

1 **Scaled boundary FEM model for interaction of short-crested waves with a** 2 **concentric porous cylindrical structure**

3 Longbin Tao*¹, M.ASCE; Hao Song ²; and Subrata Chakrabarti³, F.ASCE

4 **ABSTRACT**

5 This paper describes the development of an efficient scaled boundary finite-element model
6 (SBFEM) for the simulation of short-crested wave interaction with a concentric porous cylindri-
7 cal structure. By weakening the governing differential equation in the circumferential direction,
8 the SBFEM is able to solve analytically the weakened equation in the radial direction. Only the
9 cylinder boundary on the circumference of the exterior porous cylinder is discretized with curved
10 surface finite-elements, while a complete analytical representation is obtained for the radial differ-
11 ential equation. Comparisons of the numerical results on wave diffraction forces and surface wave
12 elevations at the cylinder to available analytical solutions demonstrate that excellent accuracy can
13 be achieved by the SBFEM with a very small number of surface finite-elements. The influence
14 of varying the wave parameters as well as the system configuration on the system hydrodynamics,
15 including the wave force, wave run-up and diffracted wave contour is examined and extensive re-
16 sults on them are presented. This parametric study will help determine the various hydrodynamic
17 effects of a concentric porous cylindrical structure.

18 **CE Database subject headings: FINITE ELEMENT METHOD; WAVE DIFFRACTION;**
19 **POROUS MEDIA; CYLINDER**

20 **INTRODUCTION**

21 Coastal and offshore structures are often constructed with one or more protective porous layers,

¹Senior Lecturer, Griffith School of Engineering, Griffith University, Queensland 4222. E-mail: l.tao@griffith.edu.au

²Post-Doc. Research Fellow, Griffith School of Engineering, Griffith University, Queensland 4222. E-mail: h.song@griffith.edu.au

³President, Offshore Structure Analysis, Inc., Plainfield, IL60544-7096, USA. E-mail: Chakrab@aol.com

22 in order to reduce the direct wave impact on them. Examples are rock-filled porous breakwaters
23 outside harbors, and concentric porous outer protective structure with the main structure in its
24 interior. One example of this application is the successful Ekofisk gravity offshore structure in the
25 North Sea (see Fig. 1). For these reasons, wave motion through a porous structure has attracted
26 considerable attention among researchers in coastal and ocean engineering (e.g., Vijayalakshmi et
27 al., 2007).

28 Investigations on waves past a porous structure have primarily been concentrated on three en-
29 gineering applications: 1) the hydrodynamic effect of a porous structure on incoming wave trains;
30 2) the use of a porous structure as a wavemaker; and 3) the use of a porous structure as a breakwa-
31 ter in a harbor (Chwang and Chan, 1998). In most cases, Darcy's law for a homogeneous porous
32 medium has been applied.

33 A brief overview of the chronological development of research on porous structures is pre-
34 sented below. Tuck (1971) and Porter (1972) derived formal solutions using potential theory for
35 the transmission of water waves through a thin plate with a small gap in infinite water depth.
36 Guiney et al. (1972) extended Tuck's (1971) theory to incorporate a finite barrier thickness and ob-
37 tained results, which were validated by a set of experiments. The thickness of the porous structure
38 was found to have a pronounced effect on reducing energy transmission. Assuming normally inci-
39 dent waves on a long homogeneous porous structure of rectangular cross section, Madsen (1974)
40 presented a simple explicit solution for the transmission and reflection coefficients using a lin-
41 earized theory. Nasser and McCorquodale (1975) studied unsteady non-Darcy flow in rectangular
42 rock-filled embankments with impervious cores, in which the effect of nonlinear, shallow water
43 waves were examined through wave tank experiments.

44 Chwang (1983) proposed a linearized porous-wavemaker theory to analyze small-amplitude
45 surface waves produced by horizontal oscillations of a porous vertical plate. The theory was later
46 applied to analyze surface waves generated by a piston-type porous wavemaker near the end of a
47 semi-infinitely long channel of constant depth (Chwang and Li, 1983). Several studies on the phys-
48 ical phenomenon of wave-trapping due to a porous plate or a concentric porous cylinder system

49 using the porous-wavemaker theory were reported (e.g., Chwang and Dong, 1984; Faltas, 1996).

50 Dalrymple et al. (1991) studied the reflection and transmission of a wave train at an oblique
51 angle of incidence by an infinitely long porous breakwater. Huang and Chao (1992) studied the
52 inertial effect of the porous breakwater based on Biot's theory of poroelasticity. Yu and Chwang
53 (1994a) investigated the resonance in a harbor with porous breakwaters with the wave entering
54 at an arbitrary angle. Yu and Chwang (1994b) performed extensive study on the transmission
55 characteristics of waves past a porous structure. The wave behavior within the porous medium
56 was also investigated. It was found that there is an optimum thickness for a porous structure
57 beyond which any further increase of the thickness may not lead to an appreciable improvement
58 in reducing its transmission and reflection characteristics. Yu and Chwang (1994c) employed the
59 boundary integral method to study wave diffraction by a horizontal porous plate submerged at a
60 distance below the free surface in a fluid of constant depth. Chwang and Wu (1994) extended this
61 study to wave diffraction by a porous disk. Wang and Ren (1993) also studied the performance
62 of a flexible and porous breakwater. Additional related work can be found in the review article of
63 Chwang and Chan (1998).

64 Though considerable research efforts have been devoted to the wave interaction with porous
65 structures, relatively limited attention has been focused on the wave diffraction by a concentric
66 bottom-mounted porous cylindrical structure, where the interior cylinder is impermeable and the
67 exterior cylinder is thin and porous. Wang and Ren (1994) investigated analytically the plane wave
68 diffraction by the above-mentioned system. They found that hydrodynamic forces on the interior
69 cylinder as well as wave amplitudes around the windward side of the interior cylinder are reduced
70 when compared to the case of a direct wave impact on the interior cylinder. As the annular spacing
71 increases, the hydrodynamic force on the interior cylinder decreases. It was further shown that,
72 as the porosity of the exterior cylinder increases, the hydrodynamic force on the interior cylinder
73 increases. Li et al. (2003) reported similar results from their experimental and numerical study
74 with only partial porosity in the circumferential direction of the exterior cylinder. Darwiche et
75 al. (1994) studied the wave diffraction by a two-cylinder system, with the exterior cylinder being

76 porous only in the vicinity of free surface. Williams and Li (1998) further extended the work by
77 mounting the interior cylinder on a storage tank. Zhong and Wang (2006) obtained solitary wave
78 solutions for a concentric porous cylinder system.

79 The understanding of the problem, however, is still far from being complete. One of the most
80 noticeable limitations in the previous studies is the two-dimensional plane wave assumption. It is
81 noted that wind-generated waves in real oceans are much better represented by short-crested waves
82 (3D) than by plane waves (2D) (Zhu, 1993; Zhu and Moule, 1994). Multi-directional waves may
83 also arise, from the oblique interaction of two travelling plane waves or intersecting swell waves,
84 from the reflection of waves at non-normal incidence off a vertical seawall or a breakwater, as
85 well as from diffraction about the surface boundaries of a structure of finite length (Fuhrman and
86 Madsen, 2006). Such waves are of paramount importance in an engineering design. Unlike the
87 plane waves propagating in a single direction, and the standing waves fluctuating vertically in a
88 confined region, short-crested waves can be doubly periodic in two horizontal directions, one in
89 the direction of propagation and the other normal to it (Tsai et al., 1994).

90 Zhu (1993) studied the diffraction problem for a circular cylinder in short-crested waves using
91 linear potential wave theory and found that the pressure distribution and wave run-up on the cylin-
92 der were quite different from those of plane incident waves. Their patterns become very complex
93 as ka (i.e., total incident wave number k times cylinder radius a .) becomes large. The hydrody-
94 namic forces on the cylinder become smaller as the spreading of the incident short-crested waves
95 increases. Subsequently, Zhu and Moule (1994) observed that the hydrodynamic force induced by
96 short-crested waves varies with the phasing of the waves along the spreading angle.

97 However, research on the short-crested wave interaction with a concentric porous cylindrical
98 structure has been relatively few. Moreover, questions may arise whether the conclusions drawn
99 by Zhu (1993) is still valid for a porous concentric cylindrical structure. Also, does the free surface
100 in the annular space remain short-crested and what about the lee region?

101 The present study aims at answering these questions in a quantitative manner by developing a
102 new numerical model. Recently, a semi-analytical method, called scaled boundary finite-element

103 method (SBFEM) for solving linear partial differential equations has found successful applica-
104 tion to soil-structure interaction problems. The SBFEM method was proposed by Song and Wolf
105 (1997) and systematically described by Wolf (2003). Combining the distinct advantages of the
106 finite-element and boundary-element methods (BEM), only the structure boundary is discretized
107 with surface finite-elements. This, in turn, transforms the governing partial differential equations
108 to a set of ordinary differential equations, and solves them analytically. The method represents
109 singularities and unbounded domains accurately and efficiently when compared to the complete
110 finite-element method and requires no fundamental solution as needed by the boundary-element
111 method. Fewer elements are required to obtain very accurate results (Wolf, 2003).

112 Deeks and Cheng (2003) developed a scaled boundary finite-element solution to the two-
113 dimensional uniform potential flow around obstacles and revealed its inherent ability and advan-
114 tages to model unbounded fluid problems, as well as, the singular points in the near field of bluff
115 obstacles.

116 Only recently has the SBFEM been applied to wave diffraction in which the radiation condi-
117 tion at infinity is required to be satisfied by both the incident and scattered waves. Li et al. (2004)
118 attempted to solve the problem of plane wave diffraction by a vertical cylinder using SBFEM with
119 only limited success in obtaining semi-analytical solution for high frequency waves. Similar to
120 the approach of Wolf (2003) in obtaining a solution for soil-structure interaction, Li et al. (2004)
121 adopted a power series in the form $(\sum_{m=0}^{+\infty} C_m \bar{\xi}^{-m})$. Since the solution obtained as asymptotic ex-
122 pansion involves sums to infinity, for large values of $\bar{\xi}$ the series approaches the exact solution
123 rapidly and only a few terms in the series need to be computed. However, this is only the case at
124 the cylinder boundary ($\bar{\xi} = ka$, where k and a denote the wave number and the radius of the cylin-
125 der respectively) for high frequency waves. For low frequency waves, the series converges very
126 slowly, hardly approaching the exact solution. Therefore, additional numerical methods, such as,
127 the Runge-Kutta scheme, were proposed to solve the radial differential equation for low frequency
128 waves. Such compromised approach, however, significantly diminishes the advantages of SBFEM
129 exhibited in dealing with bounded fluid domain problems.

130 Tao et al. (2007) applied the SBFEM to solve wave interaction with a single circular cylinder.
131 Instead of using a power series, Tao et al. (2007) chose Hankel function to solve the Helmholtz
132 equation in the unbounded domain. The radial differential equation is solved fully analytically in
133 all frequency ranges. Without relying on any other numerical schemes, the semi-analytical model
134 for the plane-wave diffraction by a single circular cylinder is shown to reproduce the analytical so-
135 lution for all the physical properties including wave run-up, effective inertia and drag coefficients,
136 and total force very accurately and at very low computational cost.

137 This paper develops a SBFEM model for simulations of short-crested wave interaction with
138 a concentric porous cylindrical structure. The presence of the wave-structure interaction in both
139 the bounded and unbounded domains imposes particular challenges to the numerical model devel-
140 opment. The following section provides description of the governing equations and summarizes
141 it for the particular physical problem. Following this section, the transformation of the above
142 governing scaled boundary finite-element equations and the corresponding boundary conditions
143 are presented and the detailed solution techniques are discussed. Only a few finite elements dis-
144 cretized on the circumference of the cylinders are shown to be sufficient to obtain accurate results.
145 Finally, detailed numerical results on wave forces and surface elevations over a broad range of
146 incident short-crested wave parameters as well as structure configurations including the porosity
147 of the exterior cylinder and the annular gap between the two cylinders are presented.

