Scattering by Resistive Strips

by

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For resistive strips of large electrical width kw illuminated by E- or H-polarized plane waves, the geometrical theory of diffraction is used to obtain expressions for the far zone scattered field through second order terms, valid for directions of incidence and observation away from grazing. The results are then cast as products of functions analogous to those appearing in the known (uniform) expansions for perfectly conducting strips. Each function involves the current on the corresponding half plane, and by invoking this connection, far field expressions are produced which are uniform in angle. In particular, for E-polarization the backscattered field at edge-on incidence is shown to consist of two terms each of which is expressible in terms of the half plane current, and for all resistivities the resulting values of the field are in excellent agreement with those found by numerical solution of the integral equation, even for kw as small as unity.

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1. Introduction

Resistive sheets are important in the design of low radar cross section targets and a specific configuration which is amenable to analytical and numerical solution is a strip illuminated by a plane wave incident in a plane perpendicular to the edges. To replace a metal strip by a resistive one significantly reduces the scattering even for modest values of the resistivity, and this could be of practical interest in the case of thin structures at near grazing incidence for E polarization when most cross section reduction techniques are ineffective. From a study of numerical data for uniform resistive strips, Senior (1979a) showed that at grazing (or edge-on) incidence, the rear edge contribution to the backscattered far field amplitude for E polarization is proportional to the square of the current on the corresponding half plane, and deduced an empirical expression for the constant of proportionality valid for a range of (real) resistivities. The desire to verify this relationship analytically motivated the present investigation.

The scattering of a plane wave by a perfectly conducting strip has been widely studied and, for strips of large electrical width kw, asymptotic techniques are available for expanding the scattered field in inverse powers of kw. Although the standard GTD approach (see, for example, Ross, 1966) fails for angles of incidence or observation close to grazing, these special cases can be treated by asymptotic solution of the integral equations for the related problem of a slit (Levine, 1957; Seshadri and Wu, 1960). Using a dual integral equation approach, Fialkovskiy (1966) and Khaskind and Vainshteyn (1964) have provided asymptotic expansions of the bistatic scattered fields for E and H polarizations respectively that are, in fact, uniform in angle.

A key feature of all these methods is the use of Babinet's principle to convert the strip to a slit which is then analyzed by considering the interaction of two half planes whose electrical separation is large. Although there is a generalization of Babinet's principle for resistive and 'conductive' strips (Senior, 1977), the Babinet equivalent of an isolated resistive strip is a conductive insert in a metallic screen, and because the two portions of the screen are no longer separated by free space, there is no obvious way to use the integral equation approach. Moreover, the scattered field representation which is the basis of Noble's (1959) method for a metallic strip is not applicable when the strip is resistive.

The purpose of the present paper is to develop asymptotic expressions for the field scattered by a uniform resistive strip of large electrical width when illuminated by an E- or H-polarized plane wave incident at any angle including edge-on. Following Bowman (1967), the geometrical theory of diffraction is used to compute the bistatic scattered field through second order terms for angles of incidence and observation away from edge-on, and the results are then cast as products of functions analogous to those in the uniform expressions of Fialkovskiy (1966) and Khaskind and Vainshteyn (1964) for perfectly conducting strips. Each function is directly related to the diffracted portion of the current on the corresponding half plane, and by inserting the representations of the currents valid for all angles of incidence, uniform expressions are obtained for the far field amplitudes of the strip that hold even for edge-on incidence and observation. In the particular case of backscattering edge-on, the rear edge contribution is indeed proportional to the square of the half plane current. The constant of proportionality involves the split function produced by the Wiener-Hopf technique and, in the case of E polarization, differs by no more than seven percent from the constant empirically derived by Senior (1979). For numerical purposes, an expression for the rear edge contribution in terms of the current is far superior to that in which the current is replaced by the leading term in its high frequency expansion. Some computed data for the half plane current are presented, and these are used to determine the edge-on backscattering from strips as a function of their width. Even for strips as narrow as a sixth of a wavelength, the results are in excellent agreement with the ones obtained by numerical solution of the integral equation.

2. Formulation

The problem considered is that of an E- or H-polarized plane wave with

$$\underline{E}^i \text{ (or } \underline{H}^i) = \hat{z} e^{-ik(x \cos \phi_0 + y \sin \phi_0)} \tag{1}$$

respectively incident on a uniform resistive strip of width w and resistivity R immersed in free space. The strip occupies the region $0 \leq x \leq w$, $-\infty < z < \infty$ of the plane $y = 0$ of a Cartesian coordinate system (x, y, z), and at large distances the scattered field can be written as

$$\underline{E}^s \text{ (or } \underline{H}^s) \sim \hat{z} \left(\frac{2}{\pi k \rho} \right)^{1/2} e^{i(k\rho - \pi/4)} \; P_{E,H}(\phi, \phi_0)$$

where ρ, ϕ are cylindrical polar coordinates with $x = \rho \cos \phi$, $y = \rho \sin \phi$. Under the assumption that the strip width is large ($kw >> 1$), the task is to develop asymptotic expressions for the far field amplitudes $P_{E,H}$ for all angles ϕ and ϕ_0, but with emphasis on the case of backscattering for edge-on incidence.

For E polarization the boundary conditions are

$$[E_z]_-^+ = 0 \ , \quad [H_x]_-^+ = -E_z/R$$

where the signs refer to the upper (positive) and lower (negative) faces of the strip. The total electric current supported by the strip is $\underline{J} = \hat{z}J_z$ with $J_z = E_z/R$. For H polarization the problem is related to that of the E-polarized plane wave (1) incident on a 'magnetically conductive' strip (Senior, 1977) at the surfaces of which the boundary conditions are

$$[H_x]_-^+ = 0 \ , \quad [E_z]_-^+ = -H_x/R^*$$

where R* is the conductivity. Such a strip supports only a total magnetic current $\underline{J}^* = \hat{x}\, J_x^*$ with $J_x^* = H_x/R^*$, and the solution for an H-polarized plane wave incident on a resistive strip then follows on making the transformation

$$\underline{E} \to H, \qquad \underline{H} \to -\underline{E}, \qquad \underline{J}^* \to -\underline{J}, \qquad R^* \to R \ .$$

An impedance strip at whose surfaces the Leontovich (or impedance) boundary condition is imposed is equivalent to the superposition of resistive and conductive strips (Senior, 1977), and if the strip is planar, the electric and magnetic currents do not interact with one another. If the normalized surface impedance is η, the total electric current is the same as for a resistive strip of resistivity $R = \eta Z/_2$ where Z is the free space impedance, and the total magnetic current is the same as for a conductive strip of conductivity $R^* = (2\eta Z)^{-1}$. We can therefore treat simultaneously the problems of resistive and conductive strips by considering the impedance strip and keeping separate the contributions of the two types of current. This is the procedure we adopt. Since key elements in our solutions are the currents on an impedance half plane, it is convenient to examine these first.

3. Half Plane Currents

For the E-polarized plane wave (1) incident on an impedance half plane occupying $0 \leq x < \infty$, $-\infty < z < \infty$, an exact analytical solution is available (Senior, 1952). The total induced electric current is given by

$$ZJ_z(x) = \frac{2\sin\phi_0}{1+\eta\sin\phi_0} e^{-ikx\cos\phi_0} - \frac{i}{\pi}\int_{S(0)} \frac{\sin^2\beta}{1+\eta\sin\beta} \frac{K(\pi-\phi_0,\eta)}{K(\beta,\eta)} \frac{e^{ikx\cos\beta}}{\cos\beta+\cos\phi_0} d\beta \quad (2)$$

where $K(\beta,\eta)$ is the split function $K_+(k\cos\beta)$ resulting from the application of a function-theoretic technique and $S(0)$ is the steepest descents path running from $\beta = -\pi/2 + i\infty$ through 0 to $\beta = \pi/2 - i\infty$. The first term in (2) is the optics contribution and vanishes for edge-on incidence, $\phi_0 = \pi$. The second is the diffracted or edge contribution and will be denoted by $j(x,\phi_0,\eta)$:

$$j(x,\phi_0,\eta) = -\frac{i}{\pi}\int_{S(0)} \frac{\sin^2\beta}{1+\eta\sin\beta} \frac{K(\pi-\phi_0,\eta)}{K(\beta,\eta)} \frac{e^{ikx\cos\beta}}{\cos\beta+\cos\phi_0} d\beta \, . \quad (3)$$

In the particular case of perfect conductivity ($\eta = 0$), $K(\beta,0) = \sqrt{2}\cos\beta/2$, and the integral can be evaluated exactly as

$$j(x,\phi_0,0) = -\frac{4}{\sqrt{\pi}} e^{-i(kx\cos\phi_0+\pi/4)} \sin\phi_0 \, G\left([2kx]^{1/2}\cos\frac{\phi_0}{2}\right) \quad (4)$$

(Bowman et al, 1969) where

$$G(\tau) = F(\tau) - \frac{i}{2\tau} e^{i\tau^2}$$

and

$$F(\tau) = \int_\tau^\infty e^{i\mu^2} d\mu$$

is the Fresnel integral. For $kx \gg 1$ and ϕ_0 bounded away from π, asymptotic expansion of the function G gives

$$j(x,\emptyset_o,0) = -\frac{1}{kx}\left(\frac{2}{\pi kx}\right)^{1/2} e^{i(kx-\pi/4)} \frac{\sin\frac{\emptyset_o}{2}}{1+\cos\emptyset_o} + O([kx]^{-5/2})$$

whereas for $\emptyset_o = \pi$

$$j(x,\pi,0) = 2\left(\frac{2}{\pi kx}\right)^{1/2} e^{i(kx+\pi/4)} \tag{5}$$

for all x.

