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Scenario Reduction and Scenario Tree Construction for Power Management Problems

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Abstract— **Portfolio and risk management problems of power utilities may be modeled by multistage stochastic programs. These models use a set of scenarios and corresponding probabilities to model the multivariate random data process (electrical load, stream flows to hydro units, and fuel and electricity prices). For most practical problems the optimization problem that contains all possible scenarios is too large. Due to computational complexity and to time limitations this program is often approximated by a model involving a (much) smaller number of scenarios. The proposed reduction algorithms determine a subset of the initial scenario set and assign new probabilities to the preserved scenarios. The scenario tree construction algorithms successively reduce the number of nodes of a fan of individual scenarios by modifying the tree structure and by bundling similar scenarios. Numerical experience is reported for constructing scenario trees for the load and spot market prices entering a stochastic portfolio management model of a German utility.**

Index Terms— **Stochastic programming, scenario reduction, scenario tree construction.**

I. INTRODUCTION

Economic needs and the ongoing liberalization of European electricity markets stimulate the interest of power utilities in developing models and optimization techniques for the generation and trading of electric power under uncertainty. Utilities participating in deregulated markets observe increasing uncertainty in electrical load (i.e., demand for electric power) and prices for fuel and electricity on spot and contract markets. Therefore, many different optimization models for the operation and planning of power utilities use scenarios to deal with uncertainty related to economic and enviromental parameters, cf. [1], [6], [7], [8], [10], [15], [18], [21], [22] and the state-of-the-art survey [24]. Each scenario corresponds to a particular outcome of the random quantity, i.e., scenarios are realizations (trajectories) of a certain multidimensional stochastic process, the data process of the optimization model. Typical components of the data process are the electrical load, stream inflows in hydro plants, and prices for fuel and electricity on wholesale markets.

The scenarios and their probabilities form an discrete approximation of the probability distribution of the data process. Clearly, the set of scenarios chosen for the optimization model might bias its solution. A survey of methods for generating sets of scenarios that form an approximation of the underlying random data process is given in [4]. Relations to the stability of optimal values and solutions of scenario-based optimization models have also been studied by several authors (see [5], Chapter 8 in [20] and the references therein). Additional features of such scenario sets in dynamic decision models are that the process is deterministic at the first time period and that it has to be *nonanticipative*. The latter means that the random data and decision processes at any time do not depend on future realizations of the data process. These requirements lead to a special form of the finite scenario set, namely, to a tree structure. A *scenario tree* may be represented by a finite set of nodes. It starts from the root node at the first period and eventually branches into nodes at the next period. Each node has a unique predecessor node, but possibly several successors. The branching continues up to nodes at the final period whose number corresponds to the number of scenarios.

Sampling from historical time series or from statistical models (e.g., time series or regression models) is the most popular method for generating data scenarios. Statistical models for the data processes entering power operation and planning models have been proposed, e.g., in [3], [10], [11], [21], [23].

The computational effort for solving scenario-based optimization models depends on the number of scenarios even if decomposition methods are used that exploit special structures. Hence, it is natural to look for scenario-based approximations of the random data process that have a small number of scenarios, but still represent reasonably good approximations. Our approach to scenario reduction controls the goodness-of-fit of the approximation by a certain distance of probability distributions, a *probability metric*. It is recommended to select the specific probability metric out of a certain family of Kantorovich or transportation metrics such that the optimal values and solution sets of the stochastic programs behave stable with respect to perturbations of the underlying probability distributions measured in terms of the specified metric. Transportation metrics represent optimal values of linear transportation problems, i.e., special linear programs. It turns out that the transportation distance between a scenario-based approximation and another one, based on a subset of scenarios and representing the best possible approximation, can be computed explicitly without solving linear programs. The latter formula trades off scenario probabilities and distances of scenarios considered as elements of Euclidean spaces.

The second part of the paper addresses the question of scenario tree generation for multiperiod dynamic decision models under uncertainty. Such dynamic stochastic programs are appropriate optimization models when decisions, such as rebalancing a power portfolio, are taken at several discrete time points called stages. For example, the portfolio manager starts with a given portfolio and a set of scenarios about future states of the system which he/she incoorporates into an investment decision. The model specifies a sequence of decisions at discrete

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time points. The precise composition of the portfolio depends on transactions at the previous stage and on scenarios realized in the interim. Hence, another set of investment decisions is made that incorporates the current status of the portfolio and new information on future scenarios.

