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A Letters Journal Exploring the Frontiers of Physics

December 2012

EPL, **100** (2012) 54003 www.epljournal.org
doi: 10.1209/0295-5075/100/54003

Scattering of the discrete solitons on the $\mathcal{PT}$-symmetric defects

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received 28 August 2012; accepted in final form 17 November 2012
published online 6 December 2012

PACS 42.25.Bs – Wave propagation, transmission and absorption
PACS 42.82.Et – Waveguides, couplers, and arrays

Abstract – We study the propagation of linear waves and solitons in an array of optical waveguides with an embedded defect created by a pair of waveguides with gain and loss satisfying the so-called parity-time ($\mathcal{PT}$) symmetry condition. We demonstrate that in the case of small soliton amplitudes, the linear theory describes the scattering of solitons with a good accuracy. We find that the incident high-amplitude solitons can excite the mode localized at the $\mathcal{PT}$-symmetric defect. We also show that by exciting the localized mode of a large amplitude, it is possible to perform phase-sensitive control of soliton scattering and amplification or damping of the localized mode.

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Introduction. – Photonic structures consisting of coupled waveguides with regions of gain and loss offer many novel possibilities for shaping optical beams in comparison with traditional conservative or low-loss structures [1–3]. Such structures can be constructed as optical analogues of the complex space-time potentials possessing the so-called parity-time ($\mathcal{PT}$)-symmetry, which can have an entirely real eigenvalue spectrum, corresponding to the energy conservation of optical eigenmodes. The beam dynamics in this case may demonstrate the properties qualitatively different from those usually observed in conservative systems [4–6]. Among them are the $\mathcal{PT}$-symmetry breaking that occurs for sufficiently large powers [7], power oscillations [4,7–9], nonmonotonous dependence of the transmission on absorbtion [10], unidirectional invisibility [7,11], conical diffraction [12], a new type of Fano resonance [13]. Such systems may support solitons [14–16], demonstrating the amplification of extended waves or solitons [13,17,18], and the nonlocal response that manifests itself through the nontrivial effect of the boundary conditions [19]. Recently, $\mathcal{PT}$-symmetric properties of couplers composed of two waveguides have been demonstrated experimentally [8,10].

Various schemes have been suggested to tailor the beam shaping and switching using $\mathcal{PT}$-symmetric structures, including introduced $\mathcal{PT}$-defects in periodic lattices [20] and self-induced refractive index change in nonlinear $\mathcal{PT}$-symmetric structures [1,21,22]. It was found, in particular, that linear waves and small-amplitude solitons can be damped or amplified as they are scattered on $\mathcal{PT}$-symmetric couplers embedded into conservative waveguide arrays [17,18,23].

In this paper, we study the interaction of solitons with a $\mathcal{PT}$-symmetric defect with balanced gain and loss (the so-called $\mathcal{PT}$-symmetric coupler, or $\mathcal{PT}$-coupler), and show that stronger nonlinear interactions may lead to the excitation of a large-amplitude defect mode localized at the $\mathcal{PT}$-coupler. Additionally, we show that by specially exciting a large-amplitude $\mathcal{PT}$-coupler mode, it becomes possible to realize phase-controlled soliton scattering, which is accompanied by damping or amplification of the localized modes.

The letter is organized as follows. First, we introduce our model. Then we summarize our recent results [17] on the reflection and transmission coefficients describing linear waves scattered by the $\mathcal{PT}$-symmetric coupler, and then identify new effects in the process of high-amplitude soliton scattering. After that we analyze the soliton interaction with a large-amplitude mode localized at the $\mathcal{PT}$-symmetric coupler. Finally, we present several conclusions stemming from our work.