148 **MATHEMATICAL FORMULATION**

149 Consider a monochromatic short-crested wave train propagating in the direction of the positive
150 x axis. A structure consisting of two concentric fixed vertical cylinders extend from the sea bottom
151 to above the free surface of the ocean along z axis. The origin is placed at the center of the
152 cylinders on the mean water surface (see Fig. 2). The exterior cylinder is made porous and the
153 interior cylinder is impermeable. The fluid domain is divided into two regions, the annular region
154 Ω_1 and the region of the outside of the exterior cylinder Ω_2 . The following notation have been used
155 in the paper: Φ_j = total velocity potential, Φ^I = velocity potential of incident wave, Φ^S = velocity
156 potential of scattered wave, k = total wave number, k_x = wave number in x direction, k_y = wave

157 number in y direction, ω = wave frequency, h = water depth, A = amplitude of incident wave,
 158 a = interior cylinder radius, b = exterior cylinder radius, t = time, ρ = mass density of water, and
 159 g = gravitational acceleration. The subscripts j ($j = 1, 2$) denote the physical parameters in the
 160 region Ω_j ($j = 1, 2$).

161 Assuming the fluid to be inviscid, incompressible and the flow to be irrotational, the fluid
 162 motion can be described by a velocity potential Φ_j satisfying the Laplace equation

$$\nabla^2 \Phi_j(x, z) = 0 \quad \text{in } \Omega_j, \quad (1)$$

163 subject to the combined free surface boundary condition

$$\Phi_{j,tt} + g\Phi_{j,z} = 0 \quad \text{at } z = 0, \quad (2)$$

164 and the bottom condition

$$\Phi_{j,z} = 0 \quad \text{at } z = -h, \quad (3)$$

165 where the comma in the subscript designates partial derivative with respect to the variable follow-
 166 ing the comma.

167 The velocity potentials can be decomposed by separating the vertical variable z and the time t
 168 from each component as

$$\Phi_j(x, y, z, t) = \phi_j(x, y)Z(z)e^{-i\omega t} \quad \text{in } \Omega_j, \quad (4)$$

$$\Phi_2^I(x, y, z, t) = \phi_2^I(x, y)Z(z)e^{-i\omega t} \quad \text{in } \Omega_2, \quad (5)$$

$$\Phi_2^S(x, y, z, t) = \phi_2^S(x, y)Z(z)e^{-i\omega t} \quad \text{in } \Omega_2, \quad (6)$$

169 where

$$Z(z) = \frac{\cosh k(z+h)}{\cosh kh}. \quad (7)$$

170 This procedure leads to the sea bottom condition being automatically satisfied, and the lin-
 171 earized free surface boundary condition is satisfied using the following dispersion relationship

$$\omega^2 = gk \tanh kh. \quad (8)$$

172 The relationship between the total velocity potential, and the scattered, and incident wave ve-
 173 locity potentials is

$$\Phi_2 = \Phi_2^I + \Phi_2^S, \quad \phi_2 = \phi_2^I + \phi_2^S \quad \text{in } \Omega_2. \quad (9)$$

174 Taylor (1956) showed that the fluid flow passing through the porous boundary can be essentially
 175 assumed to obey Darcy's law if the boundary is made of fine pores. Hence, the porous flow velocity
 176 is linearly proportional to the pressure difference between the two sides of the porous boundary,
 177 and the boundary condition on exterior porous cylinder can be expressed as

$$\phi_{1,n} = -\phi_{2,n} = iG_0k(\phi_1 - \phi_2) \quad \text{on } r = b, \quad (10)$$

178 where $G_0 = \frac{\rho\omega d}{\mu k}$ (Chwang and Li, 1983), is a measure of the porous effect, μ is the coefficient
 179 of dynamic viscosity, d is a material constant having the dimension of a length, and n denotes the
 180 normal to the boundary. Such an approach was firstly applied to wave study by Chwang (1983)
 181 and subsequently by many other researchers.

182 Thus, the problem becomes to two-dimensional at the free surface. The function $\phi_2^S(x, y)$ in
 183 Ω_2 is governed by the Helmholtz equation with the boundary condition at the interface of fluid
 184 and porous cylinder, and the radiation condition at infinity, namely, the Sommerfeld condition as
 185 follows:

$$\nabla^2 \phi_2^S + k^2 \phi_2^S = 0 \quad \text{in } \Omega_2, \quad (11)$$

$$\phi_{2,n}^S = -iG_0k(\phi_1 - \phi_2^S - \phi_2^I) - \phi_{2,n}^I \quad \text{on } r = b, \quad (12)$$

$$\lim_{kr \rightarrow \infty} (kr)^{1/2} (\phi_{2,r}^S - ik\phi_2^S) = 0 \quad \text{in } \Omega_2, \quad (13)$$

186 where r is the radial axis, and $i = \sqrt{-1}$.

187 The function $\phi_1(x, y)$ in Ω_1 is governed by the Helmholtz equation with the boundary condi-
 188 tions at the interface of fluid and interior solid cylinder at $r = a$ and exterior porous cylinder at
 189 $r = b$:

$$\nabla^2 \phi_1 + k^2 \phi_1 = 0 \quad \text{in } \Omega_1, \quad (14)$$

$$\phi_{1,n} = 0 \quad \text{on } r = a, \quad (15)$$

$$\phi_{1,n} = iG_0k(\phi_1 - \phi_2^S - \phi_2^I) \quad \text{on } r = b. \quad (16)$$

190 The velocity potential of the linear short-crested incident wave (Fuchs, 1952) travelling princi-
 191 pally in the positive x direction is given by the real part of

$$\Phi_I = -\frac{igA}{\omega} Z(z) e^{i(k_x x - \omega t)} \cos(k_y y) \quad \text{in } \Omega_2. \quad (17)$$

192 Eqs. (11)-(16) constitute two sets of the governing equation and boundary conditions for the
 193 diffraction of short-crested waves by concentric vertical porous cylindrical structure, correspond-
 194 ing to boundary value problems in a bounded domain and an unbounded domain respectively. After
 195 obtaining ϕ_2^S , Φ_2 and Φ_1 by solving the above boundary-value problems, the velocity, free surface

196 elevation and the dynamic pressure can be calculated respectively from

$$\mathbf{v}_j = \nabla \Phi_j, \quad (18)$$

$$\eta_j = \frac{i\omega}{g} \phi_j, \quad (19)$$

$$p_j = -\rho \Phi_{j,t}. \quad (20)$$

197 **SCALED BOUNDARY FINITE-ELEMENT TRANSFORMATION**

198 In this section, ϕ_1 and ϕ_2^S will both be denoted as ϕ for brevity, and the region Ω_j will be
 199 denoted as Ω . If the velocity boundary is defined by Γ_v , we have

$$\phi_{,n} = \bar{v}_n, \quad \text{on } \Gamma_v, \quad (21)$$

200 where the overbar denotes a prescribed value.

201 The finite-element method requires the weighted residuals of the governing equation to be zero.
 202 Hence Eqs. (11), (14) and (21) are multiplied by a weighting function w and integrated over the
 203 flow domain and the boundary. Performing integration by parts, the resulting equation becomes

$$\int_{\Omega} \nabla^T w \nabla \phi d\Omega - \int_{\Omega} w k^2 \phi d\Omega - \oint_{\Gamma} w \bar{v}_n d\Gamma = 0. \quad (22)$$

204 **SBFEM** defines the domain Ω by scaling a single piecewise-smooth curve S relative to a scaling
 205 center (x_0, y_0) , which is chosen at the cylinder center in this case (see Fig. 3). The circumferential
 206 coordinate s is anticlockwise along the curve S and the normalized radial coordinate ξ is a scaling
 207 factor, defined as 1 at curve S and 0 at the scaling center. The whole solution domain Ω is in the
 208 range of $\xi_0 \leq \xi \leq \xi_1$ and $s_0 \leq s \leq s_1$. The two straight sections $s = s_0$ and $s = s_1$ are called
 209 side-faces. They coincide, if the curve S is closed. For bounded domain, $\xi_0 = 0$ and $\xi_1 = 1$;

210 whereas, for unbounded domain, $\xi_0 = 1$ and $\xi_1 = \infty$. Therefore the Cartesian coordinates are
 211 transformed to the scaled boundary coordinate ξ and s with the scaling equations

$$x = x_0 + \xi x_s(s), \quad y = y_0 + \xi y_s(s). \quad (23)$$

212 By employing SBFEM, an approximate solution of ϕ is sought as

$$\phi_A(\xi, s) = \mathbf{N}(s)\mathbf{a}(\xi), \quad (24)$$

213 where $\mathbf{N}(s)$ is the shape function, the vector $\mathbf{a}(\xi)$ is analogous to the nodal values same as in FEM.
 214 The radial function $a_j(\xi)$ represents the variation of the scattered wave potential in the radial axis
 215 ξ at each node j , and the shape function $\mathbf{N}(s)$ interpolates between the nodal potential values in the
 216 circumferential axis s . The separation of variable technique has its limitation in terms of boundary
 217 surfaces (Morse and Feshbach, 1953), since a completely general boundary surface will have the
 218 value of A varying both as functions of ξ and s and these two variables may not be separated
 219 with independent functions. However, for many axisymmetric structures, e.g., which are indeed of
 220 interest for offshore applications, this technique will work well in which the two variables ξ and s
 221 may be separated as shown in Eq. (24).

222 By performing scaled boundary transformation, the operator ∇ can be expressed as (Wolf,
 223 2003):

$$\nabla = \mathbf{b}_1(s) \frac{\partial}{\partial \xi} + \frac{1}{\xi} \mathbf{b}_2(s) \frac{\partial}{\partial s}, \quad (25)$$

224 where $\mathbf{b}_1(s)$ and $\mathbf{b}_2(s)$ are dependent only on the boundary definition

$$\mathbf{b}_1(s) = \frac{1}{|J|} \begin{Bmatrix} y_s(s),s \\ -x_s(s),s \end{Bmatrix}, \quad \mathbf{b}_2(s) = \frac{1}{|J|} \begin{Bmatrix} -y_s(s) \\ x_s(s) \end{Bmatrix}, \quad (26)$$

225 and $|J|$ is the Jacobian at the boundary

$$|J| = x_s(s)y_s(s),_s - y_s(s)x_s(s),_s. \quad (27)$$

226 From Eqs. (18) and (25), the approximate velocity can be expressed as

$$v_A(\xi, s) = \mathbf{B}_1(s)\mathbf{a}(\xi),_\xi + \frac{1}{\xi}\mathbf{B}_2(s)\mathbf{a}(\xi), \quad (28)$$

227 where

$$\mathbf{B}_1(s) = \mathbf{b}_1(s)\mathbf{N}(s), \quad \mathbf{B}_2(s) = \mathbf{b}_2(s)\mathbf{N}(s),_s. \quad (29)$$

228 Applying the Galerkin approach, the weighting function w can be formulated using the same

229 shape function as in Eq. (24)

$$w(\xi, s) = \mathbf{N}(s)\mathbf{w}(\xi) = \mathbf{w}(\xi)^T\mathbf{N}(s)^T. \quad (30)$$

230 Substituting Eqs. (24), (25), (29) and (30) into Eq. (22) results in

$$\begin{aligned} & \int_{\Omega} \left[\mathbf{B}_1(s)\mathbf{w}(\xi),_\xi + \frac{1}{\xi}\mathbf{B}_2(s)\mathbf{w}(\xi) \right]^T \left[\mathbf{B}_1(s)\mathbf{a}(\xi),_\xi + \frac{1}{\xi}\mathbf{B}_2(s)\mathbf{a}(\xi) \right] d\Omega \\ & - \int_{\Omega} k^2\mathbf{w}(\xi)^T\mathbf{N}(s)^T\mathbf{N}(s)\mathbf{a}(\xi)d\Omega - \oint_{\Gamma} \mathbf{w}(\xi)^T\mathbf{N}(s)^T\bar{v}_n d\Gamma = 0, \end{aligned} \quad (31)$$

