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ELECTRODYNAMICS AND WAVE PROPAGATION

Scattering of Modes by the End of a Diaphragm-Loaded Planar Dielectric Waveguide

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Abstract—A planar dielectric waveguide loaded with an infinitely thin perfectly conducting diaphragm is considered. The problem of the guided mode reflection from the end of such a waveguide is analyzed with the help of the integral equation method and the variational method. Dependences of the fundamental mode reflection coefficient on the waveguide parameters and on the dimension of the aperture are calculated.

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INTRODUCTION

Calculation of various elements of optical and microwave circuits and, in particular, designing of radiators, sensors, excitation elements, and resonance structures often involve the problem of scattering of guided modes (GMs) by the end of a dielectric waveguide. When a simple geometry is used (i.e., a waveguide's end without additional elements) the characteristics of such a structure can be varied within a relatively small range (see below). Therefore, more complex structures are often used for practical applications. For example, multilayer dielectric coatings are deposited on the plane of the waveguide end [1, 2]. These coatings can provide for the strong frequency dependence of the mode reflection coefficient. The ends with chamfered and tapered geometries [3], and structures with the hemispherical shape of a waveguide's end are applied; moreover, lenses are placed at the output of a waveguide [4]. In all of such structures, the values of reflection and transmission coefficients can be varied within rather wide ranges (as compared to the case of a simple geometry).

In this study, the integral equation method (IEM) and the variational method (VM) are applied to analyze the problem of the lowest order GM reflection from the end of a planar dielectric waveguide (PDW) loaded with an infinitely thin perfectly conducting diaphragm, i.e., a metal plane with a slot. These or similar structures are used in various optical sensors [5, 6] as well as for releasing radiation from Cherenkov generators or amplifiers [7, 8]. In this study, we consider the symmetrical geometry[1] displayed in Fig. 1. Below, we investigate the TE problem, where all of the waves have only one nonzero component of the electric field, E_x. A single-mode PDW is considered, and it is assumed that the fundamental TE_0 GM travels towards its end.

[1] The figure displays the special geometry with the PDW thickness smaller than the dimension of the slot. The general case will be considered below.

1. THE INTEGRAL EQUATION

The thickness of the center layer is $2d$ (Fig. 1). It is assumed that the permittivities and permeabilities are step functions: these are ε_1 and μ_1 in the substrate and coating (for $|y| > d$) and ε_2 and μ_2 inside the center layer (for $|y| < d$). As usual, the time factor $\exp(-i\omega t)$ (where

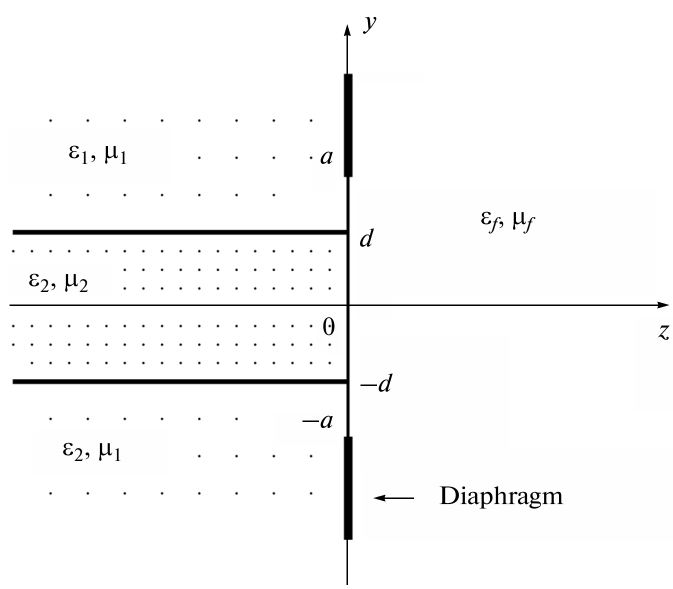

Fig. 1. Geometry of the problem in the case of a structure such that $a > d$.

$\omega = kc$, k is the wavenumber, and c is the velocity of light in free space) will be omitted. Let ε_f and μ_f denote the permittivity and permeability of the right half-space, respectively. A metal diaphragm with a slot of dimension $2a$ is located in the plane of the PDW end. We assume that all of the media are nonmagnetic, i.e., $\mu_1 = \mu_2 = \mu_f = 1$, and that the dielectric loss can be neglected. For the problem to have a single solution, we assume that wavenumber k has a vanishingly small imaginary part, $\mathrm{Im}k = +0$.

