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Scaling behaviors in differently developed markets

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Abstract

Scaling properties of four different stock market indices are studied in terms of a generalized Hurst exponent approach. We find that the deviations from pure Brownian motion behavior are associated with the degrees of development of the markets and we observe strong differentiations in the scaling properties of markets at different development stage.

1. Introduction

The scaling properties in time series have been studied in the literature by means of a great variety of techniques [1–11]. Historically, one of the most effective techniques was the rescaled range statistical analysis first introduced by Harold Edwin Hurst to describe the long-term dependence of water levels in rivers and reservoirs [12]. This analysis provides a sensitive method for revealing long-run correlations in random processes. What especially makes the Hurst analysis appealing is that all these information about a complex signal are contained in one parameter only: the *Hurst exponent*. On the other hand, one of the weaknesses of the original method is that it relies on maximum and minimum data, which makes it very sensitive to outliers. In order to study the multi-fractal features of the data, here we use an alternative method to the original approach of Hurst. This method is applied to the study of the scaling properties of

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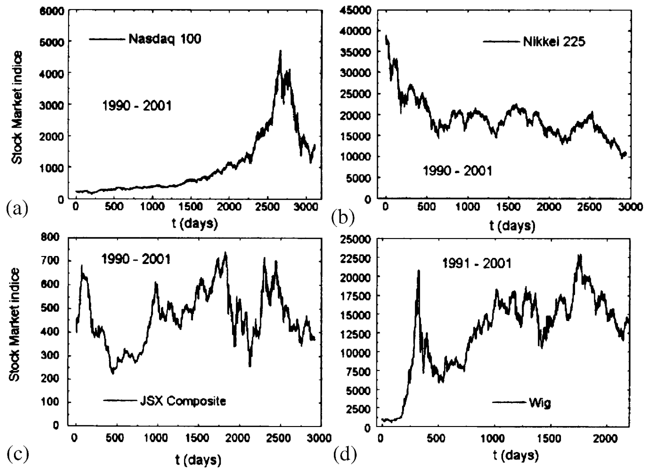

Fig. 1. The Stock Market indices as a function of time t: (a) Nasdaq 100; (b) Nikkei 225; (c) JSX Composite and (d) Wig.

returns in four different stock markets as function of the time interval on which the returns are measured. Our aim is to point out how a relatively simple statistics gives us indications on the market characteristics. To this end we compare two very developed markets such as *Nasdaq 100* (USA) and *Nikkey 225* (Japan) with two emerging markets *Wig* (Poland) and *JSX Composite* (Indonesia) (see Fig. 1).

2. Generalized Hurst exponent approach

The Hurst analysis brings to light that some statistical properties of time series $X(t)$ (with $t = v, 2v, \ldots, kv, \ldots, T$) scale with the observation-period (T) and the time-resolution (v). Such a scaling is characterized by an exponent H which is commonly associated with the long-term statistical dependence of the signal. A generalization of the approach proposed by Hurst should therefore be associated with the scaling behavior of statistically significant properties of the signal. To this purpose we analyze the q-order moments of the distribution of the increments [3,13]

$$K_q(\tau) = \frac{\langle |X(t+\tau) - X(t)|^q \rangle}{\langle |X(t)|^q \rangle} , \tag{1}$$

which is a good characterization of the statistical evolution of a stochastic variable $X(t)$. (Note that, for $q = 2$, the $K_q(\tau)$ is proportional to the autocorrelation function $a(\tau) = \langle X(t+\tau)X(t) \rangle$.)

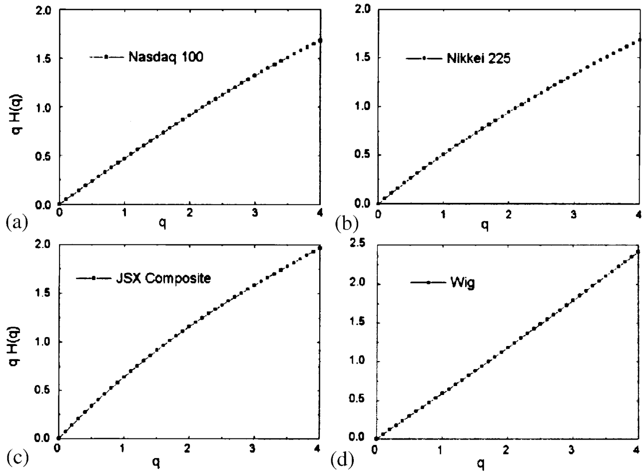

Fig. 2. Behavior of $qH(q)$ vs. q for: (a) Nasdaq 100; (b) Nikkei 225; (c) JSX Composite and (d) Wig.

