CHAPTER NINETY FIVE

SCALE-MODEL RELATIONSHIP OF BEACH PROFILE

by

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1. ABSTRACT

This paper presents a scale-model relationship for the similarity between large and small scale-models in two-dimensional equilibrium beach profiles. Taking large scale-models using large scale equipment as prototypes, the experimental scale of a medium-sized model was gradually varied keeping the grain size ratio of model to prototype constant. A similarity-comparison between large and small scale beach profiles is made by considering the degree of experimental errors. Judgement results are graphically shown, and a scale-model relationship is proposed. It is found that the scale-model relationship proposed agrees with the ones derived from the empirical formulae expressing the properties of beach profiles. Additionally, the applicability of this scale-model relationship to the reproduction test of natural beaches is examined.

2. INTRODUCTION

The similitude on the beach changes has often been studied experimentally and theoretically by several investigators. As an experimental method, the similitude of two-dimensional beach change was earlier studied using a regular wave flume in the Beach Erosion Board (presently, Coastal Engineering Research Center, US Army) in 1947. Saville (1957) and Iwagaki & Noda (1961) pointed out the scale effect as the reason that the beach profile in the model is not always in agreement with the prototype, when the wave height, wave length and grain-size scale of model are scaled down geometrically by the Froude law for the prototype. Noda (1972)

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discussed how the scale-distortion among vertical and horizontal scales, sand grain-size and specific gravity of beach material of the model has an effect on the similitude relationship. Similitudes proposed with these methods are significant only within the specialized experimental conditions, and not clearly confirmed by the verification test on a natural beach.

This paper presents a scale-model relationship for the two-dimensional equilibrium beach profiles. The scale-model relationship proposed is examined by its applicability to the verification test on the natural beach at Ogata in Niigata, Japan.

3. EXPERIMENTAL ERROR IN REPRODUCTION TEST

If a laboratory experiment on beach processes is performed several time under initially the same conditions, in general, the properties of beach profiles and breaking waves vary considerably as seen in Fig. 1, in which position and height of ↓-symbol show breaking point and relative wave height, respectively. They happen independently of with wave duration time t. It is considered that the scatter of properties depends on (i) systematic error, (ii) accidental error and (iii) mistakes. In this study the scatter of data, namely experimental error, is investigated, using the mean error ε' defined as

$$\varepsilon'\Delta x = \sum_{i=1}^{n} |x_i - \overline{x}| / n \tag{1}$$

where $\varepsilon'\Delta x$ is the experimental error of shoreline change relating to beach profiles and waves, n the number of replications, x_i the measured value at the n-th, and $\overline{x}$ the mean value.

The relationship between the experimental error of shoreline change evaluated with Eq. (1) and the wave steepness in deep water H_0/L_0 is shown in Fig. 2 including the other data. In this figure, a parameter H_0/sd usually describing a measure of the scale effect in beach profile is shown. In the parameter, H_0 is the wave steepness in deep water, s the specific gravity of beach sediment in water, and d the grain size. From Fig. 2 it is recognized that the parameter does not effect the experimental error of shoreline change. Since the values plotted in Fig. 2 scatter considerably, the solid line is drawn to envelop the highest values plotted. The solid line, namely the dimensionless experimental error of shoreline change $\varepsilon'\Delta X_{sl}/L_0$, is expressed as

$$\varepsilon'\Delta X_{sl}/L_0 = \pm 5.7 (H_0/L_0)^{1.2} \tag{2}$$

Also, the dimensionless experimental errors of position of berm crest $\varepsilon'\Delta B_x/L_0$ and breaking point $\varepsilon'\Delta X_b/L_0$ derived in the same manner are shown in Fig. 2 with broken and dotted lines, respectively.

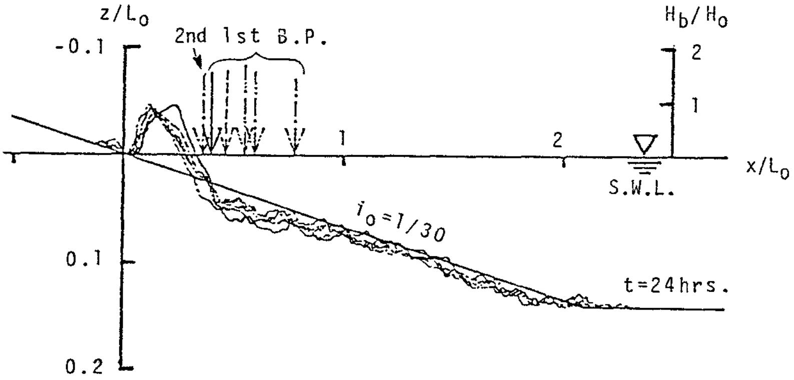


Fig. 1 Experimental error of repeating runs at the wave operation time of 24 hours in Run No. 3M-28 in Table 1.