Model. – We consider a chain of coupled optical waveguides with an embedded pair of $\mathcal{PT}$-symmetric waveguides, where one waveguide experiences gain, and the other one loss, as shown schematically in fig. 1. Light propagating through the waveguide array can be described by a set of the discrete nonlinear Schrödinger

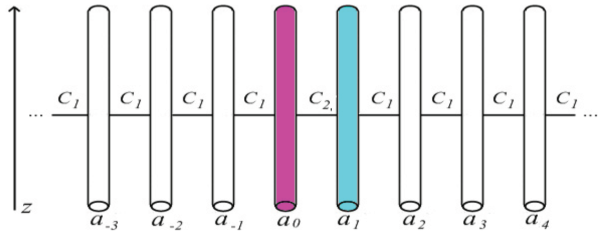

Fig. 1: (Colour on-line) Schematic illustration of an optical waveguide array composed of conservative waveguides, with the exception of a $\mathcal{PT}$-symmetric pair of waveguides with gain and loss at $j=0,1$.

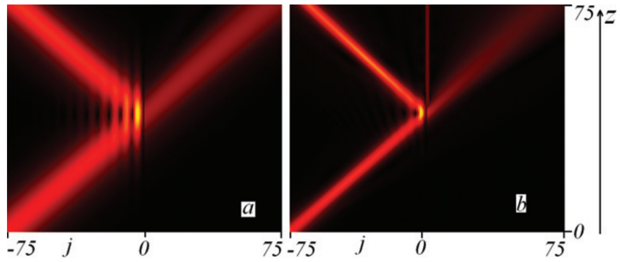

Fig. 2: (Colour on-line) Intensity distribution in the waveguide array *vs.* the propagation distance in the process of soliton scattering on a $\mathcal{PT}$-coupler. The incident soliton amplitude is $A=0.2$ (a) and $A=0.5$ (b). For both plots, $C_1=2$, $C_2=4$, $\rho=1.5$ ($\rho/\rho_{crit}=0.33$), $v=0.5$, and $j_0=-75$.

equations [17],

$$\begin{aligned} &i\frac{\mathrm{d}a_j}{\mathrm{d}z}+C_1a_{j-1}+C_1a_{j+1}+|a_j|^2a_j=0,\ j\neq 0,1,\\ &i\frac{\mathrm{d}a_0}{\mathrm{d}z}+i\rho a_0+C_1a_{-1}+C_2a_1+|a_0|^2a_0=0, \qquad (1)\\ &i\frac{\mathrm{d}a_1}{\mathrm{d}z}-i\rho a_1+C_2a_0+C_1a_2+|a_1|^2a_1=0, \end{aligned}$$

where z is the propagation coordinate along the waveguides, C_1 is the coupling constant for the conservative waveguides, C_2 is the coupling constant inside the $\mathcal{PT}$-symmetric coupler, ρ is the gain-loss coefficient, a_j are the optical mode amplitudes, and j is the waveguide number.

Soliton scattering. – Scattering of linear waves of the form $a_j\sim\exp[i(kj-\beta z)]$ on the $\mathcal{PT}$-symmetric coupler has been studied recently [17], and it was shown that the reflection coefficient, R, and transmission coefficient, T, can be written in the form,

$$\begin{aligned} T(\rho)&=-\frac{e^{-ik}2i\overline{C}_1\sin k}{D},\\ R(\rho)&=\frac{1-\overline{C}_1^2-\overline{\rho}^2+2\overline{\rho}\,\overline{C}_1\sin k}{D}, \end{aligned} \qquad (2)$$

where k is the Bloch wave number, $\beta=-2C_1\cos(k)$ is the propagation constant,

$$D=\overline{C}_1^2\exp(-2ik)+\overline{\rho}^2-1,\quad \overline{\rho}=\frac{\rho}{C_2},\quad \overline{C}_1=\frac{C_1}{C_2}. \qquad (3)$$

It was shown [19] that for $|\overline{\rho}|\geqslant(1+\overline{C}_1^2)^{1/2}\equiv\overline{\rho}_{crit}$ the $\mathcal{PT}$-symmetry breaking takes place, so that in what follows we restrict ourselves by the condition $\overline{\rho}<\overline{\rho}_{crit}$. It can be easily checked that T and R can be larger than unity meaning that the reflected and/or transmitted waves can be amplified after the scattering.

