I_{xz} & I_{yz} & I_{zz} & 0 & 0 & 0 & 0 & 0 & I_3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left\{ \begin{array}{c} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{array} \right\} = \left\{ \begin{array}{l} (I_{xx} - I_{zz})(qr) + I_{xy}(pr) - I_{xz}(pq) + I_{yz}(r^2 - q^2) + I_2(w_2 r) + \\ -I_3(w_3 q) + mgr_y \cos(\phi)\cos(\theta) - mgr_z \sin(\phi)\cos(\theta) \\ (I_{zz} - I_{xx})(pr) + I_{yz}(pq) - I_{xy}(qr) + I_{xz}(p^2 - r^2) - I_1(w_1 r) + \\ +I_3(w_3 p) - mgr_x \cos(\phi)\cos(\theta) - mgr_z \sin(\theta) \\ (I_{xx} - I_{yy})(pq) + I_{xz}(qr) - I_{yz}(pr) + I_{xy}(q^2 - p^2) + I_1(w_1 q) + \\ -I_2(w_2 p) + mgr_x \sin(\phi)\cos(\theta) + mgr_y \sin(\theta) \\ p + \tan(\theta) \left[q \sin(\phi) + r \cos(\phi) \right] \\ q \cos(\phi) - r \sin(\phi) \\ \frac{1}{\cos(\theta)} \left[q \sin(\phi) + r \cos(\phi) \right] \\ \frac{T_1}{I_1} \\ \frac{T_2}{I_2} \\ \frac{T_3}{I_3} \end{array} \right\} \tag{8}$$

which in compact form is given by

$$[M] \left\{ \dot{X} \right\} = \left\{ f(X) \right\} \quad => \quad \left\{ \dot{X} \right\} = [M]^{-1} \left\{ f(X) \right\} \tag{9}$$

where M represent the mass matrix, X the state vector of the system and $F(x)$ the function matrix given by the right hand side elements of Eq. (8).

3. Control law and parameters estimation

In order to design the control law, Eq. (9) needs to be linear. Therefore, assuming small angular displacements the equations of motion for the design purpose is

$$\begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,0 \end{bmatrix} \begin{Bmatrix} p \\ q \\ r \\ \phi \\ \theta \\ \psi \end{Bmatrix} + \begin{bmatrix} \frac{1}{I_1 - I_{xx}} & 0 & 0 \\ 0 & \frac{1}{I_2 - I_{yy}} & 0 \\ 0 & 0 & \frac{1}{I_3 - I_{zz}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} \tag{10}$$

which in state space form is given by

$$\begin{aligned} \dot{X} &= AX + Bu \\ Y &= CX \end{aligned} \tag{11}$$

where Y represents the system outputs (measures) and C the sensor location matrix.

Using a proportional control law given by

$$\{u\} = -[K]\{X\} \tag{12}$$

the control gain is obtained applying the pole allocation methods [13].

The equations of motion obtained previous are in general form and can be applied for different satellite attitude system. The identification process employed uses the equations of motion in its linear matrix form. From now on the vector X consists of the parameters to be identified, here the elements of the inertia matrix and the terms that give the location of the platform center of gravity. The elements of vector Y contain terms associated to the sensor measures and terms of the reaction wheels inertia matrix, therefore totally known. The identification problem can be solved applying an algorithm based on the least square method, in the form

$$[G]\{X\} = \{Y\} \tag{13}$$

where the matrix G contains the measured of angles, angular velocities and angular accelerations, see Ref. [14] for details. Therefore, it is function of the types of sensor used in the experiment.

Using the notation where the matrixes $[G_K]$ and $\{Y_K\}$represent the values of G and Y in the instant (or step) k, the previous matrix vector can be written

$$[\bar{G}_K] = \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_K \end{bmatrix} \quad \{\bar{Y}_K\} = \begin{Bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_K \end{Bmatrix} \tag{14}$$

Applying the regression least square method, the vector X that minimizes the error given by

$$\left\| \bar{G}_K X - \bar{Y}_K \right\| \tag{15}$$

is obtained by

$$\{X_K\} = \left(\left([\bar{G}_K]^T [\bar{G}_K] \right)^{-1} [\bar{G}_K]^T \right) \{\bar{Y}_K\} \tag{16}$$

Although the identification based on the least square regression method is relatively simple, it is important to verify the effects of the numeric errors introduced when calculating the pseudo-inverse matrix in Eq. 16, due to the need of processing a great number of points to obtain good results.