If $\eta \neq 0$ the integral in (3) cannot be evaluated exactly, but an expression asymptotic for large kx is easily obtained. For $\emptyset_o$ away from π

$$j(x,\emptyset_o,\eta) = -\frac{1}{kx}\left(\frac{2}{\pi kx}\right)^{1/2} e^{i(kx-\pi/4)} \frac{K(\pi-\emptyset_o,\eta)}{K(0,\eta)} \frac{1}{1+\cos\emptyset_o} + O([kx]^{-5/2}) \tag{6}$$

and for $\emptyset_o = \pi$

$$j(x,\pi,\eta) = 2\left(\frac{2}{\pi kx}\right)^{1/2} e^{i(kx+\pi/4)} + O([kx]^{-3/2}). \tag{7}$$

In the latter case the current is the same to the leading order in kx as that on a perfectly conducting half plane, but if $\eta > 1$ (7) does not provide an accurate estimate of the current unless $kx \gg \eta^2$. By removing only part of the integrand of (3) at the saddle point $\beta = 0$, we can also obtain an expression for the current which is uniform in angle:

$$j(x,\emptyset_o,\eta) \sim -\frac{8}{\sqrt{\pi}} e^{-i(kx\cos\emptyset_o+\pi/4)} \cos\frac{\emptyset_o}{2} \frac{K(\pi-\emptyset_o,\eta)}{K(0,\eta)} G([2kx]^{1/2}\cos\frac{\emptyset_o}{2}). \tag{8}$$

This matches the wide angle formula (6) into the edge-on value (7), and reduces to the exact result (4) when $\eta = 0$.

The solution for an impedance half plane also yields the following expression for the total induced magnetic current:

$$J_x^*(x) = -\frac{2\eta\sin\emptyset_o}{1+\eta\sin\emptyset_o} e^{-ikx\cos\emptyset_o} + \frac{2i}{\pi}\int_{S(0)} \frac{\eta\sin\beta}{1+\eta\sin\beta} \frac{K(\pi-\emptyset_o,\eta)}{K(\beta,\eta)} \frac{\cos\frac{\beta}{2}\cos\frac{\emptyset_o}{2}}{\cos\beta+\cos\emptyset_o} e^{ikx\cos\beta} d\beta \tag{9}$$

the diffracted portion of which is

$$j^*(x,\emptyset_0,\eta) = \frac{2i}{\pi} \int_{S(0)} \frac{\eta \sin\beta}{1+\eta\sin\beta} \frac{K(\pi-\emptyset_0,\eta)}{K(\beta,\eta)} \frac{\cos\frac{\beta}{2}\cos\frac{\emptyset_0}{2}}{\cos\beta+\cos\emptyset_0} e^{ikx\cos\beta}\, d\beta. \tag{10}$$

In the limit $\eta = \infty$ (9) represents the electric current on a perfectly conducting half plane for H polarization, but because of a pole of the integrand which gets ever closer to the saddle point as η increases, the current for $\eta = \infty$ behaves quite differently from that for finite η. Indeed,

$$j^*(x,\emptyset_0,\infty) = \frac{4}{\sqrt{\pi}} e^{-i(kx\cos\emptyset_0+\pi/4)} F\left([2kx]^{1/2} \cos \frac{\emptyset_0}{2}\right) \tag{11}$$

implying

$$j^*(x,\pi,\infty) = 2\, e^{ikx},$$

whereas for $kx >> 1$ and $\emptyset_0$ away from π,

$$j^*(x,\emptyset_0,\infty) = 2\left(\frac{2}{\pi kx}\right)^{1/2} e^{i(kx+\pi/4)} \frac{\cos\frac{\emptyset_0}{2}}{1+\cos\emptyset_0} + O([kx]^{-3/2}) \quad . \tag{12}$$

If $\eta \neq \infty$, $j^*(x,\emptyset_0,\eta)$ vanishes identically for edge-on incidence, and for other values of $\emptyset_0$ a steepest descents evaluation of the integral gives

$$j^*(x,\emptyset_0,\eta) = -\frac{2}{kx}\left(\frac{2}{\pi kx}\right)^{1/2} e^{i(kx-\pi/4)} \frac{K(\pi-\emptyset_0,\eta)}{K(0,\eta)} \frac{\eta^2\cos\frac{\emptyset_0}{2}}{1+\cos\emptyset_0} + O([kx]^{-5/2}). \tag{13}$$

Alternatively, by performing only a partial steepest descents evaluation, a uniform expression for the current is found to be

$$j^*(x,\emptyset_0,\eta) \sim -\frac{8}{\sqrt{\pi}} e^{-i(kx\cos\emptyset_0+\pi/4)} \eta^2(1+\cos\emptyset_0) \frac{K(\pi-\emptyset_0,\eta)}{K(0,\eta)} G([2kx]^{1/2}\cos\frac{\emptyset_0}{2}), \tag{14}$$

which clearly vanishes when $\emptyset_0 = \pi$. We observe that for $\eta \neq \infty$ the same function G characterizes the behavior of the electric and magnetic currents on an impedance half plane, as well as the electric current on a perfectly conducting half plane.

4. Perfectly Conducting Strips

For an E- or H-polarized plane wave incident on a perfectly conducting strip of electrically large width, asymptotic expressions for the far field amplitude are available.

In the case of E polarization

$$P_E(\emptyset,\emptyset_0) = \frac{i/4}{\cos\emptyset+\cos\emptyset_0}\{e^{-ikw(\cos\emptyset+\cos\emptyset_0)} A(-\cos\emptyset)\, A(-\cos\emptyset_0) - A(\cos\emptyset)\, A(\cos\emptyset_0)\} + 0\,([kx]^{-2}) \qquad (15)$$

(Fialkovskiy, 1966), where

$$A(\alpha) = \left(\frac{1+\alpha}{1-\alpha}\right)^{1/2} B(\alpha) - kw\,[2(1-\alpha)]^{1/2}\,\{H_0^{(1)}(kw) - iH_1^{(1)}(kw)\} \qquad (16)$$

$$B(\alpha) = [2(1-\alpha)]^{1/2}\, e^{-ikw\alpha}\,\{1 - 1/2(1-\alpha^2)^{1/2}\int_{kw}^{\infty} H_0^{(1)}(t)e^{i\alpha t}\,dt\} \qquad (17)$$

and $H_n^{(1)}(x)$ is the Hankel function of the first kind of order n. Consider first the function $B(\alpha)$. Since $kw >> 1$ the Hankel function can be expanded for large argument, and if $\alpha \neq -1$ the resulting Fresnel integrals can also be expanded to give

$$B(\alpha) = [2(1-\alpha)]^{1/2}\, e^{-ikw\alpha}\left\{1 - \frac{e^{i\pi/4}}{(2\pi kw)^{1/2}}\, e^{ikw(1+\alpha)}\left(\frac{1-\alpha}{1+\alpha}\right)^{1/2}\left[1 - \frac{i}{2kw}\left(\frac{1}{1+\alpha} + \frac{1}{4}\right)\right]\right\} + 0([kw]^{-5/2}) \quad . \qquad (18)$$

Similarly, by expanding the Hankel functions in (16),

$$A(\alpha) = [2(1+\alpha)]^{1/2}\, e^{-ikw\alpha}\left\{1 - \frac{e^{-i\pi/4}}{2kw(2\pi kw)^{1/2}}\, e^{ikw(1+\alpha)}\left(\frac{1-\alpha}{1+\alpha}\right)^{1/2}\left(\frac{1}{1+\alpha} - \frac{1}{2}\right)\right\} + 0\,([kw]^{-5/2}) \qquad (19)$$

and, in particular,

$$A(1) = 2e^{-ikw} \quad .$$

If $\alpha = -1$, (18) and (19) both fail because of the asymptotic expansion of the integral in (17), but as evident from (16) and (17)

$$A(-1) = -2kw\{H_0^{(1)}(kw) - iH_1^{(1)}(kw)\}. \tag{20}$$

We remark that for backscattering only the leading term in the expansion of (20) contributes to the order shown in (15).