The portfolio manager may base his/her decisions on independently generated scenarios for the parameters of the system and of the economy. Although such a fan of individual scenarios represents a very specific scenario tree, its tree structure is not appropriate for the stagewise decision process and, in addition, contains a large number of nodes. What is needed is a scenario tree where information is revealed in all stages of the model. We propose an algorithm for the construction of scenario trees that reduces the number of nodes of an original fan of individual scenarios by modifying the tree structure and by bundling similar scenarios. The whole procedure is based on a recursive reduction argument using transportation metrics.

The paper is organized as follows. In §II we give a description of our concept for the reduction of scenarios modeling the stochastic data processes of stochastic programs. In §III we present our procedure for generating scenario trees and report on numerical tests for constructing scenario trees for the load and spot market prices entering a stochastic portfolio management model of a German utility.

II. SCENARIO REDUCTION

We briefly describe a universal and general concept developed in [5], [12] for the reduction of scenarios modeling the stochastic data processes in stochastic programs. It imposes no requirements on the stochastic data processes (e.g. the dependence structure or the dimension of the process) or on the structure of the scenarios (e.g. tree-structured or not).

A. Nomenclature

$\xi,\ \{\xi_t\}_{t=1}^T$, $\tilde{\xi}, \{\tilde{\xi}_t\}_{t=1}^T$	n-dimensional stochastic processes with parameter set $\{1,\ldots,T\}$
ξ^i, $\tilde{\xi}^j$	scenarios (sample path of ξ, $\tilde{\xi}$)
p_i, q_j	scenario probabilities, i.e., $p_i \geq 0$, $q_j \geq 0$, $\sum_i p_i = \sum_j q_j = 1$
P, Q	probability distribution of the processes ξ and $\tilde{\xi}$, resp.
S	number of scenarios in the initial scenario set
J	index set of deleted scenarios
$\#J$	cardinality of the index set J; i.e., the number of deleted scenarios
$s = S - \#J$	number of preserved scenarios
ε	tolerance for the reduction
$c_t(\xi^i, \xi^j)$	distance between scenario $\{\xi^i\}_{\tau=1}^t$, $\{\xi^j\}_{\tau=1}^t$

B. Theoretical background

Assume that the probability distribution P of the n-dimensional stochastic data process $\xi = \{\xi_t\}_{t=1}^T$ (with possible components electrical load, stream flows to hydro units, and fuel and electricity prices) is approximately given by finitely many scenarios $\xi^i = \{\xi_t^i\}_{t=1}^T$, $i = 1,\ldots,S$, and their probabilities p_i, $\sum_{i=1}^S p_i = 1$.

The scenario reduction algorithms developed in [5], [12] determine a scenario subset (of prescribed cardinality or accuracy) and assign new probabilities to the preserved scenarios such that the corresponding reduced probability measure Q is the closest to the original measure P in terms of a certain probability distance between P and Q. The probability distance trades off scenario probabilities and distances of scenario values. In the context of stochastic power management models, we use the Kantorovich distance D_{K} of (multivariate) probability distributions (cf. [19], Section 5).

For discrete probability distributions with finitely many scenarios the Kantorovich distance D_{K} is just the optimal value of a linear transportation problem. Let Q be the distribution of another n-dimensional stochastic process $\tilde{\xi}$ with scenarios $\tilde{\xi}^j \in \mathbb{R}^{nT}$ and probabilities q_j, $j = 1,\ldots,\tilde{S}$. Then

$$D_{\mathrm{K}}(P,Q) = \inf\left\{ \sum_{i=1}^{S}\sum_{j=1}^{\tilde{S}} \eta_{ij} c_T(\xi^i, \tilde{\xi}^j) : \eta_{ij} \geq 0, \sum_{i=1}^{S}\eta_{ij} = q_j, \sum_{j=1}^{\tilde{S}}\eta_{ij} = p_i, \forall i, \forall j \right\},$$

where $c_t(\xi^i,\tilde{\xi}^j) := \sum_{\tau=1}^t |\xi_\tau^i - \tilde{\xi}_\tau^j|$, $t = 1,\ldots,T$, and $|.|$ denotes some norm on $\mathbb{R}^n$, i.e., c_T measures the distance between scenarios on the whole time horizon $\{1,\ldots,T\}$.
Now, let Q be the reduced probability distribution of ξ, i.e., the support of Q consists of scenarios ξ^j for $j \in \{1,\ldots,S\} \setminus J$ and J denotes some index set of deleted scenarios. For fixed $J \subset \{1,\ldots,S\}$, the scenario set Q based on the scenarios $\{\xi^j\}_{j\notin J}$ having minimal D_{K}-distance to P may be computed explicitly ([5], Theorem 3.1). The minimal distance is