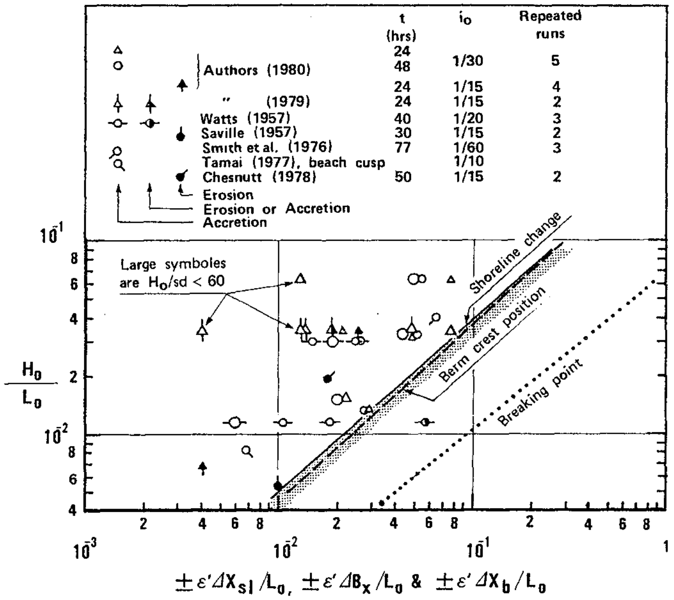


Fig. 2 Experimental errors of shoreline change berm crest position and breaking point .

4. CRITERION OF SIMILARITY

To ensure the similarity of beach processes between the prototype (i. e., large-scale model) and the model (i. e., small-scale model), phenomena such as beach profile, breaking waves and sediment transport by waves have to be similar in both cases. The comparison of the similarity between the two must be judged while considering the experimental error proposed. By the degree of agreement between the two, therefore, the following three criteria are employed:

a) Similitude : The difference in the beach processes, such as changes in shoreline and breaking point, is less than twice the experimental error. The judgement in this case is designated by circles.

b) Dissimilitude : Beach profiles to be compared are substantially different in type such as " bar type" and " step type", and the difference in the beach processes is more than four times the experimental error. This is designated by dark squares.

c) Quasi-similitude : This case is an intermediate situation between a) and b), and designated by triangles.

5. EXPERIMENTS

We performed a number of laboratory experiments in order to determine the scale-model relationship of two-dimensional beach change under the following conditions :

(1) The vertical and horizontal length scales, such as beach profile and water depth in the wave tank, are to be the same (i. e., undistorted), and both scales are to be subject to the Froude law.

(2) Wave characteristics such as wave height H and wave period T are determined by the Froude law. The wave generator operated until beach profiles reach equilibrium.

(3) The beach sediment of the small-scale models is sand or silica-sand with the same specific gravity as the sand used in the large-scale models. The grain size of the beach sediment in the model, i. e. the grain-size scale, is to be chosen independently of the experimental scales above-mentioned (1) and (2).

A large-scale model was carried out using a large-size two-dimensional wave flume of 78m long, 1m wide and 1.2m deep, and waves were generated for 60 hours on an initially even-sloping beach. We took the equilibrium beach profile and its experimental conditions to be the prototype. Additionally, those of the prototype-size experiment performed by Saville were also taken to be the prototype.

For comparison, experiments with small-scale models were carried out using the medium-sized wave flume of 28m long, 0.5m wide, and 1m deep. The grain size of the sand or silica-sand used in the model beach sediment was chosen independently of the experimental scale. The experimental scale was gradually varied keeping the grain size ratio of model to prototype constant. The initial beach slope of the model was identical to the prototype. Table 1 is an example of experimental conditions, in which d_{50} is the medium size of the beach sediment, H_0 the wave height in deep water, L_0 the wave length in deep water, h the water depth in the wave tank, g the acceleration of gravity, ν the kinematic viscosity of water and t the duration time in total. The grain size distribution of the beach sediment used in the prototype and the model is shown in Fig. 3. Median sizes of the beach sediment were used 0.15, 0.30, 0.42, 1.62mm in the model and 0.22, 0.94mm in the prototype, respectively.

Table 1 An example of experimental conditions for prototype and model.