231 where the incremental volume (Wolf, 2003) is

$$d\Omega = |J|\xi d\xi ds. \quad (32)$$

232 For convenience, coefficient matrices are introduced here as

$$\mathbf{E}_0 = \int_S \mathbf{B}_1(s)^T \mathbf{B}_1(s) |J| ds, \quad (33)$$

$$\mathbf{E}_1 = \int_S \mathbf{B}_2(s)^T \mathbf{B}_1(s) |J| ds, \quad (34)$$

$$\mathbf{E}_2 = \int_S \mathbf{B}_2(s)^T \mathbf{B}_2(s) |J| ds, \quad (35)$$

$$\mathbf{M}_0 = \int_S \mathbf{N}(s)^T \mathbf{N}(s) |J| ds, \quad (36)$$

$$\mathbf{F}_s(\xi) = \mathbf{N}(s_0)^T(-\bar{v}_n(\xi, s_0)) |J(s_0)| + \mathbf{N}(s_1)^T(-\bar{v}_n(\xi, s_1)) |J(s_1)|. \quad (37)$$

233 The above integrals Eqs. (33)-(36) can be computed element by element and assembled to-
 234 gether for the entire boundary. Expanding Eq. (31) and integrating the terms containing $\mathbf{w}(\xi)_{,\xi}$ by
 235 parts with respect to ξ using Green's theorem leads to

$$\begin{aligned} & \mathbf{w}(\xi_1)^T \left[\mathbf{E}_0 \xi_1 \mathbf{a}(\xi_1)_{,\xi} + \mathbf{E}_1^T \mathbf{a}(\xi_1) - \int_S \mathbf{N}(s)^T (\bar{v}_n(\xi_1, s)) \xi_1 ds \right] \\ & - \mathbf{w}(\xi_0)^T \left[\mathbf{E}_0 \xi_0 \mathbf{a}(\xi_0)_{,\xi} + \mathbf{E}_1^T \mathbf{a}(\xi_0) + \int_S \mathbf{N}(s)^T (\bar{v}_n(\xi_0, s)) \xi_0 ds \right] \\ & - \int_{\xi_0}^{\xi_1} \mathbf{w}(\xi)^T \left[\mathbf{E}_0 \xi \mathbf{a}(\xi)_{,\xi\xi} + (\mathbf{E}_0 + \mathbf{E}_1^T - \mathbf{E}_1) \mathbf{a}(\xi)_{,\xi} - \mathbf{E}_2 \frac{1}{\xi} \mathbf{a}(\xi) + k^2 \xi \mathbf{M}_0 \mathbf{a}(\xi) - \mathbf{F}_s(\xi) \right] d\xi \\ & = 0. \end{aligned} \quad (38)$$

236 To satisfy all sets of weighting function $\mathbf{w}(\xi)$, the following conditions must be satisfied:

$$\mathbf{E}_0 \xi_1 \mathbf{a}(\xi_1)_{,\xi} + \mathbf{E}_1^T \mathbf{a}(\xi_1) = \int_S \mathbf{N}(s)^T (\bar{v}_n(\xi_1, s)) \xi_1 ds, \quad (39)$$

$$\mathbf{E}_0 \xi_0 \mathbf{a}(\xi_0)_{,\xi} + \mathbf{E}_1^T \mathbf{a}(\xi_0) = - \int_S \mathbf{N}(s)^T (\bar{v}_n(\xi_0, s)) \xi_0 ds, \quad (40)$$

$$\mathbf{E}_0 \xi^2 \mathbf{a}(\xi)_{,\xi\xi} + (\mathbf{E}_0 + \mathbf{E}_1^T - \mathbf{E}_1) \xi \mathbf{a}(\xi)_{,\xi} - \mathbf{E}_2 \mathbf{a}(\xi) + k^2 \xi^2 \mathbf{M}_0 \mathbf{a}(\xi) = \xi \mathbf{F}_s(\xi). \quad (41)$$

237 Eq. (41) is the so-called scaled boundary finite-element equation. By introducing the shape

238 function, the Helmholtz equation has been weakened in the circumferential direction, so that the
 239 governing partial differential equation is transformed to an ordinary matrix differential equation
 240 in radial direction. The rank of matrices \mathbf{E}_0 , \mathbf{E}_1 , \mathbf{E}_2 , \mathbf{M}_0 and vector $\mathbf{a}(\xi)$ is m (where m is the
 241 number of nodes in the curve S). In the present study, the side-faces coincide so that the flow
 242 across the side-faces is equal and opposite, leading to vanishing of the term $\mathbf{F}_s(\xi)$. Therefore, the
 243 final governing equation, Eq. (41), is a homogeneous second-order ordinary matrix differential
 244 equation in terms of matrix of rank m .

245 Boundary conditions, Eqs. (12) and (13) or Eqs. (15) and (16), are weakened in the form of
 246 Eqs. (40) and (39) respectively, indicating the relationship between the integrated nodal flow on
 247 the boundary and the velocity potentials of the nodes. For the wave diffraction problem in the
 248 unbounded region Ω_2 , $\xi_0 = 1$ on the boundary of exterior porous cylinder and $\xi_1 = +\infty$ at infinity.
 249 For the boundary-value problem in the bounded annular region Ω_1 , $\xi_0 = 0$ and $\xi_1 = 1$.

250 SOLUTION PROCEDURE

251 For the exterior porous cylinder, we have

$$x_s(s) = b \cos(s/b), \quad y_s(s) = b \sin(s/b). \quad (42)$$

252 From Eqs. (23), (26), (27), (29) and (33)-(36), $x_s(s)_{,s}$, $y_s(s)_{,s}$, $\mathbf{b}_1(s)$, $\mathbf{b}_2(s)$, $|J|$, $\mathbf{B}_1(s)$, $\mathbf{B}_2(s)$,
 253 \mathbf{E}_0 , \mathbf{E}_1 , \mathbf{E}_2 , and \mathbf{M}_0 can be calculated accordingly. The following relationships hold:

$$\mathbf{E}_1 = 0 \cdot \mathbf{I}, \quad \mathbf{E}_0^{-1} \mathbf{M}_0 = b^2 \mathbf{I}, \quad (43)$$

$$\mathbf{E}_0 = \frac{1}{b} \int_S \mathbf{N}(s)^T \mathbf{N}(s) ds, \quad (44)$$

254 where \mathbf{I} is the identity matrix of rank m .

255 Using Eq. (43), pre-multiplying both sides of Eq. (41) by \mathbf{E}_0^{-1} and simplifying, we have

$$\zeta^2 \mathbf{a}(\zeta)_{,\zeta\zeta} + \zeta \mathbf{a}(\zeta)_{,\zeta} - \mathbf{E}_0^{-1} \mathbf{E}_2 \mathbf{a}(\zeta) + \zeta^2 \mathbf{a}(\zeta) = 0, \quad (45)$$

256 where

$$\zeta = kb\xi. \quad (46)$$

257 **Solution for unbounded domain Ω_2**

258 Eq. (45) is the matrix form of Bessel's differential equation. Considering the Sommerfeld
 259 radiation condition Eq. (13), it is logical to select $H_{r_j}(\zeta)\mathbf{T}_j$ as a base solution of Eq. (45) in region
 260 Ω_2 .

261 The solution for $\mathbf{a}_2(\zeta)$ is then expressed in the series form:

$$\mathbf{a}_2(\zeta) = \sum_{j=1}^m c_j H_{r_j}(\zeta) \mathbf{T}_j = \mathbf{TH}(\zeta)\mathbf{C}, \quad (47)$$

262 where \mathbf{T}_j are vectors of rank m , c_j are coefficients, $H_{r_j}(\zeta)$ are the Hankel functions of the first
 263 kind, and

$$\mathbf{T} = [\mathbf{T}_1, \mathbf{T}_2, \dots, \mathbf{T}_m], \quad (48)$$

$$\mathbf{H}(\zeta) = \text{diag}[H_{r_1}(kb\xi), H_{r_2}(kb\xi), \dots, H_{r_m}(kb\xi)], \quad (49)$$

$$\mathbf{C} = [c_1, c_2, \dots, c_m]^T, \quad (50)$$

264 where "diag" denotes a diagonal matrix with the elements in the square brackets on the main
 265 diagonal.

266 Substituting Eq. (47) into Eq. (45), and using the following properties of Hankel function

$$\zeta^2 H_{r_j}''(\zeta) = -\zeta^2 H_{r_j}(\zeta) + \zeta H_{r_{j+1}}(\zeta) - r_j H_{r_j}(\zeta) + r_j^2 H_{r_j}(\zeta), \quad (51)$$

$$\zeta H_{r_j}'(\zeta) = -\zeta H_{r_{j+1}}(\zeta) + r_j H_{r_j}(\zeta), \quad (52)$$

267 where the prime and the double prime denote the first and second derivatives with respect to the
 268 argument ζ respectively, we have

$$\sum_{j=1}^m (\mathbf{E}_0^{-1} \mathbf{E}_2 - r_j^2 \mathbf{I}) \mathbf{T}_j \cdot c_j H_{r_j}(\zeta) = 0. \quad (53)$$

269 For any $c_j H_{r_j}(\zeta)$, Eq. (53) yields

$$(\mathbf{E}_0^{-1} \mathbf{E}_2 - r_j^2 \mathbf{I}) \mathbf{T}_j = 0. \quad (54)$$

270 Let λ_j be the eigenvalues of $\mathbf{E}_0^{-1} \mathbf{E}_2$, then $r_j = \sqrt{\lambda_j}$, and \mathbf{T}_j are the eigenvectors of $\mathbf{E}_0^{-1} \mathbf{E}_2$.

271 Since the Sommerfeld radiation condition (13) or (39) has been satisfied by the Hankel func-
 272 tions, we now only consider the body boundary condition (40) of the circular cylinder

$$\mathbf{E}_0 k b \sum_{j=1}^m c_j H'_{r_j}(k b) \mathbf{T}_j = - \left[\int_S \mathbf{N}(s)^T \mathbf{N}(s) ds \right] \bar{\mathbf{v}}_{2n}^S, \quad (55)$$

273 where $\bar{\mathbf{v}}_{2n}^S$ is the vector of nodal normal velocity of scattered wave in region Ω_2 on the body bound-
 274 ary.

275 **Solution for bounded domain Ω_1**

276 Similar approach is applied to the region Ω_1 . Assume

$$\mathbf{a}_1(\zeta) = \sum_{j=1}^m [c_j^1 J_{r_j}(\zeta) + c_j^2 Y_{r_j}(\zeta)] \mathbf{T}_j = \mathbf{T} [\mathbf{J}(\zeta) \mathbf{C}^1 + \mathbf{Y}(\zeta) \mathbf{C}^2], \quad (56)$$

277 where c_j^1 and c_j^2 are coefficients, $J_{r_j}(\zeta)$ are the Bessel functions of the first kind, $Y_{r_j}(\zeta)$ are the
 278 Bessel functions of the second kind, and

$$\mathbf{C}^1 = [c_1^1, c_2^1, \dots, c_m^1]^T, \quad (57)$$

$$\mathbf{C}^2 = [c_1^2, c_2^2, \dots, c_m^2]^T, \quad (58)$$

$$\mathbf{J}(\xi) = \text{diag}[J_{r_1}(kb\xi), J_{r_2}(kb\xi), \dots, J_{r_m}(kb\xi)], \quad (59)$$

$$\mathbf{Y}(\xi) = \text{diag}[Y_{r_1}(kb\xi), Y_{r_2}(kb\xi), \dots, Y_{r_m}(kb\xi)]. \quad (60)$$

279 Again if λ_j is the eigenvalues of $\mathbf{E}_0^{-1}\mathbf{E}_2$, then $r_j = \sqrt{\lambda_j}$, and \mathbf{T} is the eigenvector of $\mathbf{E}_0^{-1}\mathbf{E}_2$.

280 Applying boundary conditions on the interior cylinder Eq. (40) and the exterior porous cylinder

281 Eq. (39), respectively, we have

$$\mathbf{C}^2 = -\mathbf{Y}_a'^{-1}\mathbf{J}_a'\mathbf{C}^1, \quad (61)$$

$$\mathbf{E}_0 kb \mathbf{T}(\mathbf{J}_b'\mathbf{C}^1 + \mathbf{Y}_b'\mathbf{C}^2) = \left[\int_S \mathbf{N}(s)^T \mathbf{N}(s) ds \right] \bar{\mathbf{v}}_{1n}, \quad (62)$$

282 where $\bar{\mathbf{v}}_{1n}$ is the vector of nodal total normal velocity in region Ω_1 on the body boundary of exterior
283 cylinder.