$$D_{\mathrm{K}}(P,Q) = \sum_{i\in J} p_i \min_{j\notin J} c_T(\xi^i,\xi^j) \tag{1}$$

and the probability q_j of the (preserved) scenarios ξ^j, $j \notin J$, of Q is given by the rule

$$q_j := p_j + \sum_{i\in J(j)} p_i, \quad \text{where} \tag{2}$$

$$J(j) := \{i \in J : j = j(i)\}\,, j(i) \in \arg\min_{j\notin J} c_T(\xi^i,\xi^j),\ \forall i \in J.$$

The interpretation of the *optimal redistribution rule* (2) is that the new probability of a preserved scenario is equal to the sum of its former probability and of all probabilities of deleted scenarios that are closest to it with respect to c_T. All deleted scenarios have probability zero.

The optimal choice of an index set J for scenario reduction with fixed cardinality $\#J$ is given by the solution of the *optimal reduction problem*

$$\min\left\{ \sum_{i\in J} p_i \min_{j\in J} c_T(\xi^i,\xi^j) : J \subset \{1,\ldots,S\}, \#J = S - s \right\}, \tag{3}$$

where $s = S - \#J > 0$ is the number of preserved scenarios. It is well-known that (3) represents a set-covering problem. It may be formulated as a 0-1 integer program and is NP-hard.

From (1) and (3) we deduce the following *maximal reduction strategy* to determine a reduced probability distribution Q of ξ such that the set of deleted scenarios has maximal cardinality and that $D_{\mathrm{K}}(P,Q) < \varepsilon$ holds, i.e., Q is close to the original distribution P with given accuracy $\varepsilon > 0$:

Maximal reduction strategy **(mrs)**:

Determine an index set J with maximal cardinality $\#J$ such that

$$\sum_{i\in J} p_i \min_{j\notin J} c_T(\xi^i,\xi^j) \le \varepsilon.$$

The redistribution rule (2) yields the probabilities q_j, $j \notin J$, of the preserved scenarios.

C. Algorithms

Since efficient solution algorithms for (3) are hardly available in general, (fast) heuristic algorithms were developed that exploit the structure of the objective. In the specific cases of $\#J = 1$ (deleting one scenario) and $\#J = S-1$ (keeping one scenario), solving (3) becomes quite easy.

Special case 1: Deleting one scenario
If $\#J = 1$, the problem (3) takes the form

$$\min_{l\in\{1,\ldots,S\}} p_l \min_{j\neq l} c_T(\xi^l,\xi^j). \tag{4}$$

If the minimum is attained at $l_* \in \{1,\ldots,S\}$, i.e., the scenario ξ^{l_*} is deleted, the redistribution rule (2) yields the probability distribution of the reduced measure Q. If $j_* \in \arg\min_{j\neq l_*} c_T(\xi^{l_*},\xi^j)$, then it holds that $q_{j_*} = p_{j_*} + p_{l_*}$ and $q_l = p_l$ for all $l \notin \{l_*, j_*\}$.

Special case 2: Optimal selection of a single scenario
If $\#J = S-1$, the problem (3) is of the form

$$\min_{u\in\{1,\ldots,S\}} \sum_{i=1}^{S} p_i c_T(\xi^i,\xi^u)\,. \tag{5}$$

If the minimum is attained at $u_* \in \{1,\ldots,S\}$, only the scenario ξ^{u_*} is kept and the redistribution rule (2) provides $q_{u_*} = p_{u_*} + \sum_{i\neq u_*} p_i = 1$.

General case
Of course, the optimal deletion of a single scenario may be repeated recursively until a prescribed number $S - s$ of scenarios is deleted. This strategy recommends a conceptual algorithm called *backward reduction* (cf. Fig. 1). If the number of preserved scenarios is small (strong reduction) the optimal selection of a single scenario may be repeated recursively until a prescribed number s of preserved scenarios is selected. This strategy provides the basic concept of a second conceptual algorithm called *forward selection*. Numerical tests in [12] have shown that the following particular variants of backward reduction (Algorithm 1) and forward selection algorithms (Algorithm 2) provide more accurate solutions of the optimal reduction problem (3) than the described ad-hoc variants.

