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SECTIONAL CURVATURE OF CONTACT CR-SUBMANIFOLDS OF AN ODD-DIMENSIONAL UNIT SPHERE

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Abstract. In this paper we study $(n+1)$-dimensional compact contact CR-submanifolds of $(n-1)$ contact CR-dimension immersed in an odd-dimensional unit sphere S^{2m+1}. Especially we provide necessary conditions in order for such a submanifold to be the generalized Clifford surface

$$S^{2n_1+1}(((2n_1+1)/(n+1))^{\frac{1}{2}}) \times S^{2n_2+1}(((2n_2+1)/(n+1))^{\frac{1}{2}})$$

for some portion (n_1, n_2) of $(n-1)/2$ in terms with sectional curvature.

1. Introduction

Let S^{2m+1} be a $(2m+1)$-unit sphere. For any point $z \in S^{2m+1}$ we put $\xi = Jz$, where J denotes the complex structure of the complex $(m+1)$-space $\mathbb{C}^{m+1}$. We consider the orthogonal projection $\pi : T_z\mathbb{C}^{m+1} \longrightarrow T_zS^{2m+1}$. Putting $\phi = \pi \circ J$, we can see that the aggregate (ϕ, ξ, η, g) is a Sasakian structure on S^{2m+1}, where η is a 1-form dual to ξ and g the standard metric tensor induced on S^{2m+1}. So S^{2m+1} can be considered as a Sasakian manifold of constant ϕ-holomorphic sectional curvature 1, that is, of constant curvature 1 (cf. [1, 2, 7]).

Let M be an $(n+1)$-dimensional submanifold tangent to the structure vector field ξ of S^{2m+1} and denote by $\mathcal{D}_x$ the ϕ-invariant subspace

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$T_xM \cap \phi T_xM$ of the tangent space T_xM of M at x in M. Then ξ cannot be contained in $\mathcal{D}_x$ at any point x in M (cf. [4, 5]). Thus the assumption $\dim \mathcal{D}_x^\perp$ being constant and equal to 2 at any point x in M yields that M can be given with a contact CR-submanifold in the sense of Yano-Kon [7], where $\mathcal{D}_x^\perp$ denotes the complementary orthogonal subspace to $\mathcal{D}_x$ in T_xM. In fact, if there exists a non-zero vector U which is orthogonal to ξ and contained in $\mathcal{D}_x^\perp$, then ϕU must be normal to M.

In this paper we shall study $(n+1)$-dimensional contact CR-submanifolds M of $(n-1)$ contact CR-dimension immersed in S^{2m+1} and, especially, provide necessary conditions in order that M is locally isometric to a Riemannian product of $M_1 \times M_2$ in terms with sectional curvature, where M_1 and M_2 belong to some $(2n_1+1)$ and $(2n_2+1)$-dimensional spheres.

Manifolds, submanifolds, geometric objects and mappings we discuss in this paper will be assumed to be differentiable and of class C^∞.

2. Preliminaries

Let $\overline{M}$ be a $(2m+1)$-dimensional almost contact metric manifold with structure (ϕ, ξ, η, g). Then by definition it follows that

$$\begin{aligned} &\phi^2 X = -X + \eta(X)\xi, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad \eta(\xi) = 1, \\ &g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \eta(X) = g(X, \xi) \end{aligned} \tag{2.1}$$

for any vector fields X, Y tangent to $\overline{M}$. We consider a Riemannian manifold isometrically immersed in $\overline{M}$ with induced metric tensor field g.

First of all we note that an n-dimensional submanifold normal to the structure vector field ξ of $\overline{M}$ is anti-invariant with respect to ϕ, that is, $\phi T_xM \subset T_xM^\perp$ for each point x of M, and $m \geq n$ (for details, see [7]).