From (19) with $\alpha = \cos \emptyset_0$,

$$A(\cos \emptyset_0) = 2 \cos \frac{\emptyset_0}{2} e^{-ikw\cos\emptyset_0} \left\{1 - \frac{e^{-i\pi/4}}{4kw(2\pi kw)^{1/2}} e^{ikw(1+\cos\emptyset_0)} \tan^3 \frac{\emptyset_0}{2}\right\} + O([kw]^{-5/2}) \tag{21}$$

and to obtain an expression uniform in angle it is necessary to retain as Fresnel integrals the terms resulting from the expansion of the integral in (17). A form consistent with (20) and (21) is then

$$A(\cos\emptyset_0) \sim 2 \cos \frac{\emptyset_0}{2} e^{-ikw\cos\emptyset_0} \left\{1 - \frac{2}{\sqrt{\pi}} e^{-i\pi/4} \sin^3 \frac{\emptyset_0}{2} G([2kw]^{1/2} \cos\frac{\emptyset_0}{2})\right\} \tag{22}$$

and we observe that the behavior is precisely that of the diffracted portion of the electric current on a perfectly conducting half plane. Thus

$$A(\cos\emptyset_0) \sim 2 \cos \frac{\emptyset_0}{2} e^{-ikw\cos\emptyset_0} + \frac{1}{2} \sin^2 \frac{\emptyset_0}{2} j(w,\emptyset_0,0) \tag{23}$$

and in particular,

$$A(-1) \sim + \frac{1}{2} j(w,\pi,0). \tag{24}$$

For backscattering ($\phi = \phi_0$)

$$P_E(\phi_0,\phi_0) = \frac{i}{8\cos\phi_0}\left\{e^{-2ikw\cos\phi_0}[A(-\cos\phi_0)]^2 - [A(\cos\phi_0)]^2\right\} + O([kw]^{-2}),$$

and without loss of generality it can be assumed that $\pi/2 \leq \phi_0 \leq \pi$. $A(-\cos\phi_0)$ can then be obtained from (21) and if ϕ_0 is away from π, (21) also suffices for $A(\cos\phi_0)$, in which case

$$P_E(\phi_0,\phi_0) = \frac{i}{4}\frac{1-\cos\phi_0}{\cos\phi_0} - \frac{i}{4}\frac{1+\cos\phi_0}{\cos\phi_0}e^{-2ikw\cos\phi_0} - \frac{e^{i\pi/4}}{2kw(2\pi kw)^{1/2}}\frac{e^{ikw(1-\cos\phi_0)}}{\sin\phi_0} + O([kw]^{-2}) . \qquad (25)$$

At broadside ($\phi_0 = \pi/2$) the first two terms on the right hand side become infinite, but the infinities cancel and

$$P_E(\pi/2,\pi/2) = -\frac{1}{2}kw - \frac{i}{2} - \frac{e^{i(kw+\pi/4)}}{2\,kw(2\pi kw)^{1/2}} + O([kw]^{-2}). \qquad (26)$$

For $\phi_0 > \pi/2$ the first term on the right hand side of (25) is the contribution of the front edge of the strip as given by GTD. The second term is the rear edge contribution and vanishes for edge-on incidence, whilst the third term is the second order contribution whose expression clearly fails when $\phi_0 = \pi$. The failure can be overcome by using the uniform expression (22) for $A(\cos\phi_0)$, and hence

$$P_E(\phi_0,\phi_0) = \frac{i}{4}\frac{1-\cos\phi_0}{\cos\phi_0}\left\{1 - \frac{e^{-i\pi/4}}{2kw(2\pi kw)^{1/2}}e^{ikw(1-\cos\phi_0)}\cot^3\frac{\phi_0}{2}\right\}$$

$$- \frac{i}{4}\frac{1+\cos\phi_0}{\cos\phi_0}e^{-2ikw\cos\phi_0}\left\{1 - \frac{2}{\sqrt{\pi}}e^{-i\pi/4}\sin^3\frac{\phi_0}{2}G([2kw]^{1/2}\cos\frac{\phi_0}{2})\right\}^2$$

$$+ O([kw]^{-2})$$

valid for $\pi/2 \leq \emptyset_0 \leq \pi$. At edge-on

$$P_E(\pi,\pi) \sim -\frac{i}{2} + \frac{i}{32}\{j(w,\pi,0)\}^2 \tag{27}$$

where the first and second terms are, respectively, the front and rear edge contributions.

For an H-polarized incident plane wave the far field amplitude is

$$P_H(\emptyset_0,\emptyset_0) = -\frac{i}{4}\frac{\mathrm{sgn}(\sin\emptyset)}{\cos\emptyset+\cos\emptyset_0}\left\{e^{-ikw(\cos\emptyset+\cos\emptyset_0)} B(-\cos\emptyset)B(-\cos\emptyset_0)\right.$$

$$\left. - B(\cos\emptyset)B(\cos\emptyset_0)\right\} + O([kw]^{-1}) \tag{28}$$

(Khaskind and Vainshteyn, 1964), where $B(\alpha)$ is defined by (17). In accordance with the order term in (28),

$$B(\alpha) = [2(1-\alpha)]^{1/2} e^{-ikw\alpha}\left\{1 - \frac{e^{i\pi/4}}{(2\pi kw)^{1/2}}\left(\frac{1-\alpha}{1+\alpha}\right)^{1/2}\right\} + O([kw]^{-3/2}) \tag{29}$$

valid for $\alpha \neq -1$, and the corresponding uniform expression is

$$B(\alpha) \sim [2(1-\alpha)]^{1/2} e^{-ikw\alpha}\left\{1 - [\frac{2}{\pi}(1-\alpha)]^{1/2} e^{-i\pi/4} F([kw(1+\alpha)]^{1/2})\right\}, \tag{30}$$

which vanishes as required when $\alpha = \pm 1$. We remark that (29) and (30) are consistent with

$$B(\cos\emptyset_0) \sim \sin\frac{\emptyset_0}{2}\{2e^{-ikw\cos\emptyset_0} - \sin\frac{\emptyset_0}{2} j^*(w,\emptyset_0,\infty)\} \tag{31}$$

where $j^*(x,\emptyset_0,\infty)$ is the diffracted portion of the electric current (11) on a perfectly conducting half plane.

In the particular case of backscattering

$$P_H(\emptyset_0,\emptyset_0) = -\frac{i}{8\cos\emptyset_0}\left\{e^{-2ikw\cos\emptyset_0}[B(-\cos\emptyset_0)]^2 - [B(\cos\emptyset_0)]^2\right\} + O([kw]^{-1})$$

where we have again assumed that $\pi/2 \leq \emptyset_0 \leq \pi$. From (29)

$$B(-\cos\phi_0) = 2\cos\frac{\phi_0}{2}\, e^{ikw\cos\phi_0} \left\{1 - \frac{e^{i\pi/4}}{(2\pi kw)^{1/2}}\, e^{ikw(1-\cos\phi_0)} \cot\frac{\phi_0}{2}\right\} + O([kw]^{-3/2}),$$

and if ϕ_0 is away from π, $B(\cos\phi_0)$ has a similar expression. The far field amplitude then is

$$P_H(\phi_0,\phi_0) = -\frac{i}{4}\,\frac{1+\cos\phi_0}{\cos\phi_0} + \frac{i}{4}\,\frac{1-\cos\phi_0}{\cos\phi_0}\, e^{-2ikw\cos\phi_0} - \left(\frac{2}{\pi kw}\right)^{1/2} e^{i\pi/4}\, \frac{e^{ikw(1-\cos\phi_0)}}{\sin\phi_0} + O([kw]^{-1}) \qquad (32)$$

and at broadside

$$P_H(\pi/2,\pi/2) = \frac{1}{2}kw - \frac{i}{2} - \left(\frac{2}{\pi kw}\right)^{1/2} e^{i(kw+\pi/4)} + O([kw]^{-1}) \ . \qquad (33)$$

For $\phi_0 > \pi/2$, the first and second terms on the right hand side of (32) are the contributions of the front and rear edges respectively, with the former vanishing for edge-on incidence. The third term is the second order contribution and clearly fails when $\phi_0 = \pi$, but the failure can be overcome by using the uniform expression (30) for $B(\cos\phi_0)$, in which case

$$P_H(\phi_0,\phi_0) = -\frac{i}{4}\,\frac{1+\cos\phi_0}{\cos\phi_0}\left\{1 - \left(\frac{2}{\pi kw}\right)^{1/2} e^{i\pi/4 + ikw(1-\cos\phi_0)} \cot\frac{\phi_0}{2}\right\}$$
$$+ \frac{i}{4}\,\frac{1-\cos\phi_0}{\cos\phi_0}\, e^{-2ikw\cos\phi_0} \left\{1 - \frac{2}{\sqrt{\pi}}\, e^{-i\pi/4} \sin\frac{\phi_0}{2}\, F([2kw]^{1/2} \cos\frac{\phi_0}{2})\right\}^2$$
$$+ O([kw]^{-1})$$

valid for $\pi/2 \le \phi_0 \le \pi$, and implying

$$P_H(\pi,\pi) = 0.$$

5. Impedance Strips

An impedance strip now occupies the region $0 \leq x \leq w$, $-\infty < z < \infty$ of the plane $y = 0$ and is illuminated by the E-polarized plane wave (1). The boundary conditions at the strip are

$$E_z = \mp \eta Z H_x = \mp \frac{\eta}{ik} \frac{\partial E_z}{\partial y} \tag{34}$$

with the upper and lower signs for $y = 0+$ and $y = 0-$ respectively. For non-grazing angles of incidence and observation, the diffracted field through second order terms can be obtained using the method described by Bowman (1967) for the particular case of backscattering at normal incidence.