284 Combining Eqs. (10), (21), (24), (44), (47), (55), (56) and (62), and noting

$$\bar{\mathbf{v}}_{2n}^I + \bar{\mathbf{v}}_{2n}^S = \bar{\mathbf{v}}_{2n} = -\bar{\mathbf{v}}_{1n}, \quad (63)$$

285 where $\bar{\mathbf{v}}_{2n}^I$ is the vector of nodal normal velocity of incident wave in the region Ω_2 on the body

286 boundary of the exterior cylinder, $\mathbf{a}_1(\xi)$ and $\mathbf{a}_2(\xi)$ are solved as

$$\mathbf{a}_1(\xi) = \mathbf{T}[\mathbf{J}(\xi)\mathbf{Y}_a' - \mathbf{Y}(\xi)\mathbf{J}_a']\mathbf{W}^{-1}(\mathbf{T}^{-1}\bar{\mathbf{a}}_2^I + \mathbf{H}_{bh}\mathbf{T}^{-1}\bar{\mathbf{v}}_{2n}^I/k), \quad (64)$$

$$\mathbf{a}_2(\xi) = \mathbf{T}\mathbf{H}_h(\xi)\mathbf{W}^{-1}[(\mathbf{J}_b'\mathbf{Y}_a' - \mathbf{Y}_b'\mathbf{J}_a')\mathbf{T}^{-1}\bar{\mathbf{a}}_2^I + \mathbf{V}\mathbf{T}^{-1}\bar{\mathbf{v}}_{2n}^I/k], \quad (65)$$

287 where

$$\mathbf{V} = -\frac{1}{iG}(\mathbf{J}'_b \mathbf{Y}'_a - \mathbf{Y}'_b \mathbf{J}'_a) + (\mathbf{J}_b \mathbf{Y}'_a - \mathbf{Y}_b \mathbf{J}'_a), \quad (66)$$

$$\mathbf{W} = \mathbf{V} - \mathbf{H}_{bh}(\mathbf{J}'_b \mathbf{Y}'_a - \mathbf{Y}'_b \mathbf{J}'_a), \quad (67)$$

288 and

$$\mathbf{J}'_a = \text{diag}[J'_{r_1}(ka), J'_{r_2}(ka), \dots, J'_{r_m}(ka)], \quad (68)$$

$$\mathbf{Y}'_a = \text{diag}[Y'_{r_1}(ka), Y'_{r_2}(ka), \dots, Y'_{r_m}(ka)], \quad (69)$$

$$\mathbf{J}_b = \text{diag}[J_{r_1}(kb), J_{r_2}(kb), \dots, J_{r_m}(kb)], \quad (70)$$

$$\mathbf{Y}_b = \text{diag}[Y_{r_1}(kb), Y_{r_2}(kb), \dots, Y_{r_m}(kb)], \quad (71)$$

$$\mathbf{J}'_b = \text{diag}[J'_{r_1}(kb), J'_{r_2}(kb), \dots, J'_{r_m}(kb)], \quad (72)$$

$$\mathbf{Y}'_b = \text{diag}[Y'_{r_1}(kb), Y'_{r_2}(kb), \dots, Y'_{r_m}(kb)], \quad (73)$$

$$\mathbf{H}_h(\xi) = \text{diag}[H_{r_1}(kb\xi)/H'_{r_1}(kb), H_{r_2}(kb\xi)/H'_{r_2}(kb), \dots, H_{r_m}(kb\xi)/H'_{r_m}(kb)], \quad (74)$$

$$\mathbf{H}_{bh} = \text{diag}[H_{r_1}(kb)/H'_{r_1}(kb), H_{r_2}(kb)/H'_{r_2}(kb), \dots, H_{r_m}(kb)/H'_{r_m}(kb)]. \quad (75)$$

289 For the limiting case of short-crested wave interacting with a hollow porous cylinder, i.e. $a = 0$,

290 $\mathbf{a}_1(\xi)$ and $\mathbf{a}_2(\xi)$ are

$$\mathbf{a}_1(\xi) = \mathbf{TJ}(\xi)\mathbf{W}^{-1}(\mathbf{T}^{-1}\bar{\mathbf{a}}_2^I + \mathbf{H}_{bh}\mathbf{T}^{-1}\bar{\mathbf{v}}_{2n}^I/k), \quad (76)$$

$$\mathbf{a}_2(\xi) = \mathbf{TH}_h(\xi)\mathbf{W}^{-1}(\mathbf{J}'_b\mathbf{T}^{-1}\bar{\mathbf{a}}_2^I + \mathbf{VT}^{-1}\bar{\mathbf{v}}_{2n}^I/k), \quad (77)$$

291 where

$$\mathbf{V} = -\frac{1}{iG}\mathbf{J}'_b + \mathbf{J}_b, \quad (78)$$

$$\mathbf{W} = \mathbf{V} - \mathbf{H}_{bh}\mathbf{J}'_b. \quad (79)$$

292 Using Eqs. (4), (5), (17), (21) and (24), $\bar{\mathbf{v}}_{2n}^I$ and $\bar{\mathbf{a}}_2^I$ can be easily determined on the exterior
 293 cylinder boundary. From Eqs. (6), (9), (24), (64) and (65), the approximation of velocity potential
 294 in both region Ω_1 and region Ω_2 can be obtained.

295 All the other physical properties of engineering interest including velocity, surface elevation,
 296 and pressure can now be determined based on the velocity potentials by Eqs. (18)-(20). The
 297 total force per unit length on the cylinder in the principal direction of wave propagation is then
 298 calculated as:

$$\frac{dF_x}{dz} = -R \int_0^{2\pi} p \cdot \cos(\theta) d\theta = 2\pi RP(k_x, k_y, k, R) \cdot \rho g A \cdot Z(z) e^{-i\omega t}, \quad (80)$$

299 where the function $P(k_x, k_y, k, R)$ is the dimensionless parameter of dF_x/dz without the term
 300 $\rho g A \cdot Z(z) e^{-i\omega t}$ and R is the radius of the cylinder (a or b).

301 The function $P(k_x, k_y, k, R)$ determines the first-order total horizontal force in x direction on
 302 the cylinder, F_x , which can be obtained by integrating Eqs. (80) with respect to z ,

$$F_x = \int_{-h}^0 \frac{dF_x}{dz} dz = 2\pi RP(k_x, k_y, k, R) \cdot \rho g A e^{-i\omega t} \cdot \tanh(kh)/k. \quad (81)$$

303 The total moment about an axis parallel to the y -axis passing through the bottom of the cylinder
 304 is

$$M_y = \int_{-h}^0 (z + h) \frac{dF_x}{dz} dz = 2\pi RP_x(k_x, k_y, k, R) \cdot \rho g A e^{-i\omega t} \cdot f(kh)/k^2, \quad (82)$$

305 where

$$f(kh) = [kh \tanh(kh) + \operatorname{sech}(kh) - 1]. \quad (83)$$

306 It can be concluded from Eqs. (81) and (82) that only the function $P(k_x, k_y, k, R)$ needs to be
 307 discussed. The effective inertia coefficient C_M and linear drag coefficient C_D per unit height are
 308 defined as

$$\operatorname{Re} \left(\frac{dF_x}{dz} \right) = \rho\pi R^2 \left(C_M \dot{U} + \omega C_D U \right), \quad (84)$$

309 where U is the velocity of the incident waves at the origin of the cylinders in their absence and the
 310 dot represents its time derivative.

311 From Eqs. (17), (80) and (84), we have

$$C_M = -\frac{2P_i}{k_x R}, \quad C_D = \frac{2P_r}{k_x R}, \quad (85)$$

312 where P_r and P_i are the real and imaginary parts of $P(k_x, k_y, k, R)$ respectively.

313 From Eq. (85), the total horizontal force without the constant terms can be expressed as

$$|2\pi RP| = \pi k_x R^2 \sqrt{C_M^2 + C_D^2}. \quad (86)$$

314 NUMERICAL MODEL VALIDATION

315 The principal steps taken to validate the present SBFEM scheme and solution procedure are (1)
 316 convergence tests using varying number of elements discretized on the surface of the cylinders; (2)
 317 comparison with the available analytical solutions (Wang and Ren, 1994) of plane-wave interaction
 318 with a concentric porous cylindrical structure; and (3) comparison with the available analytical
 319 solutions (Zhu, 1993) of short-crested waves interacting with a single circular cylinder. Taking
 320 into account the symmetry of the physical problem, only half the concentric porous structure is
 321 considered. The exterior cylinder surface is discretized with three-node quadratic elements (see
 322 Fig. 4).

323 **Wave run-up: plane waves and short-crested waves**

324 Fig. 5 is a comparison of wave run-up around the interior cylinder and exterior porous cylinder,
325 between the present SBFEM results and the analytical solutions of Wang and Ren (1994). Results
326 shown in Fig. 5 used the same parameter values as in Wang and Ren (1994) (i.e., $h = 15$ m, $b = 10$
327 m, $a/b = 0.2$, $G_0 = 1.0$ and the wave-effect parameter $C_w = g/(\omega^2 h) = 1.0$). As can be seen
328 in the figure, only two elements discretized over the cylinder boundary are sufficient to yield good
329 agreement between the present SBFEM results and the analytical solutions.

330 The run-up on a single circular cylinder due to short-crested waves is shown as solid lines in
331 Fig. 6 from the analytical solutions of Zhu (1993). In order to compare these results with the
332 SBFEM calculation, the values of G_0 is chosen properly in its limits. The run-ups on the interior
333 cylinder and exterior cylinder are calculated with two limiting values assigned to the porous effect
334 G_0 for the exterior cylinder, namely, 0 and $+\infty$, respectively. When $G_0 = 0$, the exterior cylinder
335 becomes solid, so the fluid region Ω_1 remains at rest, and only the exterior cylinder is exposed to
336 incident waves. On the other hand, the interior cylinder will be a single cylinder exposed to the
337 incident wave as $G_0 = +\infty$. Other values are chosen to match the example of Zhu (1993), i.e.,
338 $a = 1$ m, $b = 2$ m, $k_x = 1$ m⁻¹ and $k_y = 1$ m⁻¹. Again, excellent agreement with the solutions of
339 Zhu (1993) is found where the SBFEM model with only 6 elements gave almost identical solutions
340 to the analytical approach. Moreover, the convergence of the SBFEM scheme for wave run-up, as
341 the number of elements discretized on the cylinder surface increases, is clearly shown in Fig. 6,
342 and is shown to depend on the dimensionless parameters ka and kb . The relationship between the
343 wave run-up and ka in the SBFEM model was examined in more detail in Tao et al. (2007).

344 **Wave forces**

345 The dimensionless wave forces on the interior cylinder and exterior porous cylinder due to a
346 plane wave *vs.* a/b for $G_0 = 1.0$ are plotted in Fig. 7. Again, in order to compare with the
347 analytical solutions of Wang and Ren (1994), the SBFEM results shown in the figure are calculated
348 using the same parameter values as in Wang and Ren (1994). It can be seen that the SBFEM
349 results calculated with only 4 surface elements on the cylinder boundary are almost identical to

350 the analytical solutions. The essential feature that the wave force on the exterior porous cylinder
 351 reduces to zero near $a/b = 0.2$ for $C_w = 0.4$ is reproduced extremely well by the present SBFEM
 352 model. In contrast to the wave run-up calculations, where more surface elements are required as
 353 ka and kb increase, it is observed from Fig. 7, that the convergence of the SBFEM model for the
 354 wave force computation is at best weakly dependent on the dimensionless parameter ka and kb .

355 Fig. 8 shows the variation of the total force $|2\pi RP|$ with the short-crestedness (i.e., k_y/k_x)
 356 of the waves. The SBFEM results on wave forces shown in Fig. 8 were calculated either on the
 357 interior cylinder with the porosity of the exterior cylinder $G_0 = +\infty$ and $a = 1$ m, or on the
 358 exterior cylinder with $G_0 = 0$ and $b = 1$ m. As can be seen in the figure, the SBFEM model gave
 359 almost identical solutions to the analytical solutions of Zhu (1993) with merely 6 surface elements
 360 discretized on the cylinder boundary. Note that the total force decreases rapidly as the incident
 361 wave becomes more and more short-crested (i.e., higher k_y/k_x values).