Scale	Run No.	d_{50} (mm)	T (sec)	Ho (cm)	h (cm)	Ho/Lo	d_{50}/Ho	h/Lo	$\frac{\sqrt{gHo}\cdot d_{50}}{\nu}$	t/T	Remarks
							$\times 10^{-3}$	$\times 10^{-2}$		$\times 10^{4}$	
Proto.	3	0.94	3.00	20.6	100.0	0.015	5.0	7.0	925	7.2	io=1/30
1/1.82	3M-30	0.42	2.22	11.8	55.0	0.015	3.6	7.2	391	3.9	
1/2	3M-29	0.42	2.12	9.6	50.0	0.014	4.4	7.1	361	4.1	
1/3	3M-28	0.42	1.73	6.8	33.3	0.015	6.2	7.1	296	5.0	
1/4	3M-27	0.42	1.50	4.8	25.0	0.014	8.8	7.0	258	5.8	
1/5	3M-28	0.42	1.34	4.6	20.0	0.017	9.1	7.1	250	6.4	
1/6.7	3M-25	0.42	1.16	3.5	15.0	0.017	12.0	7.2	215	7.4	
1/2.9	3M-23	1.62	1.76	7.2	34.5	0.015	22.4	7.1	1194	0.1 7.2	
proto.	T-22	0.22	11.33	138.2	426.7	0.0069	0.16	2.1	664	1.3	=Saville (1957)
1/10	T-27	0.15	3.57	13.4	42.7	0.0067	1.1	2.1	200	1.3	io=1/15
1/20	T-28	0.15	2.53	6.5	21.3	0.0065	2.3	2.1	134	1.3	
1/30	T-29	0.15	2.06	4.4	14.2	0.0066	3.4	2.1	86	1.3	
1/40	T-29	0.15	1.79	3.4	10.7	0.0068	4.4	2.1	76	1.3	

d_{50}=1.62mm, sorting coef.=1.12, specific gravity=2.55
0.94 1.15 2.60 } sand
0.42 1.19 2.68
0.15 1.23 2.77 silica-sand

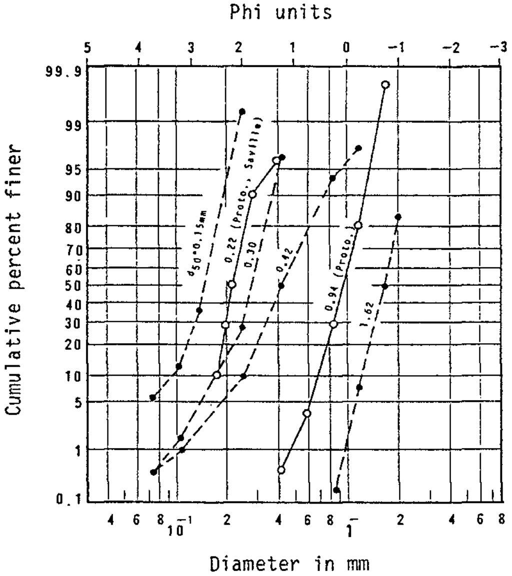


Fig. 3 Grain size distributions of beach sediments used in the model and prototype.

6. EXPERIMENTAL RESULTS AND ANALYSIS

Fig. 4 (a) shows the properties of beach profiles and breaking waves which resulted from the experiment, and expresses similarity-comparisons between the prototype (Run No. 3 in Table 1) and the models (for example, Run No. 3M-30 ~ 3M-25). In this figure, P, S and SP indicate breaker types, such as plunging breaker surging breaker and spilling breaker. The comparison shown in Fig. 4 (a) was made gradually varying the experimental scale S=1/n [= λ_{H_0} = $(H_0)_m$ / $(H_0)_p$ =1/1.82 ~ 1/6.7], keeping the grain-size scale of the model to prototype, λ_d [= $(d)_m$/ $(d)_p$ =0.45] constant. Also, the comparison between the prototype by Saville's experimental result (Run No. T-22) and the models (Run No. T-27 ~ T-29') is also shown in Fig. 4 (b). The degree of experimental error in the experiments is recognized from check run of the Run No. T-28. The judgement of the similarity-comparison between the prototypes and models was made using the criteria a), b) and c) defined already, and the judgement results are shown on the left side of the figure, respectively. Only prototypes in which the ratio of grain size to wave height satisfied the condition that $(d)_p$/ $(H_0)_p$ $\lesssim$ 0.01 are used for comparison with the models. The results judged are arranged in Fig. 5 of which ranges of experimental conditions of prototype cover $H_0/L_0 = 0.007 \sim 0.025$, $d/H_0 \lesssim 0.01$, and $i_0 = 1/10 \sim 1/30$, giving the inverse ratio of grain-size scale $1/\lambda_d$ $[(d)_p/(d)_m]$ the ordinate and the inverse ratio of experimental scale $1/\lambda_{H_0}$ $(1/S = n)$ the abscissa, respectively. Fig. 5 also includes the results which satisfy the range of the initial beach slope ranging 1/10 to 1/30 and of the deep water wave steepness ranging 0.007 to 0.025 among the experimental data from previously investigators.

