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**SCALAR SECTOR OF THE 3 3 1 MODEL
WITH THREE HIGGS TRIPLETS**

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ABSTRACT

A scalar sector of the 3 3 1 model with three Higgs triplets is considered. The mass spectrum, eigenstates and interactions of the Higgs and the SM gauge bosons are derived. We show that one of the neutral scalars can be identified with the standard model Higgs boson, and in the considered potential there is no mixing between scalars having VEV and ones without VEV.

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I. Introduction

The standard model (SM) of the electroweak interactions has been considered an extremely successful theory from the phenomenological point of view. However, this theory contains a large number of free parameters as well as a large number of unanswered questions. These drawbacks of the SM have led to a strong belief that the model is still incomplete. One of the basic elements of the SM has not been tested, that is an observation of the Higgs scalar. At present, hunting Higgs boson and looking for a new physics beyond the SM are central tasks of particle physics. Hopefully, these tasks can be done at LEP2, Tevatron, and others.

In recent years, the study of scalar sector has become one of the booming subjects in particle physics. This study has been carried out within the SM framework as well as some extensions of the SM. One of these extensions is the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ (3 3 1) models [?, ?]. These models have the following intriguing features: Firstly, the models are anomaly free only if the number of families N is a multiple of three. If further one adds the condition of QCD asymptotic freedom, which is valid only if the number of families of quarks is to be less than five, it follows that N is equal to 3. The second characteristic of these models is that one family of quarks is treated differently from the other two. This could lead to a natural explanation for the unbalancing heavy top quarks, deviations of A_b from the SM prediction etc. In addition, the models predict no very high new mass scales – a few TeV [?].

Three Higgs triplets of the minimal version [?] have firstly been analysed, then the sextet has been added [?] in the further consideration. A further study of this model in Higgs sector was done recently by Tonasse [?]. In the minimal 3 3 1 model it is necessary to have three triplets (in which there are two doubly charged and three neutral scalar states) and sextet in order for leptons to gain masses (for more details, see [?]). However, in an alternative 3 3 1 model which has *neutral* leptons such as neutrinos [5 – 8] in triplet, the Higgs sector has a quite different content, i.e. more neutral scalars (five) and no doubly charged Higgs states. Consequently, the Higgs potential constructed on these states, will have own features.

In the present paper we study the scalar sector of the model with *real* three Higgs triplets. This paper is organized as follows. In Sec. II, the main notations and the potential are introduced. Using obtained constraint equations, the mass spectrum and eigenstates in the potential are derived. In order to compare with the SM Higgs structure, Sec. III is devoted to couplings of the obtained Higgs with the SM gauge bosons such as Z , W^\pm and the photon γ . We show that under some circumstances we can get complete analogy with coupling constants in the SM. Finally, our conclusions are summarized in the last section.

II. The potential and mass spectrum

The details of the 3 3 1 models with neutral lepton in the bottom of the leptonic triplet were pointed out in Refs. [5 – 8]. Here we remind only symmetry breaking in these models

$$\begin{aligned}
& \text{SU}(3)_C \otimes \text{SU}(3)_L \otimes \text{U}(1)_N \\
& \quad \downarrow \langle \chi \rangle \\
& \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \\
& \quad \downarrow \langle \rho \rangle, \langle \eta \rangle \\
& \text{SU}(3)_C \otimes \text{U}(1)_Q,
\end{aligned} \tag{1}$$

where

$$\chi = \begin{pmatrix} \chi'^o \\ \chi^- \\ \chi^o \end{pmatrix} \sim (1, 3, -\frac{1}{3}), \eta = \begin{pmatrix} \eta^o \\ \eta^- \\ \eta'^o \end{pmatrix} \sim (1, 3, -\frac{1}{3}), \rho = \begin{pmatrix} \rho^+ \\ \rho^o \\ \rho'^+ \end{pmatrix} \sim (1, 3, \frac{2}{3}). \tag{2}$$

