Scaling Laws for Ad Hoc Wireless Networks: An Information Theoretic Approach

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Scaling Laws for Ad Hoc Wireless Networks: An Information Theoretic Approach

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Abstract

In recent years there has been significant and increasing interest in ad hoc wireless networks. The design, analysis and deployment of such wireless networks necessitate a fundamental understanding of how much information transfer they can support, as well as what the appropriate architectures and protocols are for operating them. This monograph addresses these questions by presenting various models and results that quantify the information transport capability of wireless networks, as well as shed light on architecture design from a high level point of view. The models take into consideration important features such as the spatial distribution of nodes, strategies for sharing the wireless medium, the attenuation of signals with distance, and how information is to be transferred, whether it be by encoding, decoding, choice of power level, spatio-temporal scheduling of transmissions, choice of multi-hop routes, or other modalities of cooperation between nodes. An important aspect of the approach is to characterize how the information hauling capacity scales with the number of nodes in the network.

The monograph begins by studying models of wireless networks based on current technology, which schedules concurrent transmissions to take account of interference, and then routes packets from their sources to destinations in a multi-hop fashion. An index of performance, called transport capacity, which is measured by the bit meters per second the network can convey in aggregate, is studied. For arbitrary networks, including those allowing for optimization of node locations, the scaling law for the transport capacity in terms of the number of nodes in the network is identified. For random networks, where nodes are randomly distributed, and source-destination pairs are randomly chosen, the scaling law for the maximum common throughput capacity that can be supported for all the source-destination pairs is characterized. The constructive procedure for obtaining the sharp lower bound gives insight into an order optimal architecture for wireless networks operating under a multi-hop strategy.

To determine the ultimate limits on how much information wireless networks can carry requires an information theoretic treatment, and this is the subject of the second half of the monograph. Since wireless communication takes place over a shared medium, it allows more advanced operations in addition to multi-hop. To understand the limitations as well as possibilities for such information transfer, wireless networks are studied from a Shannon information-theoretic point of view, allowing any causal operation. Models that characterize how signals attenuate with distance, as well as multi-path fading, are introduced. Fundamental bounds on the transport capacity are established for both high and low attenuation regimes. The results show that the multi-hop transport scheme achieves the same order of scaling, though with a different pre-constant, as the information theoretically best possible, in the high attenuation regime. However, in the low attenuation regime, superlinear scaling may be possible through recourse to more advanced modes of cooperation between nodes. Techniques used in analyzing multi-antenna systems are also studied to characterize the scaling behavior of large wireless networks.

Contents

1

Introduction

Over the past few years there has emerged a network information theory motivated by the twin goals of applicability as well as tractability vis-à-vis the rapidly emerging field of wireless networking. A central aspect of this theory is that the spatial aspects of the system, including locations of nodes and signal attenuation with distance, are explicitly modeled. Also, distance is intimately involved even in the performance measure of transport capacity that is analyzed. This theory has been used to develop bounds on the distance hauling capacity of wireless networks, feasibility results, scaling laws for network capacity as the number of nodes increases, and also suggest some insight into architectures. It establishes relationships for information transport in wireless networks between phenomena such as how radio signals attenuate with distance and the information hauling capacity of networks. It also connects the more recent field of networking with its emphasis on architecture and protocols with the more traditional field of communication theory with its emphasis on signals, transmitters and receivers. This text provides an account of the salient results.

