Scan Grammars: Parallel Attribute Evaluation Via Data-Parallelism

Thomas Reps
University of Wisconsin

Abstract

This paper concerns the problem of how to exploit parallelism during the phases of compilation involving syntax-directed analysis and translation. In particular, we address the problem of how to exploit parallelism during the evaluation of the attributes of a derivation tree of a non-circular attribute grammar. What distinguishes the ideas presented in this paper from earlier work on parallel attribute evaluation is the use of a *data-parallel* model: We define a new variant of attribute grammars, called *scan grammars*, that incorporates a data-parallel attribution construct.

1. Introduction

Because symbolic (non-numeric) computations often involve irregular structures, the task of devising parallel algorithms for such problems is generally very difficult and attempts are often unsuccessful. This paper addresses a certain class of symbolic computational problems and shows that for these problems speedups on the order of 64-fold with 100 processors and 100-fold with 250 processors may well be possible.

The topic that we are concerned with is how to exploit parallelism during the phases of compilation involving syntax-directed analysis and translation. Among the tasks that arise during these phases are symbol-table construction, name analysis, type checking, and either code generation or translation to an intermediate representation for subsequent processing. However, the techniques we present in the paper are not restricted to just the programming-language translation tasks listed above; they also apply to many other problems that can be posed as the translation of a derivation tree of a context-free language, such as pretty-printing, text formatting, generation of verification conditions from a program annotated with assertions, and verifying the correctness of proofs (for different kinds of mathematical and programming logics). Thus, our results contribute to the development of parallelized implementations of tools in all of these domains.

The particular question addressed is how to exploit parallelism during the evaluation of the attributes of a derivation tree of a (non-circular) attribute grammar [18] (see also [27] or [2]). Although this problem has been addressed by others [6, 13, 17, 19, 28], what distinguishes the ideas presented in this paper from earlier work on parallel attribute evaluation is the use of *data-parallelism*. The basic operation in the data-parallel model is a *scan* over a sequence with respect to an associative operator [5]. (The scan operation is sometimes called parallel-prefix or parallel-suffix.) A scan computes a new sequence whose elements are the "partial sums" of the original sequence. In general, we are given a sequence $x_1, \cdots, x_n$ and an associative binary operator $\oplus$. In a left-to-right $\oplus$-scan (or $\oplus$-parallel-prefix operation), the goal is to compute the sequence $y_1, \cdots, y_n$, such that $y_i = x_1 \oplus \cdots \oplus x_i$, for $1 \le i \le n$. For example, the result of applying the left-to-right $+$-scan operation to the sequence (1, 2, 3, 4, 5) is the sequence (1, 3, 6, 10, 15).

This paper defines a new variant of attribute grammars, called *scan grammars*, in which attributes are defined in a data-parallel fashion. Scan grammars include a *scan-attribution* construct, which defines a scan over attribute values located at a derivation tree's leaves. A scan-attribution computes the partial sums (with respect to a given associative operator) in a left-to-right or right-to-left pass over the derivation tree's leaves. Each partial sum computed by the scan is "left behind" at the appropriate leaf, where it may be an argument to other attribute equations or used in another scan-attribution operation. (Although scan-attribution is a data-parallel construct, this does not mean that scan grammars are only suitable for use on a SIMD machine; on the contrary, as discussed in Section 5.3, there are good reasons to expect that our parallel scan-grammar evaluator will perform well on a MIMD machine.)

The scan-attribution construct can be simulated with a conventional attribute grammar using attribute equations that are threaded left-to-right (or right-to-left) through the derivation tree to create the partial sums. Any of a number of sequential attribute evaluators can then be used to provide a sequential implementation of a scan-grammar evaluator. However, because scan-attributions are defined in terms of associative operators, scan grammars can be evaluated more efficiently in parallel. Instead of a simple left-to-right flow of information, the parallel-evaluation strategy uses a different pattern of information flow, which is based on the one employed in the algorithms for carry-lookahead addition [23] and the efficient parallel evaluation of scan operations [5, 20] (see Section 3.2). Given one processor per production instance in the derivation tree, a scan-attribution can be evaluated in parallel in $2D + 1$ steps, where D is the *depth* of the derivation tree. With fewer processors than one per production instance, substantial speedups can still be obtained by having each actual processor simulate the actions that need to be carried out at some number of production instances. In particular, by a theorem of Brent [7, 8], a scan-attribution can be evaluated in at most $2N/P + (2D + 1)$ steps, where N is the number of production instances in the derivation tree and $P < N$ is the number of processors.

The sequential evaluation of the left-to-right-threaded equations that implement scan-attribution requires $2N$ steps. Thus, the parallel-evaluation bounds ($2D + 1$ steps and $2N/P + (2D + 1)$ steps) suggest that the parallel evaluation of scan grammars will be substantially faster than sequential evaluation. By measuring the size of the abstract syntax trees of two (uncontrived) Pascal programs, we found that the ratio $2N/(2D + 1)$ was 119 for a 478-line Pascal pro-

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Authors' address: Computer Sciences Department, Univ. of Wisconsin, 1210 W. Dayton St., Madison, WI 53706. Email: reps@cs.wisc.edu.

gram and 174 for a 776-line Pascal program. Furthermore, the ratio $2N/(2N/P + (2D + 1))$ for the same two programs suggests that it may be possible to obtain speedups for Pascal programs on the order of 10-fold with 10 processors, 64-fold with 100 processors, and 100-fold with 250 processors. (See the table presented in Section 3.) These quantities are likely to increase with larger programs. In all previous work on parallel attribute evaluation, the speedups reported for Pascal attribute grammars have been relatively modest—in the range of 4 to 7 (having tailed off rapidly beyond about 16 processors).[1]

While these numbers are suggestive of the potential of our approach to parallel syntax-directed analysis and translation, achieving these substantial speedups will require careful attention to a number of implementation issues. These issues are discussed in Sections 3, 4, and 5.