In the sequel we assume that M is an $(n+1)$-dimensional submanifold tangent to the structure vector field ξ of a $(2m+1)$-dimensional almost contact metric manifold $\overline{M}$. We now denote by $\mathcal{D}_x$ the ϕ-invariant subspace defined by $T_xM \cap \phi T_xM$ and by $\mathcal{D}_x^\perp$ the complementary orthogonal to $\mathcal{D}_x$ in T_xM. Then the structure vector field ξ is contained in $\mathcal{D}_x^\perp$. In fact, if $\xi \in \mathcal{D}_x$, then there is a vector field X tangent to M such that $\xi = \phi X$, from which applying the operator ϕ and using (2.1), we have $X = \eta(X)\xi$. Thus it follows that $\xi = 0$, which is a contradiction. Hence $\xi \in \mathcal{D}_x^\perp$ at each point x of M. Moreover by definition we can easily see that $\phi \mathcal{D}_x^\perp \subset T_xM^\perp$ for each point x of M.

If the ϕ-invariant subspace $\mathcal{D}_x$ has constant dimension for x in M, then M is called a *contact CR-submanifold* ([1, 7]) and the constant is called *contact CR-dimension* of M([4, 5]). Especially, a contact CR-submanifold M of which contact CR-dimension is equal to $\dim M$ is called an *invariant submanifold* (cf. [1, 7]). Real hypersurfaces tangent to the structure vector field ξ are typical examples of contact CR-submanifolds.

3. Fundamental properties of contact CR-submanifolds

Let M be an $(n+1)$-dimensional contact CR-submanifold of $(n-1)$ contact CR-dimension in a $(2m+1)$-dimensional almost contact metric manifold $\overline{M}$. Then by definition $\dim \mathcal{D}_x^{\perp} = 2$ for any x in M, and so there is a unit vector field U contained in $\mathcal{D}^{\perp}$ which is orthogonal to ξ. Since $\phi\mathcal{D}^{\perp} \subset TM^{\perp}$, ϕU is a unit normal vector field to M, which will be denoted by N_1, that is,

$$N_1 = \phi U. \tag{3.1}$$

Moreover, it is clear that $\phi TM \subset TM \oplus \mathrm{Span}\{N_1\}$. Hence we have, for any tangent vector field X and for a local orthonormal basis $\{N_\alpha,\ \alpha = 1, \ldots, p\}$ $(p = 2m - n)$ of normal vectors to M, the following decomposition in tangential and normal components:

$$\phi X = FX + u^1(X)N_1, \tag{3.2}$$

$$\phi N_\alpha = -U_\alpha + PN_\alpha, \quad \alpha = 1, \ldots, p. \tag{3.3}$$

It is easily shown that F and P are skew-symmetric linear endomorphisms acting on T_xM and $T_xM^{\perp}$, respectively. Since the structure vector field ξ is tangent to M, (2.1) implies

$$g(FU_\alpha, X) = -u^1(X)g(N_1, PN_\alpha), \tag{3.4}$$

$$g(U_\alpha, U_\beta) = \delta_{\alpha\beta} - g(PN_\alpha, PN_\beta). \tag{3.5}$$

We also have

$$g(U_\alpha, X) = u^1(X)\delta_{1\alpha} \tag{3.6}$$

and consequently

$$g(U_1, X) = u^1(X), \quad U_\alpha = 0, \quad \alpha = 2, \ldots, p. \tag{3.7}$$

Furthermore from (3.2), it is clear that

$$F\xi = 0, \quad u^1(\xi) = 0, \quad FU = 0, \quad u^1(U) = 1. \tag{3.8}$$

Next, applying ϕ to (3.1) and using (2.1) and (3.3), we have

$$U_1 = U, \quad PN_1 = 0. \tag{3.9}$$

Applying ϕ to (3.2) and using (2.1), (3.2), (3.3), and (3.9), we also have

$$F^2X = -X + \eta(X)\xi + u^1(X)U, \quad u^1(FX) = 0. \tag{3.10}$$

On the other hand, it follows from (3.3), (3.7), and (3.9) that

$$\phi N_1 = -U, \quad \phi N_\alpha = PN_\alpha, \quad \alpha = 2, \dots, p \tag{3.11}$$

and moreover we may put

$$PN_\alpha = \sum_{\beta=2}^{p} P_{\alpha\beta} N_\beta, \quad \alpha = 2, \dots, p, \tag{3.12}$$

where $(P_{\alpha\beta})$ is a skew-symmetric matrix which satisfies

$$\sum_{\beta=2}^{p} P_{\alpha\beta} P_{\beta\gamma} = -\delta_{\alpha\gamma}. \tag{3.13}$$