In order to avoid the calculation of the pseudo-inverse matrix one can use the recursive form of the least square method. The main difference of the regression algorithm is that the value for the matrix X at instant $t(k)$ (step k) is done with measures obtained at instant $t(k-1)$. Therefore, considering

$$\begin{aligned} [P_0] &= \left([G_0][G_0]^T \right)^{-1} \\ \{X_0\} &= [P_0][G_0]^T \{Y_0\} \end{aligned} \tag{17}$$

the recursive form of the least square method needs to satisfy the following recursive equations

Table 1
System data used in the estimation process

Platform	Platform	Reaction wheel	External torque
$I_{xx} = 1.1667$	$I_{xy} = 0.0107$	$I_1 = 0.001792$	$\text{Mgr}_x = 0.0101$
$I_{yy} = 1.1671$	$I_{xz} = -0.0159$	$I_2 = 0.001792$	$\text{Mgr}_y = 0.0323$
$I_{zz} = 2.1291$	$I_{yz} = 0.0159$	$I_2 = 0.001792$	$\text{Mgr}_z = 0.7630$

Table 2
Parameters estimation and error

time	20s	7s	3s
	estimation	*estimation*	*estimation*
I_{xx}	1.1665	1.1666	−3.134
I_{yy}	1.1681	1.1686	−8.4305
I_{zz}	2.1295	2.1292	2.0531
I_{xy}	0.0098343	0.010854	0.14886
I_{yz}	0.016812	0.016213	−0.58618
I_{xz}	−0.018679	−0.018476	−0.11682
mgr_x	0.010133	0.010102	0.015778
mgr_y	0.032319	0.032298	0.040902
mgr_z	0.76187	0.76231	4.7281
	Error	*error*	*error*
I_{xx}	0.00021283	8.1393e-5	4.3007
I_{yy}	0.001016	0.001483	9.5976
I_{zz}	0.00036696	7.3107e-5	0.076038
I_{xy}	0.00086572	0.00015386	0.13816
I_{yz}	0.00091155	0.00031343	0.60208
I_{xz}	0.00017878	2.4408e-5	0.098317
mgr_x	3.2726e-5	1.6989e-6	0.056778
mgr_y	1.9404e-5	2.3561e-6	0.086024
mgr_z	0.0011303	0.00069486	0.039651
Total	*error*	*error*	*error*
	0.058442	0.203586	2.632287

$$
\begin{aligned}
&[L_K] = [P_{K-1}]\,[G_K]^T\left([I] + [G_K]\,[P_{K-1}]\,[G_K]^T\right)^{-1} \\
&[P_K] = ([I] - [L_K]\,[G_K])\,[P_{K-1}] \\
&\{X_K\} = \{X_{K-1}\} + [L_K]\,(\{Y_K\} - [G_K]\,\{X_{K-1}\})
\end{aligned}
\tag{18}
$$

It is observed, that the estimation of $\{X_K\}$ is obtained adding a correction to the estimation done previously in the step $(k-1)$. The correction term is proportional to the difference between the measured values of Y_k and the measures based on the estimation done in the previous step, given by $[G_K]\left\{\hat{X}_{K-1}\right\}$.

The components of the vector $\{\text{L}_k\}$ are weight factors (gain) give information about how the correction and the previous estimative should be combined. The matrix $[P_k]$ is only defined when the matrix $\left[\bar{G}_K\right]^T\left[\bar{G}_K\right]$ is not singular. To avoid singularities the recursive process must be initiated with a big matrix $[\text{P}_0]$ positive define.