Approaches based on the decomposition of fields in the eigenmode fields of open waveguides are applied to derive integral equations (IEs) and variational formulas. These approaches are similar to methods widely used in the theory of metal waveguides [9, 10] and to methods used for the analysis of the problem of scattering of GMs propagating along an infinite open PDW loaded with a metal diaphragm located in the transverse plane [11, 12].

Let us decompose unknown function $\mathscr{E} = E_x$ in the end plane z in the eigenmode fields of the free half-space and waveguide [13, 14]. The expression for the field in the right half-space (for $z \ge 0$) has the form

$$E_x = \sum_{q=0,1} \int_0^\infty \tau_{q\kappa} V_{q\kappa}(y) \exp(i\gamma_\kappa z)\, d\kappa, \tag{1}$$

where $V_{q\kappa}(y)\exp(i\gamma_\kappa z)$ are the radiation eigenmode fields in the region $z > 0$, $\tau_{q\kappa}$ are unknown coefficients, and κ is the transverse wavenumber (a continuous parameter for this problem) ranging over all values $\kappa > 0$. The indices $q = 0$ and $q = 1$ mark even (symmetric) and odd modes, respectively. With allowance for the symmetry condition, only even (symmetric) modes should be retained in all of the decompositions for the problem under study. The radiation modes (RMs) of the right half-space can easily be constructed in an explicit form. For example, the fields of even RMs ($q = 0$) have the form

$$E_x = V_{0\kappa}(y)\exp[i(\gamma_\kappa z - \omega t)], \quad V_{0\kappa}(y) = F_{0\kappa}\cos(\kappa y), \tag{2}$$

where $F_{0\kappa}$ is the amplitude coefficient. For these modes, the longitudinal wavenumber is $\gamma_\kappa = (k^2\varepsilon_f\mu_f - \kappa^2)^{1/2}$.

System of functions $\{V_{q\kappa}\}$ is orthogonal [15, 16]. The normalizing factor for the modes of the right half-space is $D_{q\kappa}^{(f)} = \pi F_{q\kappa}^2/\mu_f$. Note that, in this problem, the root branch such that $\mathrm{Im}\gamma_\kappa > 0$ when $\mathrm{Im}k > 0$ is chosen in the expression for γ_κ.

On the left of the waveguide's end (at $z \le 0$), we have

$$\begin{aligned} E_x &= [\exp(i\beta_0 z) + R\exp(-i\beta_0 z)]\Phi_0(y) \\ &+ \sum_{q=0,1}\int_0^\infty \rho_{q\kappa} U_{q\kappa}(y)\exp(-i\beta_\kappa z) d\kappa, \end{aligned} \tag{3}$$

where R is the reflection coefficient of the TE_0 fundamental GM and $\rho_{q\kappa}$ are the unknown amplitudes of the RM fields of the waveguide. In formula (3), $\Phi_0(y)\exp(\pm i\beta_0 z)$ denotes the fields of GMs (modes of the discrete spectrum) and $U_{q\kappa}(y)\exp(-i\beta_\kappa z)$ denotes the fields of RMs (modes of the continuous spectrum) that propagate away from the end. Later, as for the expansion of the fields for $z \ge 0$, we retain only the even RMs (i.e., the terms with $q = 0$) in the expressions for the fields in the waveguide region.

The general approach that can be applied for constructing eigenmodes of regular PDWs is described in [15, 17]. Note that the fields of the eigenmodes of a waveguide with piecewise constant parameters can be represented as sums of exponential functions. The coefficients of these functions are determined from the conditions on the medium interfaces $y = \pm d$ and the condition at infinity, which is derived with the help of the S-operator method [15]. The introduction of this operator makes it possible to identify RMs that are pairwise degenerate for the planar geometry. The fields of the symmetrical RMs of the waveguide have the form

$$U_{0\kappa} = \begin{cases} A_{0\kappa}\cos(g_\kappa y), & |y| < d, \\ B_{0\kappa}\{s_{0\kappa}\exp[i\kappa(|y| - d)] + \exp[-i\kappa(|y| - d)]\}, \\ |y| > d, \end{cases} \tag{4}$$

where $g_\kappa = \sqrt{\kappa^2 + k^2(\varepsilon_2 - \varepsilon_1)}$. In formula (4), the eigenvalue of the S operator is

$$s_{0\kappa} = \frac{\kappa\cos(g_\kappa d) + ig_\kappa\sin(g_\kappa d)}{\kappa\cos(g_\kappa d) - ig_\kappa\sin(g_\kappa d)}. \tag{5}$$

The eigenmodes of the discrete spectrum can be constructed with the help of a standard technique with allowance for the fact that their fields vanish at infinity (as $y \to \pm\infty$).