362 PARAMETRIC STUDY

363 Wave forces

364 *Influence of wave parameters*

365 Zhu (1993) calculated the effective inertia coefficient C_M and the linear drag coefficient C_D per
 366 unit height of a single solid cylinder, and the results were plotted against the variation of the ratio
 367 k_y/k_x . Four cases were calculated coincidental with the same $k_x a$ to show that C_M and C_D are
 368 invariants for a fixed ka . In order to examine whether this characteristic is present in a concentric
 369 porous cylindrical structure as well, four cases similar to Zhu (1993) are chosen: 1) $k_x = 0.4 \text{ m}^{-1}$,
 370 $a = 2.5$ m; 2) $k_x = 0.5 \text{ m}^{-1}$, $a = 2.0$ m; 3) $k_x = 0.8 \text{ m}^{-1}$, $a = 1.25$ m; and 4) $k_x = 1.0 \text{ m}^{-1}$,
 371 $a = 1.0$ m. In the present calculation, the ratio b/a is fixed at 2, thus giving $k_x b = 2$ and $k_x a = 1$
 372 for all the above four cases with porosity $G_0 = 1$. Two extreme cases for $G_0 = +\infty$ (inner cylinder
 373 only) and $a = 0$ (outer cylinder only) are also chosen for comparison. From Figs. 9 and 10, it is
 374 clearly seen that C_M and C_D are still invariants of ka and kb . Eq. (86) indicates that the total
 375 horizontal force is proportional to k_x while k , a and b are fixed. Thus the largest wave induced

376 forces for short-crested waves are smaller than those induced by plane waves with the same total
377 wave number because of the standing wave component.

378 The variations of the total horizontal forces $|2\pi RP|$ on the interior and exterior cylinders by
379 plane incident waves *vs.* ka are shown in Fig. 11 for $a = 1$ m, $b/a = 2$ and $G_0 = 1$. It can
380 be seen that the total horizontal forces fluctuate and peak attenuation is observed. It is interesting
381 to note that as the horizontal force on the interior cylinder (solid line) reaches a peak, the force
382 on the exterior cylinder (dashed line) reaches a trough. One the other hand, however, when the
383 horizontal force on the interior cylinder reaches a trough, the force on the exterior cylinder does
384 not correspondingly show a peak. The reduction in the amplitude of wave forces on the interior
385 cylinder in the presence of the exterior porous cylinder is clearly evident when compared to the
386 corresponding wave forces on the single impermeable cylinder (denoted by the dotted line in Fig.
387 11).

388 The variations of the total horizontal forces ($|2\pi RP|$) on the interior and exterior cylinders with
389 $G_0 = 1$ *vs.* the ratio k_y/k_x for different $k_x a$ values (k_x from 0.2 to 1.2 m^{-1}) with $a = 1$ m, $b = 2a$
390 are shown in Figs. 12 and 13 respectively. They show similar trends of peaks and troughs found in
391 Fig. 11. These variations of maximum wave forces on the interior and exterior cylinders provide a
392 means of minimizing wave loads on both cylinders.

393 Three types of variation patterns in wave forces are observed in Fig. 12. For small $k_x a$ ($k_x a =$
394 0.2), the horizontal force increases gently to a peak and then decreases mildly. For moderate
395 $k_x a$ ($k_x a = 0.4, 0.6$), the curves are seen to slowly translate leftwards (starting with larger ka).
396 Initially, the horizontal force is seen descending slightly, followed by a sharp drop off and finally
397 becoming nearly flat. For large $k_x a$ ($k_x a = 0.8, 1.0, 1.2$), the curve translates further leftwards. All
398 forces show a trend of dramatic dip to almost steady small values at large values of k_y/k_x . In Fig.
399 13, there are troughs near zero at each $k_x a$ indicating extremely low wave forces on the exterior
400 cylinder. This important characteristic in the forces can be effectively applied in a design to reduce
401 the wave impact on coastal and offshore porous structures.

402 *Influence of system configuration*

403 The effect of the annular spacing between the cylinders is investigated and the results are pre-
404 sented in terms of the total force variation on the ratio a/b in Figs. 14 and 15, where b is fixed at
405 $b = 10$ m. The wave number in x direction is taken as $k_x = 0.1 \text{ m}^{-1}$ and the ratio k_y/k_x is ranged
406 from 0 to 5.

407 Fig. 14 shows lower wave force on the interior cylinder with smaller radius a for a fixed ra-
408 dius, b , of the exterior cylinder. Also, the total horizontal force on the interior cylinder increases
409 monotonically as waves approach plane waves and fluctuates as the incident waves become more
410 short-crested. The variations, shown in Fig. 15 of the total horizontal forces on the exterior cylin-
411 der ($|2\pi bP|$) are quite different for different k_y/k_x values and no common trends can be found.
412 Fig. 15 also shows that the total horizontal forces on the exterior cylinder resulted from short-
413 crested waves fluctuate in values with a/b whereas a trend of monotonically decreasing with a/b is
414 clearly observed for plane incident waves. Generally, less short-crestedness results in larger wave
415 forces on external cylinder except in the neighborhood of troughs. These findings are considered
416 important for the designer in order to minimize the hydrodynamic loads on practical structures.

417 The influence of the porous-effect parameter G_0 on wave forces is examined in Fig. 16 with
418 diameter ratio of $b/a = 2$ and $a = 1$ m. Three cases are presented in the figure: (1) $k_x a = k_y a = 1$,
419 $ka = \sqrt{2}$; (2) $k_x a = \sqrt{2}, k_y a = 0, ka = \sqrt{2}$; and (3) $k_x a = 1, k_y a = 0, ka = 1$, corresponding
420 to a short-crested incident wave and two plane incident waves, respectively. It can be seen that the
421 total horizontal force on the interior cylinder increases monotonically, while the total horizontal
422 force of the exterior cylinder decreases monotonically as G_0 increases. The variation in forces is
423 slow at higher G_0 , approaching an asymptotic value in each case.

424 **Surface elevation**

425 It is interesting to study the changes in the wave surface elevation in the vicinity of the concen-
426 tric porous cylindrical structure for varying incident wave parameters and structure configuration.
427 Fig. 17 shows the resulting wave amplitude and corresponding phase contours resulting from
428 plane, short-crested, and standing incident wave interaction with the structure for configuration

429 parameters: $G_0 = 1.0$, $a = 1$ m and $b = 2$ m. The amplitudes shown in Fig. 17, are nondimen-
430 sionalized as $|\eta|/A$ (η and A represent free surface elevation and the amplitude of incident wave
431 respectively), and the phase values plotted in Fig. 17 are in the range of $[-\pi \sim +\pi]$.

432 It is clear that the diffracted wave patterns of short-crested waves are more complicated than
433 those of plane waves. As one will expect, all the equi-amplitude plots are symmetric with respect to
434 the longitudinal (x) axis leading to zero force in transverse (y) direction. Moreover, the diffracted
435 wave pattern resulting from a standing wave shown in Fig. 17 is symmetric in both x and y planes,
436 generating zero horizontal force in both longitudinal and transverse directions. It is seen that the
437 amplitude of the diffracted short-waves in the lee region is smaller than that of a plane wave, and the
438 region for the large amplitude waves in front of the cylinders resulting from short-crested waves
439 is also smaller than its plane wave counterpart. Further calculations on different wave number
440 ratio k_y/k_x revealed that such tendencies are more pronounced as the incident waves become more
441 short-crested.

442 The thick lines in phase contours represent changes from π to $-\pi$. The amphidromic points,
443 where equi-phase lines converge and the wave amplitude vanishes, are seen clearly formed for
444 short-crested incident waves (see Fig. 17). Similar to the feature observed by Zhu (1993) for
445 short-crested wave diffraction by a single impermeable cylinder, the phases near two adjacent am-
446 phidromic points rotate from $-\pi$ to $+\pi$ clockwise and counter-clockwise around the amphidromic
447 points respectively in the region surrounding the structure. Further calculations on different wave
448 number ratio k_y/k_x revealed that the density of the amphidromic points increases as the wave
449 crests become shorter. It is interesting to note that the equi-phase lines for the short-crested waves
450 become almost parallel to each other in the region downstream, where no obvious amphidromic
451 points formed, indicating that the waves in the lee side of the structure are no longer short-crested.
452 As one would expect for the standing incident wave component, the amplitude and phase contours
453 maintain symmetry in the x - and y -plane. The amplitudes in the transverse directions are small
454 compared to their inline values, with a faster variation in the corresponding phase contours.

455 It is worth stressing that all the numerical results presented in this paper are calculated using

456 the SBFEM model with no more than 8 surface elements discretized over the cylinder boundary.
457 The only exceptions are the results in Figs. 12 and 13, where 16 surface elements are used due
458 to larger values of ka and kb , so that the variation of the total horizontal force with varying wave
459 number ratio k_y/k_x becomes clear.

460 The computational times (recorded on a 2GHz Pentium IV PC and MATLAB 7.1) of the
461 SBFEM solutions for all the cases presented here are less than 3 sec. Thus, it clearly demonstrates
462 the superiority of the present method in significantly outperforming its counterparts in currently
463 available finite-element or boundary-element methods for similar problems. Such computational
464 efficiency and accuracy ensure a great potential of direct application of the present method to many
465 engineering problems, especially in ocean engineering.

466 The present SBFEM approach shown in this paper is valid for wave diffraction by a porous
467 circular cylinder system of bounded and unbounded domains. However, by means of domain
468 decomposition, and choosing appropriate forms of the radial function $a(\xi)$ and scaling centers for
469 different sub-domains, the authors have recently extended the SBFEM to solve other geometries
470 while maintaining dominant features of the present SBFEM (Song and Tao, 2008).

471 **CONCLUSIONS**

472 The versatility of the newly developed semi-analytical scaled boundary finite-element method
473 is demonstrated in this paper in considering the interaction of short-crested waves with a concen-
474 tric porous cylindrical structure. In contrast to the conventional boundary-element method, which
475 has been widely applied to wave-structure interaction problem in unbounded domain, the SBFEM
476 model developed for concentric porous cylinders requires no assistance from any fundamental so-
477 lutions. A unique advantage of this method is that a reduction of one in the spatial dimensions is
478 achieved. Thus only the body boundary is discretized with surface finite-elements. Excellent com-
479 putational efficiency and accuracy of the SBFEM model has been demonstrated, as the governing
480 equations are solved analytically in the radial direction.

481 Solutions for the interaction of plane and short-crested waves with a concentric porous cylin-
482 drical structure are achieved. The present numerical method is shown to reproduce the results of

483 available analytical solutions for all the physical properties including wave run-up and wave forces
484 very accurately and at very low computational cost. Detailed parametric study revealed that the
485 trends of wave forces and surface elevation patterns for short-crested waves are much more com-
486 plex than the ones for the plane incident waves. General trends of wave force on the ratio a/b are
487 discussed with potential to minimize the wave impact in a particular design. The results presented
488 here should be found useful in the design of coastal and ocean structures.

489 The present SBFEM model is demonstrated to be very efficient yielding accurate results in the
490 wave force and run-up calculations with no more than 8 elements for small and moderate ka and
491 kb values. Thus the model is believed to possess a significant advantage in its application to more
492 complicated coastal and offshore structures geometry.

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FIG. 1. Ekofisk Gravity Structure (Courtesy ConocoPhillips).