4. Simulation and results

The parameters are estimated using Eq. (13), applying both methods described in the previous section. The measures are obtainer by integrating Eq. (10). The platform and reaction wheel inertia data, and the term of the external torque are show in Table 1 in international system unit.

The parameters estimation has been done by the least square regression methods has been done taking the measure of the matrix $[A_K]$ and the vector $\{T_K\}$ for time period of 3, 7 and 20 in a simulation of 20s. Table 2 shows the parameters estimation values and their respective error with respect they real values.

Table 2 shows that the parameters estimation performed by the least square regression method is quite precise for measures taken for time period greater than 3. However, one observes some instability in calculating the pseud-inverse, when it has many elements bear zero.

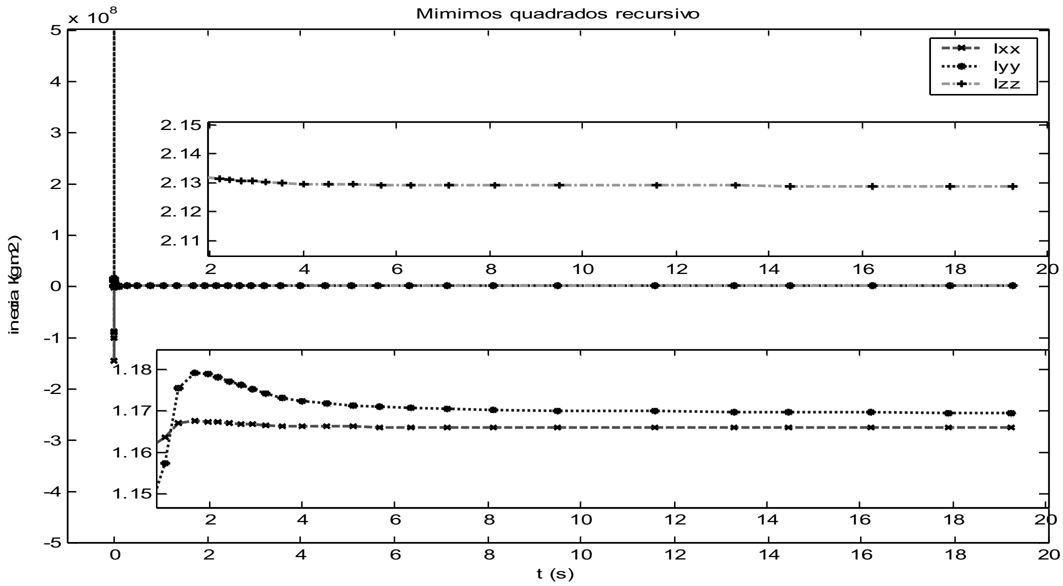

Fig. 2. Platform principal inertia parameters estimation.