For expository purposes, this paper presents the core ideas related to scan-attribution in their simplest form. Thus, while what is presented in the paper does have some limitations (in terms of the kinds of analysis and translation problems that can be easily expressed), we believe that scan-attribution represents an important advance towards creating a general attribute-grammar system that exploits parallelism. Section 6 discusses one approach to creating such a system—combining scan-attributions with certain kinds of ordinary attribute equations, while retaining the ability to perform attribute evaluation efficiently in parallel.

The remainder of this paper is organized as follows: Section 2 introduces scan grammars by means of an example. Section 3 discusses a parallel evaluation algorithm for scan grammars, which is based on the ideas used in the carry-lookahead addition algorithm (and other algorithms for computing scans quickly in parallel). Section 4 discusses the representation of lists derived from the "list nonterminals" that are typically used in context-free grammars. Section 5 considers one of the most important syntax-directed computations that arises in attribute-grammar specifications—the construction of a symbol table—and shows how to express this computation with a scan-attribution. Section 6 sketches out one way to create a more general system based on the scan-attribution paradigm. Section 7 describes how the ideas presented in this paper relate to previous work on parallel attribute evaluation.

2. Scan grammars

In this section, we introduce the components of scan grammars by means of a simple example—the computation of the value of a binary numeral.

Attribution rules are defined with respect to a set of grammar rules. Here we define the abstract syntax of binary numerals by giving a collection of operator/operand declarations:[2]

```
numeral: Numeral(bits);
bits: Pair(bits bits)
    | Bit(bit)
    ;
bit: Zero()
   | One()
   ;
```

Aside. Before giving the scan grammar's attribution rules for the binary-numeral problem, we first give rules for a conventional attribute grammar that solves the problem. In a conventional attribute grammar, two integer-valued attributes—say "position_in" and "position_out"—are used to define a right-to-left pattern of information flow through the derivation tree (so-called "right-to-left threading"):

```
bits, bit { inherited INT position_in;
            synthesized INT position_out;
          };
```

In this example, these attributes are used to determine a bit's position in the numeral. More precisely, the goal is for the value of bit.position_out to be the bit's position with respect to the right end of the numeral (where the rightmost bit is considered to be at position 1). This is arranged by the following attribute declarations:[3]

```
numeral: Numeral { bits.position_in = 0; };
bits: Pair {
        bits$3.position_in = bits$1.position_in;
        bits$2.position_in = bits$3.position_out;
        bits$1.position_out = bits$2.position_out;
      }
    | Bit {
        bit.position_in = bits.position_in;
        bits.position_out = bit.position_out;
      }
    ;
bit: Zero{bit.position_out = bit.position_in + 1;}
   | One{bit.position_out = bit.position_in + 1;}
   ;
```

From the position of a bit in the numeral (*i.e.*, the value of bit.position_out), the bit's contribution to the numeral's overall value can be determined:

```
bit { synthesized INT val; };
bit: Zero { bit.val = 0; }
   | One { bit.val = 2 ** (bit.position_out - 1); }
   ;
```

The value of the entire numeral is determined by summing the contributions from each subtree:

```
numeral, bits { synthesized INT val; };
numeral: Numeral { numeral.val = bits.val; };
bits: Pair {bits$1.val = bits$2.val + bits$3.val;}
    | Bit { bits.val = bit.val; }
    ;
```

When the derivation tree is consistently attributed according to these rules, the value of the attribute "numeral.val", which occurs at the derivation tree's root, holds the value that corresponds to the numeral.
End Aside.

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[1] Some of the speedup figures were established by measuring an implemented system [6]; others come from simulations [19, 28].

[2] This notation is a variant of context-free grammars in which the operator names (Numeral, Pair, Bit, Zero, and One) serve to identify the productions uniquely. For example, the declaration `numeral: Numeral(bits);` is the analogue of the production "numeral → bits". In general, the notation used in this paper's examples is adapted from the notation used in the Synthesizer Generator [25], a widely distributed system based on attribute grammars.

[3] In the attribute equations for a production such as

```
bits: Pair(bits bits);
```

it is necessary to distinguish between the three different occurrences of nonterminal "`bits`". We use the notation "`bits$1`", "`bits$2`", and "`bits$3`", where "`bits$1`" denotes the leftmost occurrence, *etc.* (In this case, the leftmost occurrence is the left-hand-side occurrence.)

We now show how the binary-numeral problem is specified using a scan grammar. The solution involves two scan-attributions, called "position" and "value", both of which are directed right-to-left.

A scan-attribution is declared by specifying a direction (*i.e.*, LR or RL), a type, an associative operation, and a "seed" value. For example, the declaration

```
scan position (RL, [INT]→[INT], +, 0);
```

defines a scan-attribution named "position" as a right-to-left scan-attribution that maps a sequence of INT's to a sequence of INT's via the addition function, with seed value 0. Such a declaration creates two attributes at each leaf, denoted by "position′input" and "position′output". At each leaf in the derivation tree, the value of position′output is the appropriate partial sum in a right-to-left scan, with respect to the addition function, of the sequence formed from the position′input attributes at the derivation tree's leaves. (We wish to stress that the input and output sequences are both *conceptual* objects. They are both distributed over the leaves of the derivation tree, and neither of them ever needs to be materialized as an explicit data structure.)