To the desired order in kw, the far field amplitude $P_E(\phi_0,\phi_0)$ has four contributions: the field $p_1(\phi_0,\phi_0)$ diffracted by the front edge of the strip at $x = 0$; the field $p_2(\phi_0,\phi_0)$ of the rear edge at $x = w$; the field $p_{12}(\phi_0,\phi_0)$ which reaches the observation point only after diffraction first by the front edge and then the rear; and the field $p_{21}(\phi_0,\phi_0)$ for which diffraction takes place in the reverse sequence. Expressions for the p's can be deduced from the solution for an impedance half plane in $0 \leq x \leq \infty$. As shown by Senior (1975), if ϕ is bounded away from the geometrical optics directions $\pi \mp \phi_0$, the far field amplitude of the edge-diffracted component is

$$P(\phi,\phi_0) = \frac{i}{2} \frac{1 - 2\eta \cos\frac{\phi}{2} \cos\frac{\phi_0}{2}}{\cos\phi + \cos\phi_0} K(\pi-\phi,\eta) K(\pi-\phi_0,\eta) \tag{35}$$

where the first and second terms in the numerator are generated by the induced electric and magnetic currents respectively. This is clearly the contribution of the front edge of the strip and hence

$$p_1(\phi,\phi_0) = P(\phi,\phi_0).$$

Apart from a phase factor, the contribution of the rear edge differs only in having ϕ,ϕ_0 replaced by $\pi-\phi$, $\pi-\phi_0$ respectively, and therefore

$$p_2(\phi,\phi_0) = P(\pi-\phi\ ,\pi-\phi_0)\ e^{-ikw(\cos\phi+\cos\phi_0)}.$$

We remark that

$$K(\phi,\eta) = \sin\phi\ \{(1+\eta\sin\phi)K(\pi-\phi,\eta)\}^{-1} \tag{36}$$

implying $K(\pi,\eta) = 0$ for all finite η.

Consider now the rediffracted contribution $p_{12}(\phi,\phi_0)$. From (35) and (36), the diffracted field of the half plane at a distance ρ from the edge is

$$E_z^d \sim \left(\frac{2}{\pi k\rho}\right)^{1/2} e^{i(k\rho-\pi/4)} \left\{\frac{i}{2}\ \frac{1-2\eta\cos\frac{\phi}{2}\cos\frac{\phi_0}{2}}{\cos\phi+\cos\phi_0}\ \frac{\sin\phi}{1+\eta\sin\phi}\ \frac{K(\pi-\phi_0,\eta)}{K(\phi,\eta)}\right\},$$

which vanishes on the half plane. However, from (34),

$$E_z = -\frac{\eta}{ik\rho}\ \frac{\partial E_z}{\partial\phi} \tag{37}$$

for $\phi = 0$, and when the differentiation is performed, the field proves to be non-zero to the next higher order. Thus, on the upper surface of the half plane,

$$E_z^d \sim \left(\frac{2}{\pi k\rho}\right)^{1/2} e^{i(k\rho-\pi/4)} \left\{-\frac{\eta}{2k\rho}\ \frac{1-2\eta\cos\frac{\phi_0}{2}}{1+\cos\phi_0}\ \frac{K(\pi-\phi_0,\eta)}{K(0,\eta)}\right\} \tag{38}$$

which can be attributed to a source at the edge. The source strength can be found by considering a line source of strength γ and free space field

$$E_z = \gamma H_0^{(1)}(k\rho) \sim \gamma \left(\frac{2}{\pi k\rho}\right)^{1/2} e^{i(k\rho-\pi/4)}$$

located above an impedance plane at a distance ρ and angle $\emptyset$ relative to coordinates in the plane. The field on the surface is

$$E_z \sim \gamma \left(\frac{2}{\pi k\rho}\right)^{1/2} e^{i(k\rho-\pi/4)} \frac{2\eta\sin\emptyset}{1+\eta\sin\emptyset}$$

and, to this order in $k\rho$, is zero when $\emptyset = 0$, but to the next higher order the field of a source in the plane is, from (37),

$$E_z \sim \left(\frac{2}{\pi k\rho}\right)^{1/2} e^{i(k\rho-\pi/4)} \frac{2i\eta^2\gamma}{k\rho} .$$

Comparison with (38) now shows that the field diffracted across the upper face of the strip is equivalent to that of a line source of strength

$$\gamma = \frac{i}{4\eta} \frac{1-2\eta\cos\frac{\emptyset_0}{2}}{1+\cos\emptyset_0} \frac{K(\pi-\emptyset_0,\eta)}{K(0,\eta)} \tag{39}$$

located at the front edge.

To determine the field diffracted by the rear edge when illuminated by this source, it is convenient to invoke reciprocity and examine instead the diffracted field at the front edge due to a source of strength γ at the observation point. The field incident on the rear edge is

$$E_z^i \sim \gamma \left(\frac{2}{\pi k\rho}\right)^{1/2} e^{i(k\rho-\pi/4)} e^{-ikw\cos\emptyset} \tag{40}$$

and, from (38) with $\emptyset_0$ replaced by $\pi-\emptyset$, the field which reaches the front edge after diffraction across the upper face of the strip is then

$$E_z^d \sim E_z^i \left(\frac{2}{\pi kw}\right)^{1/2} e^{i(kw-\pi/4)} \left\{-\frac{\eta}{2kw}\,\frac{1-2\eta\sin\frac{\emptyset}{2}}{1-\cos\emptyset}\,\frac{K(\emptyset,\eta)}{K(0,\eta)}\right\}$$

where E_z^i is given by (40). This is likewise the field at the observation point due to diffraction by the rear edge of the field of a source of strength γ at the front. On inserting the expression (39) for γ and doubling to take into account diffraction across the lower face of the strip, we have

$$p_{12}(\emptyset,\emptyset_0) = -\frac{e^{ikw(1-\cos\emptyset)+i\pi/4}}{2kw(2\pi kw)^{1/2}}\,\frac{1-2\eta\sin\frac{\emptyset}{2}}{1-\cos\emptyset}\,\frac{K(\emptyset,\eta)}{K(0,\eta)}\,\frac{1-2\eta\cos\frac{\emptyset_0}{2}}{1+\cos\emptyset_0}\,\frac{K(\pi-\emptyset_0,\eta)}{K(0,\eta)}\ .$$

The other second order contribution can be determined in a similar manner, and when the analysis is performed, it is found that

$$p_{21}(\emptyset,\emptyset_0) = p_{12}(\emptyset_0,\emptyset).$$

The net far field amplitude of the strip through second order terms is therefore

$$\begin{aligned}
P_E(\emptyset,\emptyset_0) = \frac{i/2}{\cos\emptyset+\cos\emptyset_0} \Bigg[& \left(1-2\eta\cos\frac{\emptyset}{2}\cos\frac{\emptyset_0}{2}\right) K(\pi-\emptyset,\eta)K(\pi-\emptyset_0,\eta) \\
& - \left(1-2\eta\sin\frac{\emptyset}{2}\sin\frac{\emptyset_0}{2}\right) e^{-ikw(\cos\emptyset+\cos\emptyset_0)}\, K(\emptyset,\eta)K(\emptyset_0,\eta) \\
& - (\cos\emptyset+\cos\emptyset_0)\,\frac{e^{i(kw-\pi/4)}}{kw(2\pi kw)^{1/2}} \Bigg\{ e^{-ikw\cos\emptyset}\,\frac{1-2\eta\sin\frac{\emptyset}{2}}{1-\cos\emptyset}\,\frac{K(\emptyset,\eta)}{K(0,\eta)} \\
& \qquad \frac{1-2\eta\cos\frac{\emptyset_0}{2}}{1+\cos\emptyset_0}\,\frac{K(\pi-\emptyset_0,\eta)}{K(0,\eta)} \\
& + e^{-ikw\cos\emptyset_0}\,\frac{1-2\eta\sin\frac{\emptyset_0}{2}}{1-\cos\emptyset_0}\,\frac{K(\emptyset_0,\eta)}{K(0,\eta)}\,\frac{1-2\eta\cos\frac{\emptyset}{2}}{1+\cos\emptyset}\,\frac{K(\pi-\emptyset,\eta)}{K(0,\eta)} \Bigg\} \Bigg] \\
& + O([kw]^{-2}) \qquad (41)
\end{aligned}$$

and embodies the solution for a resistive strip.