The focus of this text is on ad hoc wireless networks, a topic which has aroused much interest in recent years. These are wireless networks without infrastructure; see Figure 1.1. Examples of technologies

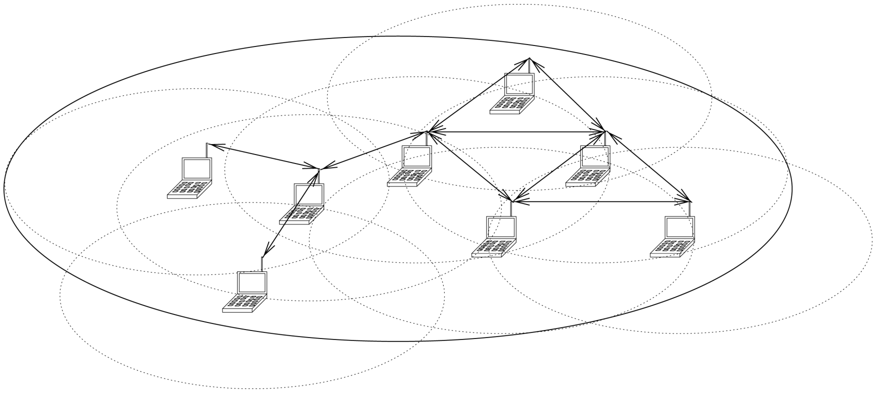

Fig. 1.1 An ad hoc wireless network.

envisioning such wireless networks are Bluetooth [29] through the use of scatter-nets, e.g., [13], IEEE802.11 [30] through the use of the Distributed Coordinated Function, e.g., [3], IEEE802.15.4 ZigBee networks [34, 33] deployed as multi-hop sensor networks, and IEEE802.16 [31, 32] deployed as mesh networks.

Such ad hoc wireless networks have been proposed to be operated in multi-hop mode: packets are relayed from node to node in several short hops until they reach their destinations. The top layer shown in the protocol stack in Figure 1.2, the *Transport Layer*, can address functionalities such as end-to-end reliable delivery of information, as well as regulating the rate at which data packets are pumped into the network so as to match it to the rate at which the network can carry information. The choice of the sequence of nodes along which to relay is the *routing* problem, and is addressed in the *Network Layer*. At each hop, a medium access control protocol is employed so that the reception at the receiver is not interfered with by another nearby transmitter, as well as to ensure that packets are retransmitted repeatedly, at least a few times, until an acknowledgement is received from the receiver. Another important functionality is *Power Control*. This addresses the power level at which a packet on a hop is transmitted. Proposals have been made to address it at the Network Layer [20, 16] or the Medium Access Control Layer [19, 15]. The *Physical Layer* addresses issues related to modulation, etc..

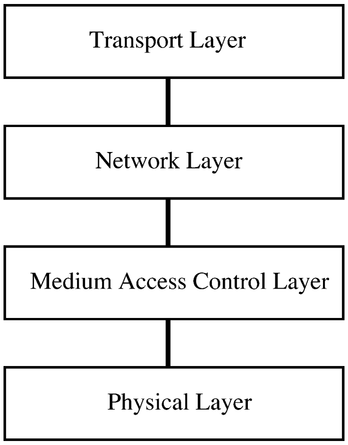

Fig. 1.2 A proposed protocol stack for wireless ad hoc networks.

One of the goals of this text is to present results on what the capacity of wireless networks is under the multi-hop model. Another question is whether the multi-hop mode of information transfer is indeed an appropriate mode. This is motivated by the fact that there are several alternative ways in which the wireless medium can be used. To study such alternatives takes us into the domain of network information theory, an area in which several apparently simple-looking problems have continued to defy characterization even after several decades of research. Another question of interest is how the information transfer capability of wireless networks scales as the number of nodes increases. This leads into the issue of scaling laws for wireless networks. Such results can help in understanding the complicated interactions in wireless networks, and to shed light on operating and designing more efficient networks.

It is to these questions that this text is addressed. At the same time, there are several issues that are excluded. Except peripherally, issues such as energy lifetime, latency, fairness, etc., are not centrally addressed.

The results presented in this text are as follows.

Sections 2–4 consider arbitrary wireless networks, where node locations are allowed to be optimized for network performance.