In this example, the goal is for the value of bit.position′output to be the bit's position with respect to the right end of the numeral. (The bit.position′output attributes serve the same function as the bit.position_out attributes in the conventional attribute grammar given earlier.) Thus, for each instance of a bit nonterminal, we want the value of bit.position′input to be 1, which is easily arranged with the following attribution equations:

```
bit: Zero { bit.position′input = 1; }
   | One  { bit.position′input = 1; }
   ;
```

The value of the entire numeral is defined by declaring a second right-to-left scan that forms the partial sums of the individual bits' contributions:

```
scan value (RL, [INT] → [INT], +, 0);
bit: Zero { bit.value′input = 0; }
   | One { bit.value′input =
               2 ** (bit.position′output - 1); }
   ;
```

As these rules illustrate, the results from one scan-attribution (*e.g.*, the values of the various bit.position′output attributes) can be used in creating the input to a second scan-attribution. The role of bit.value′output is somewhat different from that of bit.val in the conventional attribute grammar given earlier—each bit.value′output attribute defined by the rules given above represents the value the numeral would have if the left prefix of the numeral up to the leaf were discarded.

Remark. It is tempting to simplify the rules we have given, and use only one (left-to-right) scan-attribution declaration instead of two, as follows:

```
scan value (LR, [INT]→[INT], λx.λy.2*x+y, 0);
bit: Zero { bit.value′input = 0; }
   | One  { bit.value′input = 1; }
   ;
```

However, this scan-attribution declaration uses the function $\lambda x . \lambda y . 2 * x + y$, which is not an associative operation. It will become clear in the discussion of scan operations in Section 3.2 that the associativity property is of crucial importance if scans are to be evaluated efficiently in parallel. Associativity provides us with the freedom to re-group operations, and it is this freedom that makes scan-attribution amenable to parallel processing.

For scan-attributions using such non-associative operators to be well-defined, the evaluator would have to work either left-to-right or right-to-left in sequential order. This would require $2N$ steps, where N is the number of production instances in the derivation tree, rather than $2D + 1$ or $2N/P + (2D + 1)$ steps.
End Remark.

Returning to our example, closer inspection reveals that the scan grammar given above is not quite equivalent to the conventional attribute grammar that was given earlier. With the scan grammar, the numeral's overall value will be found at the leftmost leaf of the derivation tree, not at the derivation tree's root as is the case for the conventional attribute grammar. This is easily rectified by permitting interior nodes to play a role in scan-attributions directly. Conceptually, each interior node of arity-k has $k+1$ leaves, one to the left of the leftmost child, one to the right of the rightmost child, and one in between every pair of consecutive children. For example, with the declaration

```
numeral: Numeral(bits);
```

a "numeral" node has two such conceptual leaves, denoted by "numeral[0]" for the one on the left and "numeral[1]" for the one on the right. Using this notation, the following rules specify how attribute "numeral.val" receives the value that corresponds to the numeral:

```
numeral { synthesized INT val; };
numeral: Numeral(bits) {
           numeral[0].position′input = 1;
           numeral[0].value′input = 0;
           numeral.val = numeral[0].value′output;
         }
       ;
```

(We will make use of this notation in the examples in Section 5.)

It is important to note that although the example given above only makes use of the integer data type and the "+" operation, there are a wide variety of data types and operations that can be used in scan-attributions, including the following: Booleans ($\wedge$, $\vee$), integers (+, *, max, min), floating-point numbers (+, *, max, min), sequences or lists (append), sets ($\cup$, $\cap$), and finite functions (see the example in Section 5).

3. Evaluation of scan grammars

3.1. Sequential implementation of scan-attribution

It is straightforward to compile scan-attributions into conventional attribute-grammar specifications, whereupon they can be evaluated using any of a number of sequential evaluators. Each scan-attribution construct can be translated into a left-to-right or right-to-left attribute threading that creates the partial sums. For instance, in the binary-numeral example, the "position" scan would compile into essentially the same set of right-to-left-threaded attribute equations that were given for attributes position_in and position_out in the description of how the binary-numeral example is handled in a conventional attribute grammar. The "value" scan would also be translated into a right-to-left threading, as follows:

```
bits, bit { inherited INT value_in;
            synthesized INT value_out;
          };
numeral { synthesized INT val; };
numeral: Numeral { bits.value_in = 0;
                   numeral.val = bits.value_out;
```

```
                }
        ;
bits: Pair { bits$3.value_in = bits$1.value_in;
             bits$2.value_in = bits$3.value_out;
             bits$1.value_out = bits$2.value_out;
           }
    | Bit { bit.value_in = bits.value_in;
            bits.value_out = bit.value_out;
          }
    ;
bit: Zero { bit.value_out = bit.value_in; }
   | One {
       bit.value_out = 2 ** (bit.position_out - 1)
                                    + bit.value_in;
     };
```

3.2. Parallel implementation of scan-attribution

Because scan-attributions are defined in terms of associative operators, they can be evaluated efficiently in parallel. The pattern of information flow in the parallel evaluation algorithm is based on that employed in the algorithm for the efficient parallel evaluation of scan operations [5, 20] (of which carry-lookahead addition is one example [23]).

A $\oplus$-scan operation with respect to an associative operator $\oplus$ can be performed on a sequence of length k in $2\lceil \log k \rceil + 1$ steps by decomposing the problem into subproblems and arranging the subproblems in a balanced binary tree of depth $\lceil \log k \rceil$. One processing element is assigned to each leaf and interior node of the decomposition tree. Here is Blelloch's explanation of the pattern of information flow in the parallel implementation of a left-to-right scan:

> The technique consists of two sweeps of the tree, an up sweep and a down sweep . . . The values to be scanned start at the leaves of the tree. On the up sweep, each unit executes $\oplus$ on its two children units and passes the sum to its parent. Each unit also keeps a copy of the value from the left child in its memory. On the down sweep, each unit passes to its left child the value from its parent and passes to its right child $\oplus$ applied to its parent and the value stored in the memory (this value originally came from the left child). After the down sweep, the values at the leaves are the results of a scan [5].[4] .

It is actually not necessary that the problem-decomposition tree be balanced, nor is it necessary that it be a binary tree. The $\oplus$-scan computation can be carried out in parallel on any *fixed-arity* tree that represents a decomposition of the problem in $2D + 1$ steps, where D is the *depth* of the decomposition tree.