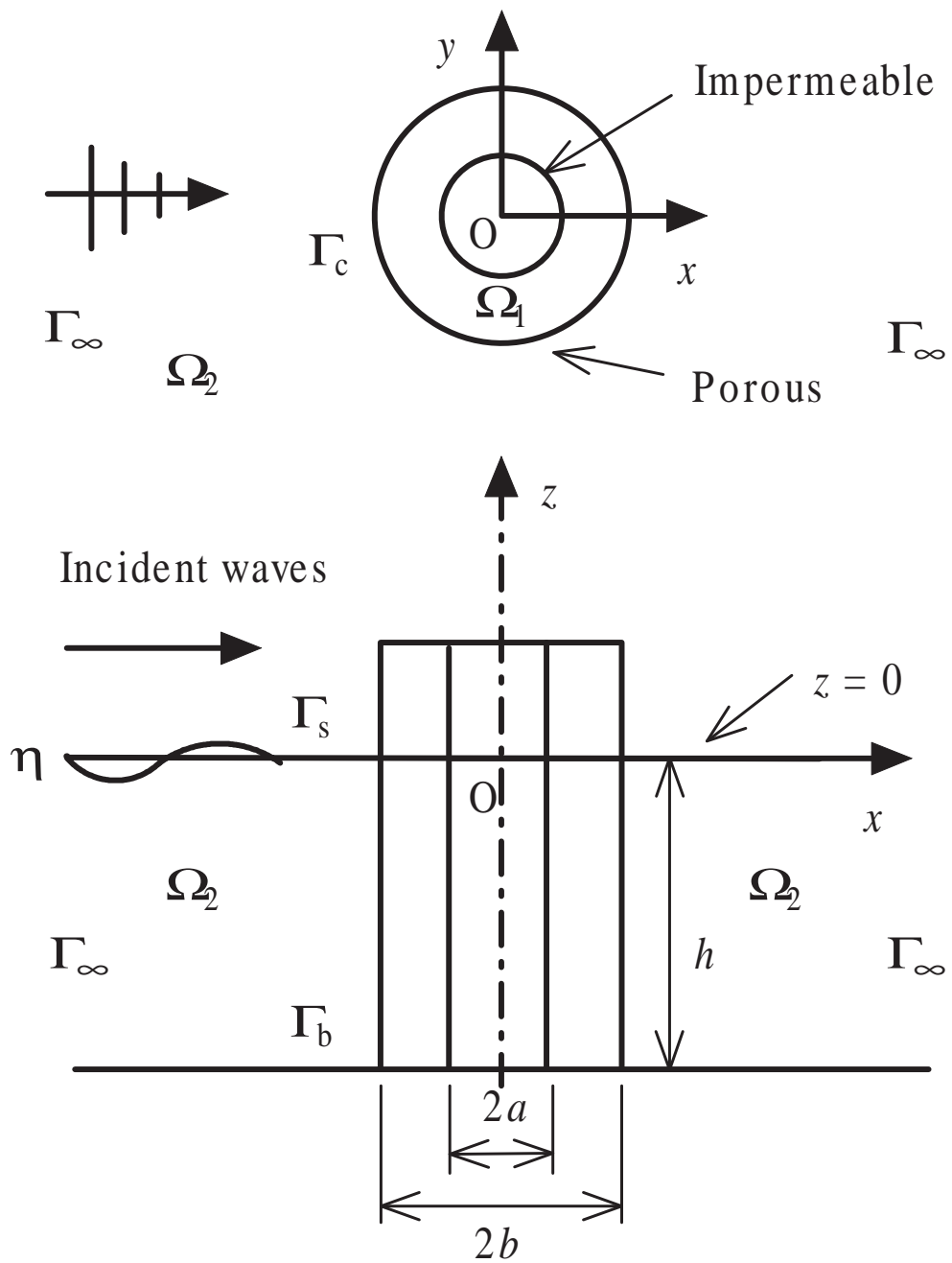


FIG. 2. Definition sketch of wave interaction with a concentric porous cylindrical structure.

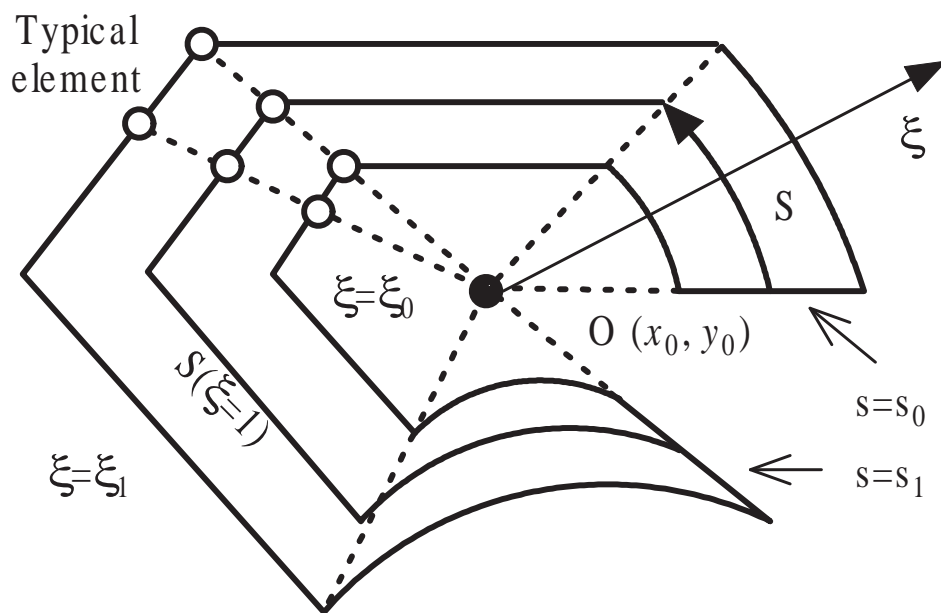


FIG. 3. The coordinate definition of SBFEM.

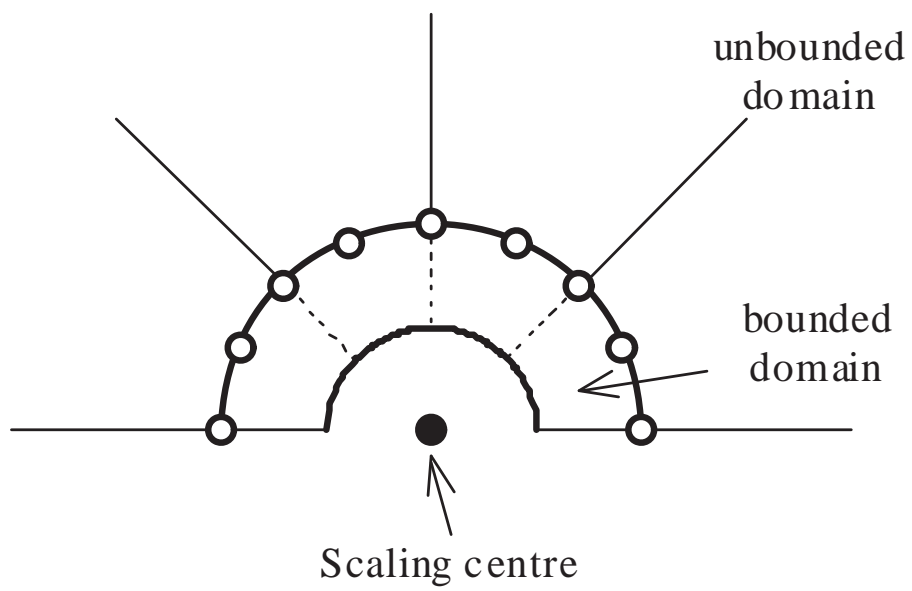


FIG. 4. Scaled boundary finite element mesh for a concentric porous cylindrical structure.

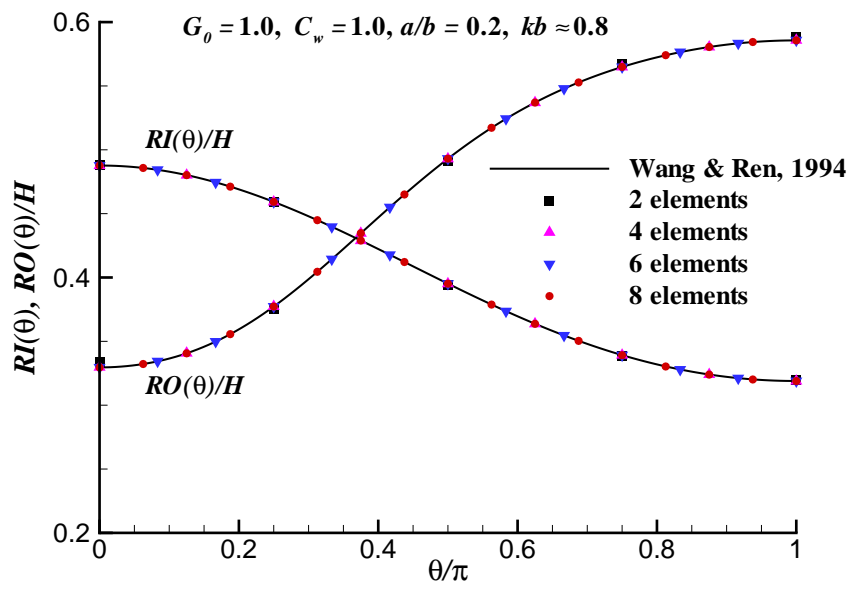


FIG. 5. Run-up of an incident plane wave around the interior and exterior cylinders.

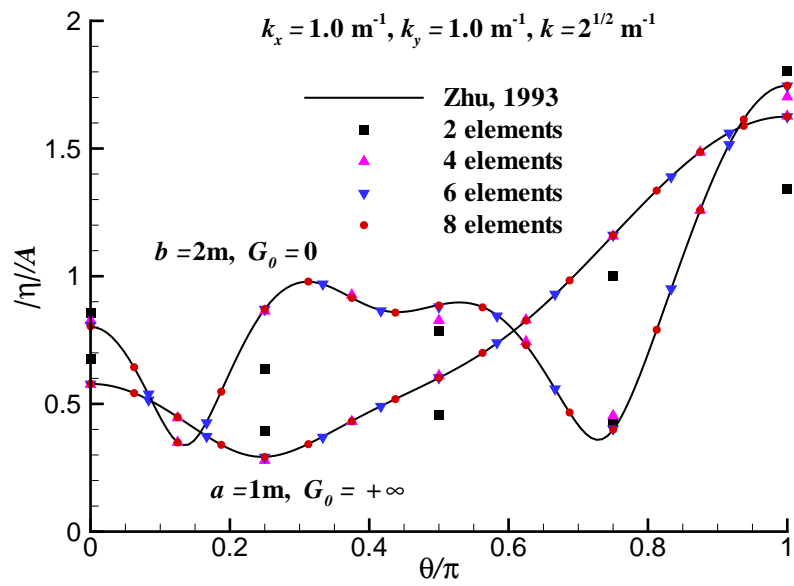


FIG. 6. Run-up of a short-crested wave on the interior cylinder at $a = 1$ m and the exterior cylinder at $b = 2$ m.

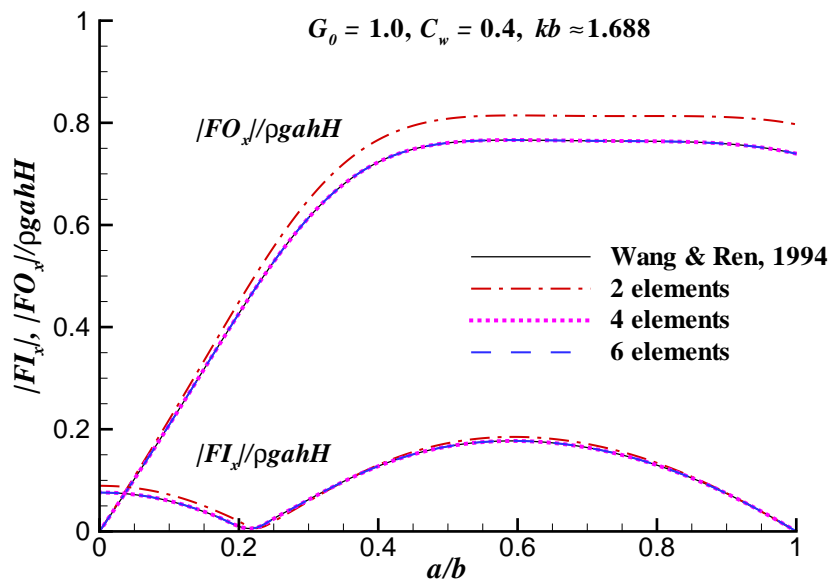


FIG. 7. Wave force due to a plane wave on the interior and exterior cylinders for $C_w = 0.4$.