From Eq. (??) we see that in our case we have two extra neutral states, therefore in the neutral scalar sector we have 5 x 5 mass matrix instead of 3 x 3 one as in [?]. Note that χ and η transform the same way, therefore the most general Higgs potential (i.e. the one including all terms consistent with the gauge invariance and renormalizability) is very complicated. However, as mentioned in [?] under assumption of the discrete symmetry $\chi \rightarrow -\chi$, the most general potential can then be written in the following form:

$$\begin{aligned}
V(\eta, \rho, \chi) &= \mu_1^2 \eta^+ \eta + \mu_2^2 \rho^+ \rho + \mu_3^2 \chi^+ \chi + \lambda_1 (\eta^+ \eta)^2 + \lambda_2 (\rho^+ \rho)^2 + \lambda_3 (\chi^+ \chi)^2 \\
&+ (\eta^+ \eta) [\lambda_4 (\rho^+ \rho) + \lambda_5 (\chi^+ \chi)] + \lambda_6 (\rho^+ \rho) (\chi^+ \chi) + \lambda_7 (\rho^+ \eta) (\eta^+ \rho) \\
&+ \lambda_8 (\chi^+ \eta) (\eta^+ \chi) + \lambda_9 (\rho^+ \chi) (\chi^+ \rho) + \lambda_{10} (\chi^+ \eta + \eta^+ \chi)^2.
\end{aligned} \tag{3}$$

The last term in (??) reflects the fact that χ and η transform similarly and this term will give specific interactions. For convenience in reading, we rewrite the expansion of the scalar fields which acquire a VEV:

$$\eta^o = v + H_\eta^o + iA_\eta^o; \rho^o = u + H_\rho^o + iA_\rho^o; \chi^o = w + H_\chi^o + iA_\chi^o. \tag{4}$$

For the sake of simplicity, here we assume that vacuum expectation values (VEVs) are real. This means that the CP violation through the scalar exchange is not considered in this work. For the prime neutral fields which do not have VEV, we get analogously:

$$\eta'^o = H_\eta'^o + iA_\eta'^o; \chi'^o = H_\chi'^o + iA_\chi'^o. \tag{5}$$

In literature, a real part H is called CP-even scalar or *scalar*, and an imaginary one A - CP-odd scalar or *pseudoscalar* field. In this paper we call them scalar and pseudoscalar, respectively.

The VEV $\langle \chi \rangle$ will generate masses for exotic 2/3 and -1/3 quarks and new heavy gauge bosons Z', X, Y , while VEV $\langle \rho \rangle, \langle \eta \rangle$ will generate masses for ordinary fermions and the SM gauge bosons Z, W^\pm . To keep the model consistent with low-energy phenomenology, the VEV $\langle \chi \rangle$ must be large enough. In this paper we will use the following approximation:

$$w \gg v, u. \tag{6}$$

Requiring that in the shifted potential V , the linear terms in fields must be absent, we get in the tree level approximation, the following constraint equations:

$$\begin{aligned}\mu_1^2 + 2\lambda_1 v^2 + \lambda_4 u^2 + \lambda_5 w^2 &= 0 \\ \mu_2^2 + 2\lambda_2 u^2 + \lambda_4 v^2 + \lambda_6 w^2 &= 0 \\ \mu_3^2 + 2\lambda_3 w^2 + \lambda_5 v^2 + \lambda_6 u^2 &= 0.\end{aligned}\tag{7}$$

Substituting Eqs. (??), (??) and (??) into Eq. (??) and diagonalizing, we will get a mass spectrum of Higgs bosons with mixings.

a. *Spectrum in neutral scalar sector*

In the $H_\eta^o, H_\rho^o, H_\chi^o, H_\eta'^o, H_\chi'^o$ basis the square mass matrix, after imposing of the constraints (??), has a quasi-diagonal form as follows:

$$M_H^2 = \begin{pmatrix} M_{3H}^2 & 0 \\ 0 & M_{2H}^2 \end{pmatrix},\tag{8}$$

where

$$M_{3H}^2 = 2 \begin{pmatrix} 2\lambda_1 v^2 & \lambda_4 v u & \lambda_5 v w \\ \lambda_4 v u & 2\lambda_2 u^2 & \lambda_6 u w \\ \lambda_5 v w & \lambda_6 u w & 2\lambda_3 w^2 \end{pmatrix},\tag{9}$$

and

$$M_{2H}^2 = (\lambda_8 + 4\lambda_{10}) \begin{pmatrix} w^2 & v w \\ v w & v^2 \end{pmatrix}.\tag{10}$$

The above mass matrix shows that the prime fields mix themselves but do not mix with others.