System of functions $\{\Phi_0, U_{q\kappa}\}$ is orthogonal. The orthogonality conditions have the form

$$\langle\Phi_0|\Phi_0\rangle = N_0, \quad \langle U_{q\kappa}|U_{q'\kappa'}\rangle = D_{q\kappa}\delta_{qq'}\delta(\kappa - \kappa'), \tag{6}$$

where N_0 is the GM norm, $D_{q\kappa}$ is the normalizing factor of the waveguide RMs, $\delta_{qq'}$ is the identity tensor, and $\delta(\kappa - \kappa')$ is the Dirac delta function. Here and below, the following notation for the integral of the product of arbitrary functions $f_1(y)$ and $f_2(y)$ over the plane $z = 0$ is used:

$$\langle f_1|f_2\rangle = \int_{-\infty}^{\infty} f_1(y) f_2(y) dy. \tag{7}$$

Equating in the plane $z = 0$ tangential field components E_x and H_y, we obtain an IE of the first kind [10, 12, 14] that, for the considered geometry, has the form

$$\hat{\Xi}[\mathscr{E}] = 2\beta_0\Phi_0(y), \quad |y| < a, \tag{8}$$

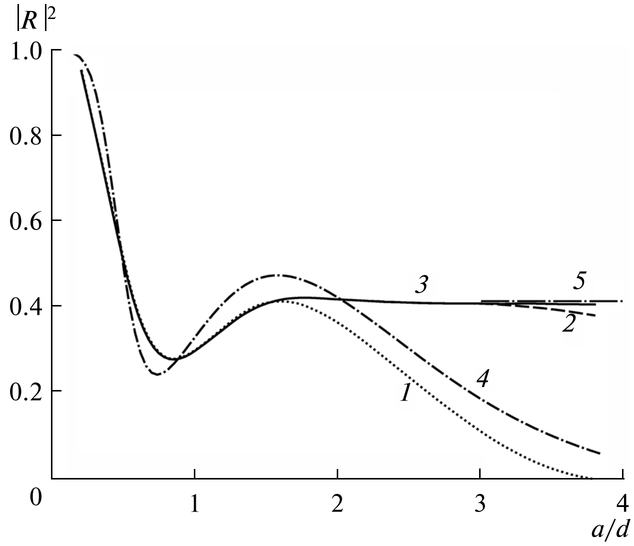

Fig. 2. Squared absolute value of the reflection coefficient for the TE_0 modes vs. the ratio a/d for the constant thickness of a weakly guiding waveguide.

where $\mathscr{E}$ is the unknown function (see above) and $\hat{\Xi}[\mathscr{E}]$ is the integral operator determined by the relationship

$$\hat{\Xi}[\mathscr{E}] = \frac{\beta_0\Phi_0}{N_0}\langle\mathscr{E}|\Phi_0\rangle + \int_0^\infty \frac{\beta_\kappa U_{0\kappa}}{D_{0\kappa}}\langle\mathscr{E}|U_{0\kappa}\rangle d\kappa + \int_0^\infty \frac{\gamma_\kappa V_{0\kappa}}{D_{0\kappa}^{(f)}}\langle\mathscr{E}|V_{0\kappa}\rangle d\kappa. \qquad (9)$$

Note that, in contrast to the end of a PDW without a diaphragm, in the problem considered, coordinate y changes within a finite interval, $|y| < a$.

The above IE is solved by means of the collocation method [11, 12]. The electric field in the diaphragm's slot is approximated with a finite series of even Chebyshev polynomials with the weighting function

$$\mathscr{E} = \sqrt{1-(y/a)^2}\sum_{m=1}^{M} C_m T_{2(m-1)}(y/a), \qquad (10)$$

where C_m are unknown coefficients and M is the number of terms. This representation of the field takes into account the field singularities [18] on sharp metal edges of an infinitely thin perfectly conducting diaphragm. Next, we equate the left- and right-hand sides of Eq. (8) at the points

$$y_j = a\cos\left(\frac{2j-1}{4M}\pi\right), \quad j = 1,2,\ldots,M. \qquad (11)$$

As a result, we obtain the system of linear algebraic equations (SLAE) for the decomposition coefficients of (10)

$$\sum_{m=1}^{M} Q_{jm}C_m = 2\beta_0\Phi_0(y_j), \quad j = 1,2,\ldots,M. \qquad (12)$$

The main steps of the calculation of matrix elements Q_{jm} are the same as those used in [12]. The algorithm of the calculation of the integrals determining Q_{jm} is described in detail in Section 3. Being rather bulky, the expressions for Q_{jm} are not presented here. After the SLAE is solved, the coefficients C_m are found, and, then, the aperture field is determined from formula (10). The reflection coefficient is calculated from the formula

$$R = \langle\mathscr{E}|\Phi_0\rangle/N_0 - 1, \qquad (13)$$

which follows from the orthogonality conditions.