It is possible to classify the data shown in Fig. 5 into three zones such as "Similitude", "Quasi-similitude" and "Dissimilitude". In Fig. 6, results in the case where $H_0/L_0 = 0.031 \sim 0.042$, which are similarly classified, are superimposed on those of Fig. 5. In Fig. 6, ranges of experimental conditions of prototype cover $d/H_0 \lesssim 0.01$ and $i_0 = 1/10 \sim 1/30$. A common similitude zone extends over both ranges $1/\lambda_d > 1$ and $1/\lambda_d < 1$ of the grain-size scale. The central position of the common similitude zone is shown with a dot-dashed line. The scale-model relationship indicated by the dot-dashed line within the range $1/\lambda_d > 1$ is expressed by

$$\frac{1}{\lambda_d} = 1.7^{a}\left(\frac{1}{H_0}\right)^{b} \tag{3}$$

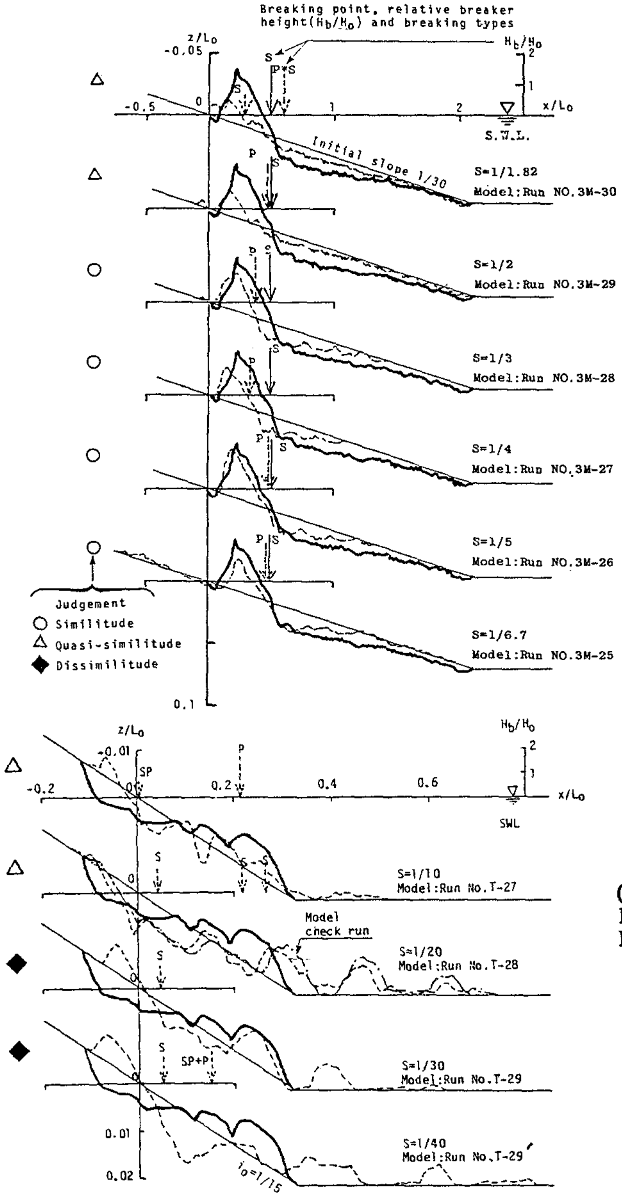
which is transformed to

$$\frac{(d)_m}{(d)_p} = \left(\frac{1}{1.7}\right)^{a}\left(\frac{1}{n}\right)^{b} \tag{4}$$

where $(d)_m/(d)_p$ is the grain-size scale, $1/n$ the experimental scale, and the values of a and b are indicated in Table 2. From Fig. 6, it is seen that the scale-model relationship of Eqs. (3) and (4) shifts slightly from the Froude law stating that $1/\lambda_d = 1/\lambda_{H_0}$ which is drawn by a dotted line.