Section 2 introduces the definition of transport capacity, which takes into account not only the throughputs supported for source–destination pairs, but also the distances between sources and their destinations. Specifically it is the sum of the rates for source–destination pairs weighted by their distance. It introduces two variants of a so-called Protocol Model. This postulates a model for successful packet reception at a receiver, by specifying either a guard zone around a receiver or an interference footprint around a transmitter. This section presents results on the scaling behavior of the transport capacity of arbitrary wireless networks under the Protocol Model. It exhibits a square-root scaling law in the number of nodes in the network for the transport capacity.

Section 3 introduces the concepts of Exclusion Region and Interference Region. Building on them, it presents improved bounds on the transport capacity of arbitrary wireless networks under the Protocol Model.

Section 4 introduces the Physical Model, which models successful reception in terms of the received signal-to-noise-plus-interference ratio at a receiver. This section shows that there is a correspondence between the Protocol Model and the Physical Model, and then presents similar results on the scaling behavior of the transport capacity for arbitrary wireless networks under the Physical Model.

Sections 5–7 study the performance of random wireless networks, where nodes are distributed randomly over a domain and destinations for sources are randomly chosen. Section 5 considers homogenous random wireless networks under the Protocol Model, where every node employs a common transmission range and wishes to transmit at a common rate. Results on the throughput capacity, the maximum common rate achievable for every source, are presented. It is shown that the common throughput that can be furnished to all the n source–destination pairs is $\Theta(\frac{1}{\sqrt{n \log n}})$. Since the factor $\sqrt{\log n}$ grows very slowly, it shows that random networks so operated are close to best case. An auxiliary consequence is that utilization of a common range for all transmissions is nearly optimal.

Section 6 considers homogenous random wireless networks under the Physical Model, where every node employs a common power for transmission. Similar results, as in Section 5, on the throughput capacity for such networks are presented for the Physical Model.

Section 7 considers random wireless networks with node locations generated by a Poisson point process, and operating under the Protocol Model. Nodes are allowed to use different transmission ranges (a common power can be used too). A constructive scheme shows that with such flexibility a better common rate for each node is achievable elimination the factor $\sqrt{\log n}$, compared to the one achieved in Section 5.

Sections 8 to Section 11 delve into an information theoretic framework, attempting to characterize fundamental limits on the performance of wireless networks, under any causal strategy, without making presuppositions about the manner in which information is sought to be communicated.

Section 8 first specifies a model for signal attenuation with distance. The central result of the section is that, in the high attenuation regime, the scaling behavior of the transport capacity of arbitrary wireless networks is similar to that of networks studied in previous sections, after an appropriate scaling of area. This shows that in this attenuation regime the proposed multi-hop information architecture towards which many current design efforts are targeted is an order-optimal architecture. In this sense information theory provides strategic guidance to designers on the architecture for information transport in wireless networks.

Section 9 addresses the scaling behavior of transport capacity in the low attenuation regime. It presents results showing that the scaling behavior can be very different compared to that in the high attenuation regime. This shows that the architecture for information transport in wireless networks under very low attenuation can indeed need to be quite different from the higher attenuation case. Thus there is a connection between the attenuation property of the medium and the architecture that needs to be adopted. Also different strategies for information transport emerge as of interest.

Section 10 studies the transport capacity for wireless networks in the presence of multi-path fading. The results show that in the high attenuation regime, for many fading cases, the scaling behavior is the same as that in the no-fading environment. So in this attenuation regime there is no difference in the transport capacity achievable, at least up to a preconstant.

Section 11 presents results showing how techniques from Multi-input Multi-output (MIMO) systems can be applied to study the performance of large wireless networks.