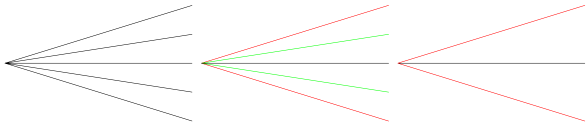

Fig. 1. Delete 2 of 5 scenarios with a backward reduction algorithm

Algorithm 1 — Simultaneous backward reduction

Step 0: Compute the distances of scenario pairs:

$$c_{kj} := c_T(\xi^k,\xi^j),\ k,j = 1,\ldots,S.$$

Sort the records $\{c_{kj} : j = 1,\ldots,S\}$, $k = 1,\ldots,S$

Step 1: Compute
$c^{[1]}_{ll} := \min_{j\neq l} c_{lj}$, $l = 1,\ldots,S$ and

$$z^{[1]}_l := p_l c^{[1]}_{ll},\ l = 1,\ldots,S.$$

Choose $l_1 \in \arg\min_{l\in\{1,\ldots,S\}} z^{[1]}_l$.

Set $J^{[1]} := \{l_1\}$.

Step i: Compute
$c^{[i]}_{kl} := \min_{j\notin J^{[i-1]}\cup\{l\}} c_{kj}$

for $l \notin J^{[i-1]}$, $k \in J^{[i-1]} \cup \{l\}$ and

$$z^{[i]}_l := \sum_{k\in J^{[i-1]}\cup\{l\}} p_k c^{[i]}_{kl},\ l \notin J^{[i-1]}.$$

Choose $l_i \in \arg\min_{l\notin J^{[i-1]}} z^{[i]}_l$.

Set $J^{[i]} := J^{[i-1]} \cup \{l_i\}$.

Step S-s+1: $J := J^{[S-s]}$ is the index set of deleted scenarios. Compute optimal probabilities for the preserved scenarios from (2).

The scenario reduction algorithms were used to reduce a ternary scenario tree for the weekly load process of a German utility. The original construction is based on an hourly discretization of the weekly time horizon with branching periods $t_k = 24k$ for $k = 1,\ldots,6$ (see [10] for a detailed description). The corresponding mean-shifted tree is illustrated in Fig. 2. Figures 3 and 4 displays the reduced trees with 15 preserved scenarios obtained by the forward and backward algorithm.

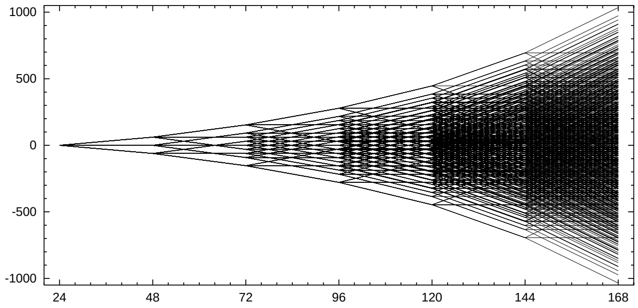

Fig. 2. Ternary scenario tree containing 729 (mean-shifted) load scenarios

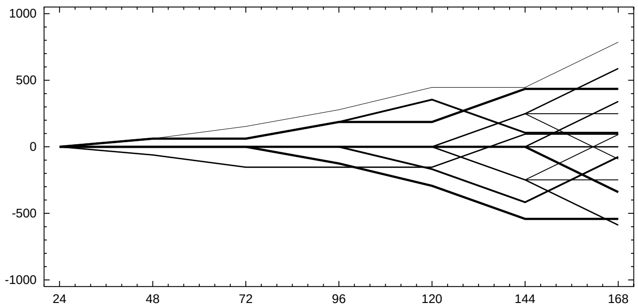

Fig. 3. Reduced load scenario tree with 15 preserved scenarios obtained by the backward algorithm

Algorithm 2 — Fast forward selection

Step 0: Compute the distances of scenario pairs:

$$c_{ku}^{[1]} := c_T(\xi^k, \xi^u),\ k, u = 1, \dots, S.$$

Step 1: Compute

$z_u^{[1]} := \sum_{\substack{k=1 \\ k \neq u}} p_k c_{ku}^{[1]},\ u = 1, \dots, S.$

Choose $u_1 \in \arg \min\limits_{u \in \{1,\dots,S\}} z_u^{[1]}$.