Because the information flow in each of the two sweeps can be expressed as an attribution of the decomposition tree according to the rules of an attribute grammar, we are able to use the scan-evaluation algorithm to evaluate a scan-attribution in parallel. The first step is to translate the scan-attribution construct to a set of attribute equations of a special form. We illustrate this translation by means of an example.

[4] Because of a slight technical difference between the way we have defined the $\oplus$-scan of a sequence and the way Blelloch defines it, we must add the following: It is also necessary for each leaf to retain a copy of its original value and to apply $\oplus$ to the value received from the parent and the original value.

Example. Suppose $\oplus$ is an associative operation of type $T \times T \to T$, and we are given a left-to-right scan-attribution of the form

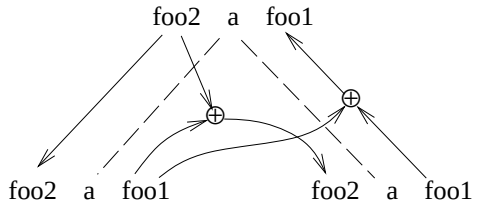
```
scan foo (LR, [T] → [T], ⊕, v);
root: Root(a);
a: Pair(a a)
 | Singleton(b)
 ;
b: Leaf() { b.foo'input =  · · · ; };
```

This scan-attribution can be translated automatically into the following equations:

```
/* Sweep I ------------------------------------ */
a, b { synthesized T foo1; };
a: Pair      { a$1.foo1 = a$2.foo1⊕a$3.foo1; }
 | Singleton { a.foo1 = b.foo1; }
b: Leaf      { b.foo1 =  · · · ; };

/* Sweep II ----------------------------------- */
a, b { inherited T foo2; };
root: Root { a.foo2 = v; };
a: Pair   { a$2.foo2 = a$1.foo2;
            a$3.foo2 = a$1.foo2⊕a$2.foo1;
          }
 | Singleton { b.foo2 = a.foo2⊕b.foo1; }
 ;
```

The dependences among the attributes in the production `a: Pair(a a)` can be depicted as follows:



We say that equations like the ones in the above example are in *scan form*. We now show what the translation to scan form produces for the two right-to-left scans of the binary-numeral example.

Example. The "position" scan is translated into the following attribute equations in scan form:

```
/* Sweep I ------------------------------------ */
bits, bit { synthesized INT position1; };
bits: Pair {
        bits$1.position1 = bits$2.position1 +
                                    bits$3.position1;
      }
    | Bit { bits.position1 = bit.position1; }
    ;
bit: Zero { bit.position1 = 1; }
   | One  { bit.position1 = 1; }
   ;

/* Sweep II ----------------------------------- */
bits, bit { inherited INT position2; };
numeral: Numeral { bits.position2 = 0; };
bits: Pair {
        bits$3.position2 = bits$1.position2;
        bits$2.position2 = bits$3.position1 +
                                    bits$1.position2;
      }
    | Bit {
        bit.position2 = bit.position1 +
                                    bits.position2;
      }
    ;
```

The "value" scan is translated as follows:

```
/* Sweep I ------------------------------------ */
bits, bit { synthesized INT value1; };
bits: Pair {
        bits$1.value1 = bits$2.value1 +
                                    bits$3.value1;
      }
    | Bit { bits.value1 = bit.value1; }
    ;
bit: Zero { bit.value1 = 0; }
   | One { bit.value1 = 2 ** (bit.position2 - 1); }
   ;

/* Sweep II ----------------------------------- */
bits, bit { inherited INT value2; };
numeral: Numeral { bits.value2 = 0; };
bits: Pair {
        bits$3.value2 = bits$1.value2;
        bits$2.value2 = bits$3.value1 +
                                    bits$1.value2;
      }
    | Bit {
        bit.value2 = bit.value1 + bits.value2;
      }
    ;
```

Given equations in scan form, there is an obvious parallel algorithm for evaluating them: Use one processor per production instance and have each processor evaluate the tree's attributes in accordance with the method quoted above. By this means, a scan-attribution can be evaluated in parallel in $2D + 1$ steps, where D is the depth of the derivation tree. When only $P < N$ processors are available, where N is the number of production instances in the derivation tree, each actual processor must simulate the actions that need to be carried out at some number of production instances. The scheduling technique used in the simulation theorem of Brent can be used [7, 8] and consequently a scan-attribution can be evaluated in at most $2N/P + (2D + 1)$ steps.

To get a feel for what this means for abstract syntax trees of actual programs, we measured two (uncontrived) Pascal programs: format.p, a simple text-formatting program taken from Kernighan and Plauger's book [16], and gradestats.p, a grading program obtained from a colleague. Figure 1 presents figures on steps, speedup, and efficiency as a function of number of processors for the two programs. Figure 1 shows that it may be possible to obtain speedups for Pascal programs on the order of 10-fold with 10 processors, 64-fold with 100 processors, and 100-fold with 250 processors.

The measurements reported in Figure 1 were taken on trees defined by the Pascal attribute grammar that is distributed with the Synthesizer Generator system for generating language-sensitive editors [25]. The numbers in Figure 1 reflect three adjustments to the raw figures:

(1) The Synthesizer Generator uses right-recursive productions to represent lists derived from "list nonterminals"; in Figure 1 "Depth" has been adjusted to reflect what the tree depths would be if such lists were represented as balanced binary trees. (See Section 4.)
(2) Because atomic leaves of Synthesizer Generator trees generally do not have attributes, in the formula for calculating "Steps(P)" the number of "Atomic leaves" is subtracted from "Nodes".
(3) Also in the formula for calculating "Steps(P)", the "Depth" is decreased by two, one for the level of atomic leaves and one for the root production.