The generalized Hurst exponent $H(q)$ can be defined from the scaling behavior of $K_q(\tau)$ which can be assumed to be given by the relation [13]

$$K_q(\tau) \sim \left(\frac{\tau}{\nu}\right)^{qH(q)} . \tag{2}$$

Within this framework, we can distinguish between two kind of processes: (i) a process where $H(q) = H$, constant independent of q; (ii) a process with $H(q)$ not constant. The first case is characteristic of uni-scaling or uni-fractal processes and its scaling behavior is determined from a unique constant H that coincides with the 'original' Hurst exponent H. This value is equal to $\frac{1}{2}$ for the Brownian motion and it is a constant different from $\frac{1}{2}$ for the fractional Brownian motion. In the second case, when $H(q)$ depends on q, the process is commonly called multi-scaling (or multi-fractal) [14] and different exponents characterize the scaling of different q-moments of the distribution. In this 'curve' of exponents $H(q)$, some values of q are associated with special features. For instance, when $q = 1$, $H(1)$ describes the scaling behavior of the absolute values of the increments. The value of this exponent is expected to be closely related to the original Hurst exponent, H, that is indeed associated with the scaling of the absolute spread in the increments. In this paper we focalize the attention on the case $q = 2$, which is associated with the scaling of the autocorrelation function and is related to the power spectrum [15]. Indeed, under suitable assumptions [11,15], one can write: $\beta = 1 + 2H(2)$ with β the exponent associated with the power spectra ($S(f) \sim f^{-\beta}$). The curves for $qH(q)$ vs. q are reported in Fig. 2. One can observe that, for all these time series, $qH(q)$ is not linear in q: a signature of deviations from pure Brownian motion and other additive models. The same behavior holds for other cases discussed elsewhere [1,11].

3. Methodology

Our empirical analysis is performed on daily time series (see Fig. 1) where we compute the returns from the logarithmic price $X(t) = \ln(P(t))$. All these variables are 'detrended' by eliminating a linear drift ηt which can be simply evaluated from

$$\langle X(t+\tau) - X(t)\rangle = \eta\tau \; . \tag{3}$$

Other more complex deviations from the stationary behavior might be present in the financial data that we analyze. In this context, the subtraction of the linear drift can be viewed as a first approximation.

We compute the q-order moments $K_q(\tau)$ (defined in Eq. (1)) with τ in the range between $v = 1$ day and $\tau_{\max}$ days. In order to test the robustness of our empirical approach, for each series we analyze the scaling properties varying $\tau_{\max}$ between 5 and 19 days. The resulting exponents computed using these different $\tau_{\max}$ are stable in their values within a range of 10%.

In order to test that our method is not biased we estimate the generalized Hurst exponents for simulated random walks produced by using three different random numbers generators. We perform 100 simulations of random walks with the same number of data points as in our samples. In all the cases, $H(2)$ have values of 0.5 within the errors. This shows that our method is robust and does not suffer of bias as other methods do.

4. Results and discussions

We calculate the values of the generalized Hurst exponent at $q = 2(H(2))$. For the following market indices we obtain:

Market	Time	$H(2)$
Nasdaq 100 (USA)	1990–2001	0.459 ± 0.009
Nikkei 225 (Japan)	1990–2001	0.463 ± 0.008
Wig (Poland)	1990–2001	0.58 ± 0.01
JSX (Indonesia)	1991–2001	0.58 ± 0.01

Moreover, the values of $H(2)$ for 32 stock markets (including the four above) in the period 1997–2001 [11] are reported in Fig. 3. From these values one can observe that there is a clear tendency for mature liquid markets to have values of $H(2)$ smaller than 0.5 whereas less developed markets shows a tendency to have $H(2) > 0.5$.