Since the array is composed of conservative waveguides on either sides of the $\mathcal{PT}$-coupler, eqs. (1) admit conventional approximate solutions for self-localized beams (solitons) that can propagate through the waveguide array [24–26] for $j<0$ and $j>0$,

$$a_j=A\mathrm{sech}[\delta(j-j_0-2C_1vz)]e^{i[v(j-j_0)+\beta_s z+\alpha]}, \qquad (4)$$

where A is the soliton amplitude, $\delta=A\sqrt{\gamma/2C_1}$ is the inverse width, v is the normalized velocity, $\beta_s=(\delta^2-v^2+2)C_1$ is the propagation constant, j_0 is the initial position, and α is the initial phase of the soliton. The soliton moving through the optical lattice can radiate energy, but such losses are negligible for small soliton velocities [24,25].

We now consider the scattering of the soliton on the $\mathcal{PT}$-coupler. It was demonstrated that in the regime of weak nonlinearity, for small soliton amplitudes, the soliton scattering is well described by the linear theory [17,27]. In this work, we aim to analyze the case of relatively strong nonlinearity, corresponding to larger soliton amplitudes. To be specific, we consider the solitons approaching the coupler from the left. For $\rho>0$, this corresponds to the situation when the incident soliton first passes through the waveguide with the loss (at $j=0$) and then with the gain (at $j=1$). We define the total intensities of the incident, reflected, and transmitted solitons as

$$\begin{aligned} P_I&=\sum_{j=-\infty}^{1}|a_j(0)|^2,\quad P_R=\sum_{j=-\infty}^{1}|a_j(L)|^2,\\ P_T&=\sum_{j=2}^{\infty}|a_j(L)|^2, \end{aligned} \qquad (5)$$

respectively. Here L is the propagation distance, when the transmitted and reflected solitons move sufficiently far away from the $\mathcal{PT}$ defect. Then the transmission and reflection coefficients for the solitons can be defined as $N_T=P_T/P_I$, $N_R=P_R/P_I$.

Figure 2 presents two examples of the soliton scattering at the $\mathcal{PT}$-coupler for the model parameters $C_1=2$, $C_2=4$, $\rho=1.5$ ($\rho/\rho_{\mathrm{crit}}=0.33$). Panels (a) and (b) correspond to the incident soliton amplitudes $A=0.2$ and $A=0.5$, respectively. In fig. 2(b) we observe that the incident soliton excites the mode localized at the coupler, while in fig. 2(a) the localized mode is not excited. The mode excitation occurs only in the regime of strong nonlinearity, and it was not reported previously in ref. [17].

Figure 3 summarizes our numerical results for the soliton scattering, where we also show for reference the linear scattering for small-amplitude plane waves. We note that