6. Resistive Strips

A resistive strip supports only an electric current whose strength is the same as that on an impedance strip of normalized surface impedance $\eta = 2R/Z$. We can therefore deduce the far field amplitude for a resistive strip with E polarization by suppressing that part of (41) contributed by the magnetic current. The result is

$$P_E(\phi,\phi_0) = \frac{i/2}{\cos\phi+\cos\phi_0}\Bigg[K(\pi-\phi,\eta)K(\pi-\phi_0,\eta) - e^{-ikw(\cos\phi+\cos\phi_0)} K(\phi,\eta)K(\phi_0,\eta)$$

$$- (\cos\phi+\cos\phi_0)\,\frac{e^{i(kw-\pi/4)}}{kw(2\pi kw)^{1/2}}\Bigg\{e^{-ikw\cos\phi}\,(1-\cos\phi)^{-1}\,\frac{K(\phi,\eta)}{K(0,\eta)}$$

$$(1+\cos\phi_0)^{-1}\,\frac{K(\pi-\phi_0,\eta)}{K(0,\eta)}$$

$$+ e^{-ikw\cos\phi_0}\,(1-\cos\phi_0)^{-1}\,\frac{K(\phi_0,\eta)}{K(0,\eta)}\,(1+\cos\phi)^{-1}\,\frac{K(\pi-\phi,\eta)}{K(0,\eta)}\Bigg\}\Bigg] + O([kw]^{-2})$$

which can be written as

$$P_E(\phi,\phi_0) = \frac{i/4}{\cos\phi+\cos\phi_0}\Big\{e^{-ikw(\cos\phi+\cos\phi_0)}\,A(-\cos\phi\quad\eta)A(-\cos\phi_0,\eta)$$

$$-A(\cos\phi,\eta)A(\cos\phi_0,\eta)\Big\} + O([kw]^{-2}) \qquad (42)$$

where

$$A(\cos\phi_0,\eta) = \sqrt{2}\,e^{-ikw\cos\phi_0}\Bigg\{K(\phi_0,\eta) - \frac{e^{-i\pi/4}}{kw(2\pi kw)^{1/2}}\,e^{ikw(1+\cos\phi_0)}$$

$$\frac{K(\pi-\phi_0,\eta)}{[K(0,\eta)]^2}\left(\frac{1}{1+\cos\phi_0} - a\right)\Bigg\} \qquad (43)$$

and a is independent of ϕ_0. The choice a = 1/2 makes $A(\cos\phi_0, 0)$ identical to the perfectly conducting function (21), and comparison with (6) then shows

$$A(\cos\phi_0,\eta) \sim \sqrt{2}\, K(\phi_0,\eta)\, e^{-ikw\cos\phi_0} + \frac{\sin^2 \frac{\phi_0}{2}}{\sqrt{2}\, K(0,\eta)}\, j(w,\phi_0,\eta) \tag{44}$$

(cf 23) where $j(w,\phi_0,\eta)$ is the diffracted portion of the electric current on a half plane, the uniform expression for which is given in (8). Using this or, alternatively, the uniform representation (22) for $A(\cos\phi_0)$, we have

$$A(\cos\phi_0,\eta) \sim \sqrt{2}\, e^{-ikw\cos\phi_0} \left\{ K(\phi_0,\eta) - \frac{2}{\sqrt{\pi}}\, e^{-i\pi/4} \sin\phi_0 \sin\frac{\phi_0}{2} \frac{K(\pi-\phi_0,\eta)}{[K(0,\eta)]^2} G\left([2kw]^{1/2} \cos\frac{\phi_0}{2}\right) \right\} \tag{45}$$

valid for all ϕ_0. For edge-on incidence

$$A(-1,\eta) \sim 2 \left(\frac{2}{\pi kw}\right)^{1/2} e^{i(kw+\pi/4)} \frac{1}{\sqrt{2}K(0,\eta)}$$

which differs from the corresponding result for perfect conductivity by the factor on the right, and in terms of the current

$$A(-1,\eta) \sim \{\sqrt{2}\, K(0,\eta)\}^{-1}\, j(w,\pi,\eta)\,. \tag{46}$$

In the particular case of backscattering

$$P_E(\phi_0,\phi_0) = \frac{i}{8\cos\phi_0} \left\{ e^{-2ikw\cos\phi_0} [A(-\cos\phi_0,\eta)]^2 - [A(\cos\phi_0,\eta)]^2 \right\} + O([kw]^{-2}) \tag{47}$$

and we shall again restrict attention to $\pi/2 \leq \emptyset_0 \leq \pi$. For incidence away from edge-on, substitution of (43) into (47) gives

$$P_E(\emptyset_0,\emptyset_0) = \frac{i}{4\cos\emptyset_0} [K(\pi-\emptyset_0,\eta)]^2 - \frac{i}{4\cos\emptyset_0} e^{-2ikw\cos\emptyset_0} [K(\emptyset_0,\eta)]^2$$

$$- \frac{e^{i\pi/4}}{kw(2\pi kw)^{1/2}} \frac{e^{ikw(1-\cos\emptyset_0)}}{\sin^2\emptyset_0} \frac{K(\pi-\emptyset_0,\eta)}{K(0,\eta)} \frac{K(\emptyset_0,\eta)}{K(0,\eta)} + O([kw]^2). \qquad (48)$$

At broadside the first two terms on the right hand side are infinite, but since the infinities cancel, the result can be found by a limiting process. From the expression for the split function (Senior, 1975)

$$K(\tfrac{\pi}{2} + \varepsilon,\eta) = K(\tfrac{\pi}{2},\eta) \left\{1 + \frac{\varepsilon}{2} \left(\frac{2x}{\pi\sin\chi} - 1\right) + O(\varepsilon^2)\right\}$$

where $\cos\chi = 1/\eta$, and by treating similarly the other factors in the first two terms and then taking the limit as $\varepsilon \to 0$, we have

$$P_E(\tfrac{\pi}{2},\tfrac{\pi}{2}) = \frac{1}{1+\eta} \left\{- \frac{1}{2} kw - \frac{i}{2} \left(1 - \frac{2\chi}{\pi\sin\chi}\right) - \frac{e^{i(kw+\pi/4)}}{kw(2\pi kw)^{1/2}} [K(0,\eta)]^{-2}\right\} + O([kw]^{-2}) \qquad (49)$$

where we have used the fact that $K(\frac{\pi}{2},\eta) = (1+\eta)^{-1/2}$. Equation (49) contains every term present in the corresponding result (26) for a perfectly conducting strip, but with a reduced amplitude.

In the expression (48) for $P_E(\emptyset_0,\emptyset_0)$ the first and second terms on the right hand side are, respectively, the contributions from the front and rear edges of the strip. The third term is the doubly diffracted contribution and its formula clearly breaks down at edge-on incidence. The failure can be overcome by using the uniform expression (45) for $A(\cos\emptyset_0,\eta)$, in which case

$$P_E(\emptyset_0,\emptyset_0) = \frac{i}{4\cos\emptyset_0}\left\{[K(\pi-\emptyset_0,\eta)]^2 - \frac{e^{-i\pi/4}}{kw(2\pi kw)^{1/2}}\, e^{ikw(1-\cos\emptyset_0)} \cot^2\frac{\emptyset_0}{2}\, \frac{K(\pi-\emptyset_0,\eta)}{K(0,\eta)}\,\frac{K(\emptyset_0,\eta)}{K(0,\eta)}\right\}$$

$$-\frac{i}{4\cos\emptyset_0}\, e^{-2ikw\cos\emptyset_0}\left\{K(\emptyset_0,\eta) - \frac{2}{\sqrt{\pi}}\, e^{-i\pi/4}\sin\emptyset_0 \sin\frac{\emptyset_0}{2}\, \frac{K(\pi-\emptyset_0-\eta)}{[K(0,\eta)]^2}\, G([2kw]^{1/2}\cos\frac{\emptyset_0}{2})\right\}^2 + O([kw]^{-2}),$$

valid for $\pi/2 \le \emptyset_0 \le \pi$. In particular,

$$P_E(\pi,\pi) = -\frac{i}{4}[K(0,\eta)]^2 - \frac{e^{2ikw}}{2\pi kw}[K(0,\eta)]^{-2} + O([kw]^{-2}). \tag{50}$$