Now we consider 3 x 3 mass matrix M_{3H}^2 of $H_\eta^o, H_\rho^o, H_\chi^o$ mixing. In its exact form it is impossible to find the acceptable solution of the characteristic equation. However, keeping only terms of the second order of w we get immediately two massless states and one physical field with mass $-4\lambda_3 w^2$.

In order to improve the solution we add to our consideration the linear terms in w . We obtained then one massless field H_1 with an eigenstate

$$H_1 \approx \frac{1}{(\lambda_5^2 v^2 + \lambda_6^2 u^2)^{1/2}} (\lambda_6 u H_\eta^o - \lambda_5 v H_\rho^o).\tag{11}$$

Two remaining states are H_2 with mass approximatively equal to zero and one massive physical state H_3 with mass:

$$m_{H_3}^2 \approx -4\lambda_3 w^2.\tag{12}$$

The conditions for orthogonality and normality allow us to get expressions for H_2 and H_3 :

$$H_2 \approx \frac{1}{(\lambda_5^2 v^2 + \lambda_6^2 u^2)^{1/2}} (\lambda_5 v H_\eta^o + \lambda_6 u H_\rho^o),\tag{13}$$

$$H_3 \approx H_\chi^o.\tag{14}$$

In order to gain mass for H_1 , we solve the characteristic equation with the exact 3 x 3 mass matrix M_{3H}^2 , and the H_1 associated with, namely:

$$\left(M_{3H}^2 - Im_{H_1}^2\right) H_1 = 0. \quad (15)$$

Solving a system of three equations (??) we get the mass for H_1

$$m_{H_1}^2 \approx \frac{v^2}{\lambda_6}(2\lambda_1\lambda_6 - \lambda_4\lambda_5) \approx \frac{u^2}{\lambda_5}(2\lambda_2\lambda_5 - \lambda_4\lambda_6). \quad (16)$$

The above equation shows that mass of H_1 depends only on v or u separately. Similarly, for H_2 we get

$$m_{H_2}^2 \approx 2\lambda_1v^2 + \frac{\lambda_4\lambda_6u^2}{\lambda_5} \approx 2\lambda_2u^2 + \frac{\lambda_4\lambda_5v^2}{\lambda_6}. \quad (17)$$

Eqs. (??) and (??) also give us relations among coupling constants and VEVs.

Diagonalization of 2 x 2 mass matrix M_{2H}^2 gives us one Goldstone G'_1 and one physical massive field H'_4 with mass:

$$m_{H'_4}^2 = -(\lambda_8 + 4\lambda_{10})(v^2 + w^2), \quad (18)$$

and mixing

$$\begin{pmatrix} H'_\eta \\ H'_\chi \end{pmatrix} = \frac{1}{(v^2 + w^2)^{1/2}} \begin{pmatrix} -v & w \\ w & v \end{pmatrix} \begin{pmatrix} G'_1 \\ H'_4 \end{pmatrix}. \quad (19)$$

b. Spectrum in neutral pseudoscalar sector

Now we consider the pseudoscalar sector. Keeping one's mind on the constraint equations (??) we have three Goldstone bosons which can be identified as follows: $G_2 \equiv A'_\eta$, $G_3 \equiv A'_\rho$, $G_4 \equiv A'_\chi$ and in the A'_η, A'_χ basis

$$M_{2A}^2 = (\lambda_8 + 4\lambda_{10}) \begin{pmatrix} w^2 & vw \\ vw & v^2 \end{pmatrix}. \quad (20)$$