Table 2 Values of a and b in Eqs. (3) and (4)

Experimental scale / Exponent	$1 > \frac{1}{n} \gtrsim \frac{1}{2.2}$	$\frac{1}{2.2} > \frac{1}{n}$
a	0	1
b	0.83	0.2



(a) Prototype : Run No. 3

(b) Prototype : Run No. T-22

Fig. 4 Similarity-comparison between prototype (solid line) and model (broken line).

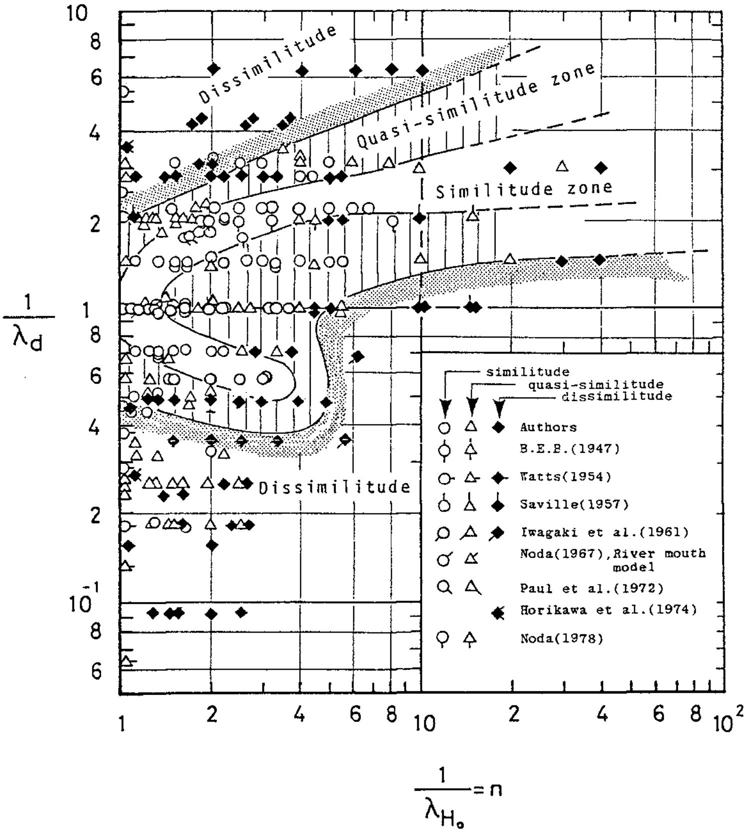


Fig. 5 Graphical representation of similitude of beach profiles.

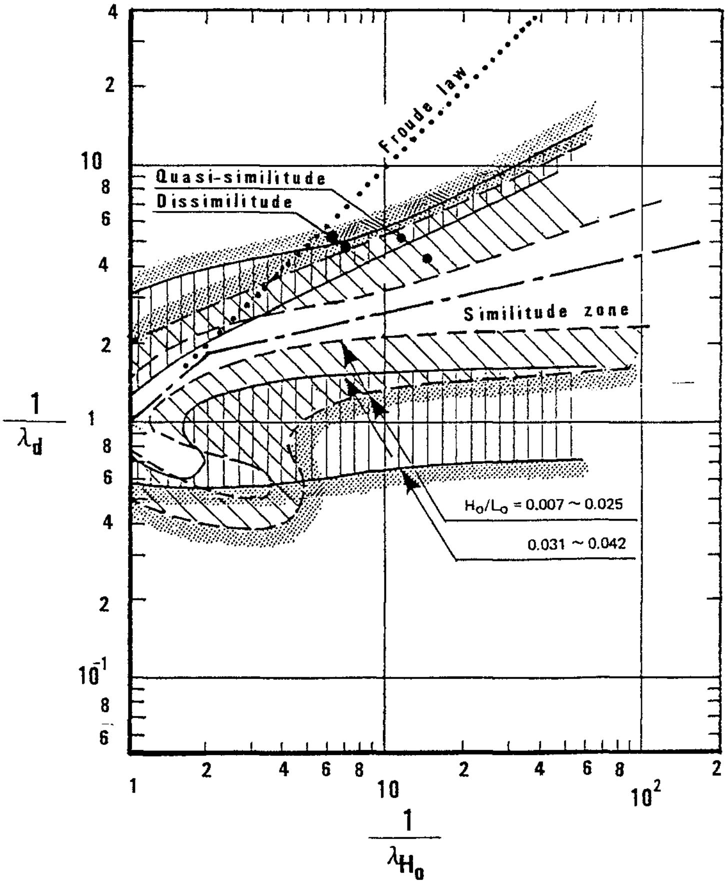


Fig. 6 Similitude of beach profile within range of H_0/L_0=0. 007 ~ 0. 042.