It is also possible to define a different parallel scan-grammar evaluation algorithm based on the same underlying idea, but in which the evaluation of attributes during Sweep II is pipelined (and evaluation is carried out "differentially"). Suppose $\oplus$ is the operation with respect to which the scan is being performed and that e is the identity element for $\oplus$:

(1) Allocate one processor per production instance.
(2) Each attribute in the scan's scan-form equations is initially assigned the value e.
(3) Sweep I is carried out in the normal fashion.
(4) During Sweep II, each processor is in charge of evaluating two attribute instances, say "`child1.foo2`" and "`child2.foo2`". Whenever new information about the value of `parent.foo2`—in the form of a change Δ in the value of `parent.foo2`—is received from the processor of the parent node in the derivation tree, the current values of `child1.foo2` and `child2.foo2` are *updated* by making assignments

    `child1.foo2` := $\Delta \oplus$ `child1.foo2`
    `child2.foo2` := $\Delta \oplus$ `child2.foo2`.

    The value Δ is then passed down the derivation tree to the processors associated with the appropriate child nodes.
(5) In a left-to-right scan, the pipelining process is initiated by setting the values of the tree's collection of `child2.foo2` attributes by making assignments

    `child2.foo2 := child1.foo1`.

With this method, information is passed to a neighboring processor whenever possible; an attribute's value steadily *accumulates* as more and more information flows down from the `parent.foo2` attribute. Because the passing of information down the derivation tree is pipelined, an attribute's final value is determined only after it has received all of the information from the collection of attributes along the path from the attribute's tree node to the root of the tree.

Remark. In the pipelined version of the evaluation algorithm, an attribute's value steadily accumulates as more and more information flows down from the Sweep II attribute of its parent node in the derivation tree. This is an unusual feature for an attribute-evaluation algorithm, but other examples of such algorithms are known.

One example of previous work in which the final value of an attribute accumulates from previous values is the "differential" algorithm of Hoover and Teitelbaum for incremental updating of aggregate-valued attributes in language-sensitive editors [10], which may consider an attribute several times during updating. This is done to compensate for the evaluator's use of imprecise information about the ordering of dependences between attributes. Because the algorithm is for a sequential processor, the multiple evaluation of attribute instances is not a desirable characteristic of the algorithm (although it does not appear to hurt the algorithm's performance in practice).

Other examples of evaluation algorithms in which the final value of an attribute is accumulated from previous values are the various algorithms proposed for evaluating circular attribute grammars [4, 9, 11, 12, 26].
End Remark.

	Program	format.p		Program	gradestats.p	
	Lines	478		Lines	776	
	Nodes	4889		Nodes	8671	
	Atomic leaves	1016		Atomic leaves	1787	
	Depth	34		Depth	41	
P	Steps(P)	Speedup(P)	Efficiency(P)	Steps(P)	Speedup(P)	Efficiency(P)
1	7811	1	1.00	13847	1	1.00
10	840	9	0.93	1456	10	0.95
20	452	17	0.86	767	18	0.90
30	323	24	0.81	538	26	0.86
40	259	30	0.75	423	33	0.82
50	220	36	0.71	354	39	0.78
60	194	40	0.67	308	45	0.75
70	176	44	0.64	276	50	0.72
80	162	48	0.60	251	55	0.69
90	151	52	0.57	232	60	0.66
100	142	55	0.55	217	64	0.64
150	117	67	0.45	171	81	0.54
200	104	75	0.38	148	94	0.47
250	96	81	0.33	134	103	0.41
500	80	97	0.19	107	130	0.26
1000	73	107	0.11	93	149	0.15
2000	69	113	0.06	86	161	0.08
3873	67	117	0.03			
4000				82	168	0.04
6884				81	171	0.02

Steps(P) = 2 * (Nodes – Atomic leaves) / P + (2 * (Depth – 2) + 1)
Speedup(P) = Steps(1) / Steps(P)
Efficiency(P) = Speedup(P) / P

Figure 1. Steps, speedup, and efficiency as a function of the number of processors P for two Pascal programs.

4. Representation of lists derived from list nonterminals

In this section, we discuss the representation of lists derived from the "list nonterminals" that are typically used in context-free grammars. For example, a programming-language grammar may have a number of different list nonterminals, such as *stmtList*, *expList*, *idList*, *etc*. In the abstract syntax trees of programming languages, there are several different ways to represent lists derived from list nonterminals using nodes of fixed-arity. (Recall that the algorithm for efficient parallel evaluation of scan operations calls for a fixed-arity problem-decomposition tree.) One way is to use right-recursive productions:

stmtList → *stmt stmtList*
stmtList → *stmt*
stmt → ···

However, in the case where N processors are available, the parallel scan-grammar evaluation algorithm requires $2D + 1$ steps, where D is the depth of the tree. If lists are represented with right-recursive productions, a list of length k has depth k, and consequently this representation is detrimental to the performance of the evaluation algorithm.

From this standpoint, it is better to replace the right-recursive productions with ambiguous binary productions and to represent lists as *balanced* binary trees:

stmtList → *stmtList stmtList*
stmtList → *stmt*
stmt → ···

Using this representation of lists can only reduce the depth of the tree; a list of length k has depth $\lceil \log k \rceil$.

To determine how much would be gained from using balanced binary trees to represent lists, we gathered some statistics on the depths of Pascal abstract syntax trees. Again, the measurements were taken on trees defined by the Pascal attribute grammar that is distributed with the Synthesizer Generator. Although the Synthesizer Generator uses right-recursive productions to represent lists, the depth counts were adjusted to reflect what the depth of the tree would have been if balanced binary trees had been used. Figure 2 presents the statistics that were obtained. The column labeled "Depth" gives the tree's depth before the adjustment; "Adjusted depth" gives the adjusted figure. Because the algorithm for efficient parallel evaluation of scan operations requires $2D + 1$ steps, the two larger examples show that the balanced-binary-tree representation would be better by a factor of 4 to 4.5 when N processors are available. This factor is likely to increase with larger programs.