Although this tendency is clearly identifiable across different stock markets, we observed that sizable fluctuations (beyond the statistical incertitude) characterize the behavior of $H(2)$ in different time periods [11]. To investigate this issue, we divide the whole time period (about 3000 days) into sub-periods of 300 days. For each of these sub-periods we calculate the corresponding values of $H(2)$ which are reported in Fig. 4. As one can see, Nasdaq 100 and Nikkei 225 fluctuate mainly below 0.5 whereas JSX and Wig fluctuate mainly above 0.5. But relevant fluctuations across 0.5 are present

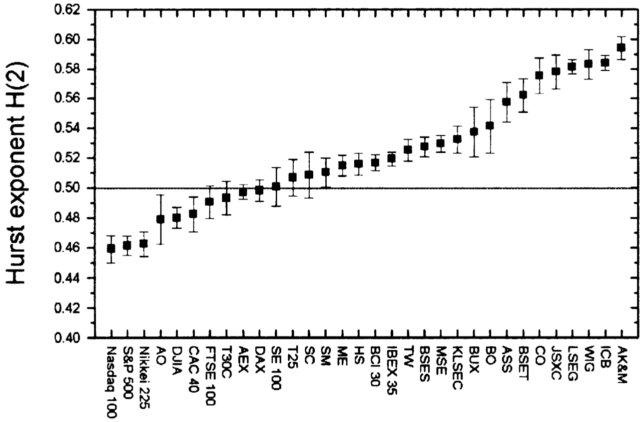

Fig. 3. The generalized Hurst exponent $H(2)$ for several Stock Market indices analyzed in the period: 1997–2001.

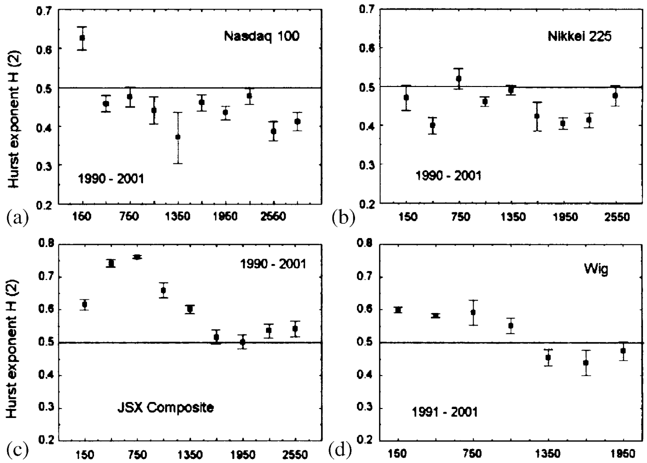

Fig. 4. (a–d) $H(2)$ computed subdividing the whole time-period in time-windows of 300 days.

in all the series. Note also that these fluctuations appear to be correlated. These features will be analyzed in details in a future work.

5. Conclusions

The analysis of scaling properties of the q-order moments in different stock indices time-series, shows that the generalized Hurst exponent $H(q)$ (Eq. (2)) is a powerful instrument to characterize and differentiate the scaling structure of such markets. The empirical analysis across a wide variety of stock markets shows that the exponent $H(2)$

is sensitive to the degree of development of the market. The robustness of the present empirical approach is tested in several ways: by varying the maximum time-step (τ_{max}); by using the Jackknife method; by varying the time-window sizes; by comparing with three distinct simulated Brownian motions [11]. We verify that the observed differentiation among different degrees of market development is clearly emerging well above the numerical fluctuations. On the other hand, the analysis over different sub-periods reveals physically significant changes in the scaling behavior of a given market in time. This indicates that the scaling structure of markets is an evolving quantity which is not only able to differentiate among markets at different development stage but can also catch the overall variability of the market conditions.

It must be noted that in this paper the scaling law is not used to conclude anything on the theoretical process but on the contrary we use it as a "stylized fact" that any theoretical model should also reproduce.

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