Note that Eq. (8) is an IE of first kind. As is known, instabilities may arise in the process of solution of such equations; i.e., a solution may start oscillating with an increasing amplitude as the number of terms taken into account in the decomposition for the sought field grows. However, in the case under consideration, instabilities practically are not manifested. This circumstance follows from the fact that the kernel of this IE is singular (has a logarithmic singularity) and, the process of solution involves the self-regularization effect (see, e.g., [19, 20]). In addition, for weakly guiding structures (see below) the series (10) rather rapidly converges, and, therefore, the instability does not have enough time to develop.[2] For the aforementioned reasons, no additional regularization algorithms are applied in this analysis.

2. COMPUTATION RESULTS

In all of the examples considered below, it is assumed that the parameters of the right half-space correspond to the parameters of free space: $\varepsilon_f = \mu_f = 1$ (i.e., $n_f = 1$). In addition, it is assumed that the wavelength in free space is $\lambda = 0:86$ μm (here, $\lambda = 2\pi/k$).

The squared absolute value of reflection coefficient $|R|^2$ is shown in Fig. 2 as a function of the ratio a/d. It is assumed that the PDW thickness is $2d = 0.25$ μm, the permittivity of the center layer is $\varepsilon_2 = 12.96$, and the permittivity of the exterior of the slab is $\varepsilon_1 = 11.664$. Curves *1–3* are plotted for the numbers of terms in the field representation $M = 2$, 4, and 6, respectively. Curve *3* graphically coincides with the curve calculated at $M = 8$. The meaning of curve *4* is explained below. Note that, when $a/d \sim 1$, there is a dip in the plotted curves. This dip is apparently related with a slot resonance of small Q-factor. As should have been expected, the absolute value of the reflection coefficient approaches unity when aperture dimension $2a$ is small.

For the aforementioned PDW parameters and for the value of a/d that is not very large, quantity $|R|^2$ is graphically stabilized even for $M \geq 6$. For wide slots

[2] This case slightly resembles the situation occurring in a calculation using asymptotic decompositions.

(when $a/d > 3$ in our case), the effect of the diaphragm is insignificant. This conclusion is illustrated in Fig. 2, where, for comparison, straight line *5* shows value $|R|^2$ in the structure without a diaphragm (i.e., when $a = \infty$). This value is calculated with the help of the accelerated iteration method [14]. It is seen that curve *3* approaches curve *5* when $a/d \sim 3$.

Thus, this technique, which is based on the representation of the solution in the form of the series of Chebyshev polynomials with weighting function (10), can be applied to calculate the coefficient of GM reflection from a simple waveguide end (without a diaphragm) when the value of the ratio a/d is chosen rather large. However, it should be taken into account that, in order to retain the acceptable accuracy of the computation results, it is necessary to increase number M of terms in the decomposition for field (10) as the ratio a/d grows (see Fig. 4 below). Rough estimation shows that number M should satisfy the condition $M \gg a/w_0 + 1$, where w_0 is the half transverse dimension of the GM field (in the order of the quantity $w_0 \sim d + 1/p_1$, where p_1 is the GM external transverse wavenumber).[3]

Let us analyze the convergence of the series for the aperture field as number M of terms in decomposition (10) grows. The computation results are presented in Fig. 3. The permittivities and permeabilities of the PDW layers are the same as those from Fig. 2, the thickness of the waveguide layer is fixed at the value $2d = 0.25$ μm, and the dimension of the slot is $a = 1$ μm (i.e., $a/d = 4$). Curves *1*–*4* are plotted for the numbers of terms in the field representation $M = 2$, 4, 6, and 8, respectively. Note that the results obtained for $M = 10$ graphically coincide with the data obtained for $M = 8$. The field of the incident TE_0 mode is shown with curves *5*. The presented data demonstrate the good convergence of decomposition (10).