(In Figure 1, the figures already reflect the use of the balanced-binary-tree representation.)

5. Symbol-table construction via scan-attribution

One of the most important syntax-directed computations that arises in attribute-grammar specifications is *symbol-table construction*. For example, attribute grammars that specify the static semantics of programming languages usually build symbol tables that map identifiers to a descriptor of scope and binding information as the starting point for name analysis (*i.e.*, the detection of undeclared and multiply declared variables) and type checking. Each node of the derivation tree is annotated with (a pointer to) a data structure that records the identifiers known in the current scope and their properties.

Program	Description	Lines	Nodes	Atomic leaves	Depth	Adjusted depth
empty.p	empty template for a program	5	15	0	4	3
primes.p	sieve of Eratosthenes	32	235	52	29	24
queens.p	eight-queens problem	56	549	116	36	28
format.p	example from Kernighan & Plauger	478	4889	1016	64	34
gradestats.p	grading program	776	8671	1787	94	41

Nodes: number of production instances in the tree
Atomic leaves: number of leaves consisting of one of the primitive types INT, CHAR, ID, STR, *etc*.
Depth: right-recursive "combs" used to represent lists
xList → *x xList* | *x*
x → ···
Adjusted depth: balanced binary trees used to represent lists
x → *xList xList* | *x*
x → ···

Figure 2. Sizes of some representative Pascal abstract syntax trees. The column labeled "Adjusted depth" is (an upper bound on) the depth of the tree obtained by replacing every list by a balanced binary tree.

In this paper, we use the term "symbol table" in a broader sense: For our purposes, a symbol table is any *finite function*—any function defined only for a finite set of argument values. For example, a text-formatting grammar might make use of a "symbol table" consisting of the current formatting properties. The ideas described below apply to any such finite function.

In this section, we show how symbol-table processing can be expressed using the scan-attribution construct (*i.e.*, as a scan-attribution with respect to a certain associative operator). The key observation is as follows: *The basic operation used for building up symbol tables—regardless of their particular implementation—is* ***associative****. Associativity provides us with the freedom to re-group the symbol-table construction operations to make symbol-table construction more amenable to parallel processing.*

In this section, our examples deal with a simple imperative language whose abstract syntax is defined as follows:

```
program: Program(declList stmtList);
declList: DeclListPair(declList declList)
        | DeclListSingleton(decl)
        ;
decl: Decl(ID type);
type: Undefined()
    | Boolean()
    | Integer()
    ;
stmtList: StmtListPair(stmtList stmtList)
        | StmtListSingleton(stmt)
        ;
stmt: Assignment(ID exp)
    | Conditional(exp stmtList stmtList)
    | WhileLoop(exp stmtList)
    ;
exp: IdExp(ID)
   |  . . .
   ;
```

The symbol tables for this language will be finite functions from identifiers to types: $env =_{df} \mathrm{ID} \to \mathrm{type}$. The domain of types contains a distinguished value "Undefined"; if *st* is an *env* and $st(i) = \mathrm{Undefined}$, then *st* is undefined on *i*.

In the presentation below, we initially address the case where there is only a single global scope. We then extend our approach to cover the case of nested scopes. Finally, we address an important performance issue: Because symbol-table attributes are not unit-sized objects, we cannot afford the overhead of shipping copies of their values from processor to processor; methods that avoid this potential overhead are presented in Section 5.3.

5.1. Scan-attribution for a single global scope

If there is only a single global scope, it is straightforward to use scan-attribution to specify the annotation of the derivation tree with appropriate symbol-table values.

In this section, rather than fix on a particular representation for symbol tables (*i.e.*, the domain *env*), we treat them in an abstract fashion. We define two operations on *env*: one, denoted by $[x : v]$, creates a symbol table defined at a single point; the other, denoted by $a \otimes b$, combines two symbol tables *a* and *b*. More precisely, the two operations behave as follows:

$$[x : v](i) = \begin{cases} v & \text{if } x = i \\ \text{Undefined} & \text{otherwise} \end{cases}$$

$$(a \otimes b)(i) = \begin{cases} b(i) & \text{if } b(i) \neq \text{Undefined} \\ a(i) & \text{otherwise} \end{cases}$$

We use *e* to denote the everywhere-undefined environment: $e(i) = \mathrm{Undefined}$, for all *i*. It is easy to see that $\otimes$ is associative and that *e* is the identity element for $\otimes$.

With this notation, we can now express symbol-table construction using the scan-attribution construct.

Example. Symbol-table construction for our simple programming language can be expressed with a left-to-right scan, with each declaration contributing a symbol table defined at a single point:

```
scan symbol_table (LR, [env]→[env], ⊗, e);
decl: Decl{decl[2].symbol_table′input = [ID:type];};
stmt: Assignment { stmt[0].symbol_table′input = e;};
exp: IdExp       { exp[0].symbol_table′input = e;};
```

Note that each use of an identifier in a statement or expression contributes a symbol table as well, but always with value *e*. As a consequence, symbol_table′output is available at that point in the tree; this is employed to access the symbol-table information for the identifier used at that point in the program:

```
stmt: Assignment {
        stmt.lhs_info =
                  stmt[0].symbol_table'output(ID);
      };
exp: IdExp {
       exp.id_info = exp[0].symbol_table'output(ID);
     };
```

Similarly, if we wish to detect duplicate declarations, then we would change the specification so that Decl contributes two symbol tables, first *e* and then [ID : type]:

```
decl: Decl {
        decl[0].symbol_table'input = e;
        decl[2].symbol_table'input = [ID:type];
        decl.id_info =
                  decl[0].symbol_table'output(ID);
      }
    ;
```

If decl[0].symbol_table′output(ID) is not equal to Undefined, then this occurrence of ID is a duplicate declaration.