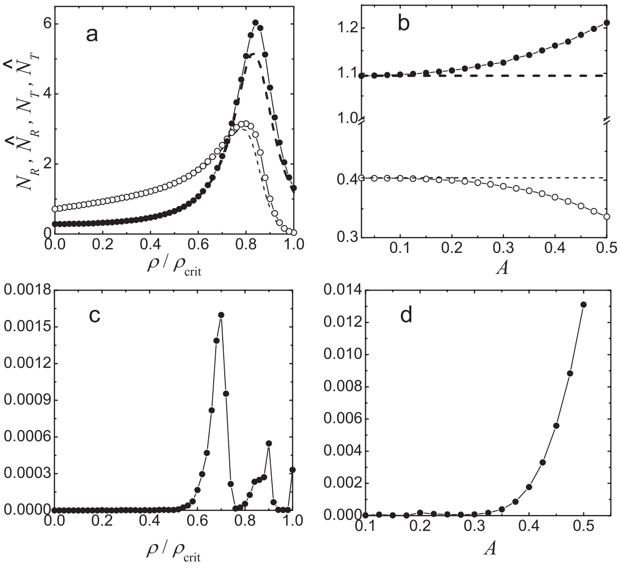

Fig. 3: (a), (b): reflection N_R (open circles) and transmission N_T (black dots) coefficients for the soliton scattering compared to the linear wave scattering $\hat{N}_R$ (thin dashed line) and $\hat{N}_T$ (thick dashed line) as functions of (a) ρ/ρ_{crit} for $A=0.2$, and (b) the soliton amplitude A for $\rho=1.5$ ($\rho/\rho_{crit}=0.335$). (c), (d): total intensity of the localized mode excited by the incident soliton (c) *vs.* ρ/ρ_{crit} for $A=0.2$ and (d) *vs.* A for $\rho=1.5$. In all the cases $C_1=2$, $C_2=4$, $v=k=0.5$.

the spatial Fourier spectrum of solitons is centered around the wave number $k=v$; therefore, in order to compare the scattering of solitons and linear waves, the waves are chosen with the propagation constant $k=v$. Reflection and transmission coefficients for the linear waves are defined by $\hat{N}_R=|R|^2$, $\hat{N}_T=|T|^2$, where R and T are determined from eqs. (2). Simulation parameters are $C_1=2$, $C_2=4$, $k=v=0.5$. In fig. 3(a) the reflection and transmission coefficients are shown as functions of $\rho/\rho_{\rm crit}$ for the incident soliton amplitude $A=0.2$ (dots connected with the lines). In this case, the linear theory (dashed curves) predicts the soliton scattering with a good accuracy in a wide range of $\rho/\rho_{\rm crit}$, and only for the values of $\rho/\rho_{\rm crit}$ around the maxima of the reflection and transmission coefficients there appears a noticeable difference between the soliton and linear scattering regimes. In fig. 3(b), we show that with the increase of the incident soliton amplitude (A) at fixed $\rho=1.5$ ($\rho/\rho_{\rm crit}=0.33$), the discrepancy between the linear wave scattering (dashed curves) and the soliton scattering (dots connected with lines) increases. We also characterize the excitation of $\mathcal{PT}$-coupler localized mode by the incident soliton. In fig. 3(c) we show the total intensity at the $\mathcal{PT}$-coupler waveguides after the passing of the soliton as a function of $\rho/\rho_{\rm crit}$. We see that the mode gets excited when the soliton scattering coefficients are close to their maximum values (cf. fig. 3(a)), although the mode amplitude shows a nontrivial dependence since the excitation process should also depend on the propagation constant mismatch between the mode and soliton. The dependence of the $\mathcal{PT}$-mode intensity on the soliton amplitude (A) is shown in fig. 3(d). We see that the larger the soliton amplitude, the higher the intensity of the excited localized mode, which contributes to the deviation from the linear scattering regime observed in fig. 3(b).

Soliton interaction with a localized mode. – Next, we study the interaction of the soliton with the mode localized on the coupler in the case of strong nonlinearity. For conservative systems, it is known that by exciting a localized mode at a defect it is possible to flexibly control the soliton scattering [28,29]. Therefore, the study of the interaction between a soliton and a localized mode in the model with gain and loss seems very important.

First, we note that the properties of modes localized at the $\mathcal{PT}$-coupler can be determined analytically in the linear regime [17]. The modes exist for $C_1 < C_2$, and their profiles are found from the linearized system (1) as

$$
\begin{aligned}
a_j(z) &= Bq^{-j}\exp(-i\beta_l z), \qquad & j \leqslant 0,\\
a_j(z) &= Bq^{j-1}\exp(-i\beta_l z + i\varphi), \qquad & j \geqslant 1,
\end{aligned} \tag{6}
$$

where

$$
\begin{aligned}
\beta_{l_{1,2}} &= \pm\left(\overline{\rho}^2 - \overline{C_1}^2 - 1\right)\left(1-\overline{\rho}^2\right)^{-1/2} C_2,\\
q_2 &= \frac{1}{q_1} = -\frac{\beta_{l_{1,2}}}{2C_1} + \left(\frac{\beta_{l_{1,2}}^2}{4C_1^2} - 1\right)^{1/2}\\
\varphi_1 &= -\arcsin\overline{\rho}, \quad \varphi_2 = \pi + \arcsin\overline{\rho}.
\end{aligned} \tag{7}
$$