In Fig. 4, the squared absolute value of reflection coefficient $|R|^2$ is shown as a function of dimensionless waveguide thickness $2d/\lambda$ for a wide slot. It is assumed that the permittivity of the center layer is $\varepsilon_2 = 12.96$, and the permittivity of the exterior of the layer is $\varepsilon_1 = 11.664$. The computation is performed for $2a = 4$ μm. Curves *1*–*4* are plotted for the numbers of terms in the field representation $M = 10$, 15, 20, and 30. The squares are the values of $|R|^2$, calculated with the help of the accelerated iteration method [14] for the case $a = \infty$ (i.e., in the absence of a diaphragm). Note that, in this example, when $2d/\lambda > 0.05$, the transverse dimension of the GM field is small as compared to a; i.e, the GM field is small when $|y| > a$. Therefore, the diaphragm affects the process of reflection of the TE_0 mode from the PDW end only slightly.

[3] This estimate is valid only for weakly guiding PDW, and it becomes rough for very large a.

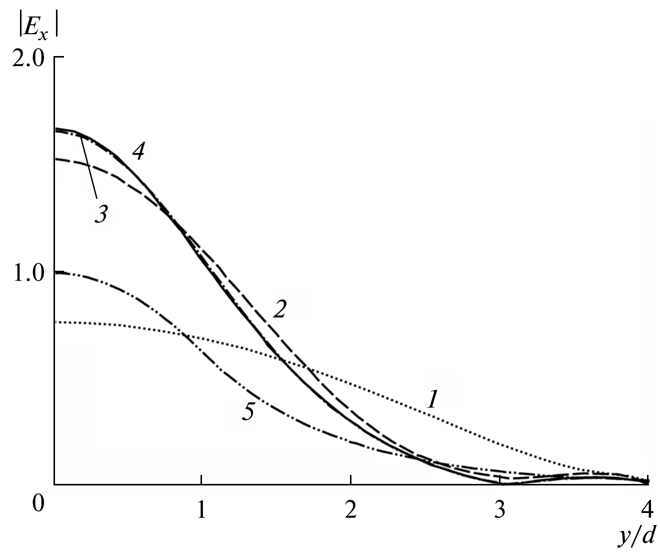

Fig. 3. Distribution of the absolute value of the electric field in the diaphragm plane for various numbers of terms in decomposition (10).

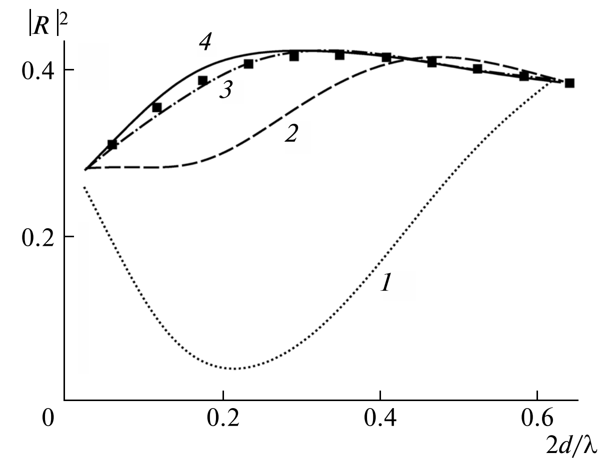

Fig. 4. Squared absolute value of the reflection coefficient for the TE_0 modes vs. the dimensionless thickness of the waveguiding layer for a wide slot.

It is reasonable to represent the solution in the form of polynomial series (10) in the case when the PDW is a weakly guiding one (when the contrast of permittivities is low: $\varepsilon_1/\varepsilon_2 \sim 1$). When the contrast of permittivities is high, then, the effect of the singularities of fields (or of their derivatives) can be pronounced near the edge points with the coordinates $z = 0$, $y = \pm d$ [18]. Therefore, decomposition (10) for the aperture field converges slowly. As has been mentioned above, numerical instabilities may occur in this case. The analysis of structures with a high contrast of refractive indexes necessitates decompositions that take into account the field behavior near edge points.

Let us discuss in brief the structure of the scattered field in the far zone (i.e., the field of a cylindrical