7. COMPARISON

Let us compare the scale-model relationship indicated in Fig. 6 or Eqs. (3) and (4) with those derived by other investigators. Previous studies of the beach processes are summarized in column No. ③ in Table 3. In table 3, d_m is the mean diameter of beach sediment, w the fall velocity of the sediment, L_0 the deep water wave length, $i_0 = \tan\beta$ the initial beach slope, l the typical length, λ_x the horizontal scale, λ_y the vertical scale, σ the density of beach sediment, ρ the water density and other notations were explained already. Now, arranging the relationship of column No. ③ using the definition,

$$\lambda_{\text{parameter}} = \frac{\text{model parameter}}{\text{prototype parameter}} \tag{5}$$

scale-model relationships as shown in column No. ④ are obtained. When the experimental conditions are restricted by the conditions (1), (2) and (3) described in 5, we may make the assumptions that $\lambda_x = \lambda_{L_0} = \lambda_y = \lambda_{H_0}$, $\lambda_{i_0} = \lambda_{\tan\beta} = 1$, $\lambda_t = 1$ and $\lambda_s = 1$ as well as the water temperature and acceleration of gravity are the same between model and prototype, $\lambda_\nu = 1$ and $\lambda_g = 1$, respectively.

Substituting these similitude conditions into the scale-model relationships of column No. ④, the relationships of column No. ⑤ are finally derived. Using the same coordinate ($1/\lambda_d$, $1/\lambda_{H_0}$) as Fig. 6, it is possible to describe relationships of column No. ⑤. Similitudes of Nos. A, B and C shown in Fig. 7 (a). The scale-model relationship of Nos. D, E and F is expressed using only the fall velocity of beach sediment w. Now, we consider the scale-model relationship between the fall velocity and the grain size of material. There are theoretical formula of Stokes, and empirical ones of Allen, Kármán and Newton for the fall velocity of sediment as reported by Tsurumi (1932). The applicable ranges of the four formulae depend on the Reynolds number $Re = wd/\nu$. Using the restricted conditions of this study, $\lambda_s = 1$, $\lambda_\nu = 1$ and $\lambda_g = 1$, the relationships of λ_w and λ_d are eventually obtained as,

Stokes' formula (In this formula, when $s=1.65$, $\rho=1.0\,\mathrm{g/cm^3}$ (water density) and $Re<4.5$ are given, the applicable sediment size to the formula is $d \leqq 0.15$mm.)

$$\lambda_w=\lambda_d^{2}\lambda_s\lambda_\nu^{-1}\lambda_g=\lambda_d^{2} \tag{5}$$

Allen's formula (similarly, $Re=4.5 \sim 300$; $d=0.15 \sim 1.1$mm)

$$\lambda_w=\lambda_d\lambda_s^{2/3}\lambda_\nu^{-1/3}\lambda_g^{2/3}=\lambda_d \tag{6}$$

Kàrmàn's formula (similarly, $Re=300 \sim 3200$, $d=1.1 \sim 5.8$mm)

$$\lambda_w=\lambda_d^{2/3}\lambda_s^{5/9}\lambda_\nu^{-1/9}\lambda_g^{5/9}=\lambda_d^{2/3} \tag{7}$$

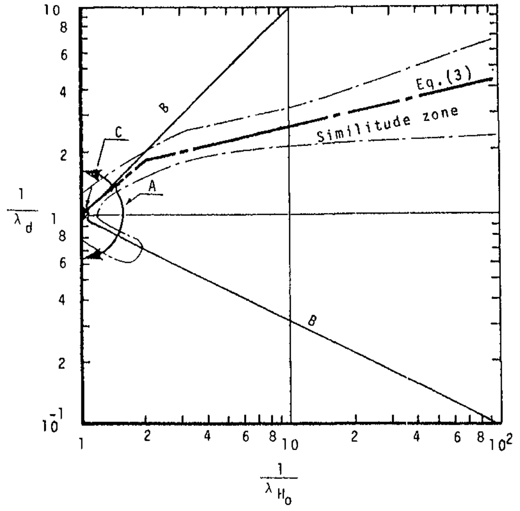
Newton's formula (similarly, $Re=3200 \sim 10000$, $d>5.8$mm)

$$\lambda_w=\lambda_d^{1/2}\lambda_s^{1/2}\lambda_g^{1/2}=\lambda_d^{1/2} \tag{8}$$