That part of the far field amplitude (41) attributable to the induced magnetic current corresponds via duality (implying $\eta \to 1/\eta$) to the solution for a resistive strip illuminated by the H-polarized plane wave (1). Thus

$$P_H(\emptyset,\emptyset_0) = -\frac{i/\eta}{\cos\emptyset+\cos\emptyset_0}\Bigg[K(\pi-\emptyset,1/\eta)K(\pi-\emptyset_0,1/\eta) - e^{-ikw(\cos\emptyset+\cos\emptyset_0)}$$

$$K(\emptyset,1/\eta)K(\emptyset_0,1/\eta) + (\cos\emptyset+\cos\emptyset_0)\,\frac{e^{i(kw-\pi/4)}}{\eta kw}\left(\frac{2}{\pi kw}\right)^{1/2}$$

$$\left\{e^{-ikw\cos\emptyset}\,\frac{\sin\frac{\emptyset}{2}}{1-\cos\emptyset}\,\frac{K(\emptyset,1/\eta)}{K(0,1/\eta)}\,\frac{\cos\frac{\emptyset_0}{2}}{1+\cos\emptyset_0}\,\frac{K(\pi-\emptyset_0,1/\eta)}{K(0,1/\eta)}\right.$$

$$\left.+\, e^{-ikw\cos\emptyset_0}\,\frac{\sin\frac{\emptyset_0}{2}}{1-\cos\emptyset_0}\,\frac{K(\emptyset_0,1/\eta)}{K(0,1/\eta)}\,\frac{\cos\frac{\emptyset}{2}}{1+\cos\emptyset}\,\frac{K(\pi-\emptyset,1/\eta)}{K(0,1/\eta)}\right\}\Bigg] + O([kw]^{-2})$$

which can be written as

$$P_H(\emptyset,\emptyset_0) = - \frac{i/4}{\cos\emptyset+\cos\emptyset_0} \Big\{ e^{-ikw(\cos\emptyset+\cos\emptyset_0)} B(-\cos\emptyset,\eta)B(-\cos\emptyset_0,\eta)$$

$$- B(\cos\emptyset,\eta)B(\cos\emptyset_0,\eta)\Big\} + 0\,([kw]^{-2}) \qquad (51)$$

where

$$B(\cos\emptyset_0,\eta) = \frac{2}{\sqrt{\eta}}\, e^{-ikw\cos\emptyset_0} \Big\{\sin\frac{\emptyset_0}{2} K(\emptyset_0,1/\eta) + \frac{e^{-i\pi/4}}{\eta kw} \left(\frac{2}{\pi kw}\right)^{1/2}$$

$$e^{ikw(1+\cos\emptyset_0)} \cos\frac{\emptyset_0}{2} \frac{K(\pi-\emptyset_0,1/\eta)}{[K(0,1/\eta)]^2} \left(\frac{1}{1+\cos\emptyset_0} - b\right)\Big\} \qquad (52)$$

and b is some constant. As in the case of $A(\cos\emptyset_0,\eta)$ the constant is chosen to produce agreement with the function for a perfectly conducting strip when $\eta = 0$, but because of the discontinuity in behavior as $\eta \to 0$, we cannot simply put $\eta = 0$ in (52). For $\eta \neq 0$, however, comparison of (52) with the expression (13) for the magnetic current on a half plane shows

$$B(\cos\emptyset_0,\eta) \sim \frac{2}{\sqrt{\eta}} \sin\frac{\emptyset_0}{2} K(\emptyset_0,1/\eta)\, e^{-ikw\cos\emptyset_0} - \sqrt{\eta}\, [K(0,1/\eta)]^{-1}$$

$$\{1 - b(1+\cos\emptyset_0)\}\, j^*(w,\emptyset_0,1/\eta).$$

As $\eta \to 0$, $\eta^{-1/2} K(\emptyset_0,1/\eta) \to 1$, and in the limit the current is that given in (12). Hence

$$B(\cos\emptyset_0,0) \sim 2\sin\frac{\emptyset_0}{2} e^{-ikw\cos\emptyset_0} - \{1 - b(1+\cos\emptyset_0)\}\, j^*(w,\emptyset_0,\infty)$$

in agreement with (31) if $b = 1/2$.

Equation (52) then becomes

$$B(\cos\emptyset_0,\eta) = \frac{2}{\sqrt{\eta}} e^{-ikw\cos\emptyset_0} \sin\frac{\emptyset_0}{2}\left\{K(\emptyset_0,1/\eta) + \frac{e^{-i\pi/4}}{\eta kw(2\pi kw)^{1/2}} e^{ikw(1+\cos\emptyset_0)} \tan\frac{\emptyset_0}{2}\frac{K(\pi-\emptyset_0,1/\eta)}{[K(0,1/\eta)]^2}\right\} \qquad (53)$$

implying

$$B(\cos\emptyset_0,\eta) \sim \eta^{-1/2} \sin\frac{\emptyset_0}{2}\left\{2K(\emptyset_0,1/\eta)\, e^{-ikw\cos\emptyset_0} - \eta \sin\frac{\emptyset_0}{2}[K(0,1/\eta)]^{-1} j^*(w,\emptyset_0,1/\eta)\right\}. \qquad (54)$$

From the uniform representation (14) for $j^*(w,\emptyset_0,1/\eta)$ or, alternatively, by employing the matching function appropriate to $A(\cos\emptyset_0,\eta)$, we can now derive an expression for $B(\cos\emptyset_0,\eta)$ which is uniform in angle, and if $\eta \neq 0$ the result is

$$B(\cos\emptyset_0,\eta) \sim \frac{2}{\sqrt{\eta}} e^{-ikw\cos\emptyset_0} \sin\frac{\emptyset_0}{2}\left\{K(\emptyset_0,1/\eta) + \frac{4}{\eta\sqrt{\pi}} e^{-i\pi/4} \sin\emptyset_0 \cos\frac{\emptyset_0}{2} \frac{K(\pi-\emptyset_0,1/\eta)}{[K(0,1/\eta)]^2} G([2kw]^{1/2}\cos\frac{\emptyset_0}{2})\right\}, \qquad (55)$$

which vanishes for $\emptyset_0 = 0,\pi$.

For backscattering

$$P_H(\emptyset_0,\emptyset_0) = -\frac{i}{8\cos\emptyset_0}\left\{e^{-2ikw\cos\emptyset_0}[B(-\cos\emptyset_0,\eta)]^2 - [B(\cos\emptyset_0,\eta)]^2\right\} + O([kw]^{-2}) \qquad (56)$$

and if $\emptyset_0$ is bounded away from π, (53) can be used to give

$$P_H(\emptyset_0,\emptyset_0) = -\frac{i}{4\eta}\frac{1+\cos\emptyset_0}{\cos\emptyset_0}\left[K(\pi-\emptyset_0,1/\eta)\right]^2 + \frac{i}{4\eta}\frac{1-\cos\emptyset_0}{\cos\emptyset_0}\left[K(\emptyset_0,1/\eta)\right]^2$$

$$-\frac{e^{i\pi/4}}{\eta^2 kw}\left(\frac{2}{\pi kw}\right)^{1/2}\frac{e^{ikw(1-\cos\emptyset_0)}}{\sin\emptyset_0}\frac{K(\pi-\emptyset_0,1/\eta)}{K(0,1/\eta)}\frac{K(\emptyset_0,1/\eta)}{K(0,1/\eta)} + O([kw]^{-2}). \tag{57}$$