Here B is the mode amplitude and β_l is the propagation constant. For $C_2 < C_1$ the localized mode is not supported because its propagation constant lies within the spectrum of extended planes waves of the waveguide array surrounding the $\mathcal{PT}$-coupler.

In order to obtain the profiles of nonlinear localized modes having large amplitudes, we use a special numerical procedure as follows. We use the initial conditions defined by eq. (6) to excite a low-amplitude mode, and then pump it up by setting in eq. (1) the gain coefficient equal to $-(\rho+\epsilon)$ and the loss coefficient equal to $(\rho-\epsilon)$, where $\epsilon > 0$ is a small quantity. Thus, the gain will exceed the loss, and the energy of the localized modes will grow. In this way, we obtain localized modes with different amplitudes. We will discuss the representative cases of the modes with amplitudes $B=\{1.2,\ 1.4,\ 1.6,\ 1.89\}$, for the model parameters $C_1=2$, $C_2=4$, $\rho=1.5$, which were obtained by setting $\epsilon=0.0025$ and the initial conditions defined by eq. (6) with $\beta_l=-4.787$, $q=0.54$, $\varphi=-0.38$, and $B=1$. We find that the modes with $B>1.89$ are unstable due to the nonlinear $\mathcal{PT}$-symmetry breaking, similar to an instability in an isolated nonlinear $\mathcal{PT}$-coupler [7]. The parameters of the incident soliton are chosen as follows: $A=0.5$, $j_0=-50v$, and $\beta_s=3.65$.

We define the parameter N_{lm} as the ratio of the localized mode energy after the passing of the soliton to its initial value. We find that N_{lm} depends on the

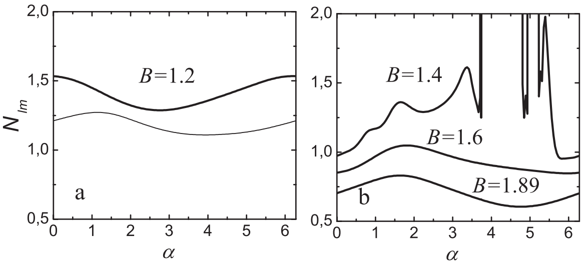

Fig. 4: Relative change of the localized mode energy due to the soliton scattering, N_{lm}, as a function of the initial phase of the incident soliton, α. The model parameters are $C_1 = 2$, $C_2 = 4$, and $\rho = 1.5$; the soliton amplitude is $A = 0.5$. Thick and thin lines correspond to the soliton incident from the left ($v = 0.5$) and from the right ($v = -0.5$). The initial amplitude of the localized mode, B, is indicated by labels.

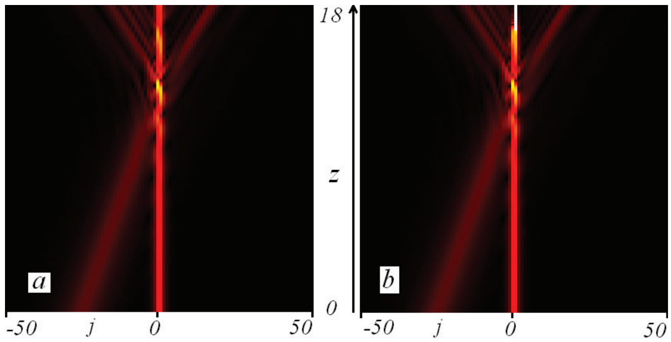

Fig. 5: (Colour on-line) Intensity distribution in the waveguide array *vs.* the propagation distance in the process of soliton scattering on a $\mathcal{PT}$-coupler, when the $\mathcal{PT}$-coupler localized mode is specially excited with the initial amplitude $B = 1.4$. The soliton velocity is $v = 0.5$ and the initial phase is $\alpha = 3.67$ (a) and $\alpha = 3.75$ (b). Other parameters correspond to those in fig. 4.