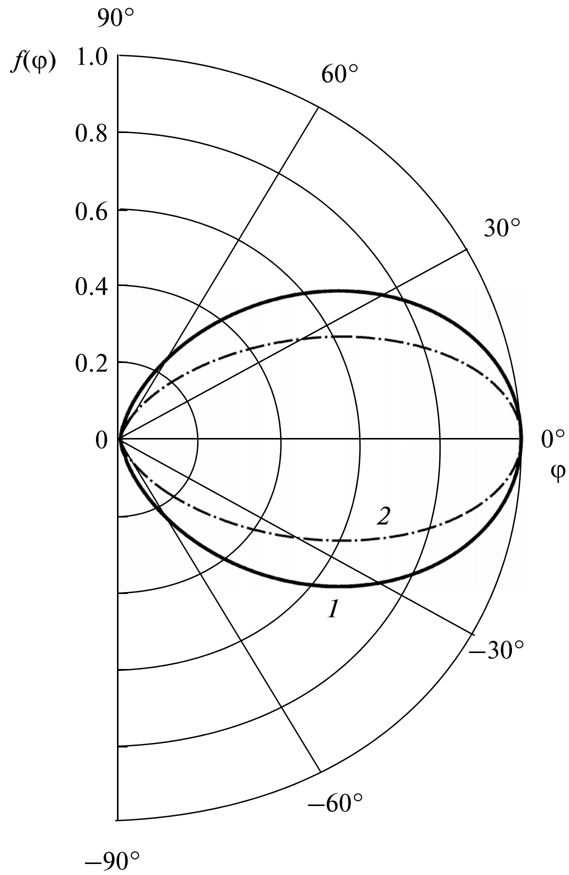

Fig. 5. Radiation field patterns for the (curve *1*) small and (curve *2*) large aperture dimensions.

wave). In the right half-space, the expression for this field can be obtained from integral representation (1) with the help of the stationary phase method. In cylindrical coordinates (r, φ), where $r = \sqrt{y^2 + z^2}$, and $\varphi = \arctan(y/z)$, the pattern is

$$f(\varphi) = r\left|E(r,\varphi)\right|^2 \sim \left|\tau_{0\kappa}\cos(\varphi)\right|^2 \quad (14)$$

as $r \to \infty$. In this formula, the notation $\tau_{0\kappa} = \langle \mathcal{E} | V_{0\kappa} \rangle$, where $\mathcal{E}$ is the aperture field (see above), is introduced and parameter κ is determined by angle φ according to the relationship $\kappa = \kappa_p = kn_f \sin(\varphi)$.

Simple estimates of pattern $f(\varphi)$ can be obtained by substituting approximate expressions for field $\mathcal{E}$ into (14), i.e., using the physical optics method. The patterns calculated with the help of this method are presented in Fig. 5 for two limit cases. When $a \ll d$, the aperture field is approximately described by the first term from decomposition (10). In this case, after some algebra, we obtain $f(\varphi) \sim [\cos(\varphi)]^2$. Curve *1* is the pattern for the waveguide end with a small slot in the diaphragm. Here and below, the fields are normalized so that $f(0) = 1$. Note that, for a narrow slot, the shape of the pattern is independent of the waveguide parameters in the first approximation.

In the other case, when the dimension of the slot is large, i.e., $a \gg d$, and the waveguide is a weakly guiding one, the aperture field is approximately described by the function $(1 + R)\Phi_0(y)$. This approximation becomes inaccurate only near the diaphragm edges (i.e., in the neighborhood of the points $y = \pm a$). Under this condition, the integral that determines function $\tau_{0\kappa}$, is calculated in terms of elementary functions. Being bulky, the corresponding expression is not presented here. Curve *2* is the pattern in the right half-space in this case. The waveguide parameters are as follows: $\varepsilon_1 = 11.664$, $\varepsilon_2 = 12.96$, and $2d = 0.25$ μm. The case when there is no diaphragm is thoroughly studied in [14]. It is not efficient to change the shape of the pattern of radiation from the PDW end with help of a diaphragm; in particular, a substantial change in the transmitted power level cannot be avoided under the variation of the slot dimension. Therefore, here, we do not consider the characteristics of the radiation field in more detail.

3. THE VARIATIONAL METHOD

We also apply a method based on the variational principle to solve the problem in question. This technique is rather universal. It has been used for investigation of various irregularities of both planar and, more general, 3D structures [13, 21]. It should be taken into account that simple VM versions are approximate and that their accuracy often cannot be determined. At present, the VM error has been estimated only for a small number of problems solved in an analytical form (see, e.g., [21, 22]). Using the results obtained with the help of the IEM, we can determine the accuracy of the VM for the geometry under consideration.