At broadside an analysis similar to that for E polarization shows

$$P_H\left(\frac{\pi}{2},\frac{\pi}{2}\right) = \frac{1}{1+\eta}\left\{\frac{1}{2}kw - \frac{i\chi}{\pi\sin\chi} - \frac{1}{\eta kw}\left(\frac{2}{\pi kw}\right)^{1/2}\frac{e^{i(kw+\pi/4)}}{[K(0,1/\eta)]^2}\right\} + O([kw]^2) \tag{58}$$

where now $\cos\chi = \eta$. As $\emptyset_0$ approaches π, the expression for the doubly diffracted contribution in (57) fails, but by using the uniform representation (55) for $B(\cos\emptyset_0,\eta)$ in place of (53) we obtain

$$P_H(\emptyset_0,\emptyset_0) = -\frac{i}{4\eta}\frac{1+\cos\emptyset_0}{\cos\emptyset_0}\left\{[K(\pi-\emptyset_0,1/\eta)]^2 + \frac{e^{-i\pi/4}}{\eta kw}\left(\frac{2}{\pi kw}\right)^{1/2}e^{ikw(1-\cos\emptyset_0)}\right.$$

$$\left.\cot\frac{\emptyset_0}{2}\frac{K(\pi-\emptyset_0,1/\eta)}{K(0,1/\eta)}\frac{K(\emptyset_0,1/\eta)}{K(0,1/\eta)}\right\}$$

$$+\frac{i}{4\eta}\frac{1-\cos\emptyset_0}{\cos\emptyset_0}\left\{K(\emptyset_0,1/\eta) + \frac{4}{\eta\sqrt{\pi}}e^{-i\pi/4}\sin\emptyset_0\cos\frac{\emptyset_0}{2}\frac{K(\pi-\emptyset_0,1/\eta)}{[K(0,1/\eta)]^2}\right.$$

$$\left.G([2kw]^{1/2}\cos\frac{\emptyset_0}{2})\right\}^2 + O([kw]^{-2})$$

valid for $\pi/2 \le \emptyset_0 \le \pi$. As expected, this vanishes for $\emptyset_0 = \pi$ regardless of η, but in order to handle the limiting case $\eta = 0$ it is necessary to express $B(\cos\emptyset_0,\eta)$ in terms of the half plane current (see 54) and then employ the known behavior of the current for perfect conductivity.

7. Discussion

In contrast to the case of H polarization, the backscattering for E polarization is non-zero at edge-on incidence and, as evident from (50), the front and rear edges of the strip both contribute to $P_E(\pi,\pi)$. Thus,

$$P_E(\pi,\pi) = P^f + P^r \tag{59}$$

where the first term on the right hand side is the front edge contribution and is proportional to the square of the current at the edge of a half plane:

$$P^f = -\frac{i\eta}{16}\,[j(0,\pi,\eta)]^2 \tag{60}$$

(Senior, 1979a). The second term is the rear edge contribution and, from (7) and (50),

$$P^r = i\gamma[j(w,\pi,\eta)]^2 \tag{61}$$

with

$$\gamma = [4K(0,\eta)]^{-2}. \tag{62}$$

A dependence of P^r on the square of the half plane current was observed by Senior (1979a), and from an examination of computed data for strips having $1 \leq \eta \leq 10$ it was concluded that

$$\gamma \simeq 0.0313 + \eta 0.0663 \;.$$

The agreement with the analytical expression (62) is remarkably good. For all real η the maximum discrepancy is only seven percent, and as η increases, the ratio of the approximate and exact values of γ decreases from 1 at $\eta = 0$ to 0.93 at $\eta \simeq 0.18$, increasing thereafter to 1 at $\eta = 2$, on up to 1.06 for $\eta = \infty$.

At edge-on incidence $j(x,\pi,\eta)$ is identical to the total induced electric current $ZJ_z(x)$. The computation of the half plane current as a function of kx for any η real or complex has been discussed by Senior (1979b), and a program written to carry out numerically the integrations, explicit and implicit, involved in (3). For $\eta = 1$, 4 and 10 the amplitudes and normalized phases of $j(x,\pi,\eta)$ as functions of x/λ are shown in Figures 1 and 2, along with the corresponding curves for a perfectly conducting half plane obtained using (5). Since $\gamma = [2\sqrt{\eta}\, j(0,\pi,\eta)]^2$, such data are sufficient to determine the backscattering from a strip as a function of its width, and in Figures 3 and 4 the resulting values of $P_E(\pi,\pi)$ are compared with those found by a numerical solution of the integral equation for a strip (Senior, 1979a). The agreement is excellent and even for kw as small as unity the discrepancy is only a few degrees in phase and less than 0.3 dB in amplitude.

For $\eta \ll 1$ the expression for $P_E(\pi,\pi)$ shown in (50), i.e., with $j(w,\pi,\eta)$ replaced by the leading term in its asymptotic expansion for large kw, provides the same accuracy, but as η increases, the superiority of using (61) in conjunction with computed values for the half plane current becomes apparent. One reason for this is the nature of the asymptotic expansion of $j(w,\pi,\eta)$. For sufficiently large kw,

$$j(w,\Pi,\eta) \sim 2 \left(\frac{2}{\pi kw}\right)^{1/2} e^{i(kw+\pi/4)} \left\{1 - \frac{i\eta^2}{2kw} \left(\frac{3}{2} + \frac{1}{\pi} [\chi\sin\chi+\cos\chi]\right) + 0\left(\left[\frac{\eta^2}{2kw}\right]^2\right)\right\}$$

and if $\eta > 1$ it is necessary that $kw \gg \eta^2$ for the leading term (which is independent of η and is the exact value of $j(w,\pi,0)$) to provide a good approximation to the current. This is otherwise evident from Figure 1. The fact that for all values of η (59) is accurate even for kw as small as unity or less suggests that the expression (61) for P^r is valid beyond just the leading term in the asymptotic expansion of $j(w,\pi,\eta)$ for $kw >> 1$.

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Legends for Figures

Fig. 1. Amplitude of the half plane current computed from (5) for $\eta = 0$ (———) and from (3) with $\emptyset_0 = \pi$ for $\eta = 1$, 4 and 10 (⊙ ⊙ ⊙).

Fig. 2. Normalized phase of the half plane current computed from (5) for $\eta = 0$ (———) and from (3) with $\emptyset_0 = \pi$ for $\eta = 1$, 4 and 10 (⊙ ⊙ ⊙).

Fig. 3. Amplitude of the edge-on backscattered field for strips of width w computed from the integral equation (———) (Senior, 1979a) and using (59) (⊙ ⊙ ⊙).

Fig. 4. Phase of the edge-on backscattered field for a strip of width w with $\eta = 4$ computed from the integral equation (———) (Senior, 1979a) and using (59) (⊙ ⊙ ⊙).