Model	**Node Locations**	**Constraint on Power/Range**	**Power Attenuation**	**Major Results**	**Chap-ters**	**Note**
Protocol model (Defined in terms of geometry)	Arbitrary in a region of area A	Arbitrary	No assumption	T(n) = $\Theta(W\sqrt{An})$ bit*meter/sec	2,3	W is the maximum single link rate
	Randomly distributed in a finite domain	Common transmission range r	No assumption	$\lambda(n) = \Theta(\frac{W}{\sqrt{n\log n}})$ bit/sec	5	Every node has a randomly chosen destination node
	Poisson process with density n in a unit domain	Different ranges are allowed	No assumption	$\lambda(n) = \Theta(W/\sqrt{n})$	7	Every node is only one destination node
Physical model (Defined in terms of SINR)	Arbitrary in a region of area A	$P_{\max} \leq \Theta((nA)^{\alpha/2})$	$1/d^{\alpha}$ with $\alpha > 2$	T(n)= $\Theta(W\sqrt{An})$	4	A Generalized Physical model is also considered
	Randomly distributed in a unit square	Common transmission power P	$1/d^{\alpha}$ with $\alpha > 2$	$\Theta(\frac{W}{\sqrt{n\log n}}) \leq \lambda(n) \leq \Theta(W/\sqrt{n})$	6	Every node has a randomly chosen destination node
	Poisson process with density n in a unit domain	No assumption	$1/d^{\alpha}$ with $\alpha > 2$	$\lambda(n) = \Theta(W/\sqrt{n})$	7	Every node is only one destination node
Information theoretical model (Defined based on physical signals)	Arbitrary, but with a minimum separation $r_{\min} > 0$	Every node is subject to a power constraint P	$Ge^{-r\gamma}/d^{\delta}$ with $\gamma > 0$ or $\delta > 3/2$	T(n)= $\Theta(\sqrt{n})$	8	
	Arbitrary, but with a minimum separation $r_{\min} > 0$		G/d^{δ} with δ small	Large T(n) can be supported with fixed total power; super-linear T(n) is achievable under individual power constraint	9	A coherent relaying with interference subtraction (CRIS) scheme is used
	Arbitrary, but with a minimum separation $r_{\min} > 0$	Every node is subject to a power constraint P	Multi-path fading with $\gamma > 0$ or $\delta > 3$	T(n)= $\Theta(\sqrt{n})$	10	
	MIMO techniques are shown to upper bound the transport capacity and study large network behavior				11	

Fig. 1.3 A summary of the main models and results.

Section 12 concludes this text.

For the convenience of readers, we summarize the major models and results in Figure 1.3. Note that we assume that there are n nodes in the network, and $T(n)$ denotes the transport capacity (bit-meters/sec), and $\lambda(n)$ the per-node throughput (bits/second).

References

[1] A. Agarwal and P. R. Kumar, "Improved capacity bounds for wireless networks," *Wireless communications and mobile computing*, vol. 4, pp. 251–261, 2004.

[2] A. Agarwal and P. R. Kumar, "Capacity bounds for ad hoc and hybrid wireless networks," *ACM SIGCOMM Computer Communications Reveiew*, vol. 34, no. 3, pp. 71–81, July 2004.

[3] D. Aguayo, J. Bicket, S. Biswas, D. S. J. D. Couto, and R. Morris, *MIT roofnet implementation*, http://www.pdos.lcs.mit.edu/roofnet/design/, August 2003.

[4] S. H. A. Ahmad, A. Jovicic, and P. Viswanath, "Upper bounds to generalized transport capacity of wireless networks," to appear in *IEEE Transactions on Information Theory*, June 2006.

[5] E. Biglieri, J. Proakis, and S. Shamai (Shitz), "Fading channels: Information-theoretic and communications aspects," *IEEE Transactions on Information Theory*, vol. 44, no. 6, pp. 2619–2692, 1998.

[6] T. M. Cover and J. A. Thomas, *Elements of information theory.* John Wiley & Sons, Inc., 1991.

[7] T. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. 25, pp. 572–584, 1979.

[8] M. Franceschetti, J. Bruck, and L. Schulman, "Microcellular systems, random walks and wave propagation," in *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 5, pp. 1304–1317, May 2004.

[9] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, "Closing the gap in the capacity of random wireless networks," in ISIT 2004, p. 438.

[10] G. R. Grimmett, *Percolation.* Springer, 1999.