5.2. Scan-attribution for nested scopes

With nested scopes, it is still possible to use scan-attribution to specify the annotation of the derivation tree with appropriate symbol-table values. In this section, it is convenient to fix on a concrete representation for symbol tables; we will use LISP-like lists whose elements are of the form CONS(ID, type) (*e.g.*, CONS(CONS(x, Integer), CONS(CONS(y, Boolean), NIL)) is an example of a symbol table in this representation). With this concrete representation, the operation $\otimes$ is $\lambda x . \lambda y .$ APPEND(y, x) (where APPEND is the function that appends two lists), and its identity element *e* is NIL. Note that the way we express symbol-table construction in the case of a single global scope changes only slightly, as follows:

```
scan symbol_table (LR, [env]→[env],
                           λx.λy.APPEND(y, x), NIL);
decl: Decl {
        decl[2].symbol_table'input =
                        CONS(CONS(ID, type), NIL);
      };
stmt: Assignment {
        stmt[0].symbol_table'input = NIL;
      };
exp : IdExp { exp[0].symbol_table'input = NIL; };
```

To model nested scopes, we extend the abstract-syntax declarations for our example language with a block construct:

```
stmt: Block(declList stmtList);
```

We also introduce two markers, denoted by BLOCK_ENTRY and BLOCK_EXIT, which will be used in symbol-table lists to bracket scopes that have been exited. In particular, the lookup function ignores all symbol-table entries when a BLOCK_ENTRY is encountered until the matching BLOCK_EXIT is found. BLOCK_ENTRY and BLOCK_EXIT are used in the scan-attribution equations for Block, as follows:

```
stmt: Block {
        stmt[0].symbol_table'input =
                           CONS(BLOCK_EXIT, NIL);
        stmt[2].symbol_table'input =
                           CONS(BLOCK_ENTRY, NIL);
      };
```

There is one drawback to this approach: The symbol table associated with a given node in the derivation tree contains symbol-table entries for all declarations to the left of the node—*including declarations from scopes that are not active*. This will cause an application of the lookup function on identifier *i* to be much less efficient whenever it does not find a symbol-table entry for *i* in the local scope.

We would like the symbol-table list to include entries only for declarations in enclosing scopes. This can be arranged by filtering symbol-table lists, removing all symbol-table information between a matching BLOCK_ENTRY/BLOCK_EXIT pair. This is permissible because all information between a matching BLOCK_ENTRY/BLOCK_EXIT pair is ignored by the lookup function. In the attribute grammar for the language given above, such filtering can be done in the Sweep I equations for the symbol-table scan.

Remark. Technically what we have done is to change the meaning of the scan operation slightly: We are given a sequence $x_1, \cdots, x_n$ and an associative binary operator $\oplus$. The goal is to compute the sequence $y_1, \cdots, y_n$, such that $y_i = x_1 \oplus \cdots \oplus x_i$, for $1 \le i \le n$, where "$a = b$" now means "equal according to what is observable in *a* and *b*". For example, in the case of symbol-table lists, "$a = b$" means "equal according to what is observable in *a* and *b* via the lookup function".
End Remark.

5.3. Pragmatic performance considerations

Because symbol-table attributes are not unit-sized objects, the performance of the parallel scan-grammar evaluator will degrade seriously from the theoretical speedup of $2N/(2N/P + (2D + 1))$ if substantial amounts of symbol-table information must be passed from processor to processor. Consequently, the amount of symbol-table information that has to be passed between processors is an important pragmatic issue.

To solve this problem, it is necessary to consider the manner in which symbol-table data is accessed. In particular, note that symbol-table construction is carried out during the evaluation of the "symbol_table" scan-attribution; only later are the symbol-table attributes accessed in order to perform lookups. Furthermore, because of the value semantics of attribute grammars (*i.e.*, no side effects are permitted in attribute equations), the symbol-table construction phase involves the use of memory in a "write-once" fashion. When lookup operations are carried out, each (virtual) processor at a node of the derivation tree accesses the symbol table in a "read-only" fashion.

Thus, there are three key aspects to reducing the cost of communicating symbol-table values from processor to processor:

(1) The multiprocessor should provide a *shared-memory* abstraction.[5]
(2) The "handle" to a symbol table needs to be represented by a small value (such as a pointer to a location in shared memory).
(3) It must be possible to access a symbol-table entry with only a few accesses to shared memory.

---

[5]There are many approaches that have been used to provide shared memory on multiprocessors, either in software (*e.g.*, shared virtual memory on loosely coupled multiprocessors [21]) or in hardware (via snooping caches [3] or directory protocols [1]).

For the case of a single global scope, one way to implement a symbol table that satisfies these conditions is to use a balanced tree (such as a 2-3 tree, B-tree, or AVL tree) that is updated applicatively (*i.e.*, the spine of the tree is copied on each insertion) [22, 24]. With such structures, the only information that needs to be passed explicitly from processor to processor during a symbol-table construction scan is a pointer to the root of the tree. A symbol-table entry in a symbol table of size k can be accessed with $O(\log k)$ references to shared memory.

To handle nested scopes, we can use a list of balanced trees. Again, the only information that needs to be passed explicitly from processor to processor during a symbol-table construction scan is a pointer, in this case to the root of the list.

6. Combining scan-attribution with other processing patterns

The examples in the previous sections show how certain syntax-directed analyses can be specified conveniently with scan-attributions. However, because scan-attributions are constrained to use associative operators, there are some limitations as to what kinds of analysis and translation problems can be easily expressed. Nevertheless, we believe that scan-attribution represents an important advance towards creating a general attribute-grammar system that exploits parallelism.