initial soliton phase, α. Figure 4(a) shows the dependence of N_{lm} on α for the localized mode with the amplitude $B = 1.2$. Thick and thin lines show the results for the soliton incident from the left ($v = 0.5$) and from the right ($v = -0.5$), respectively. In fig. 4(b) N_{lm} is shown as a function of α for the localized modes with the amplitudes $B = 1.4$, $B = 1.6$, and $B = 1.89$ for the soliton moving from the left ($v = 0.5$). For the soliton moving from the right side, the interaction with the high-amplitude localized mode leads to $\mathcal{PT}$-symmetry breaking (the fact that the $\mathcal{PT}$-symmetry breaking occurs for smaller values of the localized mode amplitudes in the case in which the incident soliton firstly hits the element with gain does not seem surprising). As the aim of this work is to investigate the case of strong nonlinearity, we concentrate on the case when the incident soliton first hits the lossy element.

As follows from fig. 4, the interaction of the soliton with a localized mode depends on the parameters of the mode and the soliton in a nontrivial way. In particular, for the localized mode amplitude $B = 1.4$, there exists a range of the soliton initial phase, α, where the $\mathcal{PT}$-symmetry breaking occurs and the mode energy diverges.

Figure 5 shows the interaction of the soliton with the localized mode with the amplitude $B = 1.4$ for two close values of the soliton initial phase, (a) $\alpha = 3.67$ and (b) $\alpha = 3.75$. We observe that in the latter case the soliton interaction leads to an unlimited growth of the light intensity in the waveguide with gain due to the $\mathcal{PT}$-symmetry breaking.

These results show that the large-amplitude soliton scattering demonstrates qualitatively new effects in comparison with the scattering of linear waves and the case of weak nonlinearity, due to the nonlinear interaction between the soliton and the mode localized at the $\mathcal{PT}$-coupler.

Conclusions. – We have analyzed the soliton scattering in waveguide arrays with an embedded $\mathcal{PT}$-symmetric defect represented by a pair of waveguides with balanced gain and loss (a $\mathcal{PT}$-coupler). We have found that for the small-amplitude solitons the linear theory predicts the results of the scattering with a good accuracy and in a wide range of the system parameters. We have demonstrated that the reflected and transmitted solitons can be substantially amplified by the $\mathcal{PT}$-symmetric coupler. In addition, the soliton can excite the mode localized at the coupler, and the soliton interaction with the large-amplitude localized mode can result in $\mathcal{PT}$-symmetry breaking. Thus, a pair of coupled waveguides with balanced gain and loss can be used as an active control element in waveguide optical systems.

* * *

The work was supported by the Russian Foundation For Basic Research grants No. 11-08-97057-p_povoljie_a and No. 12-02-31506 mol_a, by the Australian Research Council (including Future Fellowship FT100100160), and NCI National Facility.