We easily get one Goldstone G'_5 and one massive pseudoscalar boson A_1 with mass

$$m_{A_1}^2 = -(\lambda_8 + 4\lambda_{10})(v^2 + w^2), \quad (21)$$

and mixing

$$\begin{pmatrix} H'_\eta \\ H'_\chi \end{pmatrix} = \frac{1}{(v^2 + w^2)^{1/2}} \begin{pmatrix} -v & w \\ w & v \end{pmatrix} \begin{pmatrix} G'_5 \\ A_1 \end{pmatrix}. \quad (22)$$

c. Spectrum in charged scalar sector

In the charged sector we have two Goldstone bosons and two physical massive fields with mixings:

$$\begin{pmatrix} \eta^+ \\ \rho^+ \end{pmatrix} = \frac{1}{(v^2 + u^2)^{1/2}} \begin{pmatrix} -v & u \\ u & v \end{pmatrix} \begin{pmatrix} G_6^+ \\ H_5^+ \end{pmatrix}, \quad (23)$$

$$\begin{pmatrix} \rho'^+ \\ \chi^+ \end{pmatrix} = \frac{1}{(u^2 + w^2)^{1/2}} \begin{pmatrix} -u & w \\ w & u \end{pmatrix} \begin{pmatrix} G_7^+ \\ H_6^+ \end{pmatrix}. \quad (24)$$

The masses of H_5^+ and H_6^+ are given, respectively:

$$m_{H_5^+}^2 = -\lambda_7(v^2 + u^2), \quad m_{H_6^+}^2 = -\lambda_9(v^2 + w^2). \quad (25)$$

We emphasize that masses of H_5^+ and H_1, H_2 depend only on VEVs of light Higgs fields: u and v .

In the low-energy phenomenology – the limit (??), the mixing in (??) becomes small, then we have: $\rho'^+ \sim H_6^+$ and $\chi^+ \sim G_7^+$.

Requiring that square mass of the physical fields is positive (otherwise, they are Goldstone ones) and combining Eqs. (??), (??) and (??) we get the following relations among parameters of the potential:

$$\lambda_3 \lesssim 0, \quad \lambda_7 \lesssim 0, \quad \lambda_9 \lesssim 0, \quad \lambda_8 + \lambda_{10} \lesssim 0. \quad (26)$$

Now we briefly list the particle content in our Higgs sector: in the considered models we have four neutral scalars, one neutral pseudoscalar, two charged scalars and seven Goldstone bosons.

Note that the mass spectrum and eigenstates in this sector are exact.

III. Higgs – SM gauge boson couplings

In order to identify the considered above Higgs bosons with those in the SM, in this section we present the couplings of two kinds of mentioned particles in the model with right-handed neutrinos [?, ?]. For the multi-Higgs case, couplings of the Higgs bosons with the fermions (Yukawa couplings) are not equally defined. For example, to avoid unsuppressed lepton flavour violating processes [?] one uses the discrete symmetry, while other authors suggested alternative ways for heavier quarks (for details, see [?]).

Interactions among the gauge bosons and the Higgs ones arise from the following pieces:

$$(D^\mu \mathcal{H})^+(D_\mu \mathcal{H}), \quad \mathcal{H} = \eta, \rho, \chi, \quad (27)$$

in which the covariant derivatives are defined [?]:

$$D_\mu = \partial_\mu + ig \sum_{a=1}^8 W_\mu^a \frac{\lambda_a}{2} + ig_N \frac{\lambda_9}{2} NB_\mu. \quad (28)$$