With the help of standard techniques [9, 10, 13], we can obtain stationary functionals for the main characteristics of the problem using the IE derived above. For TE_0 mode reflection coefficient R, the stationary functional has the form

$$\frac{1-R}{1+R} = \frac{N_0}{\beta_0 \langle \mathcal{E} | \Phi_0 \rangle^2}(I_1 + I_2), \quad (15)$$

where

$$I_1 = \int_0^\infty \beta_\kappa \langle \mathcal{E} | U_{0\kappa} \rangle^2 \, d\kappa / D_{0\kappa}, \quad (16)$$

$$I_2 = \int_0^\infty \gamma_\kappa \langle \mathcal{E} | V_{0\kappa} \rangle^2 \, d\kappa / D_{0\kappa}^{(f)}. \quad (17)$$

Recall that, owing to the stationarity property, formula (15) yields the results whose error is the next-order infinitesimal relative to that of a trial (approximate) field $\mathscr{E}_a$ which is substituted into equation (15). The following function is used as such a distribution:

$$\mathscr{E}_a = C_e\sqrt{1-(y/a)^2}, \quad (18)$$

where C_e is a certain constant. Note that this constant is cancelled out on the right-hand side of (15), and, therefore, the value of reflection coefficient R depends only on the shape of the distribution of the aperture field rather than its amplitude. The aforementioned test field takes into account the singularities on the diaphragm edges but disregards the field structure near the axis, especially, in the case when the mode incident on the waveguide end is extremely slow. Therefore, it is evident that this distribution can be suitable for comparatively small slots whose dimension usually does not exceed wavelength λ. The exact boundary of the applicability and the error of formula (15) are determined below with the help of simulation.

Let us describe in brief the method of calculation of the reflection coefficient from formula (15). Note that the integrands in integrals I_1 and I_2 comparatively slowly approach zero as $\kappa \to \infty$. Therefore, the main task of the calculation is accelerating their convergence.

First, consider second term I_2. For an arbitrary value of κ, the inner integral from I_2 can be calculated analytically:

$$\langle \mathscr{E}_a | V_{0\kappa} \rangle = a\sqrt{\pi}[J_1(\kappa a)/(\kappa a)], \quad (19)$$

where $J_m(\kappa a)$ is the Bessel function ($m = 1$). Thus, we obtain

$$I_2 = \pi a \int_0^\infty \gamma_\kappa [J_1(\kappa a)/(\kappa a)]^2 d\kappa. \quad (20)$$

For $\kappa \gg kn_f$, we have $\gamma_\kappa \approx i(\kappa - 0.5k^2\varepsilon_f/\kappa)$. Next, taking into account the formula

$$\int J_1^2(t)\,dt/t = -0.5[J_0^2(t) + J_1^2(t)], \quad (21)$$

for the tail of integral I_2, we obtain the following estimate:

$$\pi a \int_{\kappa_b}^\infty \gamma_\kappa \langle \mathscr{E}_a | V_{0\kappa} \rangle^2 d\kappa / D_{0\kappa}^{(f)} \approx 0.5\pi i[J_0^2(\kappa_b a) + J_1^2(\kappa_b a)] - i0.25(ka)^2\varepsilon_f/(\kappa_b a)^2, \quad (22)$$

where $\kappa_b \gg kn_f$ is a certain intermediate large value of the argument. The integral is calculated numerically over the interval $(0, \kappa_b)$ and, according to the above formula, over the semi-infinite interval (κ_b, ∞). Partition point κ_b is chosen such that the omitted terms of the order $(k/\kappa_b)^3$ are small.

An analytical representation in the above form cannot be obtained for integral I_1. Nevertheless, the tail of this integral can be estimated rather accurately when it is taken into account that, for $\kappa \gg kn_1$, the waveguide RM fields are close to the RM fields of a homogeneous space where the refractive index is n_1. This conclusion can readily be drawn from formulas (2), (4), and (5) if it is taken into account that, for $\kappa \gg kn_1$, $g_\kappa^2 = \kappa^2 + k^2(\varepsilon_2 - \varepsilon_1) \approx \kappa^2$, and, hence $s_{0\kappa} \approx \exp(2i\kappa d)$ and $U_{0\kappa} \approx V_{0\kappa}$. Note that waveguide RMs especially rapidly become the modes of the homogeneous space when the PDW is weakly guiding[4], i.e., when $\varepsilon_2 \approx \varepsilon_1$. In this case, the second term from the expression for g_κ^2 is small within a wide range of number κ. Thus, the tail of integral I_1 can be estimated from formula (22), where ε_f should be replaced by ε_1. As a rule, the accuracy necessary for the calculation of I_1 is provided when the value of partition point κ_b is larger than that for integral I_2.

The variational principle can be applied to obtain a series of analytical relationships. For example, under the conditions $ka \ll 1$ and $a \ll d$, the following estimate can be derived: $R + 1 \sim (ka)^2$, where the proportionality coefficient is dropped to simplify the representation. The above relationship can be derived if it is taken into account that, for small a, the quantities I_1 and I_2 approach finite limits and that the quantity $\langle \mathscr{E}_a | \Phi_0 \rangle^2$ from the denominator of formula (15) is proportional to a^2. This estimate can be obtained with the help of a quasi-static method. However, since the permittivities of the media on the left and right of the diaphragm slot are different, apparently, its polarizability can be calculated only numerically.