Set $J^{[1]} := \{1, \dots, S\} \backslash \{u_1\}$.

Step i: Compute

$c_{ku}^{[i]} := \min\{c_{ku}^{[i-1]}, c_{ku_{i-1}}^{[i-1]}\},\ k, u \in J^{[i-1]}$

and

$$z_u^{[i]} := \sum_{k \in J^{[i-1]} \backslash \{u\}} p_k c_{ku}^{[i]},\ u \in J^{[i-1]}.$$

Choose $u_i \in \arg \min\limits_{u \in J^{[i-1]}} z_u^{[i]}$.

Set $J^{[i]} := J^{[i-1]} \backslash \{u_i\}$.

Step s+1: $J := J^{[S-s]}$ is the index set of deleted scenarios. Compute optimal probabilities for the preserved scenarios from (2).

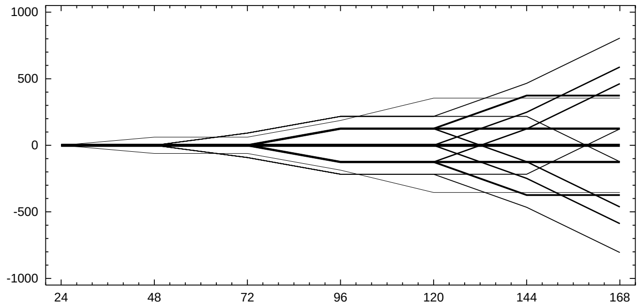

Fig. 4. Reduced load scenario tree with 15 preserved scenarios obtained by the forward algorithm

III. SCENARIO TREE CONSTRUCTION

A scenario tree represents the abstract structure of scenarios. It shows how the uncertainty unfolds over time. A simple example is illustrated in the scenario tree of Fig. 5. Each complete

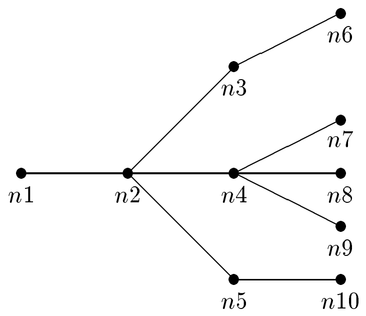

Fig. 5. Scenario tree with 5 scenarios and 10 nodes

path from the root node $n1$ to one of the leaves $n6, \dots, n10$ represents a scenario, i.e. the tree consists of 5 scenarios. Approximations of stochastic processes in form of scenario trees are useful for the formulation of multiperiod dynamic decision models as multistage stochastic programs. A multistage stochastic programming model will determine an optimal decision for each node of the scenario tree, given the information available at that point. As there are multiple succeeding nodes the optimal decisions will not exploit hindsight, but they should anticipate future events.

Presently, a number of approaches to the generation of scenario trees is available. Here, we mention only those that are not reviewed in [4]. The paper [2] uses approximations based on conditional expectations in order to be able to use bounds for generating scenarios. The approach in [14] is based on solving certain regression models to match certain presribed moments of the original measure. Although moment matching is a widespread method, Example 1 in [13] shows that it may lead to strange results. In [16] modern quadrature formulas are proposed for conditional sampling and the papers [17], [13] propose algorithms for determining scenario trees that are best approximations with respect to certain probability distances. The latter idea also serves as a motivation for the following tree construction based on successive reduction.

We assume that finitely many individual paths or scenarios $\xi^i = \{\xi_t^i\}_{t=1}^T$ and corresponding probabilities p_i, $i = 1, \dots, S$

of an n-dimensional stochastic process $\xi = \{\xi_t\}_{t=1}^T$ are given, e.g., obtained from nonparametric or parametric models for the underlying process. Further we assume that all scenarios coincide at $t = 1$, i.e., $\xi_1^1 = \ldots = \xi_1^S =: \xi_1^*$. This means that $t = 1$ may be regarded as the root node of a scenario tree consisting of S branches or that the paths ξ^i, $i = 1, \ldots, S$, form a fan of scenarios. The general tree generation approach described in [4] recommends the use of a recursive cluster analysis method to bundle similar scenarios at all stages.