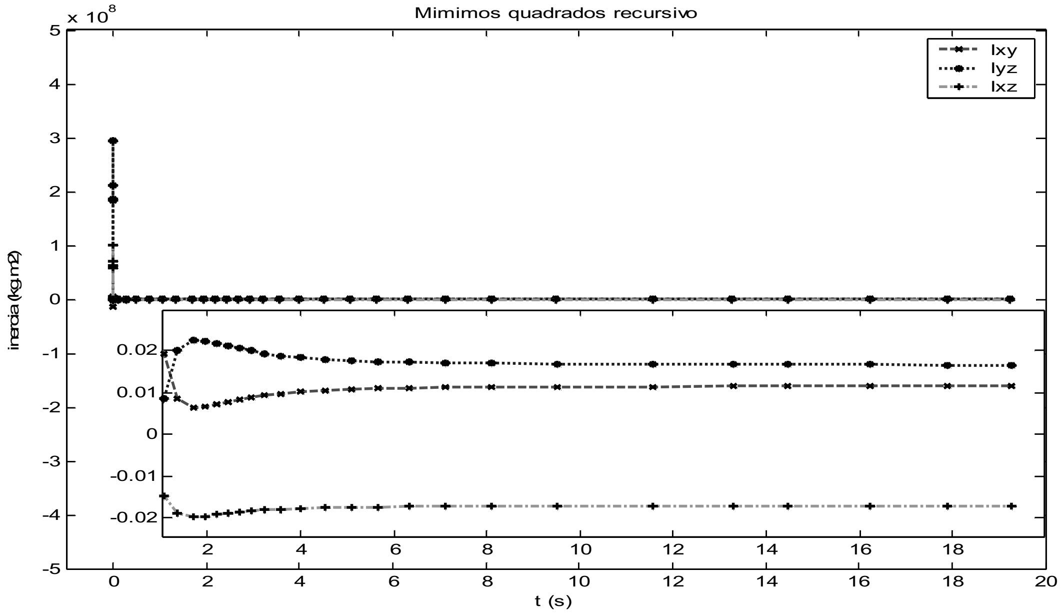

Fig. 3. Platform cross inertia parameters estimation.

In the parameters estimation applying the least square recursive method the measures are taken in time interval of 5 s for a simulation period of 20 s. Figures 2, 3 and 4 show the platform inertias and external torques parameters estimations, respectively.

From Figs 2, 3 and 4 one observes that although it is necessary a relatively great numbers of interaction to obtain an accurately estimative, the recursive procedure is more stable, once the error tend to decrease with time. That behavior shows that the recursive least square method is more reliable than the regression to estimate the system parameters.

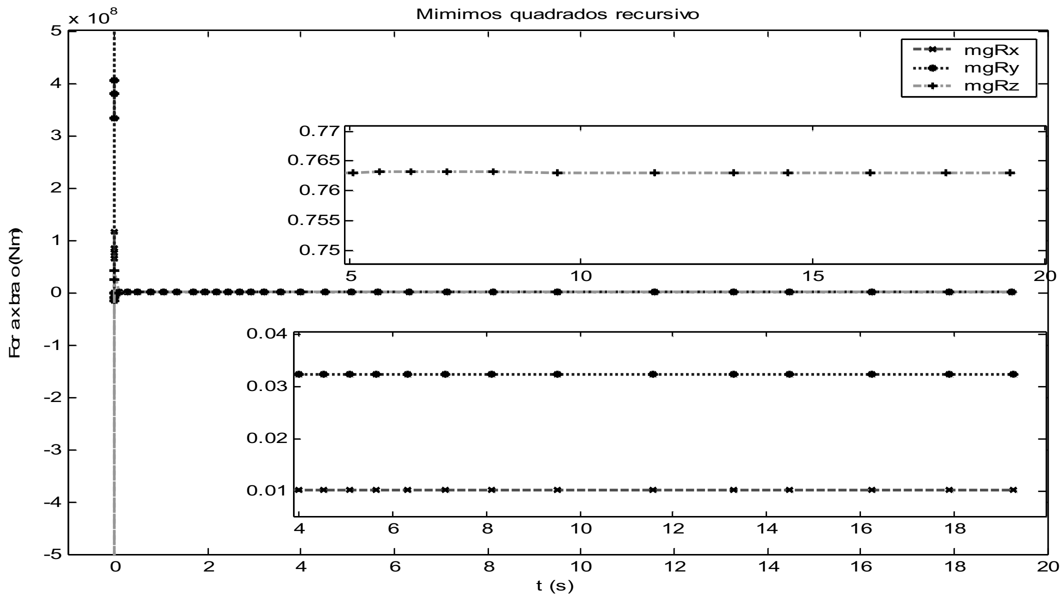

Fig. 4. External torque parameters estimation.

5. Conclusion

This paper presents the development of a Satellite Attitude Control System Simulator. The equations of motions were obtained for a platform with free rotation in three axes, which can be simplified for simpler configuration. The control system was designed using the pole allocation methods, assuming that the states are available for feedback, resulting in a robust control laws, since it was tested to control the nonlinear system The parameters estimation was initially done applying the least square regression method. In the sequence, the parameters were estimated applying the least square recursive method. Both methods have estimated the system parameters with small error. However, the least square recursive methods have performance more adequate for the SACSS objectives.

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