One approach to creating a more general system based on the scan-attribution paradigm is to exploit the fact that some sets of ordinary attribute equations have dependence patterns that allow them to be evaluated in parallel in $O(D)$ steps (given a sufficient number of processors). Consequently, when these attributes are combined with scan-attributions, efficient parallel attribute evaluation is still possible. Examples of such dependence patterns include

(1) Purely local attribution phases, during which some attribute instances of each production instance are given values.
(2) Purely bottom-up attribution phases, during which information flows from the leaves toward the root.
(3) Purely top-down attribution phases, during which information flows from the root to the leaves.

Because we represent lists derived from list nonterminals as balanced binary trees, it is necessary to impose certain restrictions on the right-hand-side functions used in the attribute equations of these productions. In particular, in the rules for a bottom-up pass, the right-hand-side functions must be associative. In the rules for a top-down pass, the right-hand-side functions must be idempotent; in addition, the function used to pass information to the left child and the function used to pass information to the right child must commute.

The reasons why these restrictions need to be imposed can be seen with the following example. Suppose the attribute grammar contains the following definitions:

```
xList { synthesized T foo;
        inherited T bar;
      };
xList: Op(xList  xList) {
         xList$1.foo = xList$2.foo⊕xList$3.foo;
         xList$2.bar = F(xList$1.bar);
         xList$3.bar = G(xList$1.bar);
       }
     ;
```

Only when operation $\oplus$ is associative can we guarantee that the xList.foo attribute at the root of Op(a, Op(b, c)) is equal to the xList.foo attribute of Op(Op(a, b), c). Without associativity, the attribution of lists represented as balanced binary trees would not be uniquely defined. Similarly, only when functions F and G are idempotent (*i.e.* F(F(x)) = F(x), for all x) and commute (*i.e.* F(G(x)) = G(F(x)), for all x) can we guarantee that the three xList.bar attributes at the leaves of Op(a, Op(b, c)) are equal to the corresponding xList.bar attributes at the leaves of Op(Op(a, b), c). (In effect, these restrictions mean that there are only three different values of the xList.bar attribute in any given xList: If x is the value of xList.bar at the root of the list, then the xList.bar value at the leftmost member of the list is $F(x)$, the value at the rightmost member of the list is $G(x)$, and the values at all other members of the list are $F(G(x))$.)

For all three of the dependence patterns listed above, evaluation can be carried out in parallel in at most D steps (assuming N processors, where N is the number of production instances in the derivation tree). When only $P < N$ processors are available, they can be evaluated in at most $N/P + D$ steps. When attribute equations with such dependence patterns are combined with scan-attributions, evaluation can be carried out as a sequence of parallel-evaluation passes over the tree, where each pass requires either $N/P + D$ steps or $2N/P + (2D + 1)$ steps. Thus, the total cost of parallel evaluation would be $O(N/P + D)$.

(The question of whether it is possible to have an efficient parallel evaluator that handles a combination of scan-attributions and ordinary attribute equations whose dependence patterns are not in the three classes listed above is left for future work. However, from a pragmatic standpoint this may not be necessary. My conjecture is that for almost all problems that arise in practice scan-attributions plus the above three patterns suffice.)

7. Relation to previous work

Other work on parallel attribute evaluation includes [6, 13, 17, 19, 28]. Kaplan and Kaiser present a distributed evaluator for the problem of incremental attribute updating in language-sensitive editors [13]. Boehm and Zwaenepoel describe an implemented parallel attribute-grammar evaluator that runs on a network multiprocessor of six SUN-2 workstations connected by an Ethernet network [6]. In [28], Zaring presents parallel algorithms for ordered attribute grammars [14]; Zaring gives algorithms for both tightly coupled and loosely coupled multiprocessor architectures. The approach presented in this paper is much different from these previous approaches in two respects: (1) we focus on attribute specifications that employ a data-parallel construct; (2) the essence of our technique is to exploit associativity, which provides us with the freedom to re-group the computation, thereby making it more amenable to parallel processing.

Kuiper's work is somewhat closer to ours in spirit in that he makes use of an attribute-grammar transformation to restructure computations [19]. When his transformation is applicable, the attribute equations of the transformed attribute grammar specify a computation equivalent to the original attribute equations, but the derivation tree's dependence chains are shorter, which increases the amount of potential parallelism. However, Kuiper's transformation modifies the attribute equations only, leaving the underlying context-free grammar (and derivation trees) unchanged.

Kuiper's transformation applies to certain types of threadings over lists defined as right-recursive "combs" (*i.e.*, with a production of the form *xList* $\rightarrow$ *x xList*). For a

list of length k, it transforms the dependence graph of the threading from a chain of length $2k$ into two chains, each of length k. In the case of the construction of a symbol-table attribute via a thread running through a list of declarations, Kuiper's transformation applies due to the fact that the list-concatenation operation he uses to specify the construction of a symbol table is an associative operator. A key difference between our work and Kuiper's is that for this important kind of attribute computation, Kuiper's transformation does not exploit the associativity property to the fullest. His transformation is less powerful than the idea introduced in this paper, which calls for transforming both the derivation tree and the attribute equations. In particular, with our technique a right-recursive comb is transformed into a balanced binary tree; the original dependence chain of length $2k$ is replaced by a dependence graph in which the length of the longest path is $2\lceil \log k \rceil + 1$.

Klaiber and Gokhale also make use of an attribute-grammar transformation to restructure computations, in their case a "list-flattening" transformation [17]. However, their work is more in the spirit of Zaring's work, which deals with the parallel evaluation of "plans" for Kastens's class of ordered attribute grammars. Klaiber and Gokhale's work has similar goals, but is couched in terms of Katayama's translation of attribute grammars to mututally recursive procedures [15]. Klaiber and Gokhale address the problem of determining which calls to evaluation procedures can be scheduled in parallel. The list-flattening transformation is a normalization step that can uncover additional parallelization opportunities for their scheduling algorithm.

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