Our scenario construction method fits into this general scheme by implementing a backward strategy using the scenario reduction principle (**mrs**) on the time horizon $\{1, \ldots, t\}$ at each $t \in \{1, \ldots, T\}$ as a similarity measure. This means, the algorithm recursively reduces the number of nodes of the fan $\{\xi^i\}_{i=1}^S$ of individual scenarios by modifying the tree structure and by bundling scenarios according to a successive scenario reduction technique (cf. Section II). The idea is to compare the Kantorovich distance of original and reduced (sub)trees on $\{1, \ldots, t\}$, $t = T, T-1, \ldots, 2, 1$, and to delete scenarios if the reduced tree is still close enough to the original one. By J_t we denote the scenario sets deleted at t and by I_t the set of scenarios that is preserved at t. Algorithm 3 describes a particular variant of the method. Fig. 6 highlights the interplay between the reduction and bundling steps.

Algorithm 3 — Scenario tree construction

Let tolerances $\varepsilon_t > 0$, $t = 1, \ldots, T$, be given.

Step k=1: Apply the maximal reduction strategy (**mrs**) and Alg. 1 to determine the index set $J_T \subset \{1, \ldots, S\} = I_{T+1}$ such that

$$\sum_{i \in J_T} p_i \min_{j \notin J_T} c_T(\xi^i, \xi^j) \leq \varepsilon_T$$

Set $I_T := I_{T+1} \setminus J_T$ and $\xi^i_{\text{app}} := \xi^i$, $i \in I_T$. Calculate from (2) optimal probabilities π^i_T, $i \in I_T$, for the (preserved) scenarios.

k=T–t+1: **Reduction:**
Apply (**mrs**) and Alg. 1 to determine the index set $J_t \subset I_{t+1}$ such that

$$\sum_{i \in J_t} p_i \min_{j \in I_{t+1} \setminus J_t} c_t(\xi^i, \xi^j) \leq \varepsilon_t \, .$$

Set $I_t := I_{t+1} \setminus J_t$.

Scenario bundling:
For each $j \in J_t$ select an index
$i^* \in \arg\min_{i \in I_t} c_t(\xi^i, \xi^j)$, add π^j_{t+1} to $\pi^{i^*}_{t+1}$ and bundle scenario j with i^*, i.e.,
$\xi^j_{t,\text{app}} := \xi^{i^*}_t$ for $\tau = 2, \ldots, t$,
$\xi^j_{t,\text{app}} := \xi^j_t$ for $\tau = t+1, \ldots, T$.
Set $\xi^i_{t,\text{app}} := \xi^i_{t+1,\text{app}}$, $\pi^i_t := \pi^i_{t+1}$, $i \in I_t$.

Step k=T: Set $\xi^i_{1,\text{app}} := \xi^*_1$ and consider the tree consisting of the scenarios $\{\xi^i_{t,\text{app}}\}_{t=1}^T$ for $i \in I_T$.

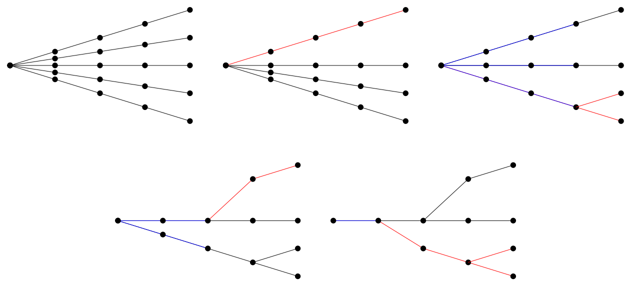

Fig. 6. Construction of a scenario tree by successive scenario reduction

IV. GAMS/SCENRED

The General Algebraic Modeling System (GAMS) is a high-level modeling system for mathematical programming problems. It is specifically designed for modeling linear, nonlinear and mixed integer optimization problems. GAMS consists of a language compiler and a battery of integrated high-performance solvers. GAMS is tailored for complex, large scale modeling applications. More information can be obtained from (<`www.gams.com`>).

Algorithm 1 and 2 and a fast backward method for huge scenario sets are contained in the library SCENRED. GAMS/SCENRED [9] was introduced to the GAMS Distribution 20.6 (May 2002). It takes the original scenarios from the modeler, along with parameters controlling the reduction, and returns a reduced scenario set for use in subsequent solves or data manipulation.

