

# Scheduling Algorithms for Multiprogramming in a Hard-Real-Time Environment

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Presenters: Forrest Landola and Ilge Akkaya  
(thanks to Marco Di Natale and Tarek  
Abdelzaher)

C.L. Liu<sup>1</sup>, James W. Layland<sup>2</sup>  
<sup>1</sup> MIT and UIUC  
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## (The Liu-Layland Bound)

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# Outline

## Goals

- Synthesize Marco Di Nitale's discussion of the Liu-Layland bound
- Review the key takeaways on this topic

## Roadmap

- State the Liu-Layland bound
- Do some short examples, build up intuition
- ...and move on!

# Is a task set schedulable?

## Schedulability analysis strategies

- Utilization bounds (easy but pessimistic)
  - Lower utilization -> easier to meet deadlines
  - Higher utilization -> harder to meet deadlines
- Exact analysis (optimal, but NP-hard)
- Heuristics

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# The Liu-Layland Bound

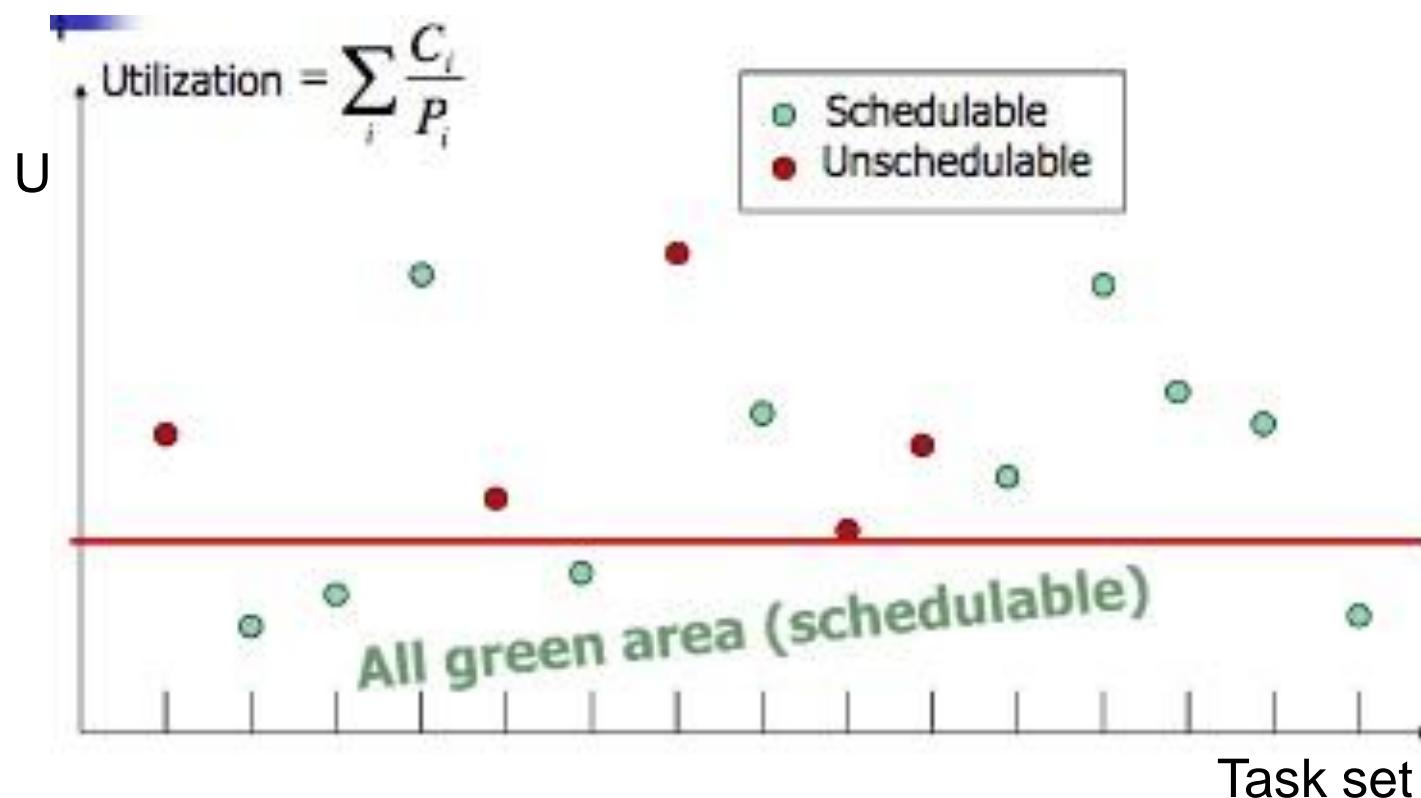
- Utilization bound for  $n$  periodic tasks:

$$U = n \left( 2^{\frac{1}{n}} - 1 \right)$$

- For  $n=2$  tasks,  $U = 0.83$
- As  $n$  goes to infinity,  $U = 0.69$
- Assumption: fixed-priority scheduling

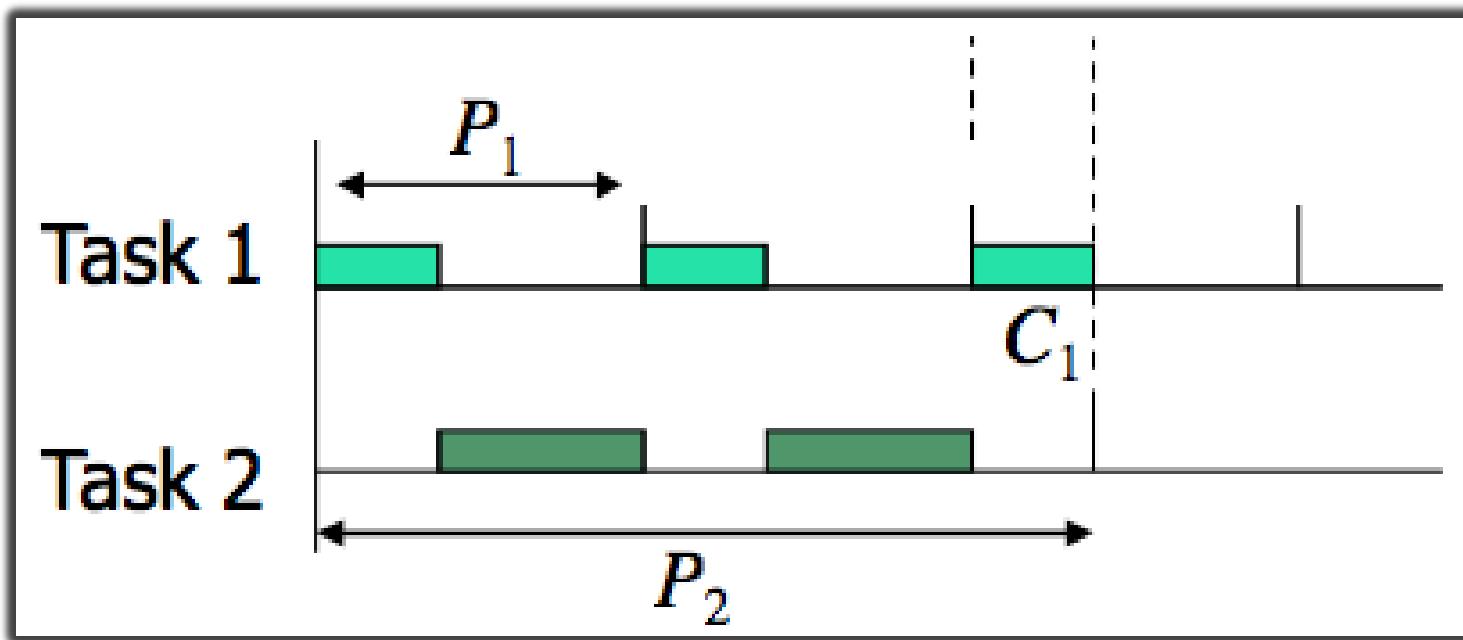
# The bound is NOT “if and only if.”

- If system utilization is within the bound, it is *guaranteed* to be schedulable
- If system utilization exceeds the bound, it *may* be schedulable



# Key insight: Blocking time

- 2 tasks, both arrive at the same time
- Notice that the lower-priority task waits for a long time



# Conclusion

- Liu-Layland bound:

$$U = n \left( 2^{\frac{1}{n}} - 1 \right)$$

- Assuming worst-case arrival and blocking times leads to less-than-100% utilization bound
- If a system exceeds the bound, it may still be schedulable