Now, we present results obtained with the help of the VM. First, we return to the example analyzed with the help of the IEM. In Fig. 2, curve *4* is the dependence of $|R|^2$ on the ratio a/d for the waveguide with the parameters indicated in the foregoing. It is seen that the results obtained by means of the VM and IEM are in good agreement when $a/d < 1.5$. Obviously, for larger values of a/d, it is necessary to apply other test functions $\mathscr{E}_a$, substituted into variational relationship (15). For example, we can use the function that coincides with the GM field when $|y| < a$ and is zero when $|y| > a$. For this distribution, the values of integrals I_1 and I_2 can readily be estimated. The analysis of this problem is beyond the framework of this study; therefore, we restrict ourselves to the above remarks.

In the considered example, the PDW is a weakly guiding one, i.e., having a small contrast of permittivities, $\varepsilon_2/\varepsilon_1 \sim 1$. The VM can also be applied for analyzing structures with rather a high contrast of these parameters. As an example, the dependence of $|R|^2$ on the ratio a/d for such a problem is shown in Fig. 6 (curve *1*). The structure parameters are as follows: $\varepsilon_1 =$

[4] Note that this property was earlier successfully used in the development of the free space radiation mode (FSRM) method [2, 23], which was widely applied for solution of various problems of wave scattering.

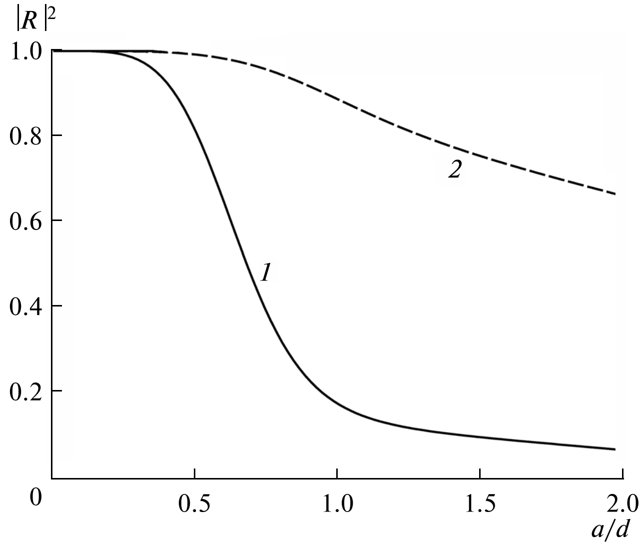

Fig. 6. Squared absolute value of the reflection coefficient for the TE_0 modes vs. the ratio a/d for various values of the permittivity of the waveguiding layer.

1.0, $\varepsilon_2 = 2.1092$ ($n_2 = 1.4523$), $2d = 0.8$ μm, and $\lambda = 0.86$ μm.

Curve *2* shows the same dependence for a waveguide with $\varepsilon_1 = 1.0$, $\varepsilon_2 = 12.96$ ($n_2 = 3.6$)[5], and $2d = 0.12$ μm. Note that, for the considered problems, when $a/d > 1$, the magnetic fields at the edge points $z = 0$, $y = \pm d$ can have noticeable singularities that are disregarded in simplified representation (18). Nevertheless, the analysis of similar problems [10] shows that these singularities do not substantially affect the calculation of integral characteristics.

CONCLUSIONS

In the study, a PDW loaded with a thin metal diaphragm in the plane of the waveguide end has been considered. For the fundamental TE mode the coefficients of reflection from the PDW end have been calculated by means of the IEM and VM. The computation has shown that the reflection coefficient of GMs can be varied within a wide range with the help of the diaphragm.

The IEM makes it possible to develop rather a simple algorithm for the numerical solution of the problem. The calculation has shown that, when the PDW is weakly guiding, the solutions, including field distributions, converge rather rapidly within a wide range of the problem parameters. In particular, this method can be applied to analyze even the case of wide slots for a single-mode waveguide.

The VM used in the study is an approximate technique. Nevertheless, as has been shown, this method provides for a good accuracy of the reflection coefficient calculation. This approach is universal. In particular, it can be generalized to the case of a PDW whose core is formed by several dielectric layers and to the case of anisotropic structures.

[5] The values of n_2 used in these examples are close to the refractive indexes of sintered quartz and gallium arsenide.

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