We denote by $\overline{\nabla}$ and ∇ the Levi-Civita connection on $\overline{M}$ and M, respectively and denote by $\nabla^\perp$ the normal connection induced from $\overline{\nabla}$ in the normal bundle $TM^\perp$ of M. The Gauss and Weingarten equations are

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{3.14}$$

$$\overline{\nabla}_X N_\alpha = -A_\alpha X + \nabla^\perp_X N_\alpha, \quad \alpha = 1, \dots, p \tag{3.15}$$

for any tangent vector fields X, Y to M. Here h denotes the second fundamental form and A_α is the shape operator corresponding to N_α. They are related by

$$h(X,Y)=\sum_{\alpha=1}^{p} g(A_\alpha X,Y)N_\alpha.$$

Furthermore, we put

$$\nabla^\perp_X N_\alpha=\sum_{\beta=1}^{p} s_{\alpha\beta}(X)N_\beta, \tag{3.16}$$

where $(s_{\alpha\beta})$ is the skew-symmetric matrix of connection forms of $\nabla^\perp$.

Finally the equation of Gauss, Codazzi and Ricci are

$$\begin{aligned} g(\overline{R}(X,Y)Z,W) &= g(R(X,Y)Z,W)\\ &\quad+\sum_\alpha\{g(A_\alpha X,Z)g(A_\alpha Y,W)\\ &\quad- g(A_\alpha Y,Z)g(A_\alpha X,W)\}, \end{aligned} \tag{3.17}$$

$$\begin{aligned} g(\overline{R}(X,Y)Z,N_\alpha) &= g((\nabla_X A_\alpha)Y-(\nabla_Y A_\alpha)X,Z)\\ &\quad+\sum_\beta\{g(A_\beta Y,Z)s_{\beta\alpha}(X)\\ &\quad- g(A_\beta X,Z)s_{\beta\alpha}(Y)\}, \end{aligned} \tag{3.18}$$

$$g(\overline{R}(X,Y)N_\alpha,N_\beta)=g(R^\perp(X,Y)N_\alpha,N_\beta)+g([A_\beta,A_\alpha]X,Y) \tag{3.19}$$

for any tangent vector fields X,Y,Z to M, where $\overline{R}$ and R denote the Riemannian curvature tensors of $\overline{M}$ and M, respectively, and $R^\perp$ is the curvature tensor of the normal connection $\nabla^\perp$ (cf. [2]).

4. The special case of an ambient Sasakian manifold $\overline{M}$

In this section we specialize to the case of an ambient Sasakian manifold $\overline{M}$, that is,

$$\overline{\nabla}_X\xi=\phi X, \tag{4.1}$$

$$(\overline{\nabla}_X\phi)Y=-g(X,Y)\xi+\eta(Y)X. \tag{4.2}$$

Then, by differentiating (3.2) and (3.11) covariantly and by comparing the tangential and normal parts, we have

$$(4.3)\quad (\nabla_Y F)X = -g(Y,X)\xi + \eta(X)Y - g(A_1Y,X)U + u^1(X)A_1Y,$$

$$(4.4)\quad (\nabla_Y u^1)X = g(FA_1Y,X),$$

$$(4.5)\quad \nabla_X U = FA_1X,$$

$$(4.6)\quad g(A_\alpha U,X) = -\sum_{\beta=2}^{p} s_{1\beta}(X)P_{\beta\alpha}, \quad \alpha = 2,\ldots,p$$