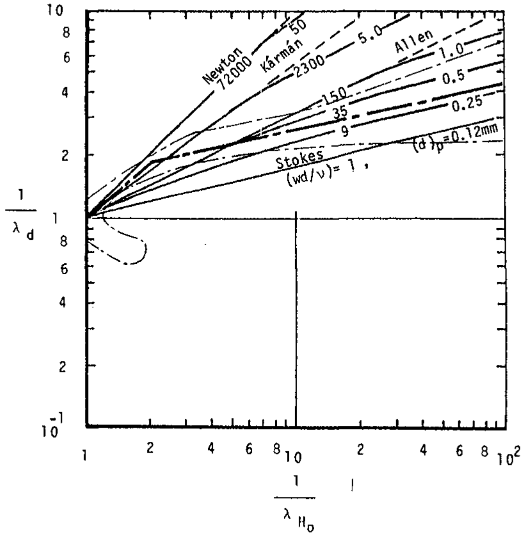
In Table 3 all the relationship of No. D, E and F in column No. ⑤ can be expressed in Fig. 7 (b) taking the Reynolds number of the prototype as a parameter. Since the scale-model relationship in Fig. 6 is comparable with those shown in Fig. 7, it can be seen that the scale-model relationship proposed in this study agrees well with those derived from the relationships which express the properties of beach processes using the fall velocity of beach sediment.

Table 3 List of similitudes derived from characteristics of beach profile.

① No.	② Investigator	③ Equation or figure expressing the properties of the beach profile.	④ Similitude-relationship derived from ③.	⑤ Relationship of column No. ④ when $\lambda_x=\lambda_y=\lambda_{H_0}=\lambda_{L_0}=1$, $\lambda_s\doteq 1$, $\lambda_{i_0}=\lambda_{\tan\beta}=1$ $\lambda_T=\lambda_x^{1/2}$, $\lambda_\nu=1$ and $\lambda_g=1$.	⑥ Remarks
A	J.W.Johnson (1949)	* $\frac{H_0}{L_0}$ >0.03 (Storm beach) $=0.025\sim0.03$ (Transition) <0.025 (Ordinary beach)	$\lambda_{H_0}\lambda_{L_0}^{-1}=1$	Relationship between λ_d and λ_{H_0} is satisfied always.	* The effect of grain-diameter is not considered.
B	Iwagaki & Noda (1961)	$\frac{l}{L_0}=f[\frac{H_0}{L_0}, \frac{d_m}{H_0}, \frac{t}{T}, (\frac{\sigma}{\rho}-1), \frac{\nu}{d_m\sqrt{gH_0}}, i_0]$	$\lambda_{H_0}\lambda_{L_0}^{-1}=1$, $\lambda_d\lambda_{H_0}^{-1}=1$, $\lambda_t\lambda_T^{-1}=1$, $\lambda_s=1$, $\lambda_d^{-1}\lambda_{H_0}^{-1/2}\lambda_\nu\lambda_g^{-1/2}=1$, and $\lambda_{i_0}=1$	$\lambda_d=\lambda_{H_0}$ $\lambda_d=\lambda_{H_0}^{-1/2}$	
C	E.K.Noda (1972)		** $\lambda_d\lambda_s^{1.85}=\lambda_y^{0.55}$ $\lambda_x=\lambda_y^{1.32}\lambda_s^{-0.386}$	Only the prototype satisfies in these conditions.	** Similitude-relationship can apply only in the vicinal region of a shoreline.
D	R.G.Dean (1973)	$\frac{H_0}{L_0} \gtrless 1.7\frac{w}{gT}$ (Storm beach) (Ordinary beach)	$\lambda_w\lambda_{H_0}^{-1}\lambda_{L_0}^{-1/2}\lambda_g^{-1/2}=1$	$\lambda_w=\lambda_{H_0}^{1/2}$	
E	R.A.Dalrymple (1976)	Similitude-relationship derived from H_0/L_0, Froude law and the equation of fall velocity	$\lambda_x=\lambda_y$ $\lambda_T=\lambda_y^{1/2}$ $\lambda_w=\lambda_y^{1/2}$	$\lambda_w=\lambda_y^{1/2}$	Seven kinds of similitude relationships were proposed. This is one selected from among the seven.
F	Hattori & Kawamata (1980)	$\frac{(H_0/L_0)\tan\beta}{wgT}$ $>$ (Accretive profile) $=0.5$ (Equlibrium P.) $<$ (Erosive P.)	$\lambda_w\lambda_{H_0}^{-1}\lambda_{L_0}^{1/2}\lambda_{\tan\beta}^{-1}\lambda_g^{-1/2}=1$	$\lambda_w=\lambda_{H_0}^{1/2}$	Beach processes within the breaker zone.