REFERENCES

[1] Chen Y. J., Snyder A. W. and Payne D. N., *IEEE J. Quantum Electron.*, **28** (1992) 239.
[2] Ruschhaupt A., Delgado F. and Muga J. G., *J. Phys. A*, **38** (2005) L171.
[3] El-Ganainy R., Makris K. G., Christodoulides D. N. and Musslimani Z. H., *Opt. Lett.*, **32** (2007) 2632.
[4] Makris K. G., El-Ganainy R., Christodoulides D. N. and Musslimani Z. H., *Phys. Rev. Lett.*, **100** (2008) 103904.
[5] Longhi S., *Phys. Rev. Lett.*, **103** (2009) 123601.
[6] Zheng M. C., Christodoulides D. N., Fleischmann R. and Kottos T., *Phys. Rev. A*, **82** (2010) 010103.
[7] Sukhorukov A. A., Xu Z. and Kivshar Yu. S., *Phys. Rev. A*, **82** (2010) 043818.
[8] Ruter C. E., Makris K. G., El-Ganainy R., Christodoulides D. N., Segev M. and Kip D., *Nat. Phys.*, **6** (2010) 192.

[9] Barashenkov I. V., Suchkov S. V., Sukhorukov A. A., Dmitriev S. V. and Kivshar Yu. S., *Phys. Rev. A*, **86** (2012) 053809.
[10] Guo A., Salamo G. J., Duchesne D., Morandotti R., Volatier-Ravat M., Aimez V., Siviloglou G. and Christodoulides D., *Phys. Rev. Lett.*, **103** (2009) 093902.
[11] Ramezani H., Kottos T., El-Ganainy R. and Christodoulides D. N., *Phys. Rev. A*, **82** (2010) 043803.
[12] Ramezani H., Kottos T., Kovanis V. and Christodoulides D. N., *Phys. Rev. A*, **85** (2012) 013818.
[13] Miroshnichenko A. E., Malomed B. A. and Kivshar Yu. S., *Phys. Rev. A*, **84** (2011) 012123.
[14] Suchkov S. V., Malomed B. A., Dmitriev S. V. and Kivshar Yu. S., *Phys. Rev. E*, **84** (2011) 046609.
[15] Alexeeva N. V., Barashenkov I. V., Sukhorukov A. A. and Kivshar Yu. S., *Phys. Rev. A*, **85** (2012) 063837.
[16] Driben R. and Malomed B. A., *Opt. Lett.*, **36** (2011) 4323.
[17] Dmitriev S. V., Suchkov S. V., Sukhorukov A. A. and Kivshar Yu. S., *Phys. Rev. A*, **84** (2011) 013833.
[18] Suchkov S. V., Dmitriev S. V., Malomed B. A. and Kivshar Yu. S., *Phys. Rev. A*, **85** (2012) 033825.
[19] Sukhorukov A. A., Dmitriev S. V., Suchkov S. V. and Kivshar Yu. S., *Opt. Lett.*, **37** (2012) 2148.
[20] Zhou K. Y., Guo Z. Y., Wang J. C. and Liu S. T., *Opt. Lett.*, **35** (2010) 2928.
[21] Musslimani Z. H., Makris K. G., El-Ganainy R. and Christodoulides D. N., *Phys. Rev. Lett.*, **100** (2008) 030402.
[22] Dmitriev S. V., Sukhorukov A. A. and Kivshar Yu. S., *Opt. Lett.*, **35** (2010) 2976.
[23] D'Ambroise J., Kevrekidis P. G. and Lepri S., *J. Phys. A: Math. Theor.*, **45** (2012) 444012.
[24] Kivshar Yu. S. and Agrawal G. P., *Optical Solitons. From Fiber to Photonic Crystals* (Academic Press, San Diego) 2003.
[25] Yulin A. V., Skryabin D. V. and Russel P. S. J., *Phys. Rev. Lett.*, **91** (2003) 260402.
[26] Lederer F., Stegeman G. I., Christodoulides D. N., Assanto G., Segev M. and Silberberg Y., *Phys. Rep.*, **463** (2008) 1.
[27] Cao X. D. and Malomed B. A., *Phys. Lett. A*, **206** (1995) 177.
[28] Kivshar Yu. S., Fei Z. and Vazquez L., *Phys. Rev. Lett.*, **67** (1991) 1177.
[29] Forinash K., Peyrard M. and Malomed B., *Phys. Rev. E*, **49** (1994) 3400.