FIGURE 1

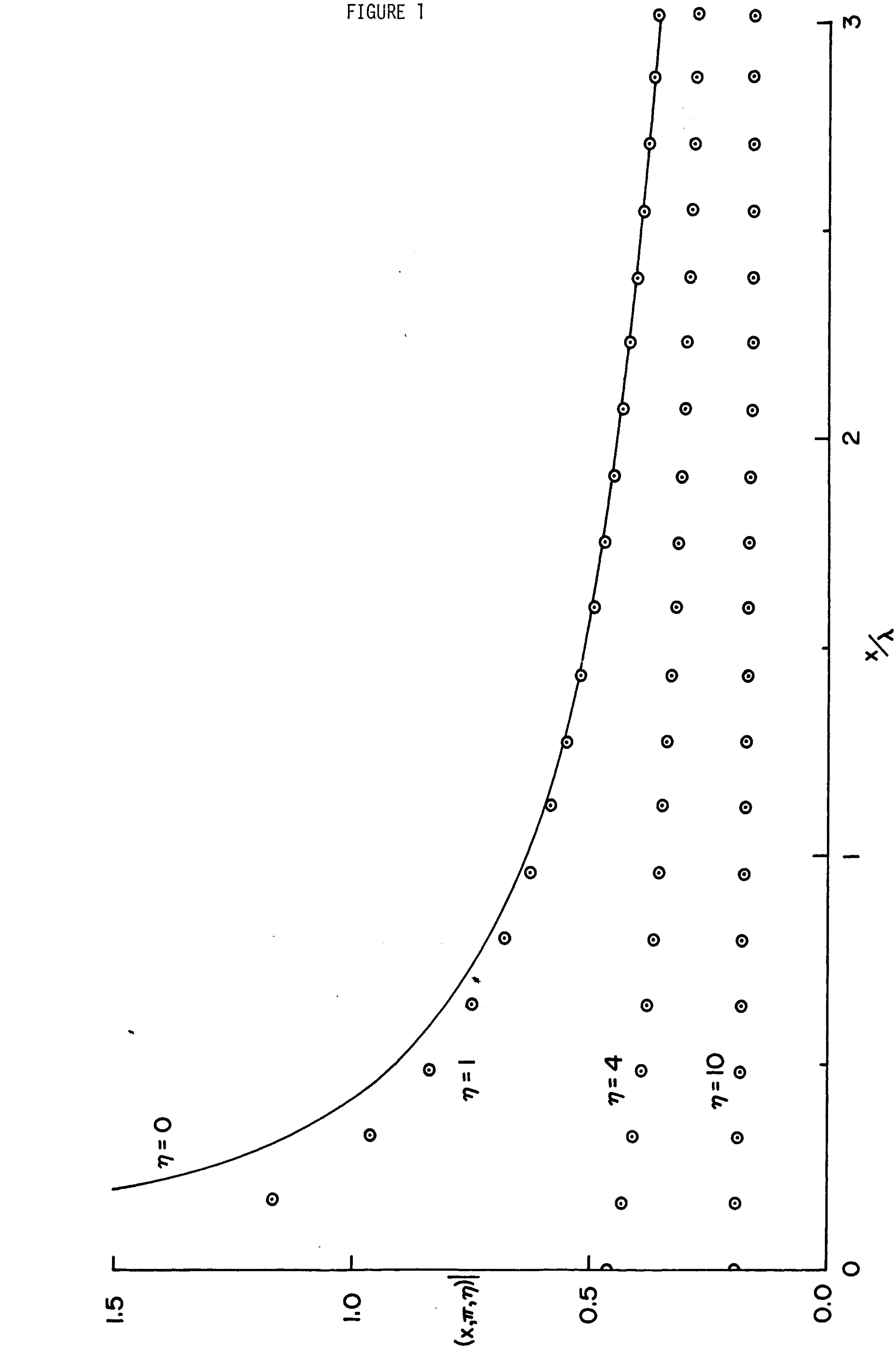

FIGURE 2

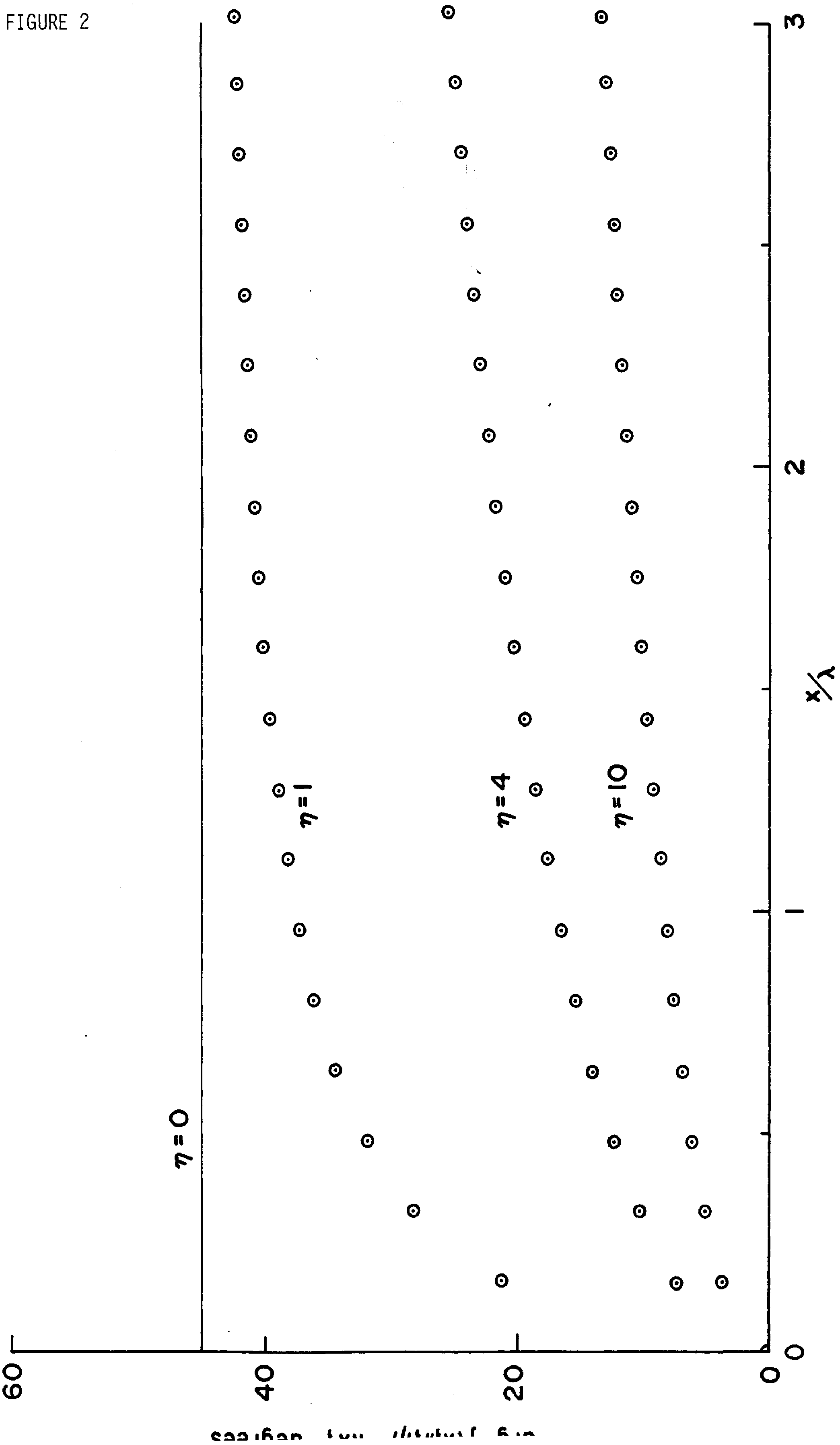

FIGURE 3

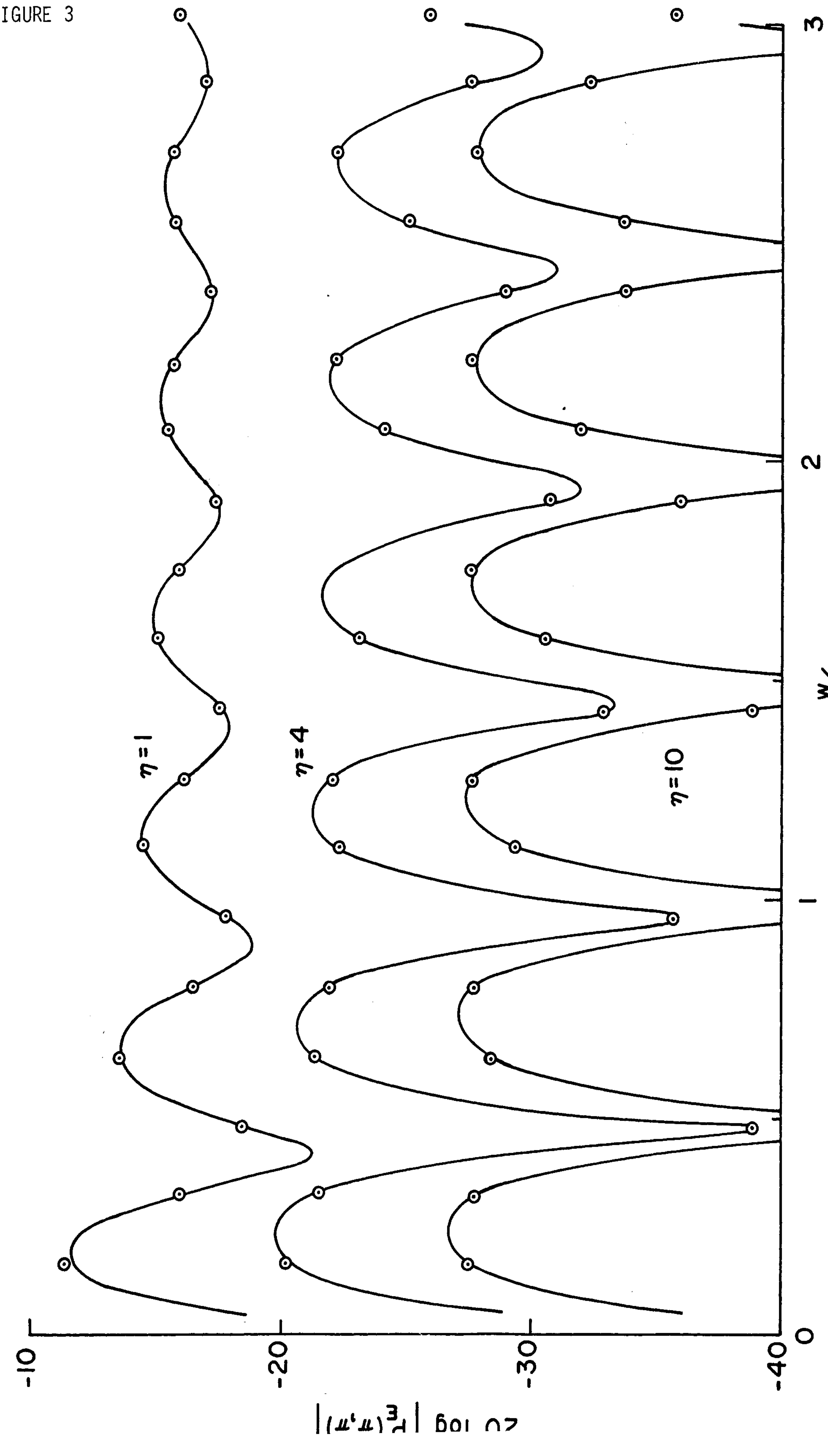

FIGURE 4