On the other hand, since ξ is tangent to M, (4.1) gives

$$(4.7)\quad \nabla_X\xi = FX,$$

$$(4.8)\quad g(A_1\xi,X) = u^1(X), \text{ that is, } A_1\xi = U,$$

$$(4.9)\quad A_\alpha\xi = 0, \ \alpha = 2,\ldots,p.$$

In what follows we assume that $\overline{M}$ *is a Sasakian manifold of constant curvature* 1 *and that* N_1 *is parallel with respect to the normal connection* $\nabla^\perp$. Hence it follows from (3.16) that

$$(4.10)\quad s_{1\beta} = 0, \quad \beta = 2,\ldots,p,$$

which and (4.6) give

$$(4.11)\quad A_\alpha U = 0, \quad \alpha = 2,\ldots,p.$$

Next, since the curvature tensor $\overline{R}$ has the form

$$\overline{R}(\overline{X},\overline{Y})\overline{Z} = g(\overline{Y},\overline{Z})\overline{X} - g(\overline{X},\overline{Z})\overline{Y}$$

for $\overline{X}$, $\overline{Y}$, $\overline{Z}$ tangent to $\overline{M}$, the equations (3.17), (3.18), and (3.19) imply

$$(4.12)\quad \begin{aligned} g(R(X,Y)Z,W) &= g(Y,Z)g(X,W) - g(X,Z)g(Y,W) \\ &\quad + \sum_\alpha\{g(A_\alpha Y,Z)g(A_\alpha X,W)\} \\ &\quad - g(A_\alpha X,Z)g(A_\alpha Y,W), \end{aligned}$$

$$(4.13)_{(a)}\quad (\nabla_X A_1)Y - (\nabla_Y A_1)X = 0,$$

$$(4.13)_{(b)}\quad \begin{aligned} (\nabla_X A_\alpha)Y - (\nabla_Y A_\alpha)X &= \sum_{\beta=2}^{p}\{s_{\beta\alpha}(Y)A_\beta X - s_{\beta\alpha}(X)A_\beta Y\}, \\ &\qquad \alpha = 2,\ldots,p, \end{aligned}$$

$$(4.14)\quad [A_1, A_\alpha] = 0, \quad \alpha = 2,\ldots,p$$

with the help of (4.10) and (4.11).

Finally we introduce some lemmas for later use :

LEMMA 4.1. *Let M be an $(n+1)$-dimensional contact CR-submanifold M of $(n-1)$ contact CR-dimension immersed in a Sasakian manifold of constant curvature 1 and let the distinguished normal vector field N_1 be parallel with respect to the normal connection. Then the commutativity condition*

$$A_1F = FA_1$$

holds on M if and only if

$$\nabla A_1 = 0.$$

Moreover, in this case

$$A_1{}^2 = u^1(A_1U)A_1 + I, \quad A_1U = \xi + u^1(A_1U)U \tag{4.15}$$

and the function $u^1(A_1U)$ is locally constant.

Proof. We first assume that $\nabla A_1 = 0$. Differentiating (4.8) covariantly along M and using (4.5), (4.7), and $\nabla A_1 = 0$, we can easily see that $A_1F = FA_1$ holds on M.

The proofs of the converse and (4.15) have been given in [4, Lemma 5.1, p.433].

LEMMA 4.2. *Let M be as in Lemma 4.1. Then*

$$FA_\alpha + A_\alpha F = 0, \quad trA_\alpha = 0, \quad \alpha = 2, \dots, p. \tag{4.16}$$

Proof. Differentiating (4.9) covariantly and using (4.7), we have

$$(\nabla_X A_\alpha)\xi + A_\alpha FX = 0,$$

or equivalently

$$g((\nabla_X A_\alpha)Y, \xi) + g(A_\alpha FX, Y) = 0 \tag{4.17}$$

for any vector fields X, Y tangent to M. By means of (4.9) and $(4.13)_{(b)}$, it can be easily verified from (4.17) that

$$(A_\alpha F + FA_\alpha)X = 0, \quad \alpha = 2, \dots, p$$

hold on M. Inserting FX back into the above equation and using (3.10), (4.9) and (4.11), we have $A_\alpha X = FA_\alpha FX$, which implies $\mathrm{tr}A_\alpha = 0$.