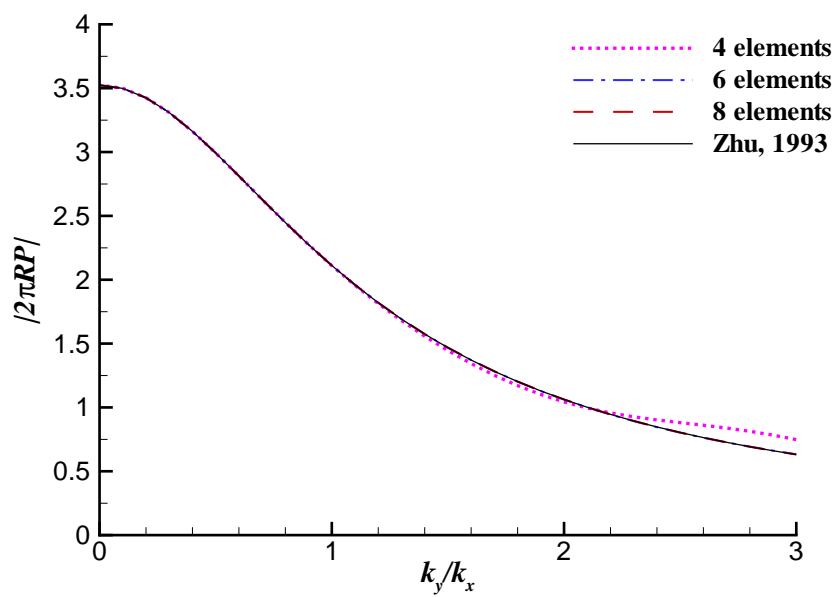


FIG. 8. Total wave force $|2\pi RP|$ due to short-crested waves on a single solid cylinder at $k_x = 2 \text{ m}^{-1}$, $R = 1 \text{ m}$.

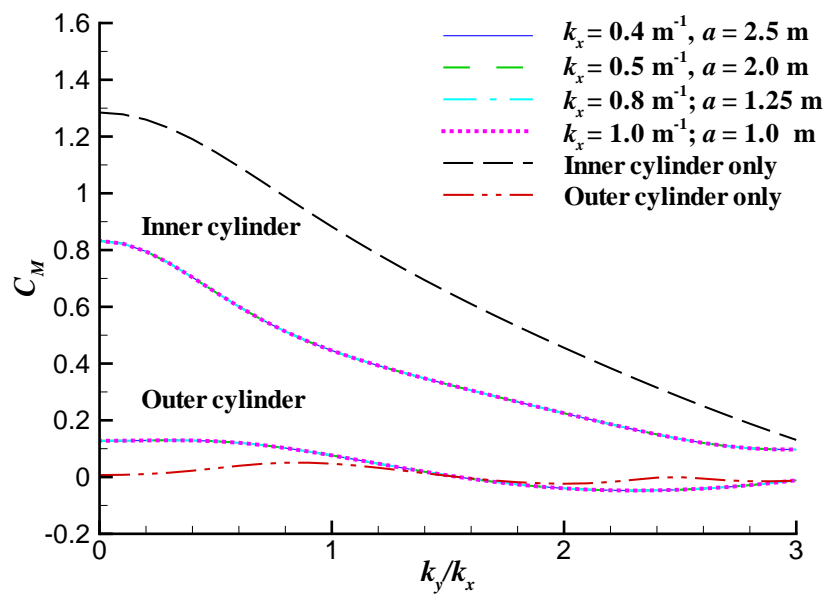


FIG. 9. Variation of the effective inertial coefficient C_M vs. the ratio k_y/k_x at $k_x a = 1$, $b/a = 2$ and $G_0 = 1$.

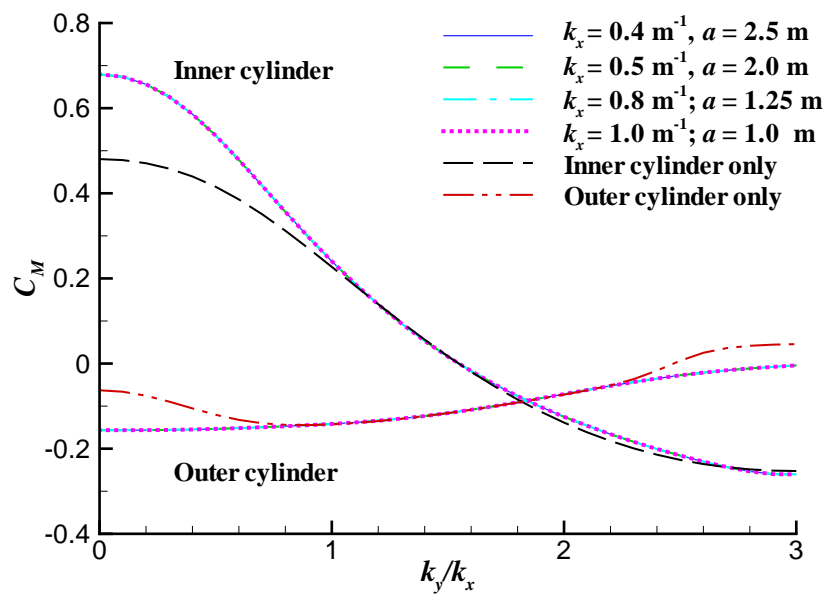


FIG. 10. Variation of the effective drag coefficient C_D vs. the ratio k_y/k_x at $k_x a = 1$, $b/a = 2$ and $G_0 = 1$.

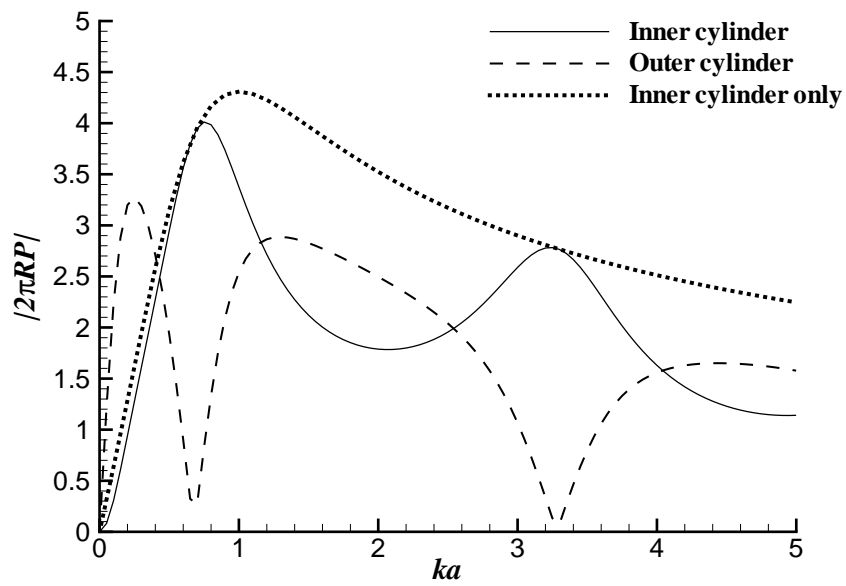


FIG. 11. Variation of the total horizontal forces on the interior and exterior cylinder by plane incident wave *vs.* ka at $a = 1\text{m}$, $b/a = 2$ and $G_0 = 1$.

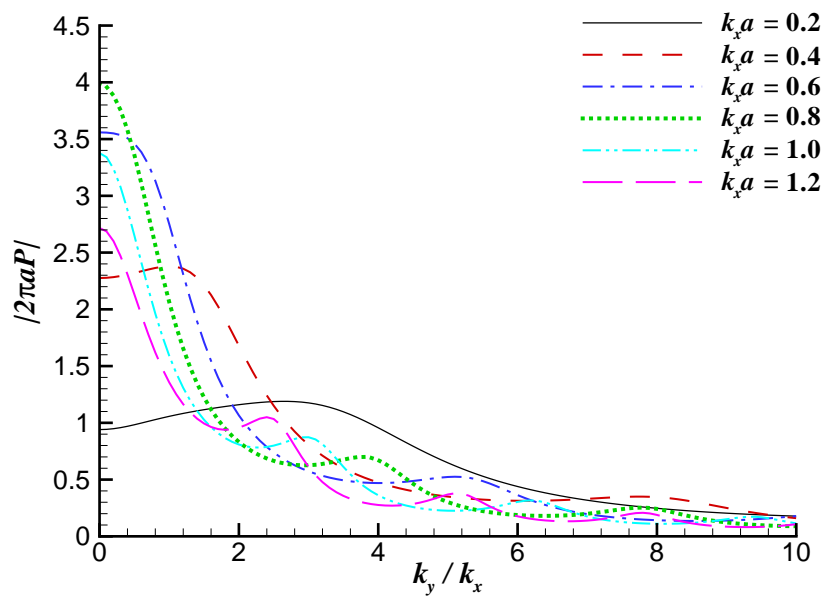


FIG. 12. Variation of the total horizontal force on the interior cylinder *vs.* the ratio k_y/k_x at $a = 1\text{m}$, $b/a = 2$ and $G_0 = 1$.

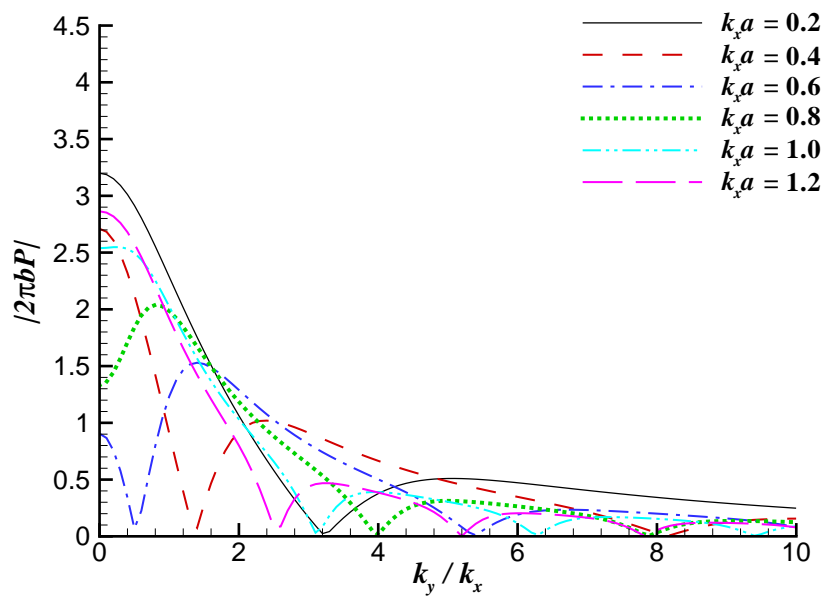


FIG. 13. Variation of the total horizontal force on the exterior cylinder *vs.* the ratio k_y/k_x at $a = 1\text{m}$, $b/a = 2$ and $G_0 = 1$.

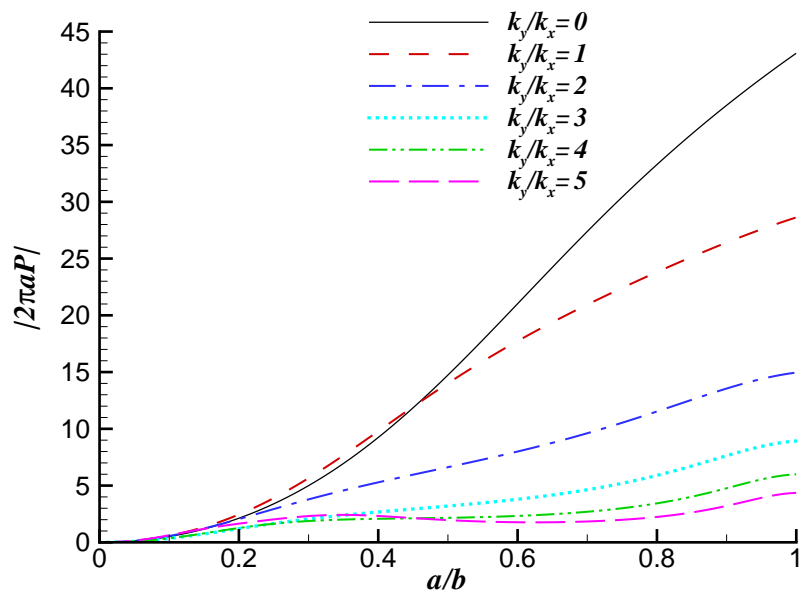


FIG. 14. Variation of the total horizontal force on the interior cylinder *vs.* the ratio a/b at different k_y ($b = 10$ m, $k_x = 0.1$ m⁻¹).