V. Portfolio Management for a Hydro-Thermal Power System

To test our approach to scenario reduction and scenario tree construction, we consider the following instances of the portfolio management problem of a hydro-thermal generation (sub)system of a German utility. The optimization model determines trading activitities and the production decisions of the generation system such that the (expected) revenue is maximized. A full description of the model and the Lagrangian relaxation algorithms for its solution is given in [10], [11].

A. *Uncertain electrical load and spot market price*

The first experiment was designed to test the link between GAMS and the scenario reduction algorithms. The GAMS model for the weekly portfolio management problem was solved with CPLEX 7.5 for a hydro-thermal subsystem comprising 4 thermal generation units and two pumped-storage hydro units.

A fan of scenarios served as initial approximation of the stochastic data process with components electrical load and spot market price. To extract scenarios for the bivariate data process we were given historical load profiles and market data of the European Energy Exchange (EEX). Graphical and clustering methods selected 54 scenarios with identical probabilities to model the distribution of the bivariate stochastic process for an hourly discretized time horizon of one week in summer.

Figures 7 and 8 display the components of an reduced tree for the scenario reduction algorithm. Fig. 9 shows the relative accuracy of the reduced scenario trees depending on the number of preserved scenarios. The optimal value of the power management model having different numbers of preserved scenarios and nodes is given in Fig. 10.

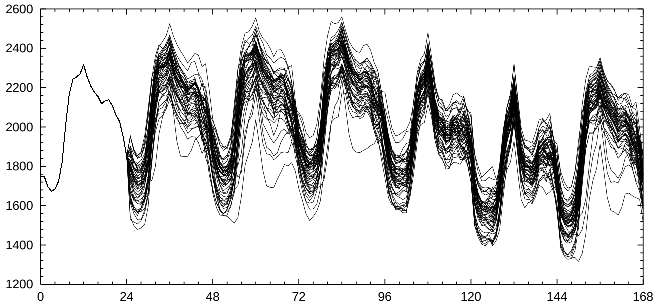

Fig. 7. A tree with 54 scenarios for the component series electrical load

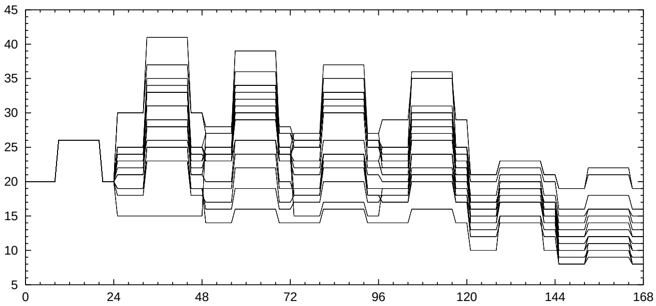

Fig. 8. A tree with 54 scenarios for the component series spot market price

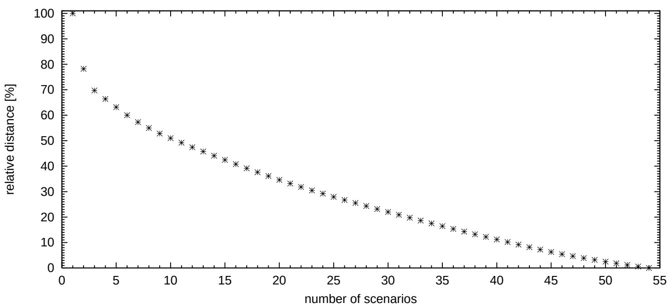

Fig. 9. Relative accuracy for the reduced scenario trees with components electrical load and spot market price

B. Uncertain electrical load

Another experiment was designed to test the performance of the link between the Lagrangian relaxation algorithm and the scenario tree construction algorithm. The portfolio management problem was now solved for 25 thermal generation units and 7 pumped-storage hydro units using the Lagrangian relaxation algorithm described in [10]. The tree construction started with an initial fan $\{d^i\}_{i=1}^{100}$ of load scenarios. They were simulated from the statistical model for the load process developed in [11]. It combines a time series model for the

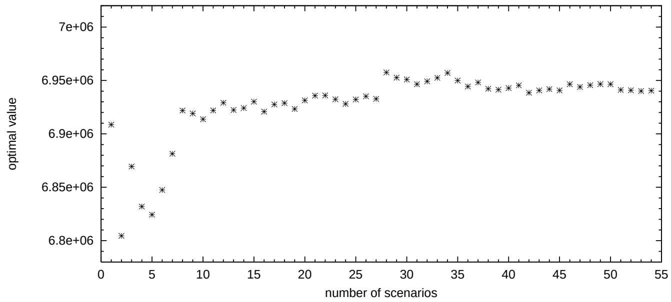

Fig. 10. Optimum of the portfolio management model based on scenario trees with components electrical load and spot market price with different relative accuracy