5. Main theorems

In this section we assume that the ambient manifold is a $(2m+1)$-dimensional unit sphere S^{2m+1}. Suppose that the distinguished normal vector field N_1 is parallel with respect to the normal connection $\nabla^{\perp}$ and that the trace of the shape operator A_1 vanishes, that is,

$$\mathrm{tr} A_1 = 0. \tag{5.1}$$

Then, from $(4.13)_{(a)}$ and (5.1), we have

$$\sum (\nabla_i A_1) e_i = 0, \tag{5.2}$$

where $\{e_i\}_{i=1,\ldots,n+1}$ is an orthonormal basis of tangent vectors to M and $\nabla_i := \nabla_{e_i}$. Hence it follows from $(4.13)_{(a)}$ and (5.2) that

$$\sum (\nabla_i \nabla_i A_1) X = \sum (R(e_i, X) A_1) e_i$$

for any vector X tangent to M, and consequently we have

$$g(\nabla^2 A_1, A_1) = \sum_{i,j} g((R(e_i, e_j) A_1) e_i, A_1 e_j). \tag{5.3}$$

Thus we have

THEOREM 5.1. *Let M be an $(n+1)$-dimensional compact contact CR-submanifold of $(n-1)$ contact CR-dimension immersed in S^{2m+1} and let the distinguished normal vector field N_1 be parallel with respect to the normal connection. Suppose that the trace of the shape operator A_1 in direction of N_1 vanishes and that the minimum of sectional curvatures of M is zero. Then M is minimal and $\nabla A_1 = 0$ on M.*

Proof. The minimality of M is easily followed by our assumptions and Lemma 4.2.

Taking account of the Laplacian of $\mathrm{tr} A_1^2$, we have

$$\int_M \|\nabla A_1\|^2 * 1 = - \int_M g(\nabla^2 A_1, A_1) * 1,$$

which together with (5.3) yields

$$0 \le \int_M \|\nabla A_1\|^2 * 1 = - \int_M \sum_{i,j} g((R(e_i, e_j) A_1) e_i, A_1 e_j) * 1. \tag{5.4}$$

Now we choose an orthonormal frame $\{e_j\}$ of M such that

$$A_1 e_j = \lambda_j e_j \quad (j = 1, \dots, n+1).$$

Then it is clear that

$$\begin{aligned} &\sum_{i,j} g((R(e_i, e_j)A_1)e_i, A_1 e_j) \\ &= \sum_{i,j} \{g((R(e_i, e_j)A_1 e_i, A_1 e_j) - g(A_1 R(e_i, e_j)e_i, A_1 e_j)\} \\ &= \frac{1}{2}\sum_{i,j} (\lambda_i - \lambda_j)^2 K_{ij}, \end{aligned}$$

where K_{ij} denotes the sectional curvature of the plane section spanned by $\{e_i, e_j\}$. Hence, if the minimum of the sectional curvature of M is zero, the above equation and (5.4) imply $\nabla A_1 = 0$.

By means of Theorem 5.1 we can obtain the following theorem under additional condition:

THEOREM 5.2. *Let M be an $(n+1)$-dimensional compact contact CR-submanifold of $(n-1)$ contact CR-dimension in S^{2m+1} and assume that there exists an orthonormal basis $\{N_1, N_\alpha\}_{\alpha=2,\dots,p}$ of normal vectors to M each of which is parallel with respect to the normal connection. If the trace of the shape operator A_1 in direction of N_1 vanishes and if the minimum of sectional curvatures of M is zero, then there is an $(n+2)$-dimensional totally geodesic unit sphere S^{n+2} of S^{2m+1} such that $M \subset S^{n+2}$.*

Proof. Under our assumptions it follows from Theorem 5.1 that

$$\mathrm{tr} A_\alpha = 0, \quad \alpha = 2, \dots .p.$$

Moreover, it is clear from $(4.13)_{(b)}$ that, for any vector fields X, Y tangent to M,

$$(\nabla_X A_\alpha)Y - (\nabla_Y A_\alpha)X = 0$$

since $s_{\alpha\beta} = 0$, $1 \leq \alpha, \beta \leq p$, and consequently

$$\sum (\nabla_i A_\alpha) e_i = 0,$$

where $\{e_i\}_{i=1,\dots,n+1}$ is an orthonormal basis of tangent vectors to M. Taking account of the Laplacian of $\mathrm{tr}A_\alpha^2$ and using the quite similar method as shown in the proof of Theorem 5.1, we can easily see that

$$\nabla_X A_\alpha = 0, \quad \alpha = 2, \dots, p \tag{5.5}$$

for any vector field X tangent to M.