- [11] P. Gupta and P. R. Kumar, "Critical power for asymptotic connectivity in wireless networks," in *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W.H. Fleming*, (W. M. McEneany, G. Yin, and Q. Zhang, eds.), pp. 547–560, Boston, MA: Birkhauser, March 1998. ISBN 0-8176-4078-9.
- [12] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, pp. 388–404, March 2000.
- [13] P. Johansson, M. Kazantzidis, R. Kapoor, and M. Gerla, "Bluetooth: An enabler for personal area networking," *IEEE Network*, vol. 15, pp. 28–37, September/October 2001.
- [14] A. Jovicic, P. Viswanath, and S. R. Kulkarni, "Upper bounds to transport capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 50, no. 11, pp. 2555–2565, November 2004.
- [15] E.-S. Jung and N. Vaidya, "A power control MAC protocol for ad hoc networks," in *ACM International Conference on Mobile Computing and Networking (MobiCom)*, September 2002.
- [16] V. Kawadia and P. R. Kumar, "Principles and protocols for power control in ad hoc networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 5, pp. 76–88, January 2005.
- [17] O. Leveque and I. E. Telatar, "Information-theoretic upper bounds on the capacity of large extended ad hoc wireless networks," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 858–865, March 2005.
- [18] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and its Applications,* Series in Mathematics in Science and Engineering. vol. 143, New York-London: Academic, 1979.
- [19] J. P. Monks, V. Bharghavan, and W.-M. Hwu, "A power controlled multiple access protocol for wireless packet networks," in *Proceedings of INFOCOM*, pp. 219–228, 2001.
- [20] S. Narayanaswamy, V. Kawadia, R. S. Sreenivas, and P. R. Kumar, "Power control in ad-hoc networks: Theory, architecture, algorithm and implementation of the COMPOW protocol," *Proceedings of European Wireless 2002. Next Generation Wireless Networks: Technologies, Protocols, Services and Applications*, pp. 156–162, Florence, Italy, February 25–28 2002.
- [21] M. D. Penrose, "The longest edge of the random minimal spanning tree," *The Annals of Applied Probability*, vol. 7, no. 2, pp. 340–361, 1997.
- [22] J. G. Proakis, *Digital Communications.* McGraw-Hill, fourth ed., 2001.
- [23] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, vol. 10, pp. 585–595, November/December 1999.
- [24] D. Tse and P. Viswanath, *Fundamental of wireless communications.* Cambridge UK: Cambridge University Press, 2005.
- [25] V. V. Veeravalli, in Wideband Multi-Antenna Wireless Channels: Statistical Modeling, Analysis, and Simulation, IEEE VTC 2001 Tutorial, October 2001. Tutorial #7.

[26] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation," *IEEE Transactions on Information Theory*, vol. 50, no. 5, pp. 748–767, February 2004.

[27] L.-L. Xie and P. R. Kumar, "On the path-loss attenuation regime for positive cost and linear scaling of transport capacity in wireless networks," submited to *IEEE Transactions on Information Theory*, February 2005.

[28] F. Xue and P. R. Kumar, "The transport capacity of wireless networks under fading channels," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 834–847, March 2005.

[29] Bluetooth SIG Inc., "Specification of the Bluetooth system: Core," http://www.bluetooth.org/, 2001.

[30] IEEE 802.11 Standard, "IEEE Local Area Network Working Group," IEEE Standard for Information Technology.

[31] IEEE P802.16-REVd/D5-2004, "Part 16: Air interface for fixed broadband wireless access systems," May 13, 2004.

[32] Intel White Paper, "IEEE802.16 and WiMAX: broadband wireless access for everyone," July 2003.

[33] LAN-MAN Standard Committee of the IEEE Computer Society, *Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for Low-Rate Wireless Personal Area Networks (LR-WPANs)*, IEEE, 2003.

[34] ZigBee Alliance, http://www.caba.org/standard/zigbee.html.