$\varepsilon_{\mathbf{rel}}$	S	N	Variables		Nonzeros	time[s]
			binary	continuous		
0.6	1	168	4200	7728	44695	7.83
0.1	67	515	12875	23690	137459	17.09
0.05	81	901	22525	41446	240233	37.82
0.01	94	2660	66500	122360	708218	150.14
0.005	96	3811	95275	175306	1014398	291.65
0.001	100	9247	231175	425362	2460402	1176.38

Fig. 11. Test results for solving the stochastic dual based on a reduced load scenario tree of relative tolerance $\varepsilon_{\mathbf{rel}}$

daily mean load with regression models for the intra-day behaviour of the load series. Figure 11 reports the computing times for solving the stochastic dual based on different load scenario trees, each having a different numbers of scenarios (S) and of nodes (N). The test runs were performed on an HP 9000 (780/J280) computer with 180 MHz frequency and 768 MByte main memory under HP-UX 10.20. The trees are constructed by Algorithm 3 with $\varepsilon_t := \frac{\varepsilon}{2^{T-t+1}}$, $t = 1, \ldots, T$, and for different relative tolerances $\varepsilon_{\text{rel}} := \frac{\varepsilon}{\varepsilon_{\max}}$, where $\varepsilon_{\max}$ is the best possible Kantorovich distance D_{K} of the probability distribution having scenarios d^i, $i = 1, \ldots, 100$, with identical probabilities $p_i = 0.01$, to one of its scenarios endowed with unit mass. Figure 12 and 13 show the scenario tree structure and the improved accuracy of the dual optimum, respectively, for decreasing relative tolerances.

VI. Conclusions

We described algorithms for the reduction and scenario tree construction to approximate the random data processes of multiperiod dynamic decision models under uncertainty. The numerical results for the solution of a portfolio management model illustrate the usefulness of our reduction concept. The optimal value of the optimization model can be well approximated using a small number of scenarios.

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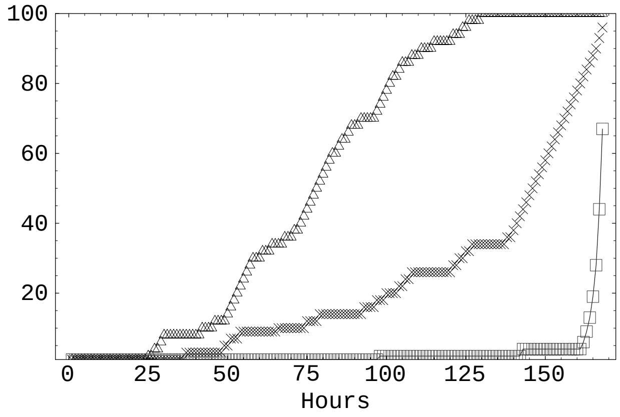

Fig. 12. Number of scenario bundles $|I_t|$ $(t = 1, \ldots, T)$ for scenario trees with relative tolerance $\varepsilon_{\mathrm{rel}} = 0.001$ ($\triangle$), 0.005 ($\times$), 0.01 ($\square$)

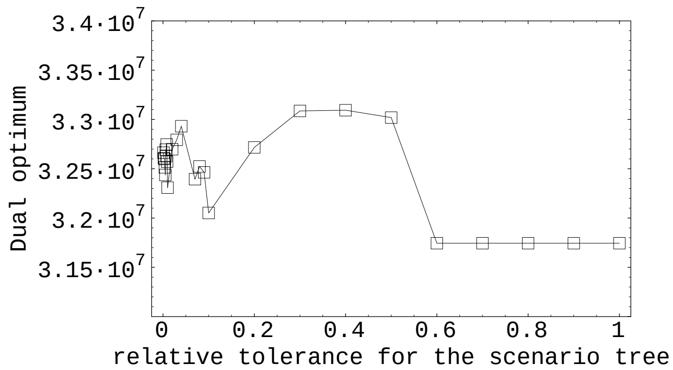

Fig. 13. Optimum for the portfolio management model for scenario trees with different relative tolerance $\varepsilon_{\mathrm{rel}}$

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