Differentiating (4.9) covariantly and using (4.7) and (5.5), we have

$$A_\alpha F X = 0$$

for any vector fields X, Y tangent to M. Inserting FX instead of X in this equation and using (3.10), (4.9), and (4.11), we have

$$A_\alpha = 0, \quad \alpha = 2, \dots, p.$$

Hence the first normal space of M is contained in $\mathrm{Span}\{N_1\}$, which is invariant under parallel translation with respect to the normal connection from our assumption. Thus we may apply Erbacher's reduction theorem [3], which gives the proof of our theorem.

Combining Theorem 5.2 and a theorem provided in [4, Theorem 6.2, p.436], we have

THEOREM 5.3. *Let M be an $(n+1)$-dimensional compact contact CR-submanifold of $(n-1)$ contact CR-dimension in S^{2m+1} and assume that there exists an orthonormal basis $\{N_1, N_\alpha\}_{\alpha=2,\dots,p}$ of normal vectors to M each of which is parallel with respect to the normal connection. If the trace of the shape operator A_1 in direction of N_1 vanishes and if the minimum of sectional curvatures of M is zero, then M is isometric to a generalized Clifford surface:*

$$S^{2n_1+1}(((2n_1+1)/(n+1))^{\frac{1}{2}}) \times S^{2n_2+1}(((2n_2+1)/(n+1))^{\frac{1}{2}})$$

for some portion (n_1, n_2) of $(n-1)/2$.

Proof. By means of Theorem 5.2, M can be regarded as a real minimal hypersurface of S^{n+2} which is a totally geodesic invariant submanifold of S^{2m+1}. Moreover, it is clear from (4.15) that M has exactly two constant eigenvalues ρ_1, ρ_2 with

$$\rho_1 = (\lambda + \sqrt{\lambda^2+4})/2, \quad \rho_2 = (\lambda - \sqrt{\lambda^2+4})/2,$$

where $\lambda = g(A_1U, U)$. In fact, since $\rho_k{}^2 - \lambda\rho_k - 1 = 0$ $(k = 1, 2)$, (4.8) and (4.15) imply

$$A_1(\rho_1 U + \xi) = \rho_1(\rho_1 U + \xi), \quad A_1(\rho_2 U + \xi) = \rho_2(\rho_2 U + \xi).$$

Therefore, since $\nabla A_1 = 0$ and $A_1 F = F A_1$, we can easily see that M is isometric to a Riemannian product of odd-dimensional spheres, that is,

$$S^{2n_1+1}(r_1) \times S^{2n_2+1}(r_2)$$

for some portion (n_1, n_2) of $(n-1)/2$ and some r_1, r_2 with $r_1^2 + r_2^2 = 1$ (for details, see [4]). In fact, since M is minimal, $r_1 = ((2n_1+1)/(n+1))^{\frac{1}{2}}, r_2 = ((2n_2+1)/(n+1))^{\frac{1}{2}}$. Taking account of (4.15), we can verify that the minimum of sectional curvatures of those hypersurfaces is zero.

COROLLARY 5.4. *Let M be a compact, minimal real hypersurface tangent to the structure vector fields ξ of an odd-dimensional unit sphere S^{n+2} . If the minimum of sectional curvatures of M is zero, then M is isometric to*

$$S^{2n_1+1}(((2n_1+1)/(n+1))^{\frac{1}{2}}) \times S^{2n_2+1}(((2n_2+1)/(n+1))^{\frac{1}{2}})$$

for some portion (n_1, n_2) of $(n-1)/2$.

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