Substituting Eq. (??) into (??) then (??) we get the following trilinear couplings:

$$g(WWH_1) = \frac{g^2 uv(\lambda_6 - \lambda_5)}{(\lambda_5^2 v^2 + \lambda_6^2 u^2)^{1/2}}, \quad (29)$$

$$g(WWH_2) = \frac{g^2(\lambda_5 v^2 + \lambda_6 u^2)}{(\lambda_5^2 v^2 + \lambda_6^2 u^2)^{1/2}}, \quad (30)$$

$$g(ZZH_1) = \frac{g^2 uv(\lambda_6 - \lambda_5)}{2c_W^2(\lambda_5^2 v^2 + \lambda_6^2 u^2)^{1/2}}, \quad (31)$$

$$g(ZZH_2) = \frac{g^2(\lambda_5 v^2 + \lambda_6 u^2)}{2c_W^2(\lambda_5^2 v^2 + \lambda_6^2 u^2)^{1/2}}, \quad (32)$$

where $c_W \equiv \cos \theta_W$.

From Eqs. (??, ??, ??) and (??) we see that if

$$\lambda_5 = \lambda_6, \quad (33)$$

the interactions among H_1 and Z, W vanish, while the interactions with H_2 are found to be:

$$g(WWH_2) = \sqrt{2}g m_W, \quad g(ZZH_2) = \sqrt{2}g m_Z/c_W, \quad (34)$$

where $m_W^2 = \frac{g^2(v^2+u^2)}{2}$, $m_Z = \frac{m_W}{c_W}$.

Looking at Eq. (??) we see that the condition (??) means that quartic interactions among light Higgs states η, ρ and heavy one χ are the same. Therefore this assumption is not badly based.

The quartic couplings are determined to be:

$$\begin{aligned} g(WWH_1H_1) &= g(WWH_2H_2) = g(WWG_2G_2) = g(WWG_3G_3) = \frac{g^2}{2}, \\ g(ZZH_1H_1) &= g(ZZH_2H_2) = g(ZZG_2G_2) = g(ZZG_3G_3) = \frac{g^2}{4c_W^2}. \end{aligned}$$

For charged Higgs bosons we have

$$\begin{aligned} g(\gamma\gamma G_5^+ G_5^-) &= g(\gamma\gamma H_5^+ H_5^-) = e^2, \\ g(ZZ G_5^+ G_5^-) &= g(ZZ H_5^+ H_5^-) = \frac{g^2}{4c_W^2}(1 - 2s_W^2). \end{aligned}$$

Keeping one's mind that the VEVs and Higgs fields in this paper are redefined (see for example, [?]) we recognize complete analogy with the SM interactions.

Summarizing, from couplings of the SM gauge bosons with Higgs bosons in this model we can conclude that the H_2 can be identified with the SM neutral Higgs boson, while charged ones G_5^\pm – with charged Goldstone boson. Another neutral Higgs boson is very light and interacts weakly with light (i.e. the SM) gauge bosons.

IV. Summary

In this paper we have considered the mass spectrum, eigenstates of the potential specialized for the 3 3 1 models with three Higgs triplets. It is shown that in the considered model there are two light neutral Higgs bosons and one of them can be identified with the SM Higgs. In the some circumstance the coupling becomes identical to that in the SM. Other charged Higgs bosons such as scalar H_5^\pm and pseudoscalar G_5^\pm interact to the SM gauge bosons including the photons γ , with the same coupling constants.

We emphasize again that the analysis here (excluding the neutral scalar sector) are exact.

The light neutral Higgs bosons play an important role in the testing of new physics beyond the SM [?].

In this letter we have analysed the scalar sector in the 3 3 1 model with three Higgs triplets, especially in the two parts scalar and pseudoscalar sector. An application in new physics beyond the SM will be a subject of further study.

The most general potential for this kind of 3 3 1 models is very complicated and it is impossible to get an acceptable solution. However, with the help of the discrete symmetry, we obtained rather simple solutions, because there are no mixings between scalars having VEV and ones without VEV.

In concluding, the considered here 3 3 1 model with three Higgs triplets has a rather simple Higgs structure. In addition, the data from neutrino neutral current scatterings gives a lower bound for mass of the heaviest new neutral gauge boson Z' of few hundreds GeV (and hence the VEV w) [?]. With no very high new mass scales the 3 3 1 models are interesting, and they can be confirmed or ruled out in the near future.

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