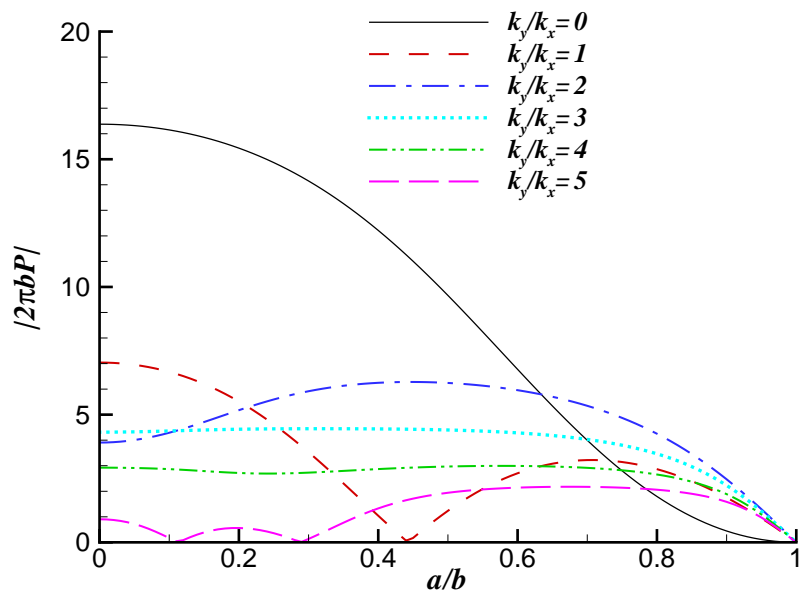


FIG. 15. Variation of the total horizontal force on the exterior cylinder *vs.* the ratio a/b at different k_y ($b = 10$ m, $k_x = 0.1$ m⁻¹).

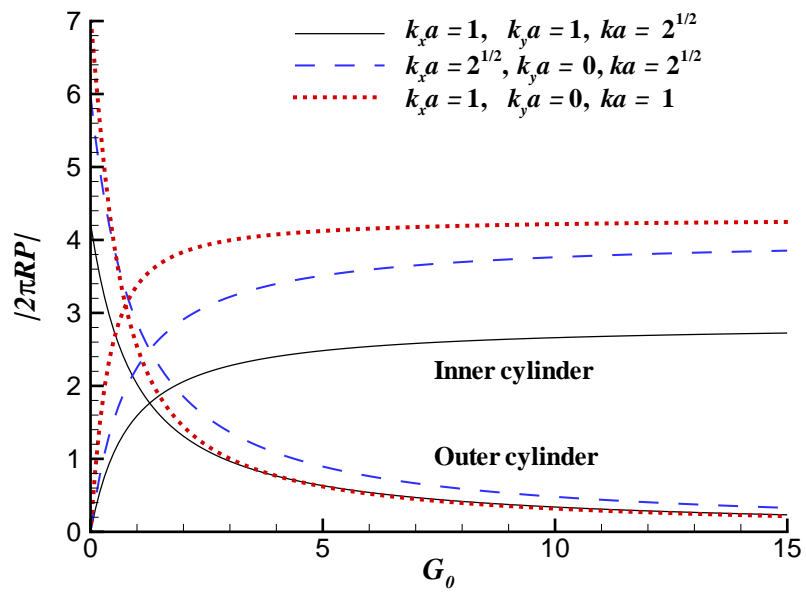
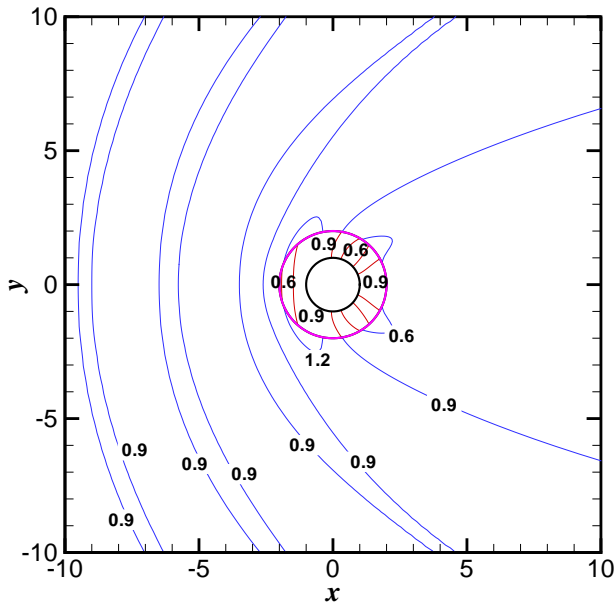
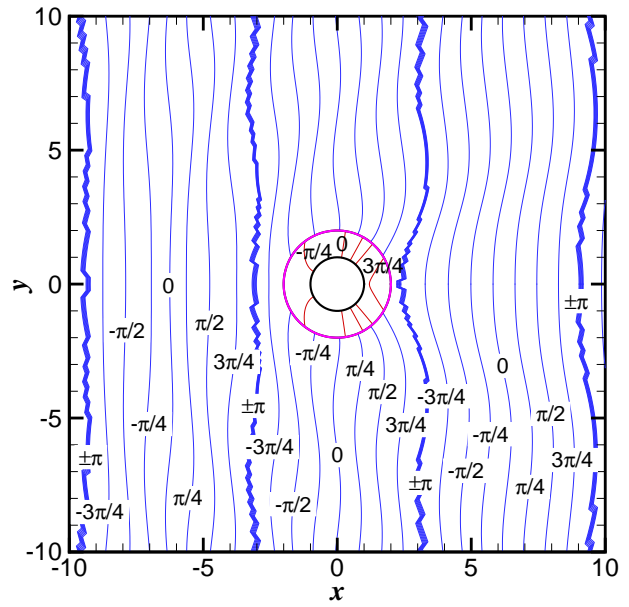


FIG. 16. Variations of the total horizontal forces of the cylinders *vs.* the porous-effect parameter G_0 ($b/a = 2$ and $a = 1$ m).

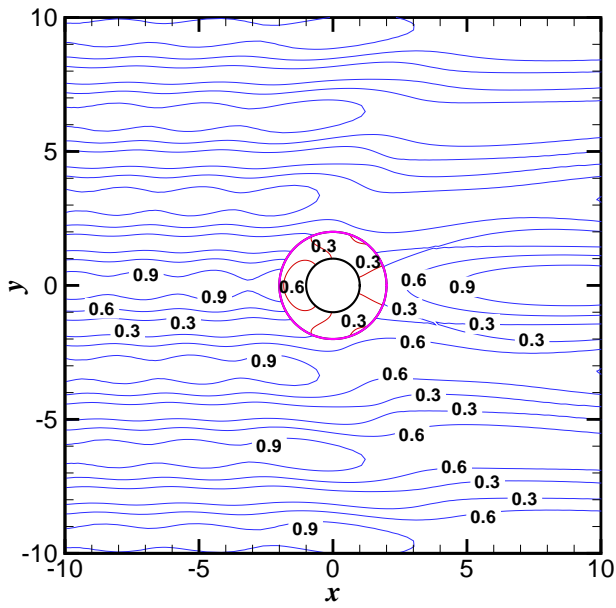
Co-amplitude: Plane waves ($k_x a = 1, k_y a = 0$)



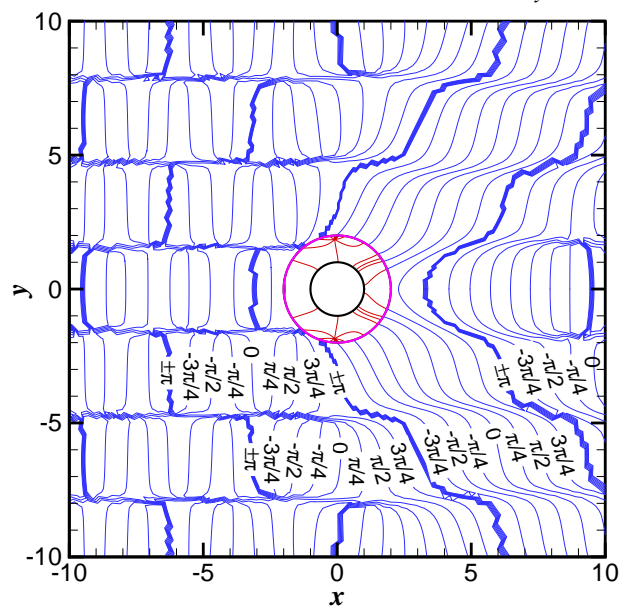
Co-phase: Plane waves ($k_x a = 1, k_y a = 0$)



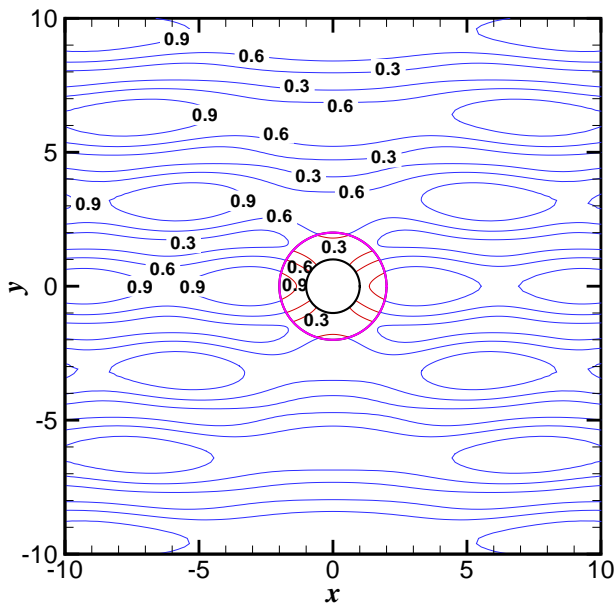
Co-amplitude: Short-crested waves ($k_x a = 1, k_y a = 1$)



Co-phase: Short-crested waves ($k_x a = 1, k_y a = 1$)



Co-amplitude: Standing waves ($k_x a = 0, k_y a = 1$)



Co-phase: Standing waves ($k_x a = 0, k_y a = 1$)

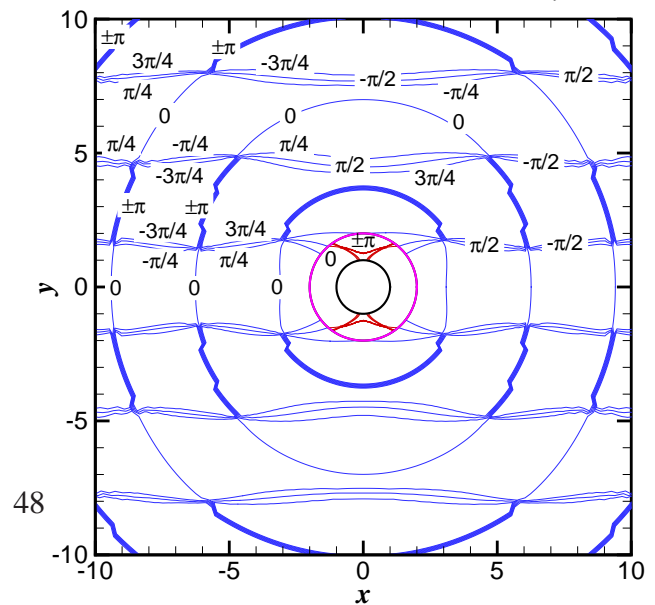


FIG. 17. Equi-amplitude contours (left) and Equi-phase contours (right) for the incident plane, short-crested, and standing waves, respectively.