(a) Nos. A, B and C



(b) Nos. D, E and F

Fig. 7 Comparisons of scale-model relationship proposed with those in column No. ⑤ in Table 3.

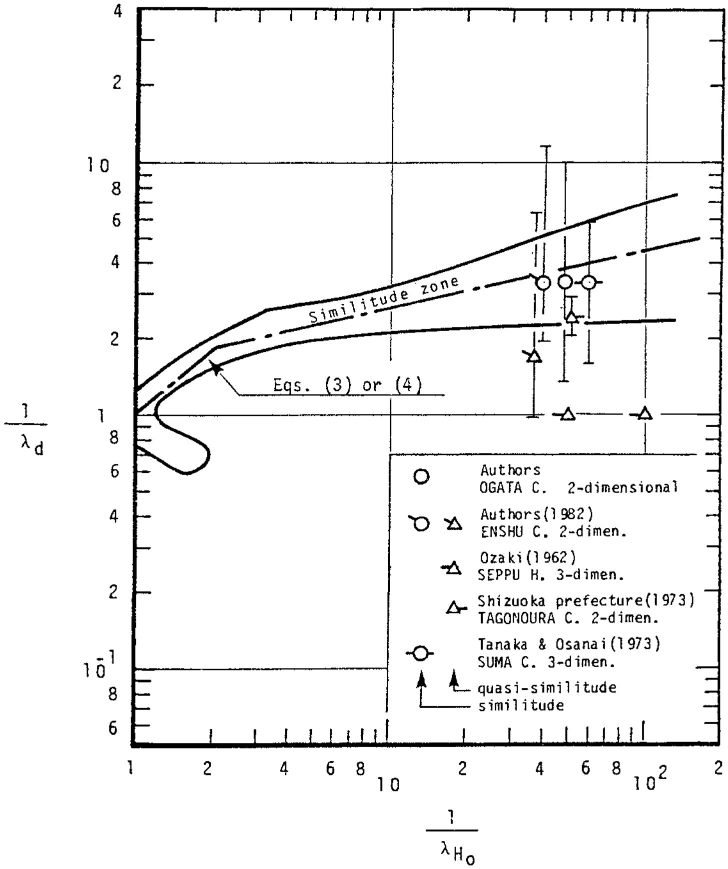


Fig. 8 Verification of proposed scale-model relationship by reproduction of natural beaches in scale models.

8. VERIFICATION TEST

To verify the scale-model relationship proposed in this study, experiments with a two-dimensional beach model of the Ogata coast in Niigata were carried out. On this coast, the predominant waves strike the shore orthogonally. Therefore, it may be considered that the coast has a two-dimensional property. The experimental conditions of the verification test were set with an experimental scale of S=1/50 and a grain-size scale of $\lambda_d \approx 1/3.2$ using the scale-model relationship. The properties of waves in the model reproduced the significant waves off Ogata, and the temporal wave change was chosen according to the Froude time scale. It was found that the verification test well reproduced the beach change at the coast. The grain-size scale and the experimental scale used in the verification test agree well with the similitude zone shown in Fig. 8. This figure, as a comparison, also includes other results of two-dimensional wave flume experiments by the first authors (1982) and Sizuoka prefecture (1973), and of the three-dimensional wave basin by Ozaki (1962) and by Tanaka·Osanai (1973). Consequently, it is concluded that the model can reproduce very well beach changes when the experimental conditions are set so as to be contained within the similitude zone proposed.

9. CONCLUSIONS

The main conclusions are summarized as;

1) The experimental errors of the shoreline change, berm crest position and breaking point in the two-dimensional beach processes are shown as a function of wave steepness in deep water. The tendencies of the three experimental errors on it are similar.

2) The three criteria for classifying the degree of similarity between prototype and model are defined from the degree of experimental error.

3) The similarity-comparison between prototype and model is made with reference to the similarity criteria. The scale-model relationship of beach changes is proposed from the Fig. 6 showing the result of similarity-comparison.

4) The scale-model relationship proposed agrees well with the one derived from the relationship expressing the properties of beach changes using the fall velocity of beach sediment.

5) It is found that the proposed scale-model relationship is applicable to reproduce changes in natural beaches in laboratory experiments.

10. ACKNOWLEDGMENTS

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